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# Python, Open-Source, and Numerical Solutions of Conservation Laws

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Sheng-Tao John Yu

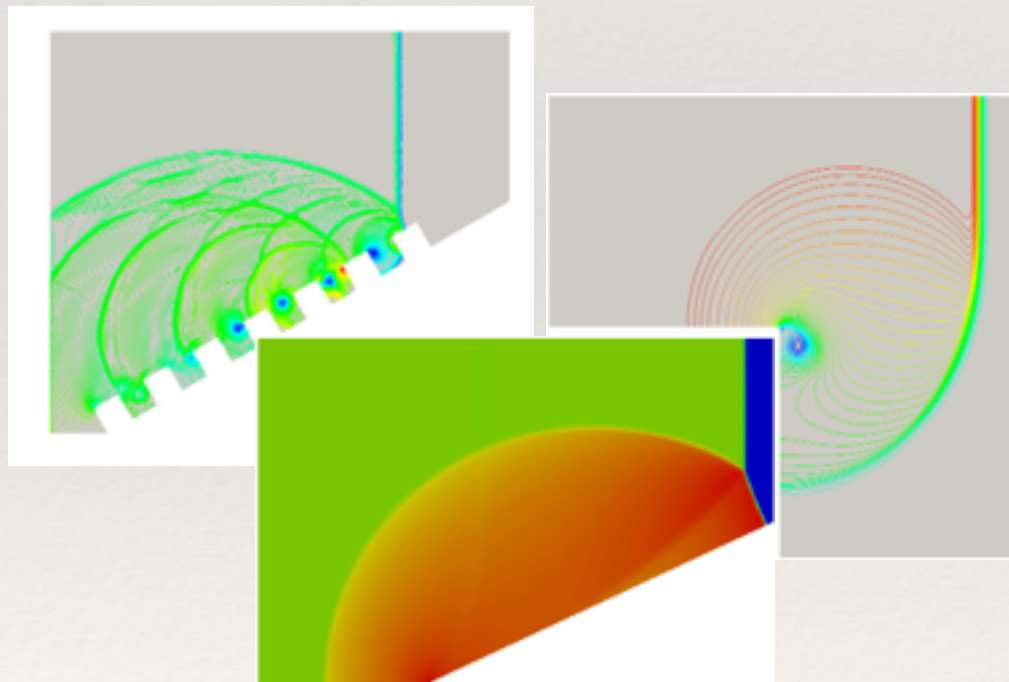
SOLVCON Project

@ ISGC 2014 March 28

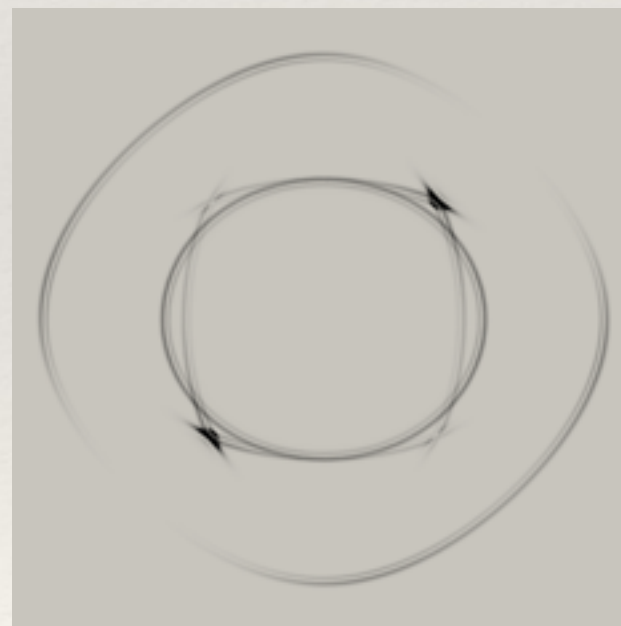
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# Introduction

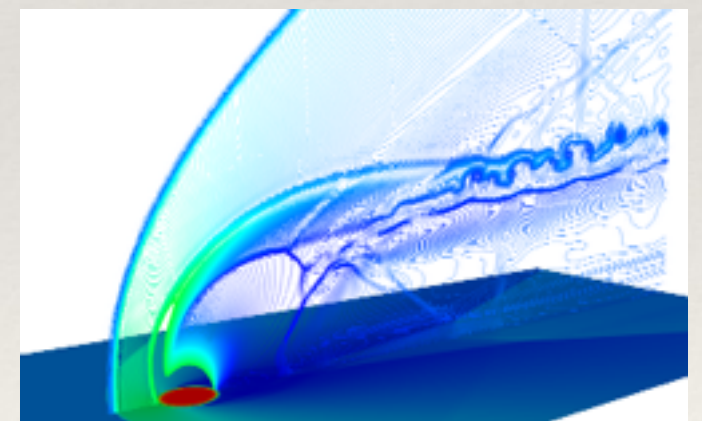
- ❖ Explore how to engineer numerical simulations of conservation laws and build a software framework (SOLVCON).
- ❖ Examples of the simulations:



Aerodynamics Benchmarks



Stress Waves in  
Anisotropic Solids



Supersonic Jet in Cross Flow

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# Goals

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- ❖ We want multiple solvers for physical processes governed by the equations taking the quasilinear form:

$$\frac{\partial \mathbf{u}}{\partial t} + \sum_{k=1}^3 A^{(k)}(\mathbf{u}) \frac{\partial \mathbf{u}}{\partial x_k} = \mathbf{s}(\mathbf{u})$$

- ❖ We want an extensible system to modularize physical processes.
  - ❖ It is a merit provided by the space-time conservation element and solution element (CESE) method, and we want to explore its applicability.
  - ❖ Plan ahead for interoperation.
- ❖ We want parallel code (scale from 1,000 cores to more). We want hybrid parallelism for heterogeneous architecture.
- ❖ We want the code to be well maintained. We don't want spaghetti code patched from paper to paper. We want to code systematically.



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# It's an Engineering Problem

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- ❖ The goals can be generalized to developing SOLVCON, a software framework for the CESE method. It's an engineering problem for scientific research.
- ❖ Predecessors of SOLVCON began as a mesh processing utility.
  - ❖ Two-/three-dimensional mesh and solution data need to be shared among gas-dynamics CFD (computational fluid dynamics) tools for reuse and analyses.
- ❖ The predecessors are redeveloped to a software framework for the space-time conservation element and solution element (CESE) method since 2008. We
  - ❖ wanted to explore the applicability of the CESE method, and thus
  - ❖ needed to develop solvers for multiple physical processes.
- ❖ All goals (extensibility, modularity, parallel computing, and maintainability) are achieved, but with many rough ends.

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# Python and Open-Source

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- ❖ Python: Build everything upon the scientific Python ecosystem.
  - ❖ Various built-in and third party packages. More than 41,392 on <https://pypi.python.org/pypi> now.
  - ❖ ndarray (N-dimensional array) of NumPy.
  - ❖ Cython for interfacing between Python and C.
- ❖ Open-source: Open up everything, from source code to documents.
  - ❖ SOLVCON online documents can be a better vehicle than sole papers for propagating knowledge.
  - ❖ Frictionless collaboration.



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# Development Activities

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1. Documenting for reproduction.
  - ❖ Dense knowledge in both natural and computer languages.
  - ❖ Propagate the knowledge with reproducibility.
2. Architecting and developing code.
  - ❖ Before writing it, we don't know what to expect.
  - ❖ Agile methodology must be employed for the uncertain nature.
3. Testing and validation.
  - ❖ We need unit tests and answer tests for development.
  - ❖ Running answer tests can take hours. We need a server farm.

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# Documenting

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- ❖ SOLVCON documents should contain what's in conventional research papers.
- ❖ What we want is more than conventional papers.
  - ❖ Bi-directional linking between the code and the documents.
  - ❖ Some part of the documents should be runnable. It's related to validation.

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# Sphinx

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- ❖ Sphinx (<http://sphinx-doc.org/>) is the default tool for Python documentation.
- ❖ ReadTheDocs (<https://readthedocs.org/>) is a service to host Sphinx documents.
- ❖ SOLVCON authors and organizes its documents with Sphinx.
- ❖ The documents are published at <http://www.solvcon.net/> (ReadTheDocs is behind the scenes).





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- Hyperbolicity
- Riemann Invariants
- Diffusion Term Treatment

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# Formulations (Under Development)

The governing equations of the hydro-acoustic wave include the continuity equation

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^3 \frac{\partial \rho v_i}{\partial x_i} = 0 \quad (1)$$

and the momentum equations

$$\frac{\partial \rho v_j}{\partial t} + \sum_{i=1}^3 \frac{\partial (\rho v_i v_j + \delta_{ij} p)}{\partial x_i} = \frac{\partial}{\partial x_j} \left( \lambda \sum_{k=1}^3 \frac{\partial v_k}{\partial x_k} \right) + \sum_{i=1}^3 \frac{\partial}{\partial x_i} \left[ \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right], \quad j = 1, 2, 3 \quad (2)$$

where  $\rho$  is the density,  $v_1, v_2$ , and  $v_3$  the Cartesian component of the velocity,  $p$  the pressure,  $\delta_{ij}$ ,  $i, j = 1, 2, 3$  the Kronecker delta,  $\lambda$  the second viscosity coefficient,  $\mu$  the dynamic viscosity coefficient,  $t$  the time, and  $x_1, x_2$ , and  $x_3$  the Cartesian coordinate axes. Newtonian fluid is assumed.

The above four equations in (1) and (2) have five independent variables  $\rho, p, v_1, v_2$ , and  $v_3$ , and hence are not closed without a constitutive relation. In the `bulk` package, the constitutive relation (or the equation of state) of choice is

$$K = \rho \frac{\partial p}{\partial \rho}$$

where  $K$  is a constant and the bulk modulus. We chose to use the density  $\rho$  as the independent variable, and integrate the equation of state to be

$$p = p_0 + K \ln \frac{\rho}{\rho_0} \quad (3)$$

where  $p_0$  and  $\rho_0$  are constants. Substituting (3) into (2) gives

$$\frac{\partial \rho v_j}{\partial t} + \sum_{i=1}^3 \frac{\partial}{\partial x_i} \left[ \rho v_i v_j + \delta_{ij} \left( p_0 + K \ln \frac{\rho}{\rho_0} \right) \right] = \sum_{i=1}^3 \frac{\partial}{\partial x_i} \left[ \delta_{ij} \lambda \sum_{k=1}^3 \frac{\partial v_k}{\partial x_k} + \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right], \quad j = 1, 2, 3 \quad (4)$$

## Jacobian Matrices

v: latest

We proceed to analyze the advective part of the governing equations (1) and (4). Define the conservation variables

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# Authoring, Editing, and Reviewing

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- ❖ We use Mercurial for version control and BitBucket (<https://bitbucket.org/solvcon/solvcon>) to host the project and its source code.
- ❖ Reviewing the documents goes through the pull-request flow of the code review.
- ❖ We treat the reStructuredText source files of the documents like the program source code.



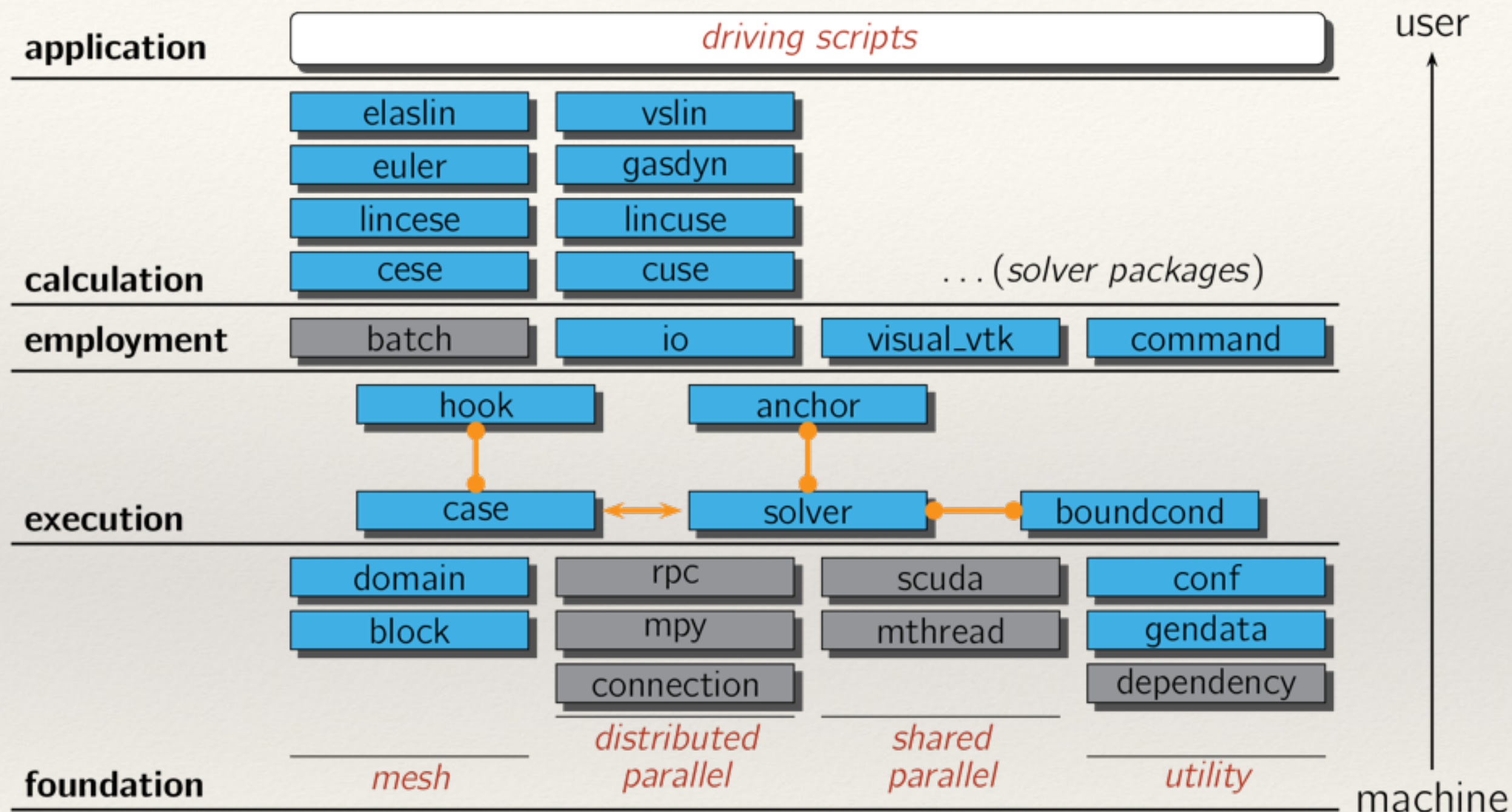
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# SOLVCON Architecture

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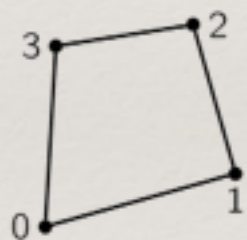
- ❖ Fundamentals of SOLVCON:
  1. Unstructured meshes of mixed elements. Data structure affects everything of the system.
  2. The space-time CESE method. It insists a two-nested-loop structure.
  3. Hybrid implementation of Python and C. It's for usability and flexible parallel computing.
- ❖ Components are organized into 5 layers (next page).



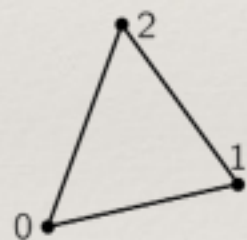


# Flexible Geometry

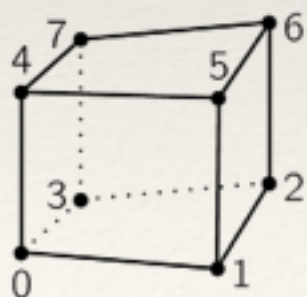
- ❖ We want the ability to model the most complex geometry.
- ❖ We chose to employ the unstructured meshes of mixed elements for spatial discretization.



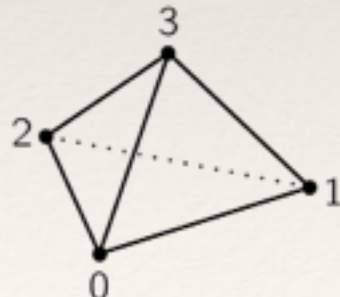
quadrilateral



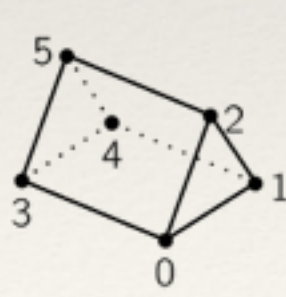
triangle



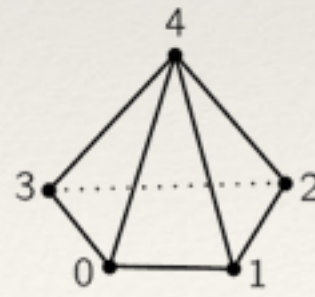
hexahedron



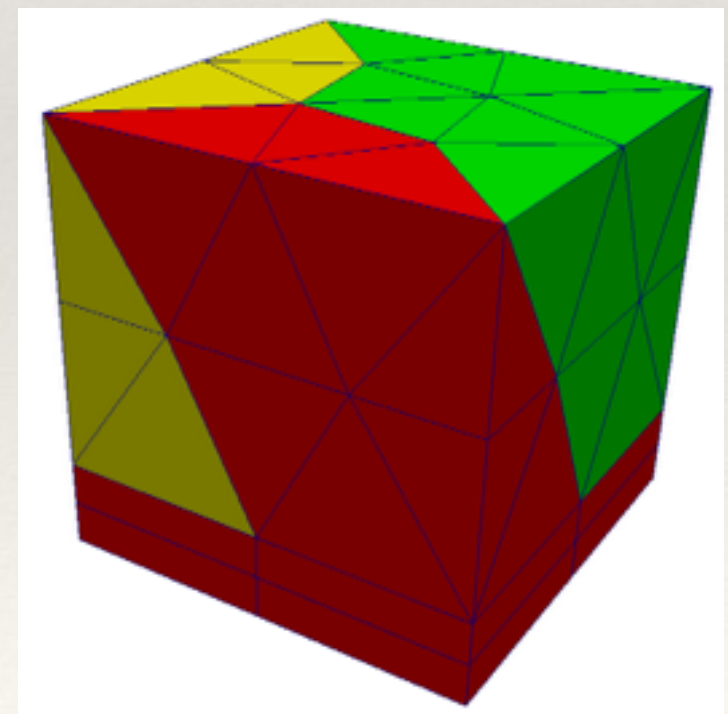
tetrahedron



prism



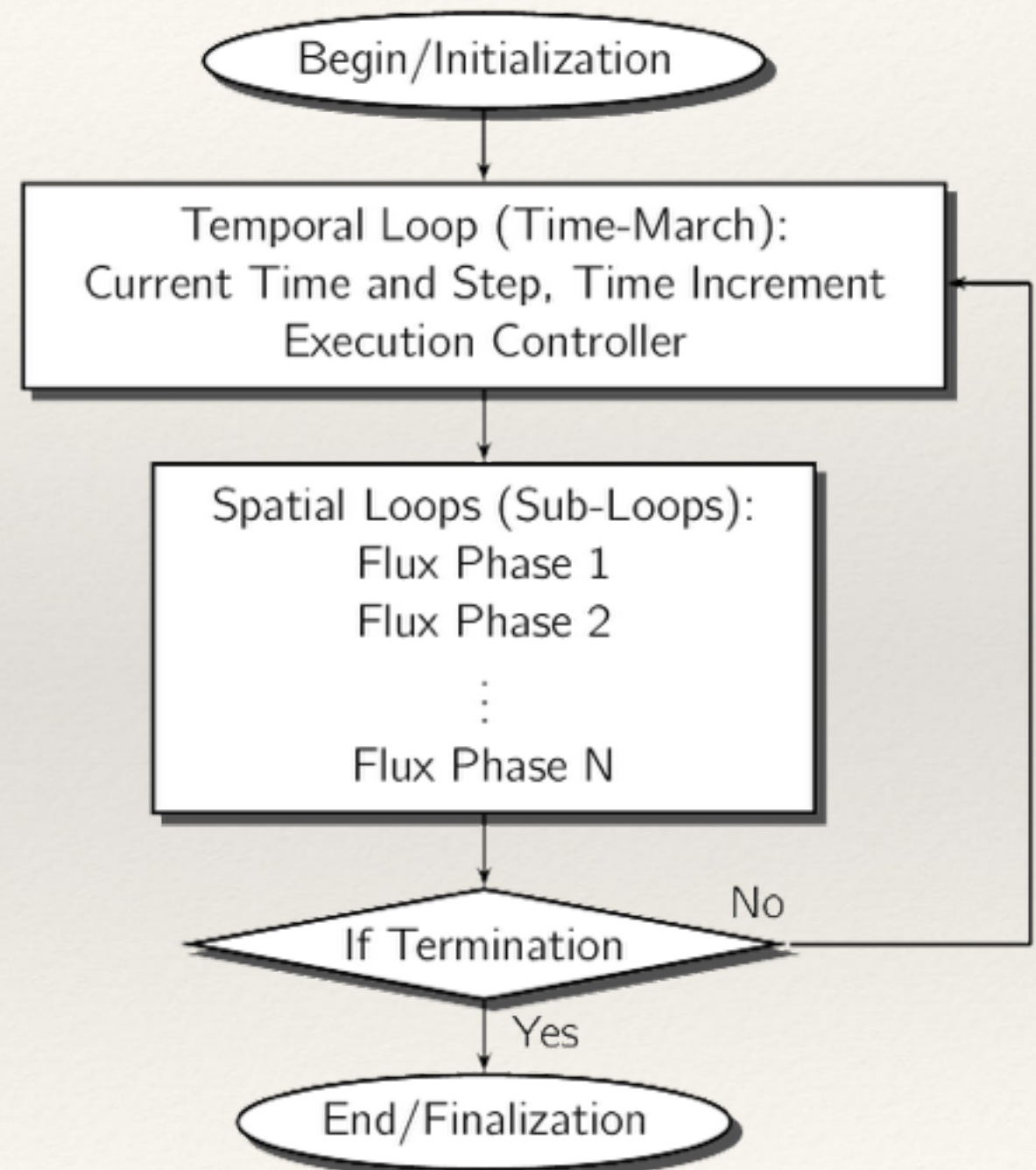
pyramid





# Time-Accurate Solutions

- ❖ SOLVCON exclusively uses the space-time Conservation Element and Solution Element (CESE) method for time-accurate solutions.
- ❖ The pursuit of time-accurate solutions mandates the two-nested-loop structure shown at the right-hand side.
- ❖ All facilities can be built around the execution flow.





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# Python: Scripting Is a Must

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- ❖ There are too many parameters and analyses required.
- ❖ No interface is sufficient for the simulations: SOLVCON needs to be a library.
- ❖ Controlling a library by using low-level languages like C and C++ is too painful to be realistic.

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# Python: Architecting

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- ❖ The outstanding capability to interface to C makes Python an ideal manager for low-level, high-performance C code.
  - ❖ C++ works as well but SOLVCON uses only C to avoid the complexity.
  - ❖ SOLVCON uses Cython (<http://cython.org/>) for interfacing.
  - ❖ NumPy (<http://www.numpy.org/>) is the key component for efficient data access through **arrays**.
- ❖ The hybrid approach allows us to architect the system with the high-level Python and still have the high performance of C.
  - ❖ Ideas can be quickly prototyped with Python. After results get validated, the fast C version can then be developed iteratively.

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# Integrated Testing

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- ❖ Code without verification can't be taken as working code, no matter how carefully we wrote it.
- ❖ With the testing system, refactoring and quick prototyping become possible.
  - ❖ Many research codes don't have a regression testing system, so while coding the developers can't be sure how the changes impact the system.
- ❖ Two kinds of tests: unit tests and answer tests. Both need automation.



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# Unit Tests

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- ❖ Unit tests are short and quick, and check for simple output of elementary API behaviors.
- ❖ SOLVCON uses standard Python unittest module (<http://docs.python.org/2/library/unittest.html>).
- ❖ nose (<https://nose.readthedocs.org/>) is used as the automatic test runner.

2. bash

yungyuc@nanoha:~/work/coding/solvcon

\$ nosetests

.....  
.....  
.....  
.....

-----  
Ran 279 tests in 5.290s

OK

yungyuc@nanoha:~/work/coding/solvcon

\$

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# Answer Tests

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- ❖ Answer tests check for the correctness of simulations. Usually an answer test is a meaningful simulation, and can serve as an example to how to use SOLVCON to simulate other problems.
- ❖ Jenkins (<http://jenkins-ci.org/>) is installed at <http://ci.solvcon.net/> to automatically run the answer tests for every commit of SOLVCON in a testing farm.



People

Build History

Build Queue (5)

- legacy\_examples.vslin
- legacy\_examples.elaslin
- legacy\_examples.visout
- legacy\_examples.gasdyn
- legacy\_examples.misc

Build Executor Status

#	Status
srv03	
1	Building examples.linear cvg #40
2	Building legacy_examples.euler #38
3	Building examples.bulk_air #10
4	Building examples.vewave cvg #39

w101 (offline)

w102 (offline)

w103 (offline)

w104 (offline)

w105 (offline)

w128 (offline)

w129 (offline)

w130 (offline)

All	legacy_examples	solution_tests			
S	W	Name ↓	Last Success	Last Failure	Last Duration
		<a href="#">examples.bulk_air</a>	3 days 2 hr - <a href="#">#9</a>	N/A	2 min 52 sec
		<a href="#">examples.linear_cvg</a>	3 days 2 hr - <a href="#">#39</a>	19 days - <a href="#">#30</a>	11 min
		<a href="#">examples.vewave_cvg</a>	3 days 1 hr - <a href="#">#38</a>	3 days 2 hr - <a href="#">#37</a>	6 min 1 sec
		<a href="#">legacy_examples.elaslin</a>	3 days 1 hr - <a href="#">#37</a>	N/A	7 min 40 sec
		<a href="#">legacy_examples.euler</a>	3 days 2 hr - <a href="#">#37</a>	N/A	7 min 35 sec
		<a href="#">legacy_examples.gasdyn</a>	3 days 2 hr - <a href="#">#37</a>	N/A	13 min
		<a href="#">legacy_examples.misc</a>	3 days 2 hr - <a href="#">#37</a>	N/A	2 min 37 sec
		<a href="#">legacy_examples.visout</a>	3 days 1 hr - <a href="#">#37</a>	N/A	3 min 29 sec
		<a href="#">legacy_examples.vslin</a>	3 days 1 hr - <a href="#">#37</a>	N/A	8 min 36 sec
		<a href="#">release</a>	2 min 17 sec - <a href="#">#42</a>	5 days 0 hr - <a href="#">#39</a>	2 min 1 sec
		<a href="#">sdev</a>	N/A	N/A	N/A
		<a href="#">solvcon_cese</a>	2 mo 13 days - <a href="#">#56</a>	3 mo 0 days - <a href="#">#44</a>	10 min
		<a href="#">solvcon_cuse</a>	2 mo 13 days - <a href="#">#56</a>	3 mo 0 days - <a href="#">#45</a>	11 min
		<a href="#">solvcon_misc</a>	1 yr 0 mo - <a href="#">#15</a>	2 mo 13 days - <a href="#">#57</a>	58 sec
		<a href="#">solvcon_test</a>	2 mo 13 days - <a href="#">#82</a>	N/A	35 sec
		<a href="#">solvcon_visout</a>	2 mo 13 days - <a href="#">#57</a>	3 mo 0 days - <a href="#">#44</a>	34 sec

# Current Work: Hydroacoustics

- ❖ Governing equations: conservation of mass and momentum.

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^3 \frac{\partial \rho v_i}{\partial x_i} = 0$$
$$\frac{\partial \rho v_j}{\partial t} + \sum_{i=1}^3 \frac{\partial (\rho v_i v_j + \delta_{ij} p)}{\partial x_i} =$$
$$\frac{\partial}{\partial x_j} \left( \lambda \sum_{k=1}^3 \frac{\partial v_k}{\partial x_k} \right) + \sum_{i=1}^3 \frac{\partial}{\partial x_i} \left[ \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right], \quad j = 1, 2, 3$$

- ❖ Water passes a cylinder at  $Re = 89,000$ ; decibel contour at RHS.
- ❖ Ongoing investigation.





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# Challenges (Conclusions)

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- ❖ The simulations were done with older version of SOLVCON; it takes a lot of efforts to upgrade (ongoing).
- ❖ Unfamiliarity of the collaborators to the authoring system (Sphinx): <http://www.solvcon.net/en/latest/bulk/theory.html>
  - ❖ Unification of code and document needs extra efforts.
- ❖ Feeding back to the main framework.
- ❖ Server farm for continuous integration.
- ❖ And more. **I'd love to learn your comments.**