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Online Portfolio Selection

Follow-the-Winner Approach

中心思想：increasing market weights of more successful experts/ stocks.

1. Universal Portfolios(UP):
    - UP 和 BAH strategy 類似，不同點在 BAH 投資的是 single stock，而 UP 投資的是 a set of stocks(multiple classes)。
    - 1991 年 Cover 版本：最初給每個投資組合經理投資 dμ (b)，期末 wealth 會變成$S_n(b)\mathrm{d}\mu(\mathrm{b})$，再加上 exponential rate of $W_n(\mathrm{b})$。
    $$S_n(b) = \mathrm{e}^{\mathrm{n}W_n(\mathrm{b})}$$
    - 1996 年 Cover and Ordentlich 版本：UP 被改良為μ-Weighted Universal Portfolio，(μ代表 a given distribution on the space of valid portfolio △m)，a historical performance weighed average of all valid constant rebalanced portfolios(根據每位 CRP 經理以前的績效做加權平均的動作)
2. Exponential Gradient(EG):
    - 主要是解決$b_{t+1}\arg\max_{b\epsilon\triangle m} \quad \eta \log \mathrm{b} \cdot X_t - \mathrm{R}(\mathrm{b}, b_t)$ 的最佳化問題。
    - 論文中 Helmbold 提到兩種解決方法: Gradient Projection (GP)採用的是 L2-norm regularization 和 Expectation Maximization (EM)採用的是X^2 regularization
    - 這個演算法重要的一個參數是 <u>learning rate η > 0</u>，為了要達到特定的 regret bound，需要最小化 η。但當 η 趨近於 0 時，EG 會 reduce 成 UCRP
    - EG 後來被 Das and Banerjee 改良為 OGU
3. Follow the Leader(FTL):
    - try to track the Best Constant Rebalanced Portfolio (BCRP) until time t
    - 依據前面的 benchmark BCRP 是有納入 exponential growth rate 的
    - Gaivoronski and Stella 假設在 <u>stationary markets</u> 的狀況下，提出 Successive Constant Rebalanced Portfolios (SCRP) 和 Weighted Successive Constant Rebalanced Portfolios (WSCRP)
    - 但在 the historical market is <u>non-stationary</u> 的狀況下，Gaivoronski and Stella 提出 Variable Rebalanced Portfolios (VRP)，這裡提到藉由 <u>a specified window size</u> 計算 BCRP。
    - 後來持續修正成可以處理 three types of portfolio selection task 的 Adaptive Portfolio Selection (APS)、處理交易成本的 Threshold Portfolio Selection (TPS)，當新投資組合的 expected return 比舊的還高時(超過門檻)，進行 rebalance。

4. Follow the Regularized Leader(FTRL):
    - 比 FTL 多了 a regularization term
    - the regularization term only concerns the next portfolio
    - Online Newton Step (ONS), by solving the optimization problem with L2-norm regularization via online convex optimization technique
    - the Geometric Brownian Motion (GBM) model, which is a probabilistic model of stock returns
    - Exp-Concave-FTL, follows a slightly different form of optimization problem with L2-norm regularization
    - Online Newton Update (ONU), which guarantees that the overall performance is no worse than any convex combination of its underlying experts
5. Aggregating-type Algorithms:
    - 前提 the i.i.d. assumption is controversial in real markets
    - Aggregating Algorithm (AA), the general setting for AA is to define a countable or finite set of base experts and sequentially allocate the resource among multiple base experts in order to achieve a good performance that is no worse than any fixed combination of underlying experts
    - Switching Portfolios (SP) to track a changingmarket, in which the stock’s behaviors may change frequently
    - Gaussian Random Walk (GRW) strategy, which switches among the base experts according to Gaussian distribution

HELMBOLD 的 Exponential Gradient(EG)演算法

On-Line Portfolio Selection Using Multiplicative Updates

前提假設

- Consider a portfolio containing N stocks
- price relatives X (Xi=the next day's opening price of the th stock divided by its opening price on the current day)
- portfolio weight vector W (Wi 總和=1)

The ith entry of a portfolio w is the proportion of the total portfolio value invested in the ith stock.

總結$\mathrm{W} \cdot \mathrm{X} = \sum_{i}^{N} X_i Y_i$

Exponential Gradient(EG)

類似 Kivinen and Warmuth 的 framework

Kivinen and Warmuth 希望找到W^{t+1}趨近於W^{t}，而 HELMBOLD 修改成W^{t+1}是$\mathrm{F}(W^{t+1}) = \eta \log(W^{t+1} \cdot X^{t}) - d\,(W^{t+1}, W^{t})$的 maximum

註：

- learning rate η > 0，為控制因素(如果W^{t+1}趨近於W^{*}，η 又是 large，那就會偏離)
- d = a distance measure that serves as a penalty term(設定 penalty term 是為了使W^{t+1}趨近於W^{t})

a distance function for motivating updates(不同 distance function 會產生不同 update rules)

$$D_{RE}(\mathrm{u} \parallel \mathrm{v}) \stackrel{\text{def}}{=} \sum_{i=1}^{N} u_i \log \frac{u_i}{v_i}$$

second-order Taylor approximation(u=v)，X^2-distance

$$D_{X^2}(\mathrm{u} \parallel \mathrm{v}) \stackrel{\text{def}}{=} \frac{1}{2} \sum_{i=1}^{N} \frac{(u_i - v_i)^2}{v_i}$$

因為無法 maximize F 所以改 maximize $\mathcal{F}$(上面有一個箭頭，將 the N partial derivatives 設為 zero)

再加上 additional constraint$\sum_{i=1}^{N} W_i^{t+1} = 1$，就稱為 exponentiated gradient(EG(η)) update

$$W_i^{t+1} = \frac{W^{t} \exp(\eta X_i^{t} / W^{t} \cdot X^{t})}{\sum_{j=1}^{N} W_i^{t} \exp(\eta X_i^{t} / W^{t} \cdot X^{t})}$$

另外如果納入考量X^2-distance，就稱為X^2(η)- update