Bootstrap and Resampling Methods: TD 1

Exercise 1

Consider the Data Generating Process (DGP) about the linear model we have seen in Session 2. The linear model was

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon \text{ with } \mathbb{E}\varepsilon X_j = 0 \text{ for } j = 1, 2$$

- Extract a single sample from such a DGP
- Given such a sample construct the 95 percent confidence interval for each parameter of the linear model based on a White-Huber estimator of the asymptotic variance.
- By using the same sample, construct the 95 percent confidence interval based on the jackknife estimator of the asymptotic variance.
- Which one of the two confidence interval is larger? What is the connection between the width of the confidence interval and the type 1 error of a t-test? What can we say about the ability of the jackknife variance estimator and the White-Huber estimator in estimating the asymptotic variance?

Exercise 2

In this exercise we want to assess the performances of the Jackknife in reducing the Bias and the Mean-Squared Error of an estimator.

Let $X \sim \mathcal{N}(0, 6)$ (so that $Var(X) = 6$), and assume we observe an iid sample $\{X_i\}_{i=1}^n$ with $n = 30$. We are interested in estimating the parameter

$$\theta_0 := \exp(\mathbb{E}\{X\}) .$$

1. Propose an estimator for θ_0, say $\widehat{\theta}$. Construct an R function for it.
2. Propose a Jackknife estimator of $Bias(\widehat{\theta})$ and use it to construct a bias corrected estimator of θ_0. Call this bias-corrected estimator $\widehat{\theta}_{jack}$ and construct an R function for it.
3. Run a Monte Carlo experiment to compute the bias and mean square errors of $\widehat{\theta}$ and $\widehat{\theta}_{jack}$.
4. Comment on the results of point 3.