

Question 1

For 1st connected component which is a complete graph with 100 vertices, there are $99 * 100 / 2 = 4950$ edges. For 2nd connected component which is a complete bipartite graph, there are $50 * 50 = 2500$ edges. In total, **there are 7450 edges**. There is no triangle in 2nd component; **there are** $\binom{100}{3} = 161700$ **triangles**.

Question 2

The maximally possible global clustering coefficient equals to **1**. This is achieved in a **complete graph**, there is no open triplet.

Question 3

The eigenvector corresponding to the smallest eigenvalue of L_{rw} is **vector 1** (whose all components equal to 1). If we ignore this eigenvector, the performance of spectral clustering algorithm would not be affected, as this eigenvector offers no information.

Question 4

It depends on how we initialize K-means. For example, if we use K-means++, the output of the spectral clustering is stochastic.

Question 5

In both cases, the number of edges in graph $m = 13$, the number of community $n_c = 2$.

In figure 1.(a), $l_{green} = 6, l_{blue} = 6, d_{green} = 13, d_{blue} = 13$, therefore, $Q = 0.42$.

In figure 1.(b), $l_{green} = 2, l_{blue} = 4, d_{green} = 11, d_{blue} = 15$, therefore, $Q = -0.05$.

Question 6

For path graph P_4 , its feature map of the shortest path kernel is $\phi(P_4) = [3, 2, 1, 0, \dots, 0]$. For cycle graph C_4 , its feature map of the shortest path kernel is $\phi(C_4) = [4, 2, 0, 0, \dots, 0]$.

Consequently, under shortest path kernel, $(C_4, C_4) = \phi(C_4) * \phi(C_4) = 20, (C_4, P_4) = \phi(C_4) * \phi(P_4) = 16, (P_4, P_4) = \phi(P_4) * \phi(P_4) = 14$.