

# Optimization and Decision Making Case study: Automatic Game Balancing

## Problem

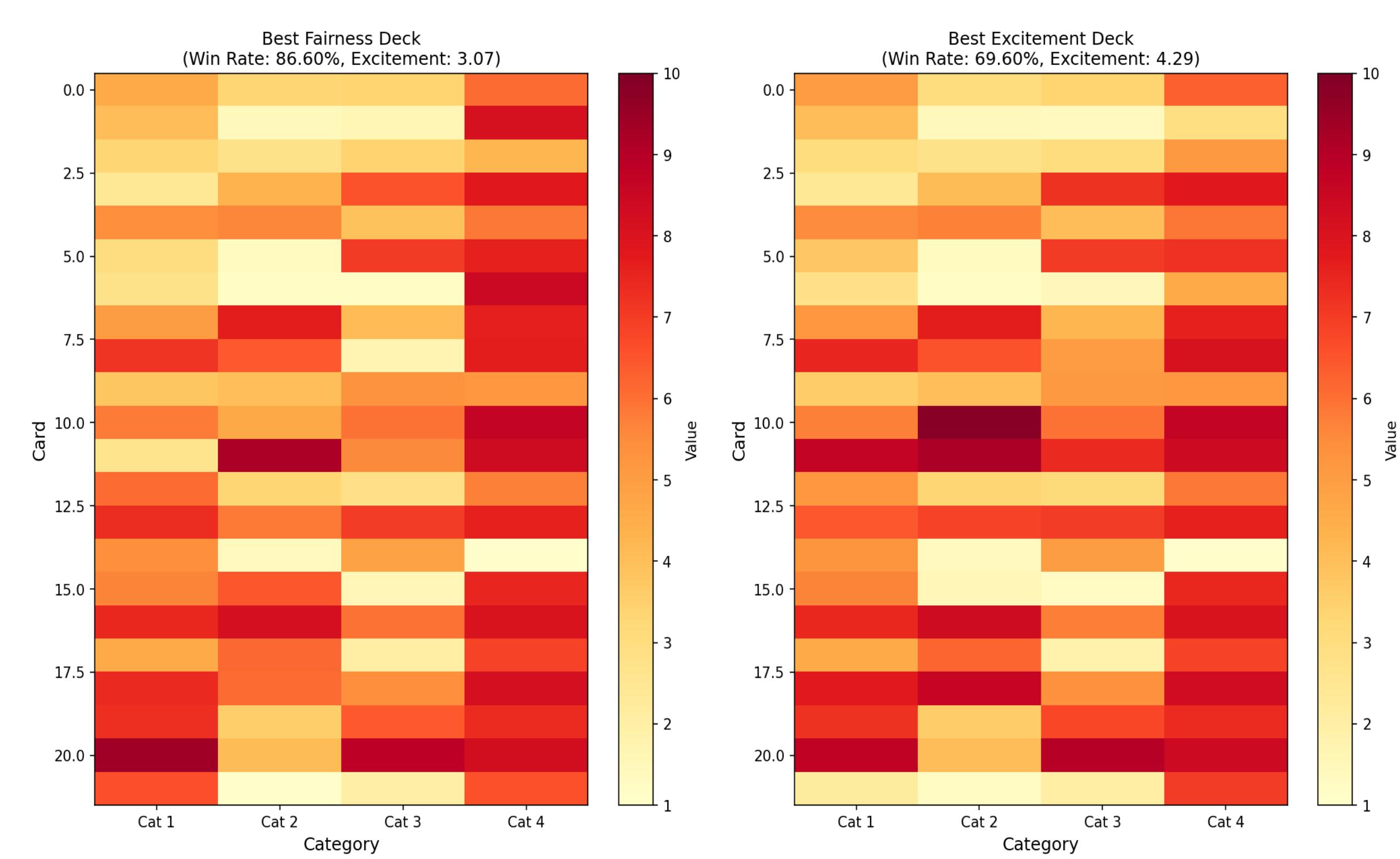
- Objective:** Automatically balance a Top Trumps deck by optimizing **fairness** and **excitement**.
- Problem size:** 22 cards  $\times$  4 categories  $\rightarrow$  **88 continuous decision variables**.
- Evaluation:** Each candidate deck is assessed using **1,000 stochastic game simulations**.
- Challenge:** High-dimensional search space combined with **noisy** objective evaluations.
- Goal:** Approximate the **Pareto front** capturing trade-offs between fairness and excitement.

## Approach

- Optimizer:** NSGA-II (multi-objective evolutionary algorithm)
- Population size:** **100** candidate decks per generation
- Generations:** **100** evolutionary iterations
- Search budget:** **10,000** total evaluations ( $100 \times 100$ )
- Procedure:** Evolve decks via **selection, crossover, and mutation** to approximate the fairness–excitement Pareto front

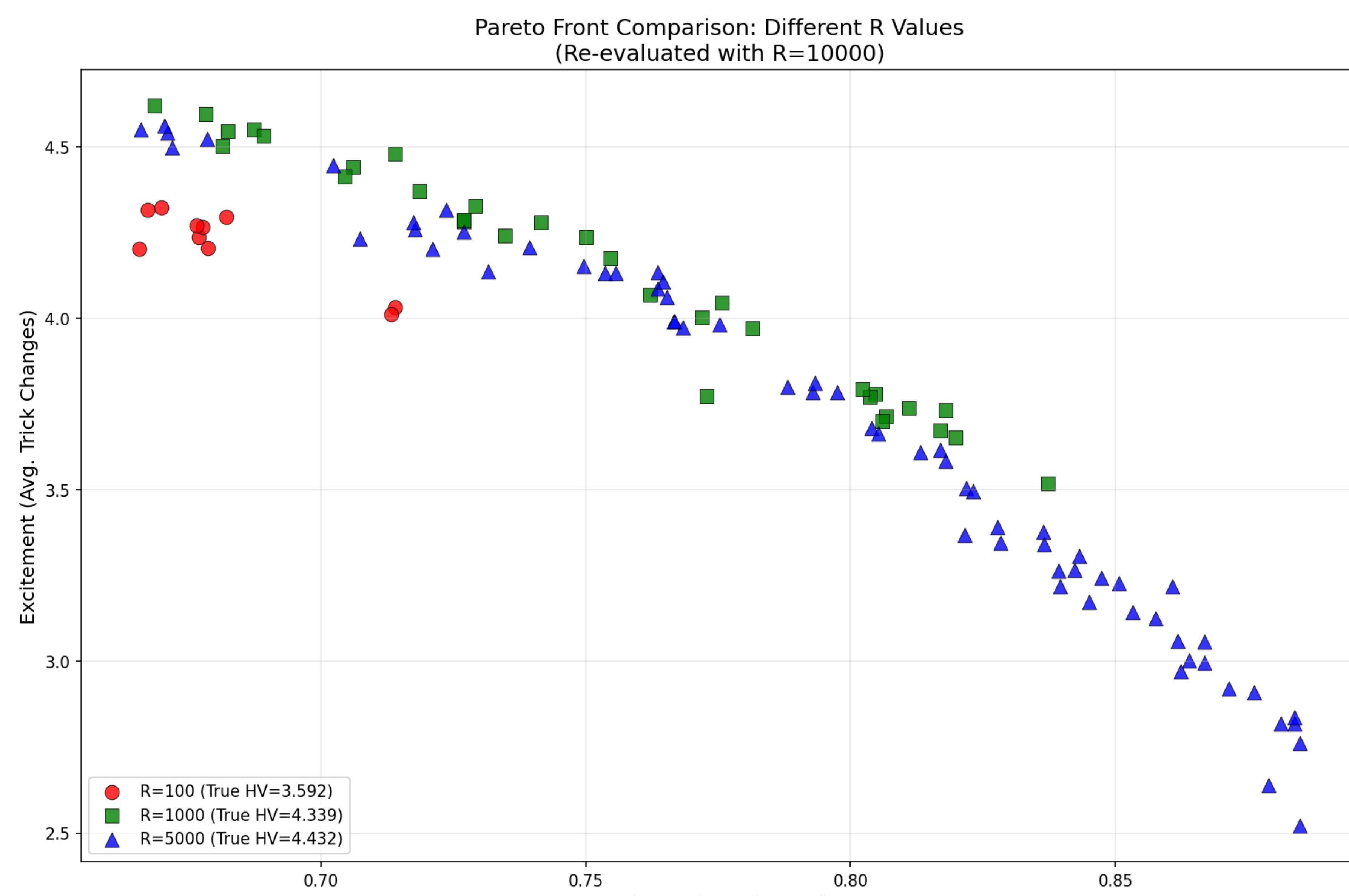
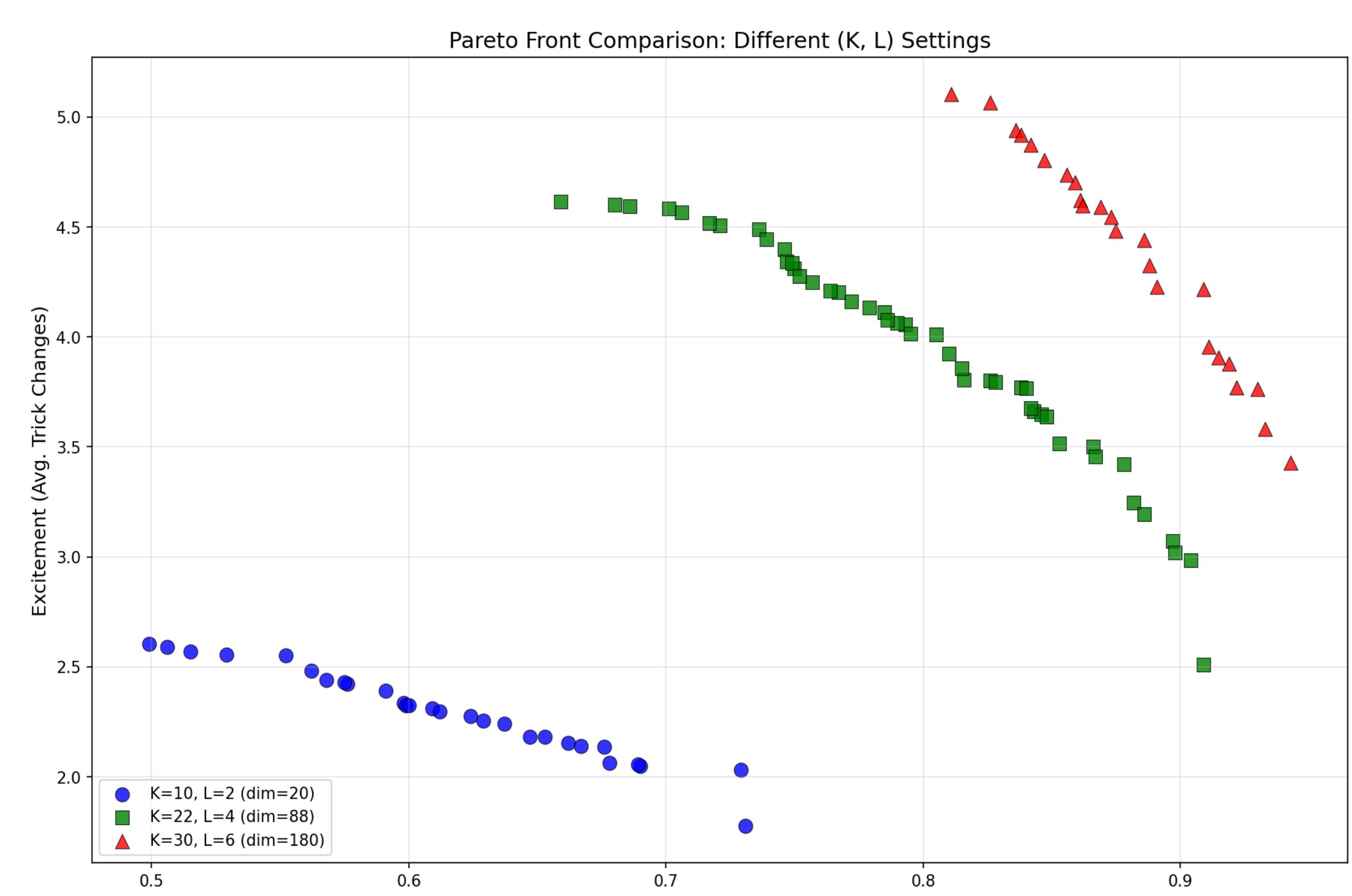
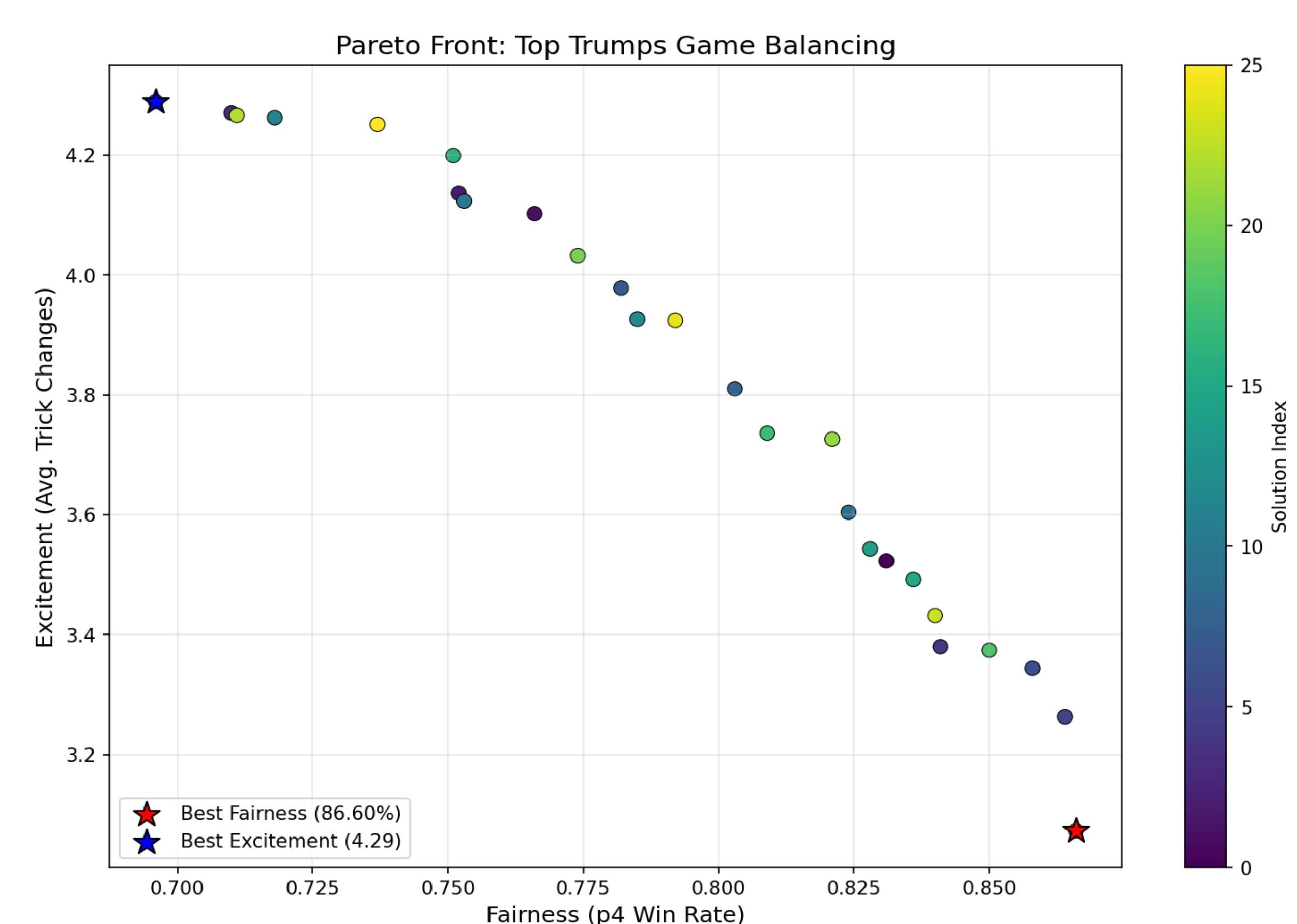
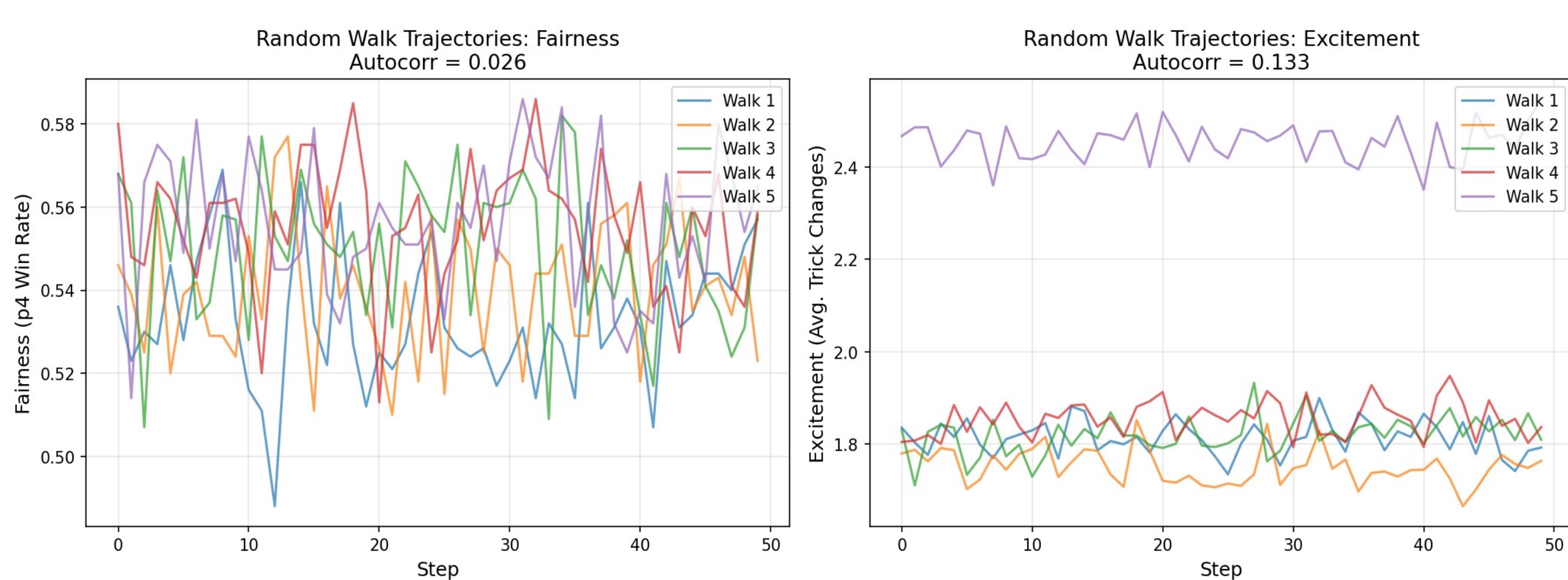
## Results

- Solution Quality:** Hypervolume = 4.170591 (reference point [0.1, 0.1]).
- Pareto Front:** 26 non-dominated solutions discovered.
- Objective Ranges:**
  - \* **Fairness (p4 win rate):** 0.696 – 0.866
  - Excitement (trick changes):** 3.07
- Extreme Trade-offs:**
  - Best Fairness:** 86.60% win rate, 3.07 excitement.
  - Best Excitement:** 69.60% win rate, 4.29 excitement.
- Runtime Performance:** Total runtime of 4.2 minutes (average 0.025s per evaluation).



## Impact of Card (K) and Category (L) Scaling

- Search Space Complexity:**
  - Increasing K or L expands the decision variables **linearly**, but the search space grows **exponentially**. A standard 52-card deck with 6 categories would involve **312 variables**, requiring a larger population size (>100) to maintain diversity.
- Pareto Front Density:**
  - Higher K allows for more **granular "balancing"**. Smaller decks (like your K=22) result in more **distinct "steps"** on the Pareto front, whereas larger decks would produce a **smoother, more continuous curve**.



## Problem Landscape Properties

- High-Dimensional Epistasis:**
  - The fitness of a single card is not independent; it depends entirely on the values of the other 21 cards in the deck. This makes the landscape **highly interdependent**.
- Ruggedness:**
  - Small adjustments to a single category value can cause a "cascade" effect in trick-taking logic, creating a rugged landscape where **local search algorithms would likely fail**, justifying the use of **NSGA-II**.
- Objective Correlation:**
  - Fairness and Excitement are generally **negatively correlated**. Maximizing excitement (trick changes) often requires "swingy" cards that inherently disrupt perfectly equal win rates.

## Influence of Simulation Repetitions (R)

- The "Noise" Factor:**
  - Because Top Trumps is stochastic (due to the shuffle), R acts as a **signal-to-noise filter**.
- Solution Reliability:**
  - With low R (e.g., R=100), a deck might appear on the Pareto front simply because it got a "lucky" series of shuffles.
- Convergence Trade-off:**
  - Increasing R to 1,000 significantly improves the **robustness** of the results but increases the computational runtime from seconds to minutes.