Urban Informatics

Fall 2018

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Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
- Basic statistics: distributions and their moments
- Hypothesis testing: p-value, statistical significance

Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
- Basic statistics: distributions and their moments
- Hypothesis testing: *p*-value, statistical significance

This class:

- How to choose the right statistical test
- Z, t, F test and tests for correlation
- Correlation vs Causation

hypothesis testing

did we detect a phenomenon
(e.g. as a result of an implemented policy)?

null hypothesis: no relationship between (two) measured phenomena
or no difference between (two) groups
if you have a test control sample: test sample and control sample are the same - no effect

falsify the null hypothesis: do you see an effect?
do you see a difference b/w samples?

A simple (too simple?) answer

did we detect a phenomenon
(e.g. as a result of an implemented policy)?

p-value a measure of the probability that the result you observed could have been observed by chance under the *Null hypothesis*

CUSP

Steps in Null-Rejection Hypothesis Testing

1. Formulate Null (and alternative) Hypothesis
2. Choose a significance level α

3. Measure a *statistic* for *a sample* to be compared to the *parameter of a population*

OR

Measure a *statistic* for *two or more samples* to be compared to *each other*

4. Assess if your statistics is significant or not. In practice: compare the statistics (Z, t, F, chisq) with a distribution table

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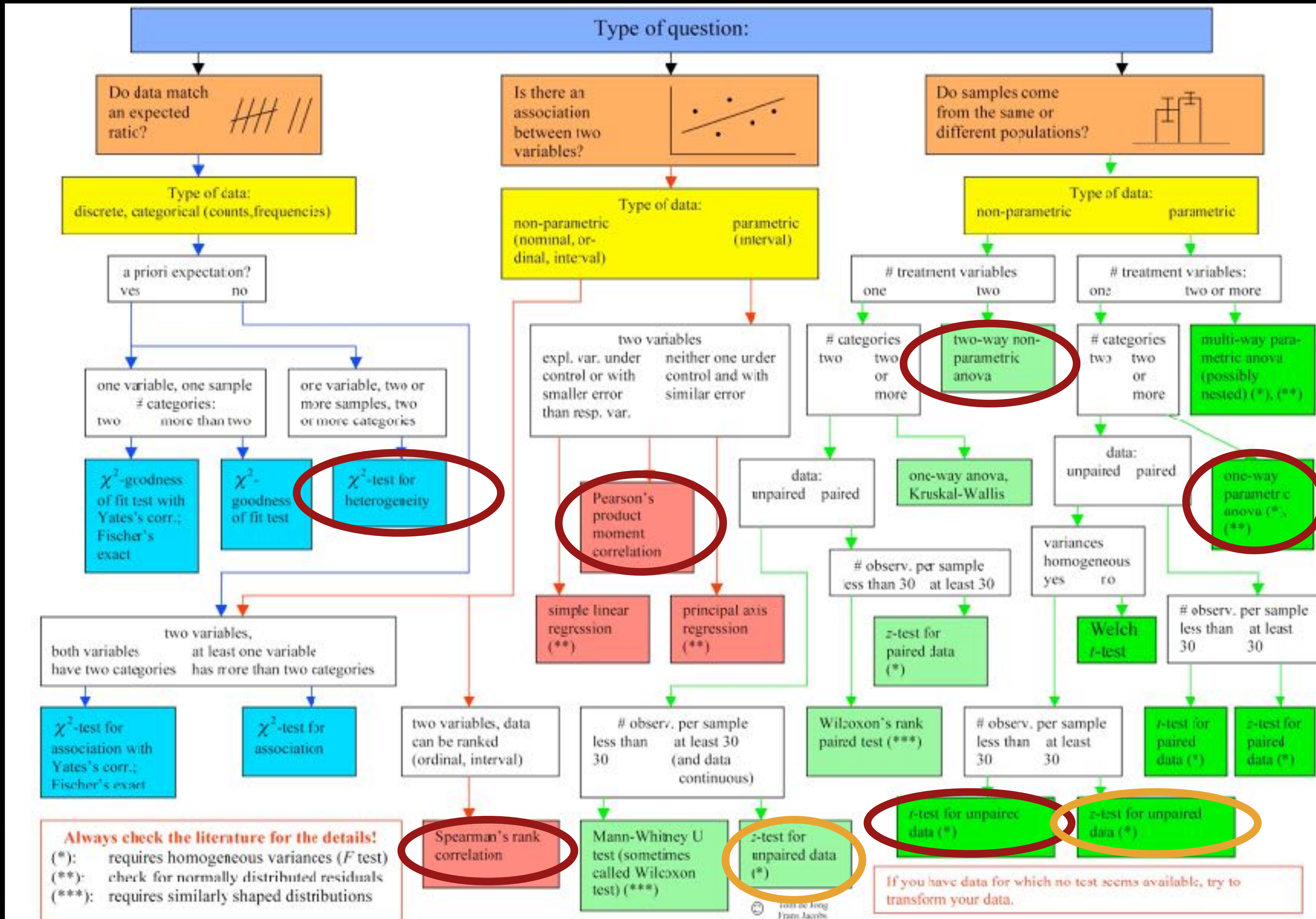
Steps in Null-Rejection Hypothesis Testing



which statistics?

how to compare it?

https://stats.idre.ucla.edu/other/mult-pkg/whatstat/

how to choose the right statistical test

http://www.csun.edu/~amarenco/Fcs%20682/When%20to%20use%20what%20test.pdf

Statistical Analyses	Independent Variables		Dependent Variables		Control Variables	Question Answered by the Statistic
	# of IVs	Data Type	# of DVs	Type of Data		
Chi square	1	categorical	1	categorical	0	Do differences exist between groups?
***t*-Test**	1	dichotomous	1	continuous	0	Do differences exist between 2 groups on one DV?
ANOVA	1 +	categorical	1	continuous	0	Do differences exist between 2 or more groups on one DV?
ANCOVA	1 +	categorical	1	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on one DV?
MANOVA	1 +	categorical	2 +	continuous	0	Do differences exist between 2 or more groups on multiple DVs?
MANCOVA	1 +	categorical	2 +	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on multiple Dvs?
Correlation	1	dichotomous or continuous	1	continuous	0	How strongly and in what direction (i.e., +, -) are the IV and DV related?
Multiple regression	2 +	dichotomous or continuous	1	continuous	0	How much variance in the DV is accounted for by linear combination of the IVs? Also, how strongly related to the DV is the beta coefficient for each IV?
Path analysis	2 +	continuous	1 +	continuous	0	What are the direct and indirect effects of predictor variables on the DV?
Logistic Regression	1 +	categorical or continuous	1	dichotomous	0	What is the odds probability of the DV occurring as the values of the IVs change?

how to choose the right statistical test

Assignment 1:

Browse PlosOne's abstracts to find 1 paper for each of the 3 tests:

http://www.csun.edu/~amarenco/Fcs%20682/When%20to%20use%20what%20test.pdf

choose 1

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how to choose the right statistical test

Assignment 1:

Example: ANCOVA

Statistical Analyses	Dependent Variables	Independe nt Variables	Control Variables	Question Answered
ANCOVA	Ratings about their values ***type: ordinal***	did Self Affirmation or not ***type: categorical***	age ***type: continuous***	self-affirmation group rates the value significantly more than control group

https://journals.plos.org/plosone/article/figure?id=10.1371/journal.pone.0062593.t001

how to choose the right statistical test

Assignment 1: Prepare a markdown table describing the use of the test in each of the papers you chose (3 rows in your table)

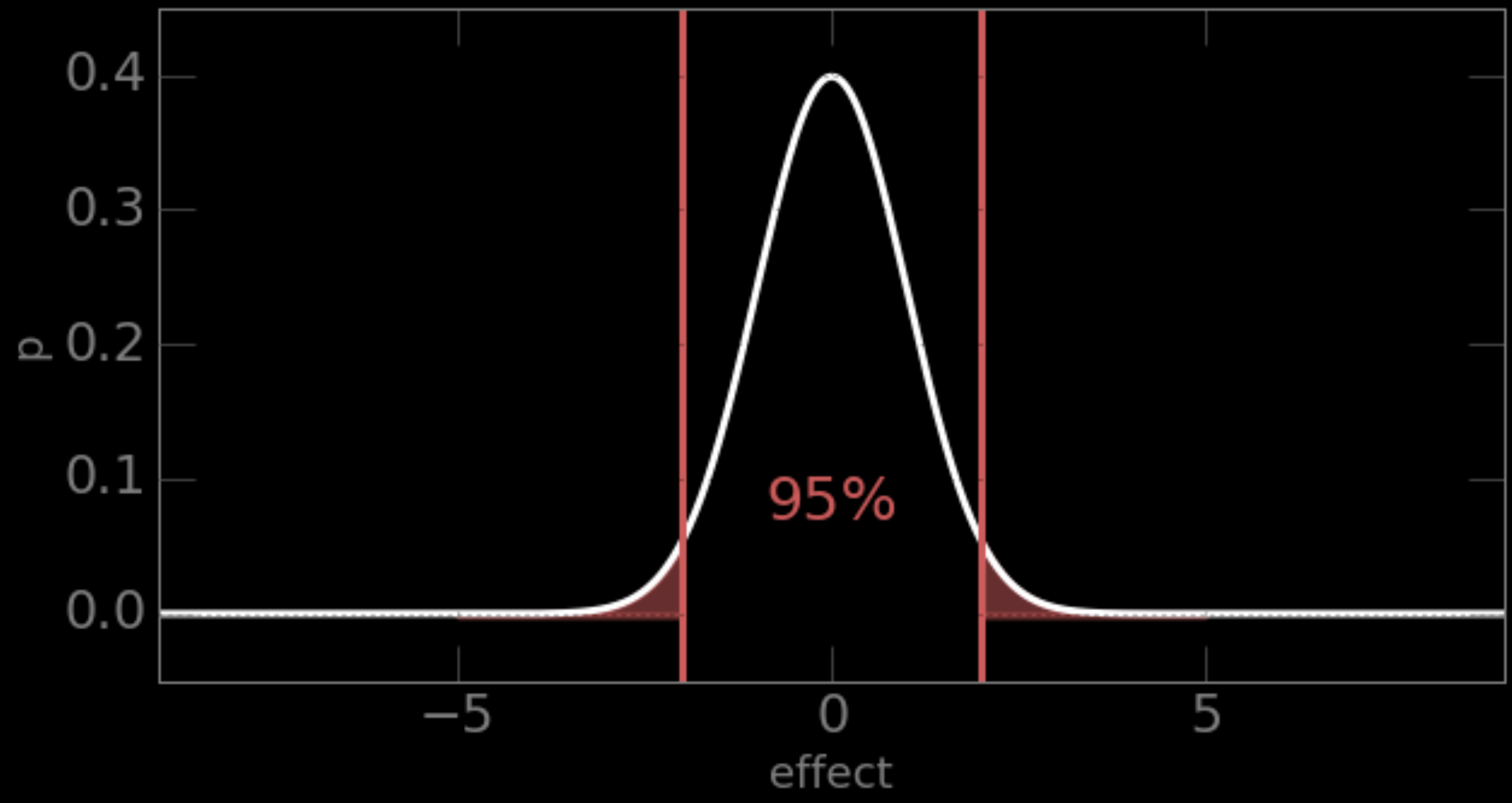
Example: ANCOVA

Statistical Analyses	**Dependent Variables**	**Independent Variables**	**Control Variables**	**Question Answered**
ANCOVA	Ratings about their values ***type: ordinal***	did Self Affirmation or not ***type: categorical***	age ***type: continuous***	self-affirmation group rates the value significantly more than control group

https://journals.plos.org/plosone/article/figure?id=10.1371/journal.pone.0062593.t001 *if you knew how your statistics should be distributed…*

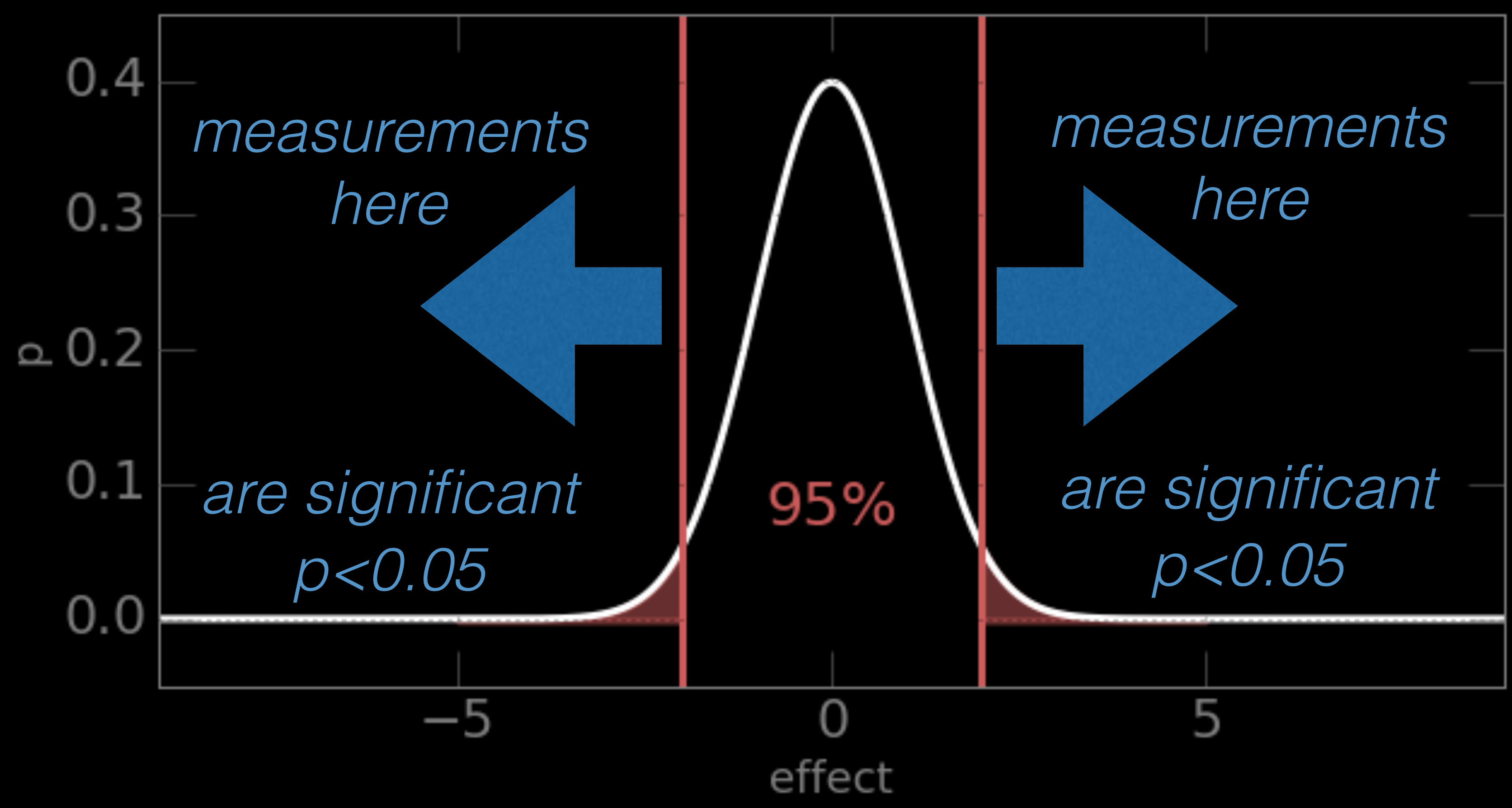
$\alpha = 0.05$

$1 - 0.05 = 0.95 \Rightarrow 95\%$



$\alpha = 0.05$

$1-0.05 = 0.95 \quad => 95\%$



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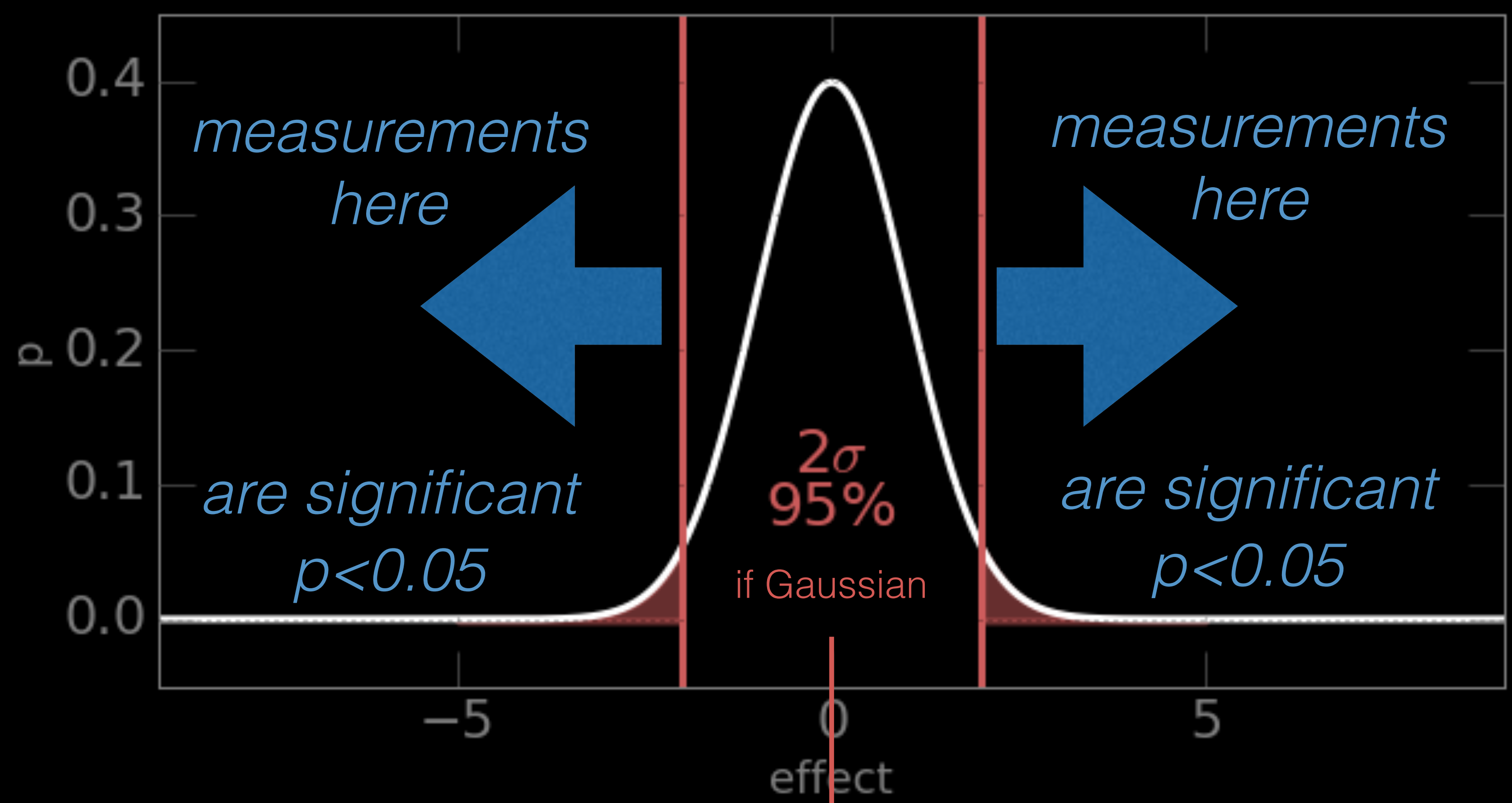
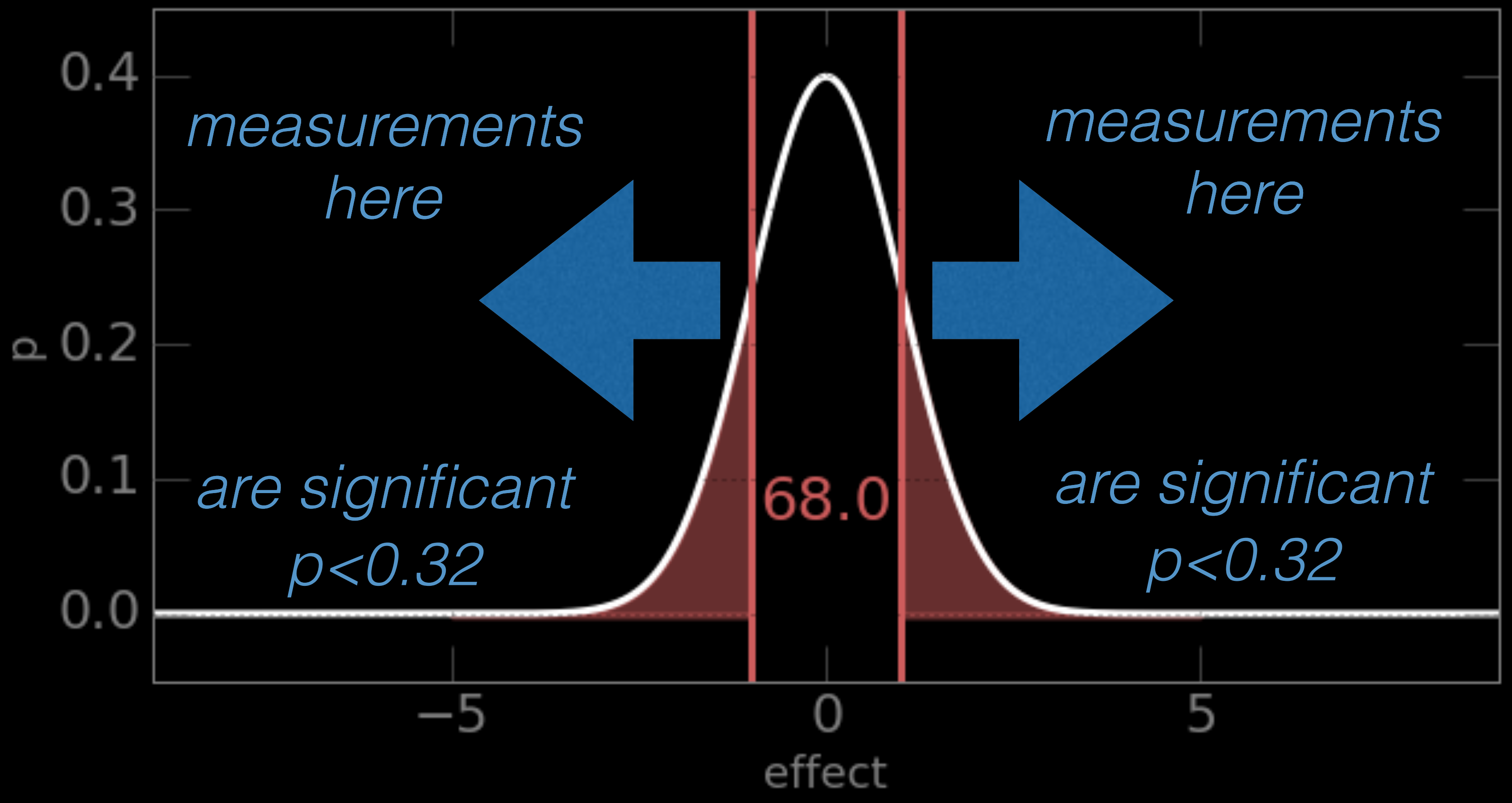


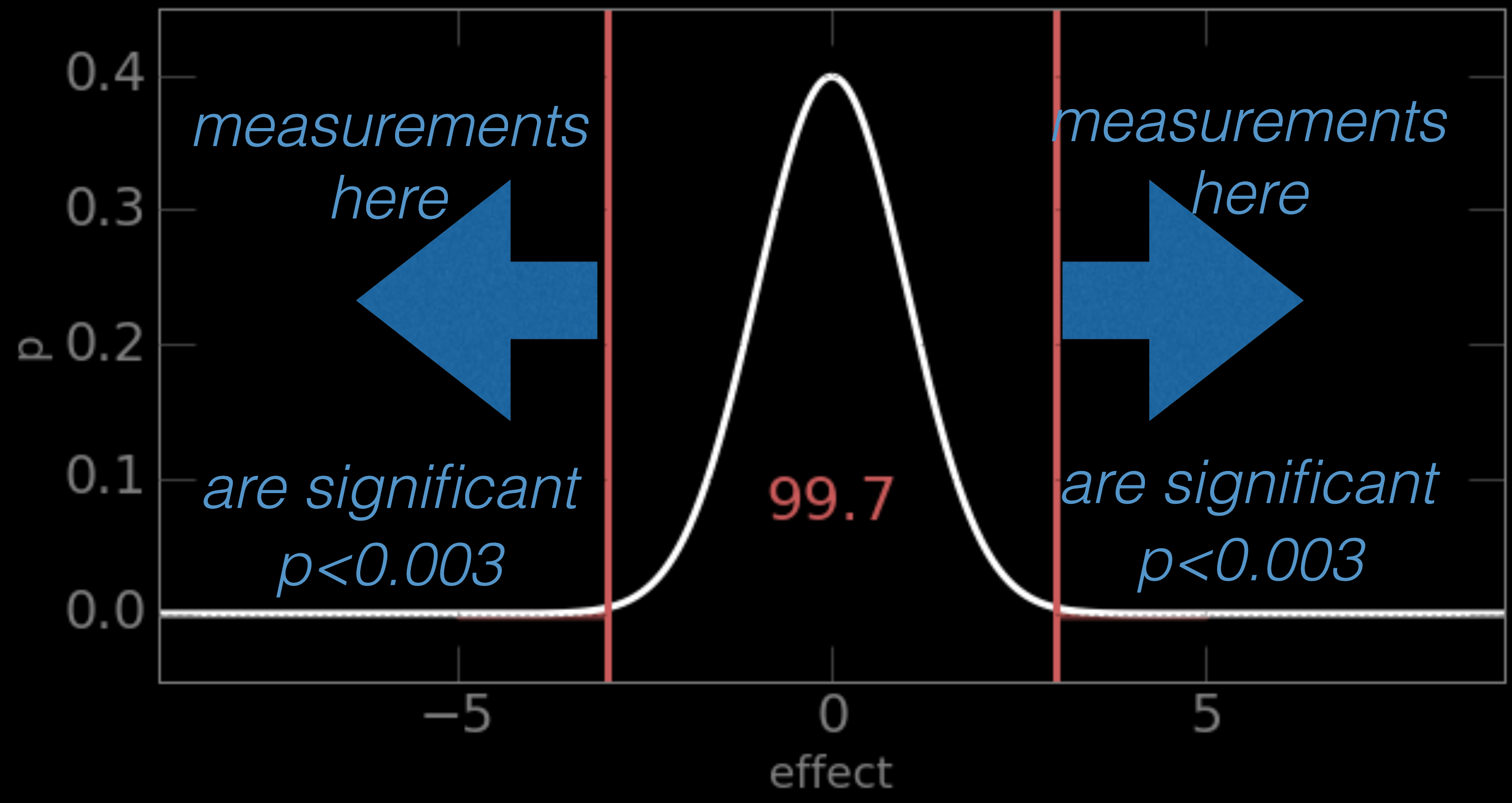
Z test

$\alpha = 0.32$



0.4
0.3
0.2
0.1
0.0
p
measurements here
are significant p<0.32
68.0
measurements here
are significant p<0.32
−5
0
5
effect

$\alpha = 0.003$



0.4
0.3
0.2
0.1
0.0
p
measurements here
are significant p<0.003
99.7
measurements here
are significant p<0.003
−5
0
5
effect

What is the *probability distribution* of a *statistics*?

To measure the probability of the value of a statistics that was measured after an experiment we need to know *how the statistics is distributed* under the null hypotesis.

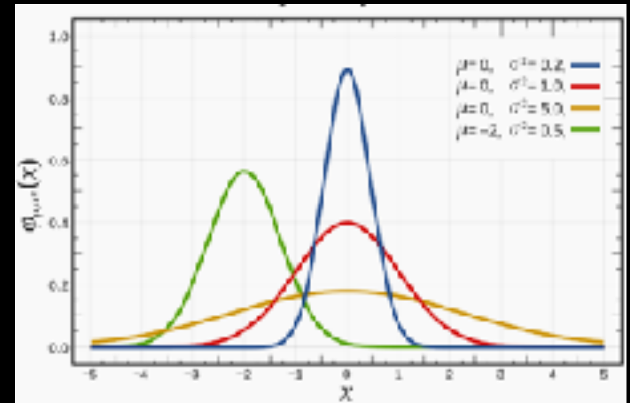
Quantities that follow a specific distribution under the null hypothesis are called *pivotal*.

Each *statistics* follows some distribution.

Which one though?

Z statistics Gaussian

$$Z = \frac{\mu - \bar{x}}{\sigma/\sqrt{n}}$$

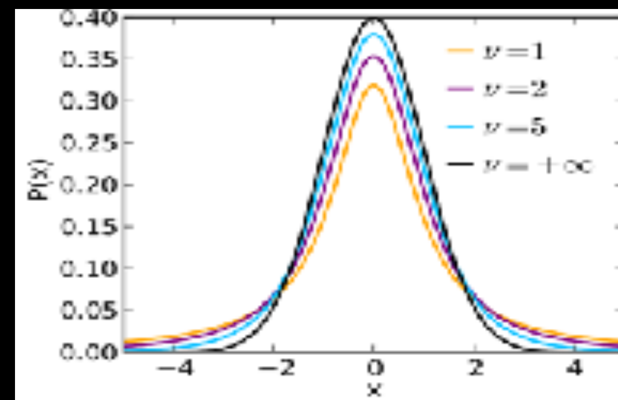


Notation	$\mathcal{N}(\mu,\sigma^2)$
Parameters	$\mu \in \mathbb{R}$ — mean (location) $\sigma^2 > 0$ — variance (squared scale)
Support	$x \in \mathbb{R}$
PDF	$\frac{1}{\sqrt{2\sigma^2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
CDF	$\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$

Quantile	$\mu+\sigma\sqrt{2}\,\operatorname{erf}^{-1}(2F-1)$
Mean	μ
Median	μ
Mode	μ
Variance	σ^2

Student's *t*

$$t = \frac{\mu - \bar{x}}{s/\sqrt{n}}$$

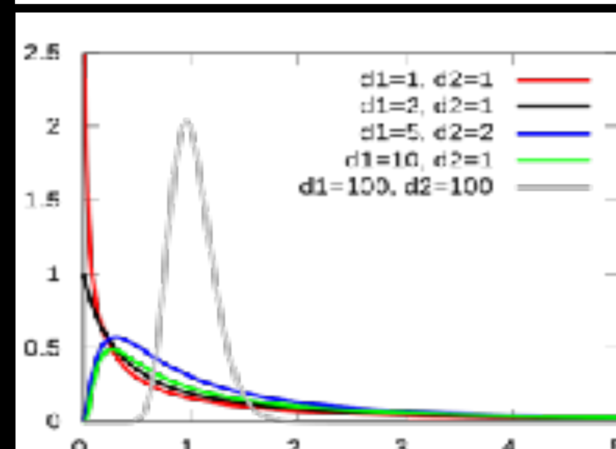


Parameters	$\nu > 0$ degrees of freedom (real)
Support	$x \in (-\infty; +\infty)$
PDF	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
CDF	$\frac{1}{2}+x\Gamma\left(\frac{\nu+1}{2}\right)\times\frac{{}_2F_1\left(\frac{1}{2},\frac{\nu+1}{2};\frac{3}{2};-\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)}$ where ${}_2F_1$ is the hypergeometric function

Mean	0 for $\nu > 1$, otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$, ∞ for $1 < \nu \le 2$, otherwise undefined

F statistics

$$F = \frac{\sum_i n_i(\bar{x}_i - \bar{x})^2/(K-1)}{\sum_{ij}(x_{ij} - \bar{x}_i)^2/(N-K)}$$

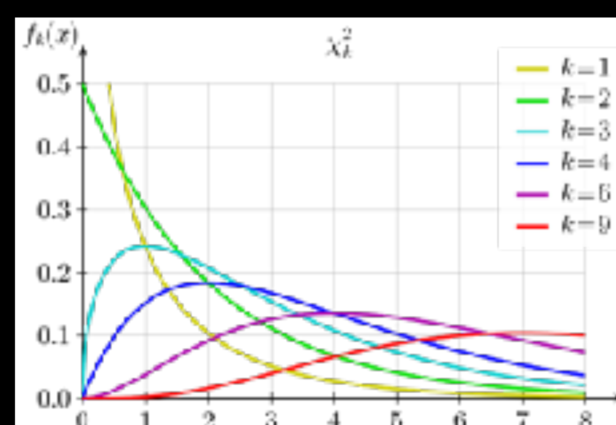


Parameters	$d_1, d_2 > 0$ deg. of freedom
Support	$x \in [0, +\infty)$
PDF	$\frac{\sqrt{\frac{(d_1 x)^{d_1}\, d_2^{d_2}}{(d_1 x+d_2)^{d_1+d_2}}}}{x\,\mathrm{B}\left(\frac{d_1}{2},\frac{d_2}{2}\right)}$
CDF	$I_{\frac{d_1 x}{d_1 x+d_2}}\left(\frac{d_1}{2},\frac{d_2}{2}\right)$

Mean	$\frac{d_2}{d_2-2}$ for $d_2 > 2$
Mode	$\frac{d_1-2}{d_1}\,\frac{d_2}{d_2+2}$ for $d_1 > 2$
Variance	$\frac{2\,d_2^2\,(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}$ for $d_2 > 4$
Skewness	$\frac{(2d_1+d_2-2)\sqrt{8(d_2-4)}}{(d_2-6)\sqrt{d_1(d_1+d_2-2)}}$ for $d_2 > 6$

Pearson's χ^2

$$\chi^2_P = \sum_i \frac{(O_i - E_i)^2}{E_i}$$



Notation	$\chi^2(k)$ or χ^2_k
Parameters	$k \in \mathbb{N}_{>0}$ (known as "degrees of freedom")
Support	$x \in [0, +\infty)$
PDF	$\frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)}\, x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$
CDF	$\frac{1}{\Gamma\left(\frac{k}{2}\right)}\,\gamma\left(\frac{k}{2},\frac{x}{2}\right)$

Mean	k
Median	$\approx k\left(1-\frac{2}{9k}\right)^3$
Mode	$\max(k-2, 0)$
Variance	$2k$
Skewness	$\sqrt{8/k}$

goodness of fit χ^2

$$\chi^2_F = \sum_i \frac{(m_i - x_i)^2}{e_i}$$

see
Statistics in a Nutshell - IV: Statistical analysis

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https://documents.software.dell.com/statistics/textbook/distribution-tables

Is there a difference between means or population and sample, difference between proportion in 2 samples?

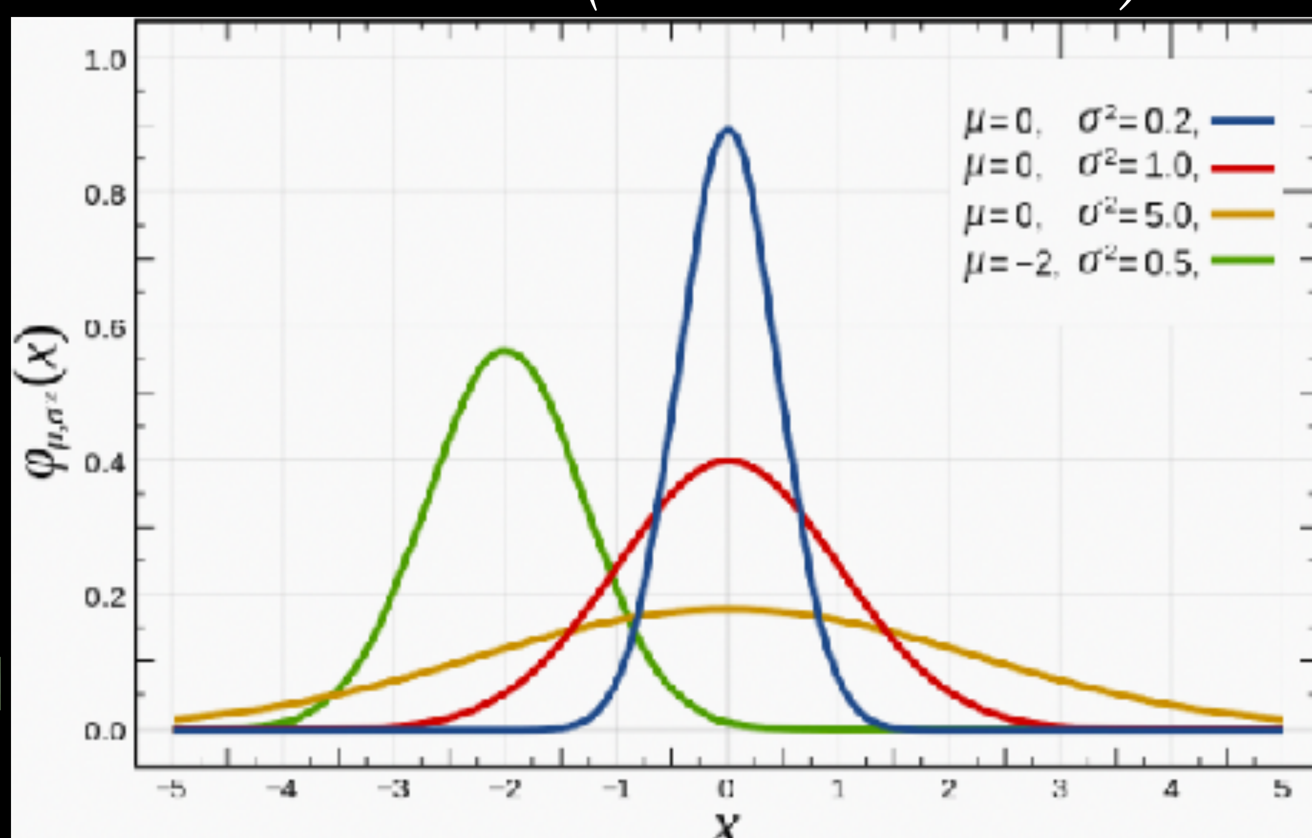
Z statistics

$$Z = \frac{\mu - \bar{x}}{\sigma / \sqrt{n}}$$

In absence of effect (i.e. under the Null)

Z *is distributed according to a* ***Standard Normal*** N***(μ=0, σ=1)***

(i.e. Gaussian)



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Mode	μ
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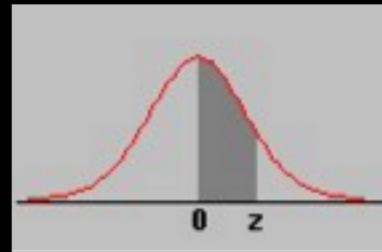
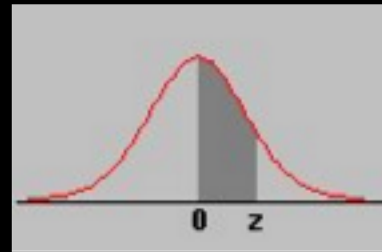


$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{N}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md



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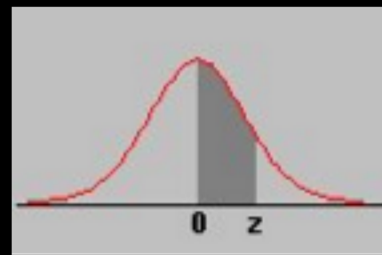
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{N}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
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3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

1 sided test $1 - 0.4948 - 0.5 = 0.0052$

$p < 0.05$

H_0 IS REJECTED ($p<0.05$)

$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{N}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

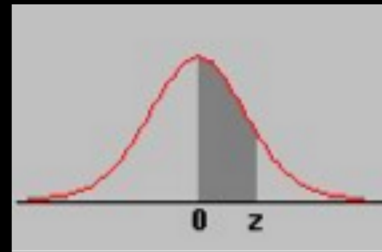
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

1 sided test $1 - 0.4948 - 0.5 = 0.0052$

$p < 0.05$

H_0 IS REJECTED ($p<0.05$)



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{N}} = 2.55$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

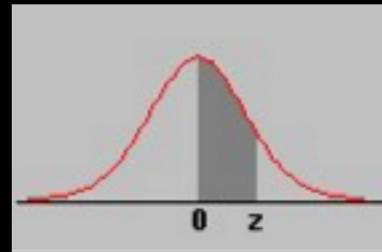
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

1 sided test   1 - 0.4946 - 0.5 = 0.0054

$p < 0.05$

H_0 IS REJECTED ($p<0.05$)



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{N}} = 1.57$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	**0.4418**	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

2 sided test $1 - 0.4418 * 2 = 0.1164$

$p > 0.05$

CUSP

H_0 CANNOT BE REJECTED

Is there a difference between means of 2 sample?

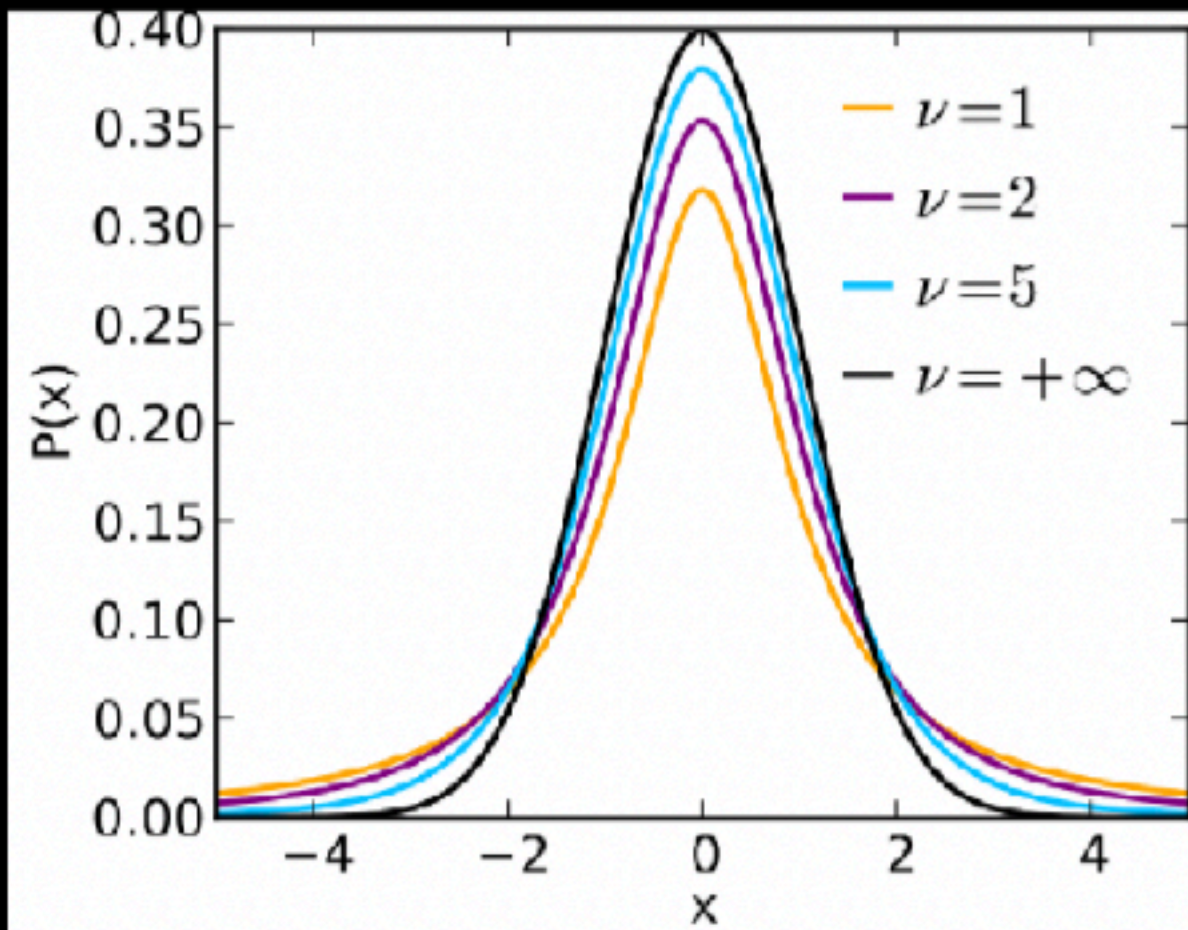
In absence of effect (i.e. under the Null)

Student's *t*

$$t = \frac{\mu - x}{s/\sqrt{n}}$$

t test for 1 small samples and a population

t *is distributed according to a* ***t-distribution*** *with* ***v*** *= degrees of freedom of the problem*



Parameters	$\nu > 0$ degrees of freedom (real)
Support	$x \in (-\infty; +\infty)$
PDF	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
CDF	$\frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \times \frac{{}_2F_1\left(\frac{1}{2},\frac{\nu+1}{2};\frac{3}{2};-\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)}$ where ${}_2F_1$ is the hypergeometric function
Mean	0 for $\nu > 1$, otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$, ∞ for $1 < \nu \le 2$, otherwise undefined

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

t test between 2 samples (*unpaired t test*)

Comparing to population

Standard DEVIATION of Sample Estimates

Sample mean, $\overline{x}$	$Z = \dfrac{\mu_{pop} - \overline{x}}{\sigma / \sqrt{n}}$	$\sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}}$
Sample proportion, p	$z = \dfrac{p - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$	$\sigma_p = \sqrt{\dfrac{p_0(1-p_0)}{n}}$
Difference between proportions, p1 - p2	$z = \dfrac{(p_2 - p_1)}{\sqrt{p(1-p)(\dfrac{1}{n_2} + \dfrac{1}{n_1})}}, p = \dfrac{p_2 n_2 + p_1 n_1}{n_2 + n_1}$	$\sigma_{p_1 - p_2} = \sqrt{\dfrac{P_1(1-P_1)}{n_1} + \dfrac{P_2(1-P_2)}{n_2}}$

Use if you know the population *parameters*

e.g. Z - test

Comparing samples

Standard ERROR of Sample Estimates

Sample mean, $\bar{x}$	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$
Sample proportion, p	$SE_p = \frac{p(1-p)}{n}$
Difference between means, x1 - x2	$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
Difference between proportions, p1 - p2	$SE_{p_1-p_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

Use if you DO NOT know the population *parameters*

e.g. t - test

Is there a difference between means of 2 sample?

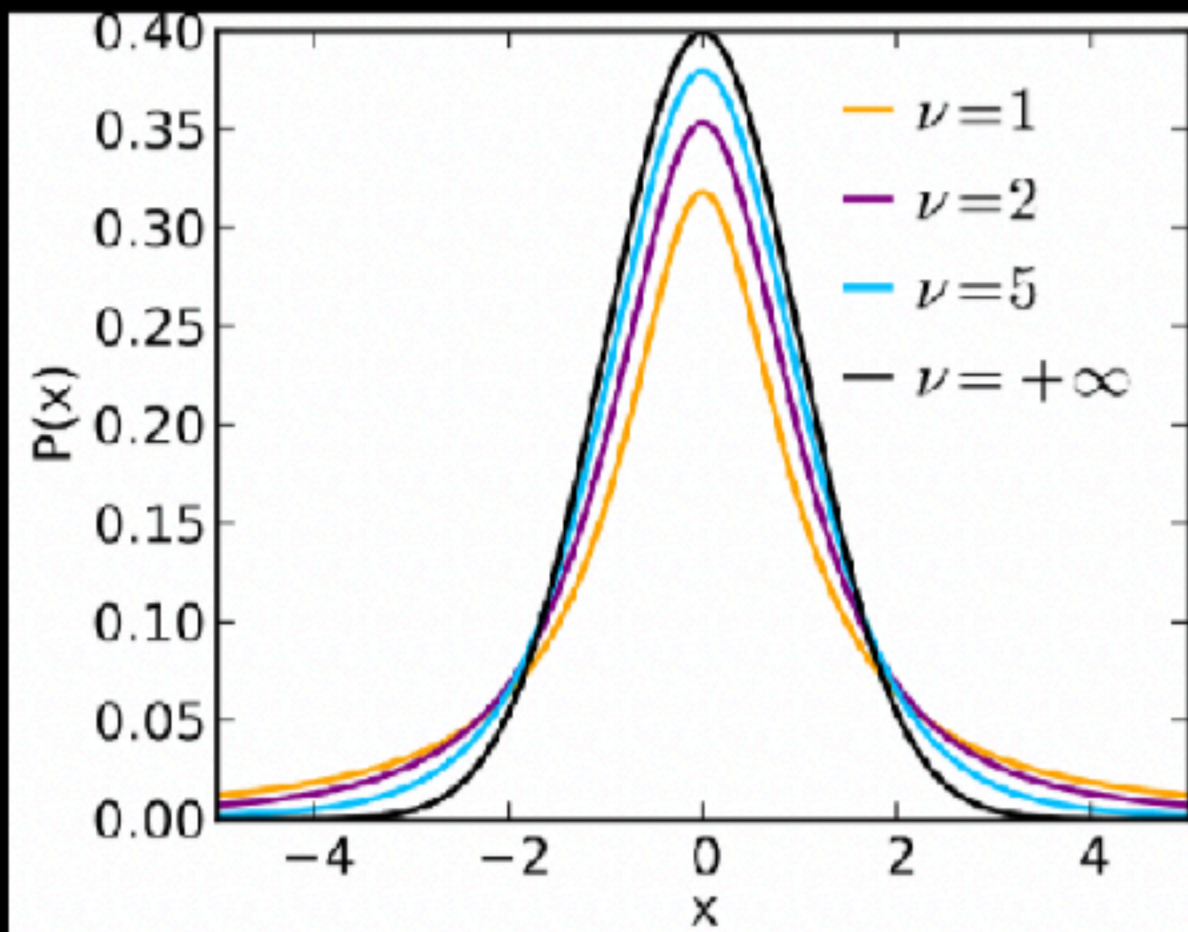
In absence of effect (i.e. under the Null)

Student's *t*

$$t = \frac{\mu - x}{s/\sqrt{n}}$$

t test for 1 small samples and a population

t *is distributed according to a* ***t-distribution*** *with* **v** *= degrees of freedom of the problem*



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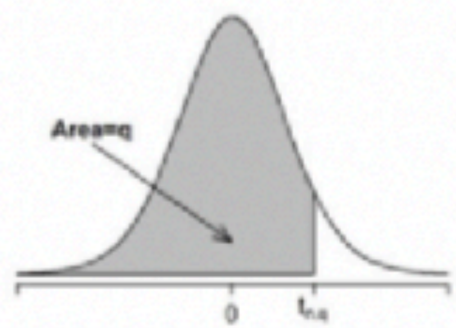
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

t test between 2 samples (*unpaired t test*)

Is there a difference between means of 2 sample?

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$



	$q = 0.6$	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$n = 1$	0.3249	1.0000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.2887	0.8165	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.2767	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.2707	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.2672	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.2648	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.2632	0.7111	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.2619	0.7064	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.2610	0.7027	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.2602	0.6998	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.2596	0.6974	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.2590	0.6955	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.2586	0.6938	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.2582	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140

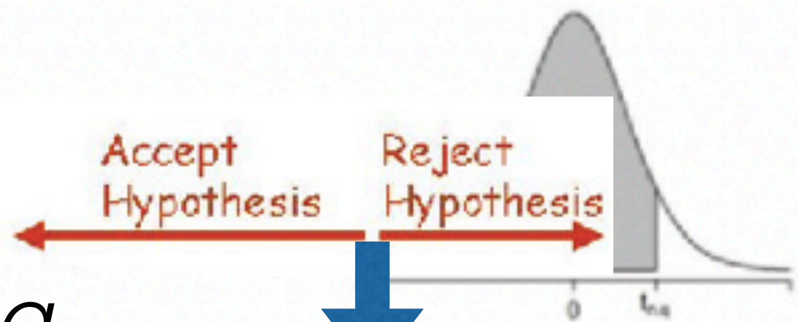
← *1-a*

d.o.f *: n - 2 (1 sample) or n1 + n2 - 2 (2 samples)*

Generally d.o.f = *n_observations* - 1

Is there a difference between means of 2 sample?

$$t_{\substack{n=13,\\ q=.95}} = \frac{\mu - x}{s/\sqrt{n}} = 1.75$$



q=1-a

	q = 0.6	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
n = 1	0.3249	1.0000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.2887	0.8165	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.2767	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.2707	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.2672	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.2648	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.2632	0.7111	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.2619	0.7064	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.2610	0.7027	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.2602	0.6998	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.2596	0.6974	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.2590	0.6955	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.2586	0.6938	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.2582	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140

← 1-a

d.o.f

Generally d.o.f $= n\_{observations} - 1$

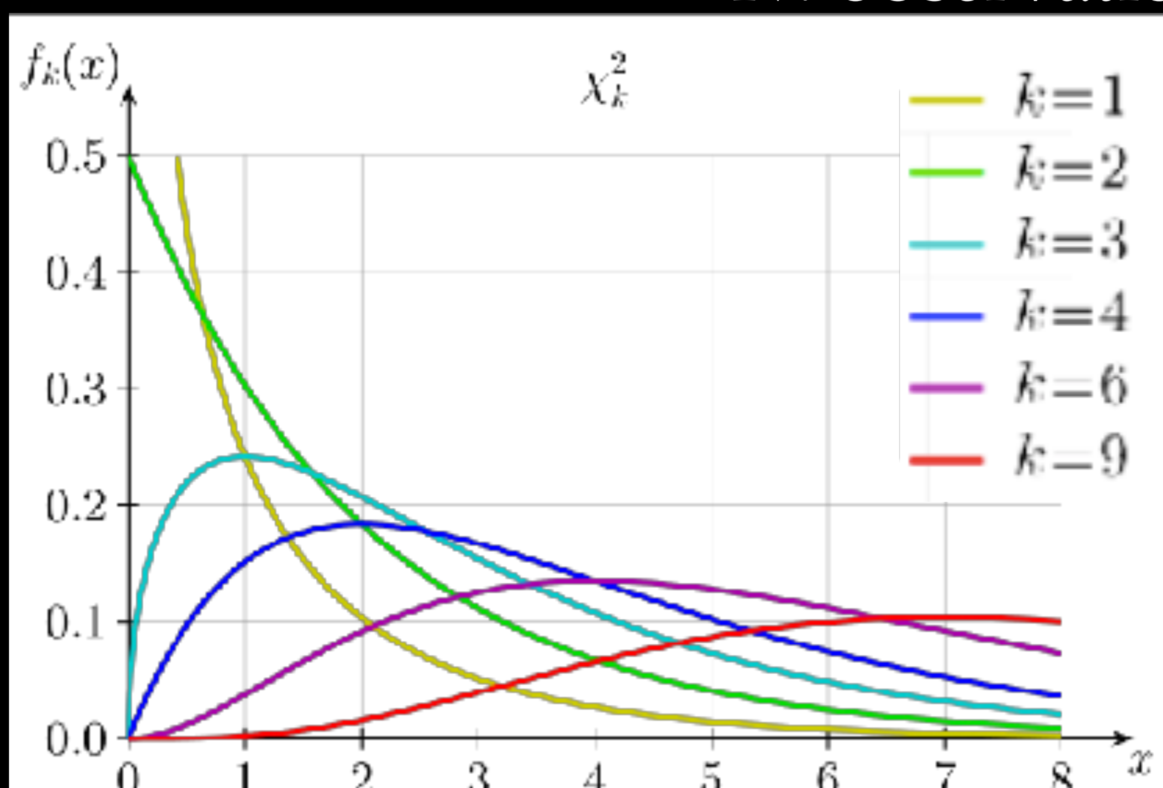
Is there a difference between means or population and sample, difference between proportion in 2 samples?

χ^2 statistics

$$\chi_P^2 = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}$$

In absence of effect (i.e. under the Null) χ^2 is distributed according to a χ^2 distribution with k=degrees of freedom

O: observed
E: expected (also model prediction)
N: observations



Notation	$\chi^2(k)$ or χ_k^2
Parameters	$k \in \mathbb{N}_{>0}$ (known as "degrees of freedom")
Support	$x \in [0, +\infty)$
PDF	$\frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$
CDF	$\frac{1}{\Gamma\left(\frac{k}{2}\right)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$
Mean	k
Median	$\approx k\left(1 - \frac{2}{9k}\right)^3$
Mode	$\max\{k-2, 0\}$
Variance	$2k$
Skewness	$\sqrt{8/k}$

observed → $\chi^2_P = \sum_i \frac{(O_i - E_i)^2}{E_i}$ ← expected, expected

For test of proportion

4 observations - 1 independent variable = 3 degreed of freedom

Accept Hypothesis ← | → Reject Hypothesis

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09

$\alpha = 0.05$

$p < 0.05$

H_0 CAN BE REJECTED

For goodness of fit

model data

$$\chi_F^2 = \sum_{i=1}^{4} \frac{(m_i - x_i)^2}{e_i}$$

error

4 observations - 1 independent variable =
3 degreed of freedom

Accept Hypothesis Reject Hypothesis

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
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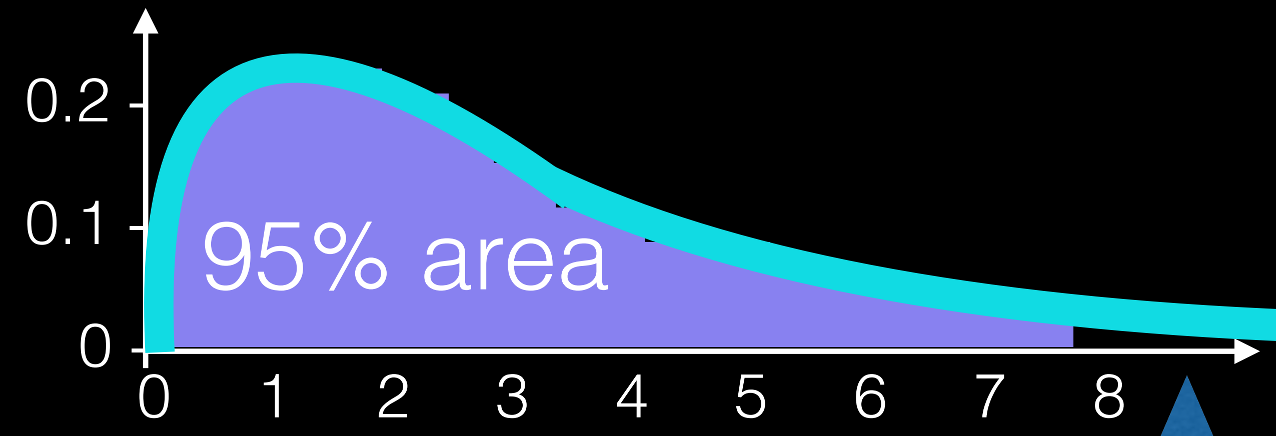
$\alpha = 0.05$

$p < 0.05$

H_0 CAN BE REJECTED



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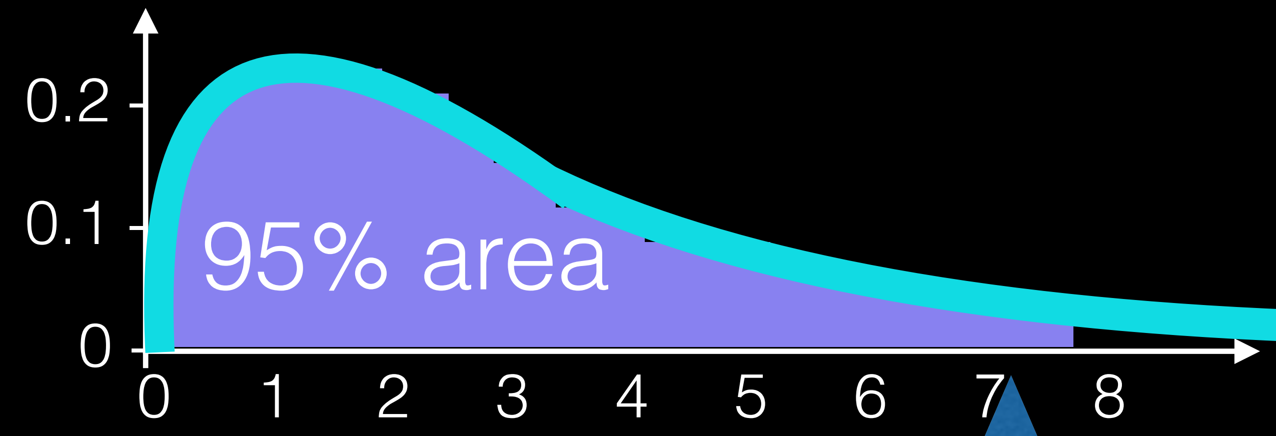
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09

$\alpha = 0.05$

$$\chi^2_P = \sum_i \frac{(O_i - E_i)^2}{E_i} = 8.57 \qquad 8.57 > 7.81$$

$p < 0.05$

H_0 CAN BE REJECTED



4 observations - 1 independent variable =
3 degreed of freedom

Accept Hypothesis ← → Reject Hypothesis

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
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5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09

$\alpha = 0.05$

$$\chi_P^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} = 7.11 \qquad 7.11 < 7.81$$

$p >= 0.05$

H_0 CANNOT BE REJECTED

Example: ***NULL* HYPOTHESIS**: the % of former prisoners employed 3 years after release is *the same or lower* for candidates who participated in the program as for the control group,

significance level p=0.05

What Strategies Work for the Hard-to-Employ?

Final Results of the Hard-to-Employ Demonstration and Evaluation Project and Selected Sites from the Employment Retention and Advancement Project

OPRE Report 2012-08

March 2012

https://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf

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significance level p=0.05

The Enhanced Services for the Hard-to-Employ Demonstration and Evaluation Project

Table 2.1

Summary of Impacts, New York City Center for Employment Opportunities

Outcome	Program Group	Control Group	Difference (Impact)	P-Value
Employment (Years 1-3) (%)	P_1	P_0		
Ever employed	83.8	70.4	13.4 ***	0.000
Ever employed in a CEO transitional job[a]	70.1	3.5	66.6 ***	0.000
Ever employed in an unsubsidized job	63.7	69.0	-5.3 *	0.078
Postprogram unsubsidized employment (Years 2-3)				
Ever employed in an unsubsidized job (%)	53.3	52.1	1.2	0.713
Employed in an unsubsidized job, average per quarter (%)	28.2	27.2	1.1	0.618
Employed for six or more consecutive quarters (%)	14.7	11.9	2.8	0.195
Total UI-covered earnings[b] ($)	10,435	9,846	589	0.658
Sample size (total = 973)[c]	564	409		

SOURCES: MDRC earnings calculations from the National Directory of New Hires (NDNH) database and employment calculations from the unemployment insurance (UI) wage records from New York State, MDRC calculations using data from the New York State Division of Criminal Justice Services (DCJS) and the New York City Department of Correction (DOC).

NOTES: Statistical significance levels are indicated as: *** = 1 percent; ** = 5 percent; * = 10 percent.
The p-value indicates the likelihood that the difference between the program and control groups arose by chance.

H_0: $P_0 - P_1 >= 0$

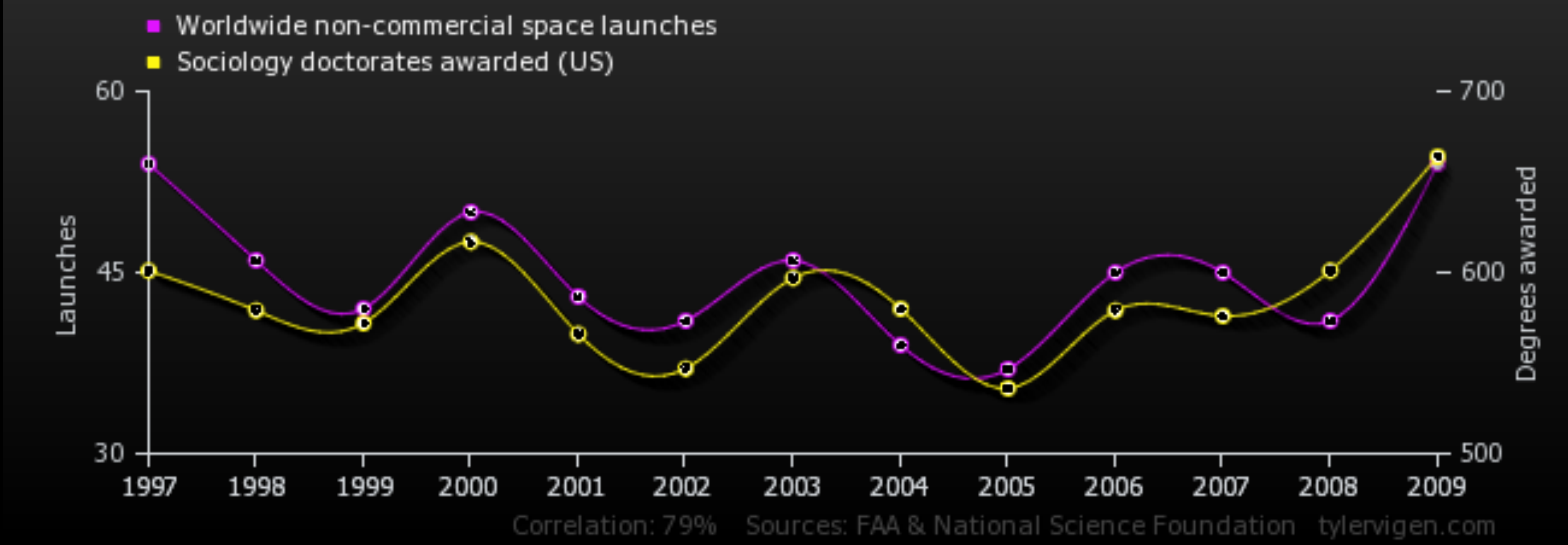
H_a: $P_0 - P_1 < 0$

$\alpha = 0.05$

http://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf

CUSP

https://github.com/fedhere/PUI2018_fb55/blob/master/Lab5_fb55/effectivenes%20of%20NYC%20Post-Prison%20Employment%20Programs.ipynb

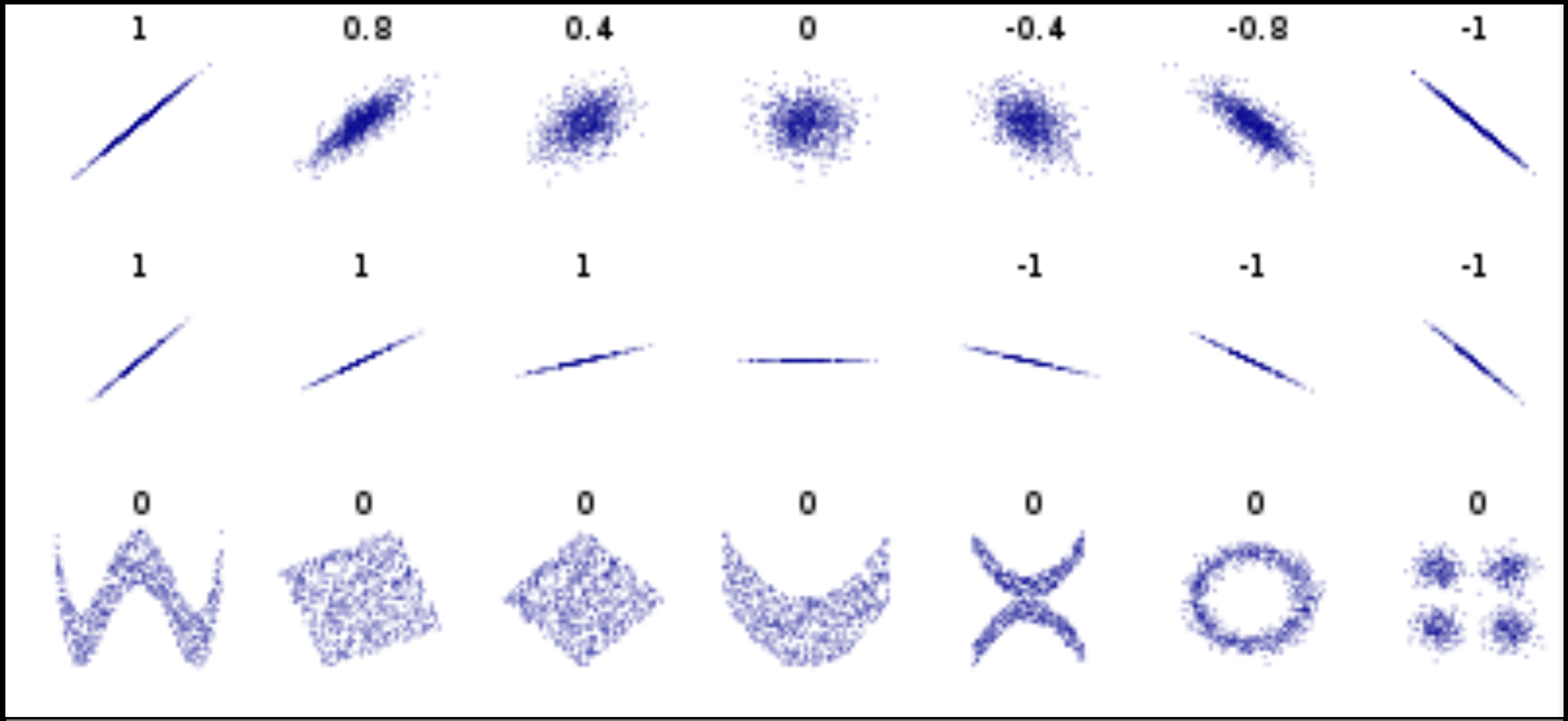


Correlation

Pearson's test:

$$r_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

$$s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$



Pearson's test:

$$r_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

Spearman's test:
(Pearson's for ranks)

$$\rho = 1 - \frac{6 \sum (x_i - y_i)^2}{n(n^2 - 1)}$$

Choosing the test

Use the table below to choose the test. See below for further details.

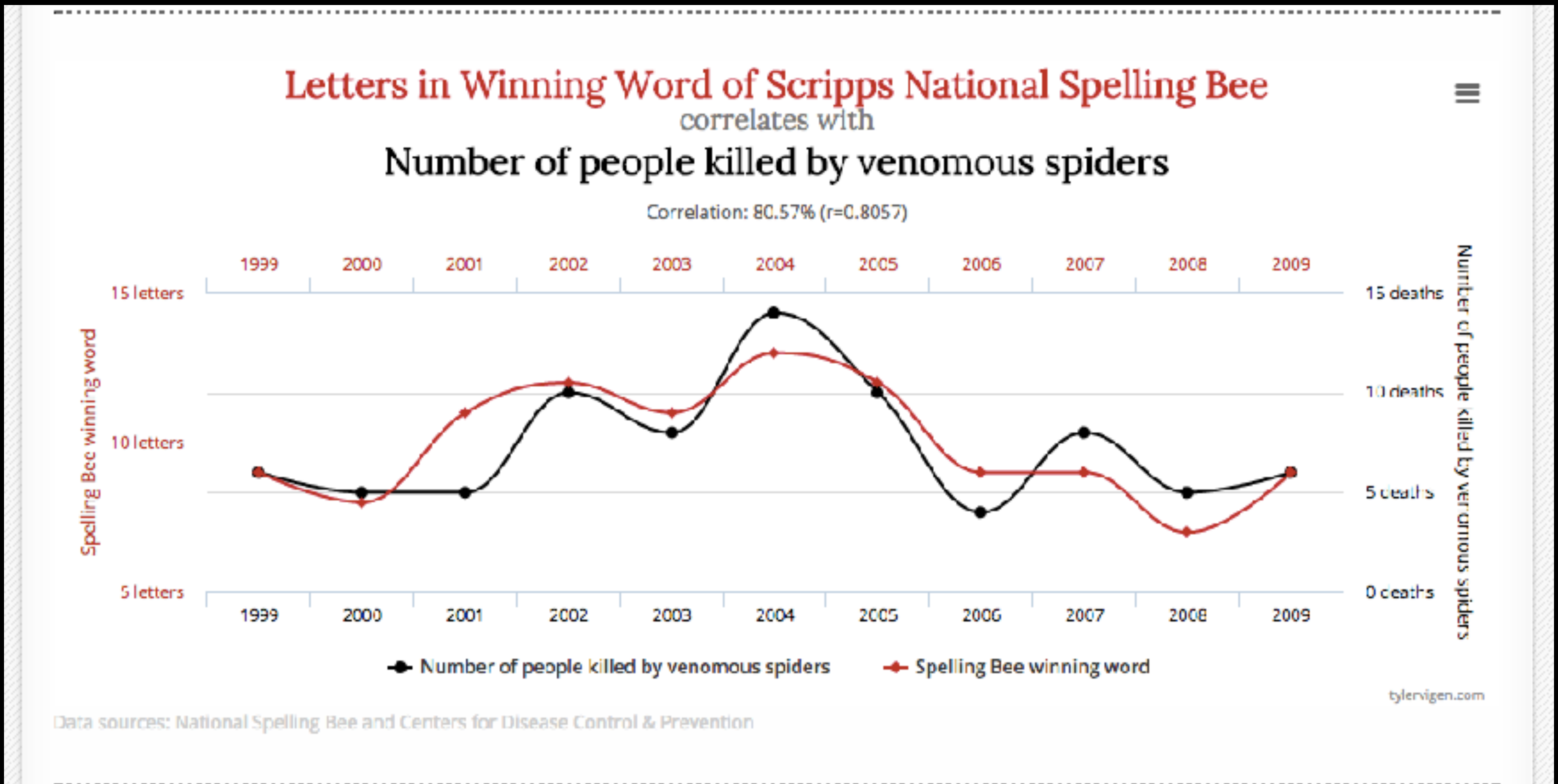
How many dichotomous[+] (binary) variables?				
0	Both variables interval or ratio?			
	Y	Measures are linear? (No = monotonic[*])		
		Y	Pearson correlation	
		N	Spearman correlation	
	N	Both variables are ordinal?		
		Y	Kendall correlation	
		N	Both variables can be ranked?	
			Y	Kendall correlation
			N	Convert to frequency data and use Chi-square test for independence
1	Biserial Correlation Coefficient			
2	2 x 2 table?			
	Y	Phi		
	N	Cramer's V		

Data has frequency values for each category?	
Y	Chi-square test for independence

[+]dichotomous = 'can have only two values' (eg. yes/no or 0/1).
[*]monotonic = constantly increasing or decreasing.

http://changingminds.org/explanations/research/analysis/choose_correlation.htm

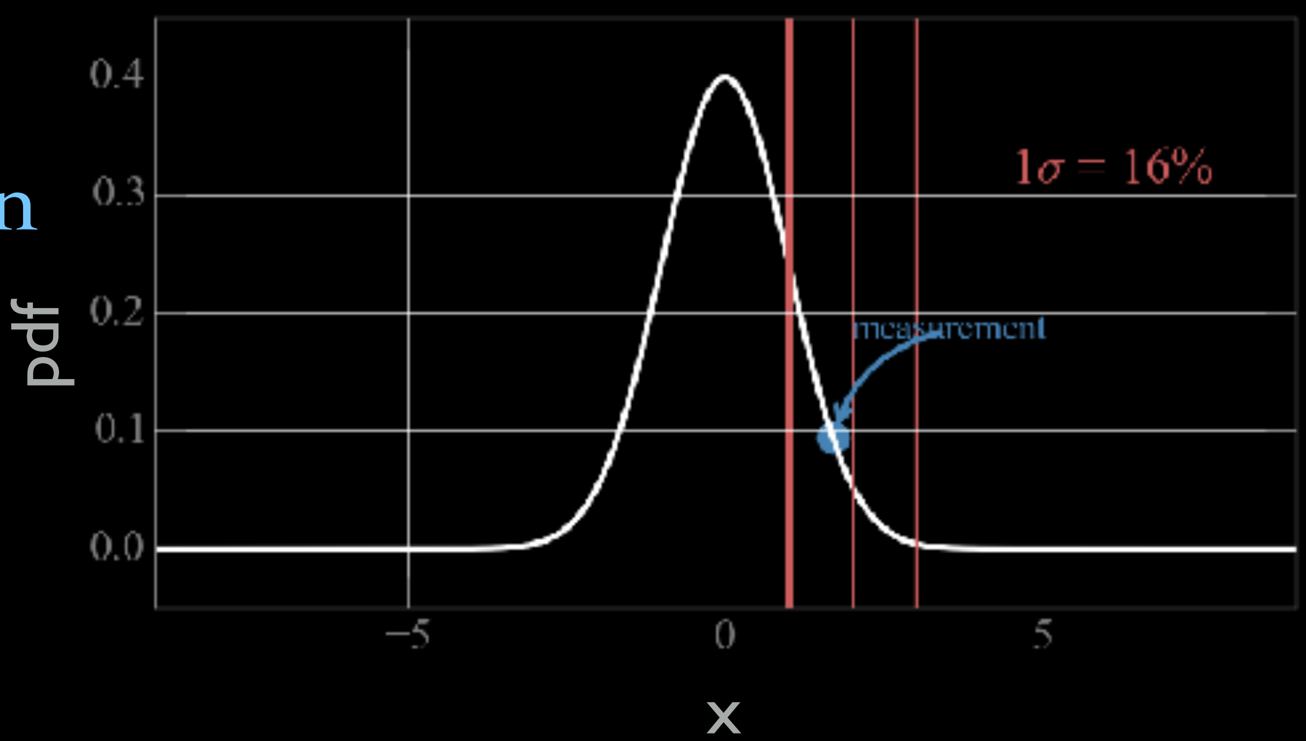
WARNING: Correlation is not causation!



http://www.tylervigen.com/spurious-correlations

Tests for correlation and independence (continuous variables)

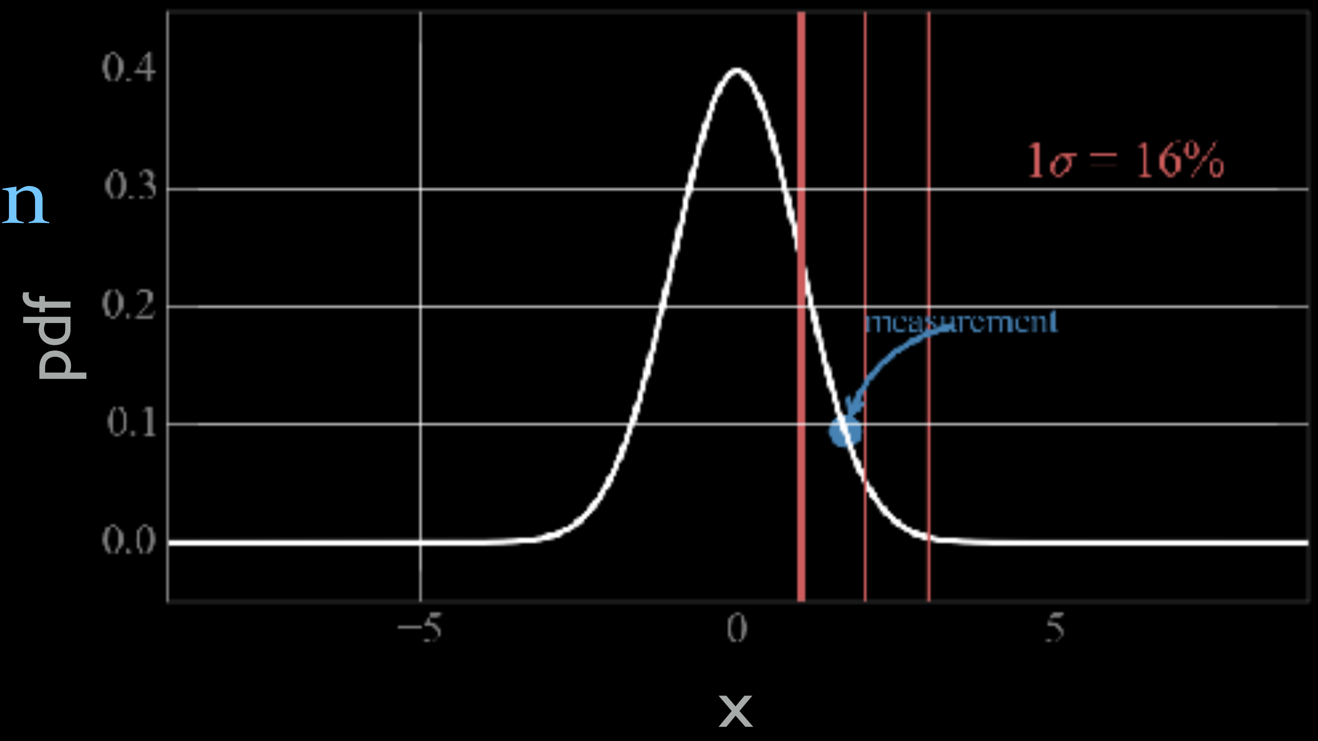
Probability Distribution Function



$$f_{x_0}(x) \sim \; p(x=x_0)$$

$$f_{x_0}(x) \sim \; p(x>x_0-dx) \bigcap p(x<x_0+dx)$$

Probability Distribution Function

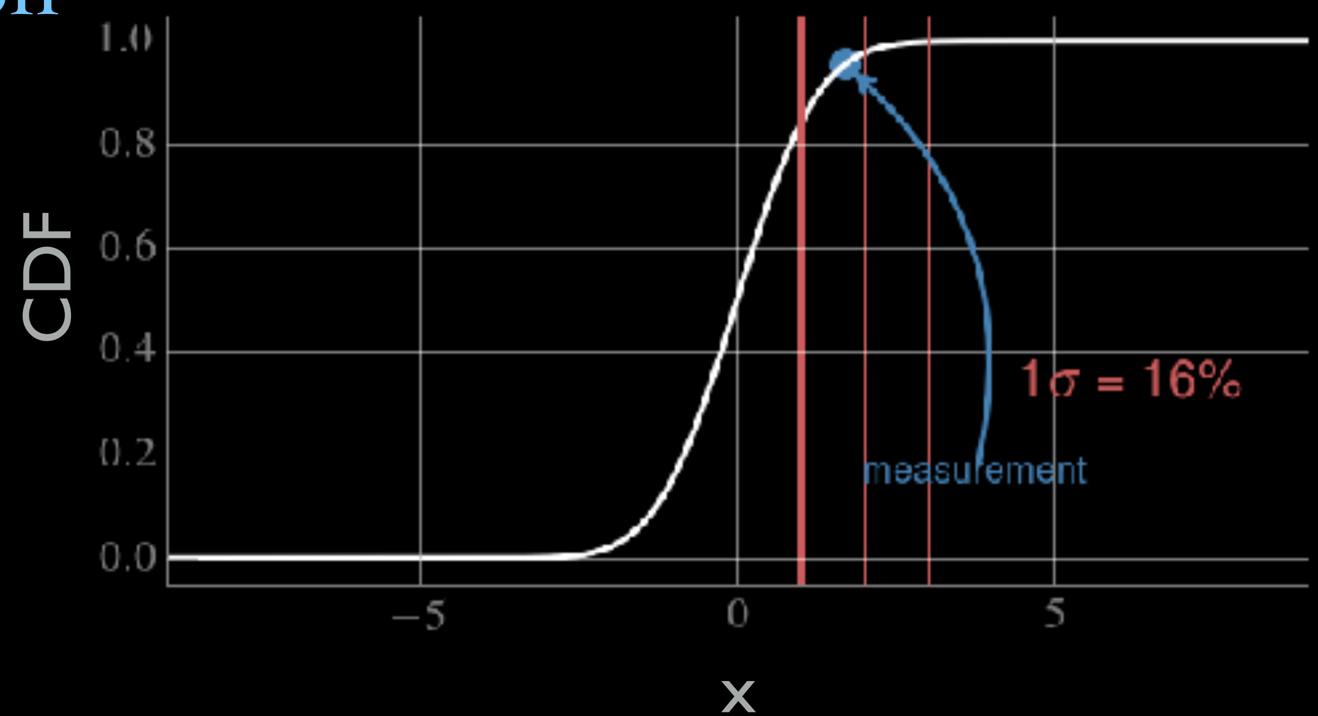


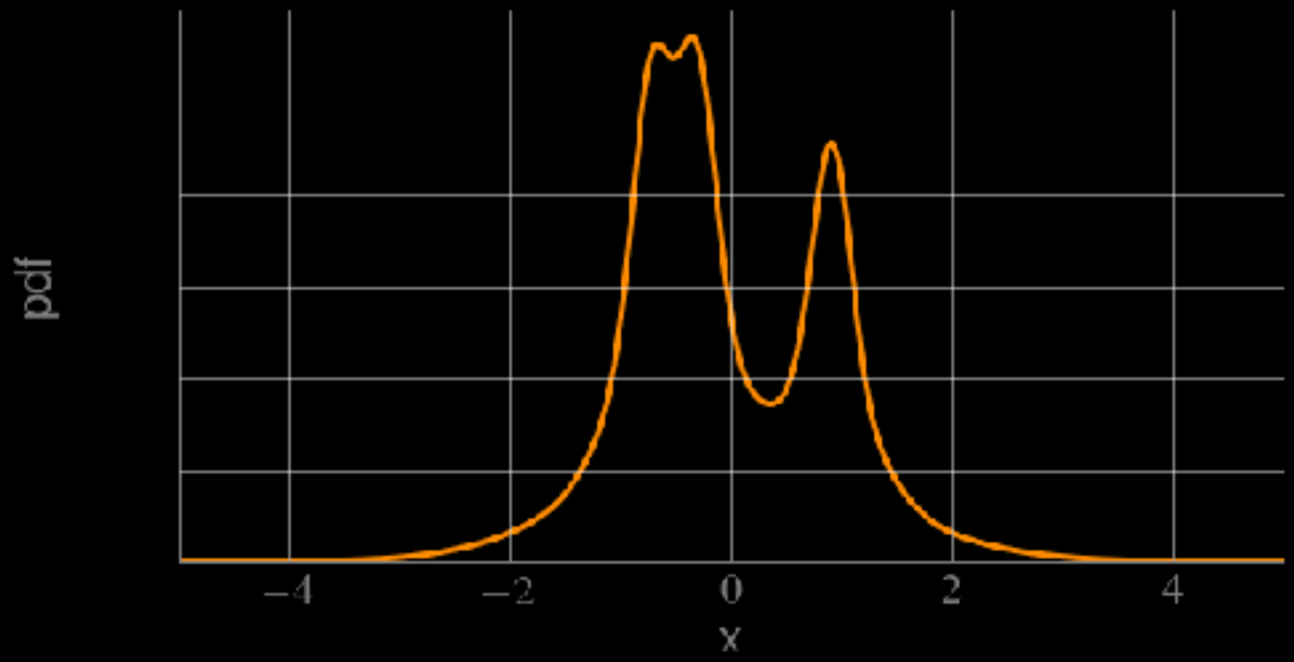
$$f_{x_0}(x) \sim p(x = x_0)$$

$$f_{x_0}(x) \sim p(x > x_0 - dx) \bigcap p(x < x_0 + dx)$$

Cumulative Distribution Function

$$F_{x_0}(x) = P(x < x_0)$$

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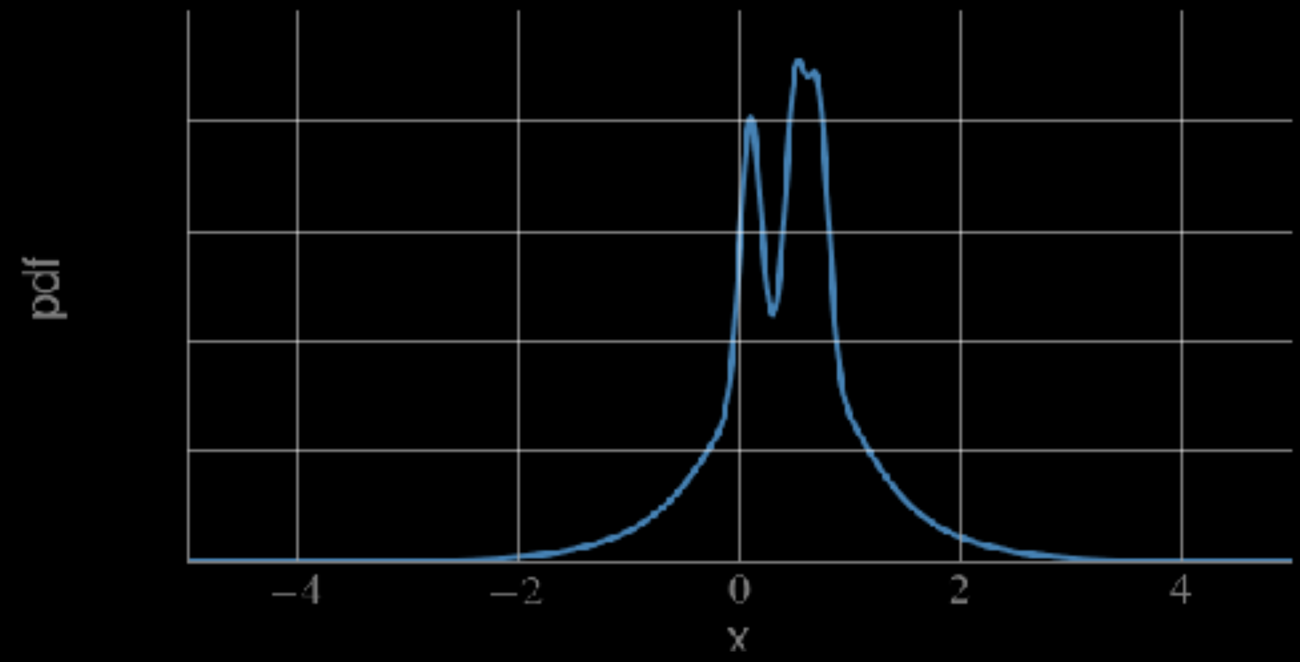
−2
0
2
4
x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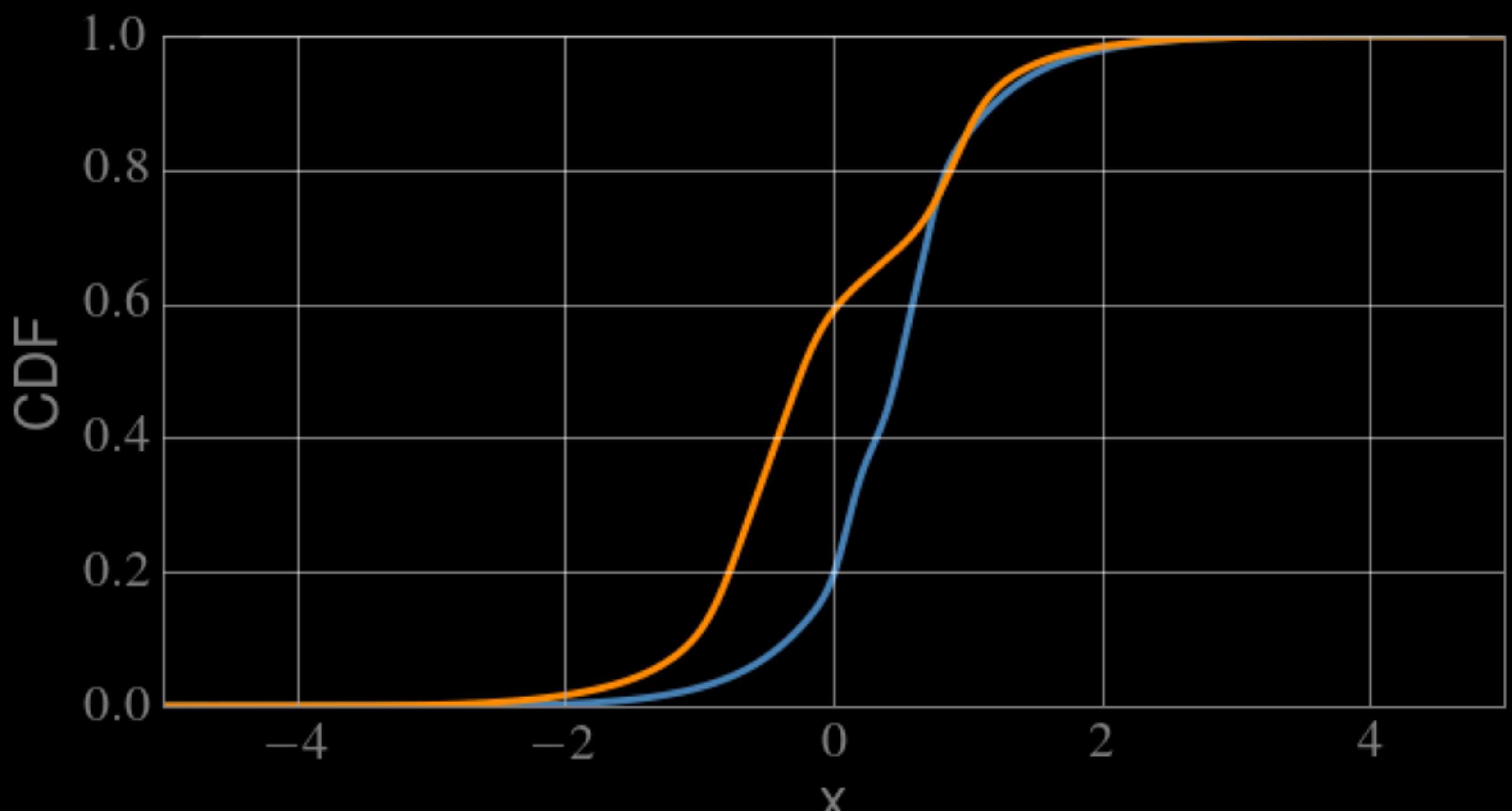
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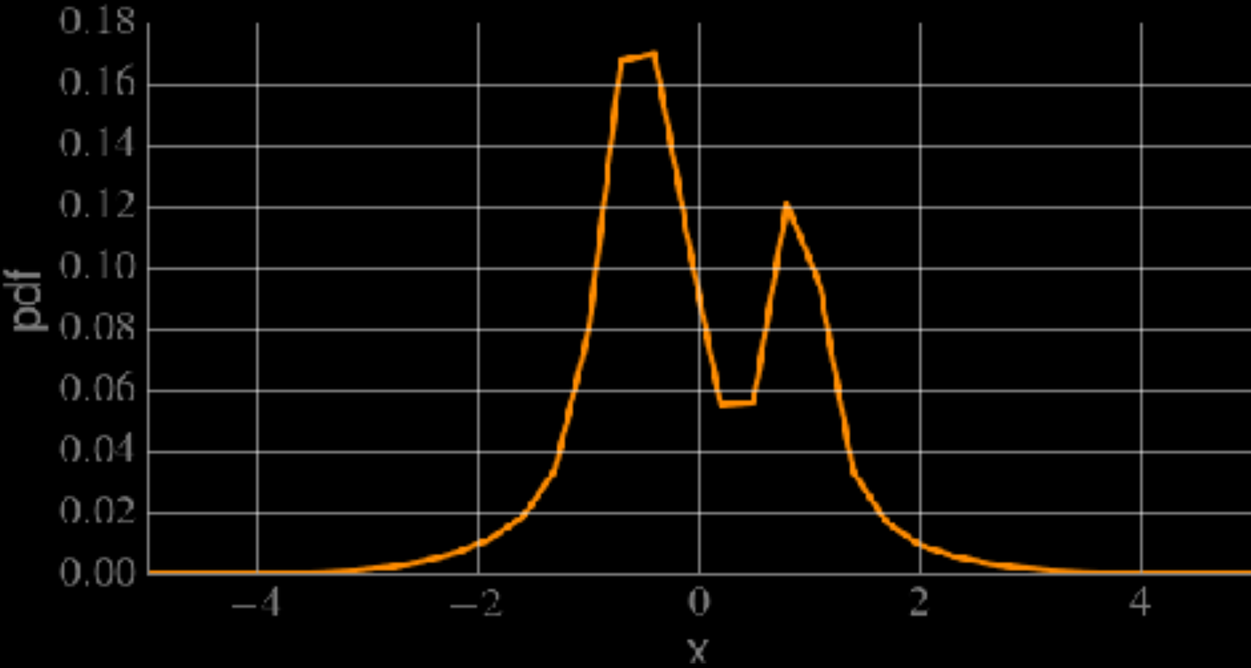
0.8
0.6
0.4
0.2
0.0
CDF
−4
−2
0
2
4
x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x
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0.02
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0.12
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0.18
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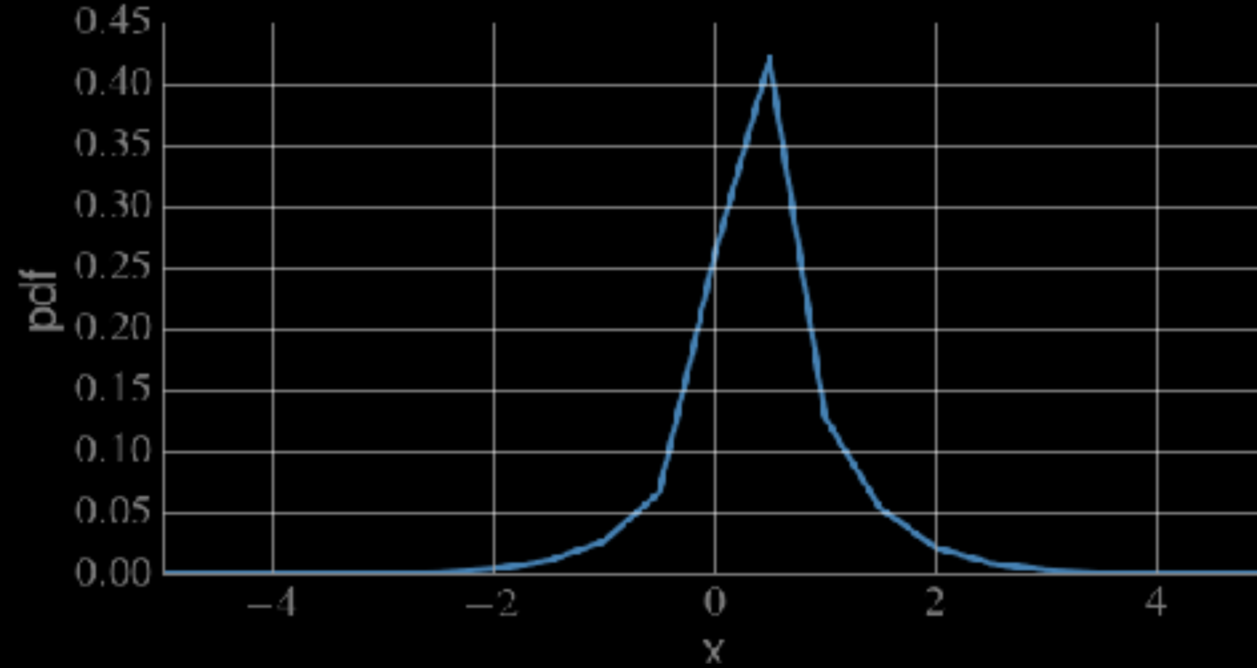
−2
0
2
4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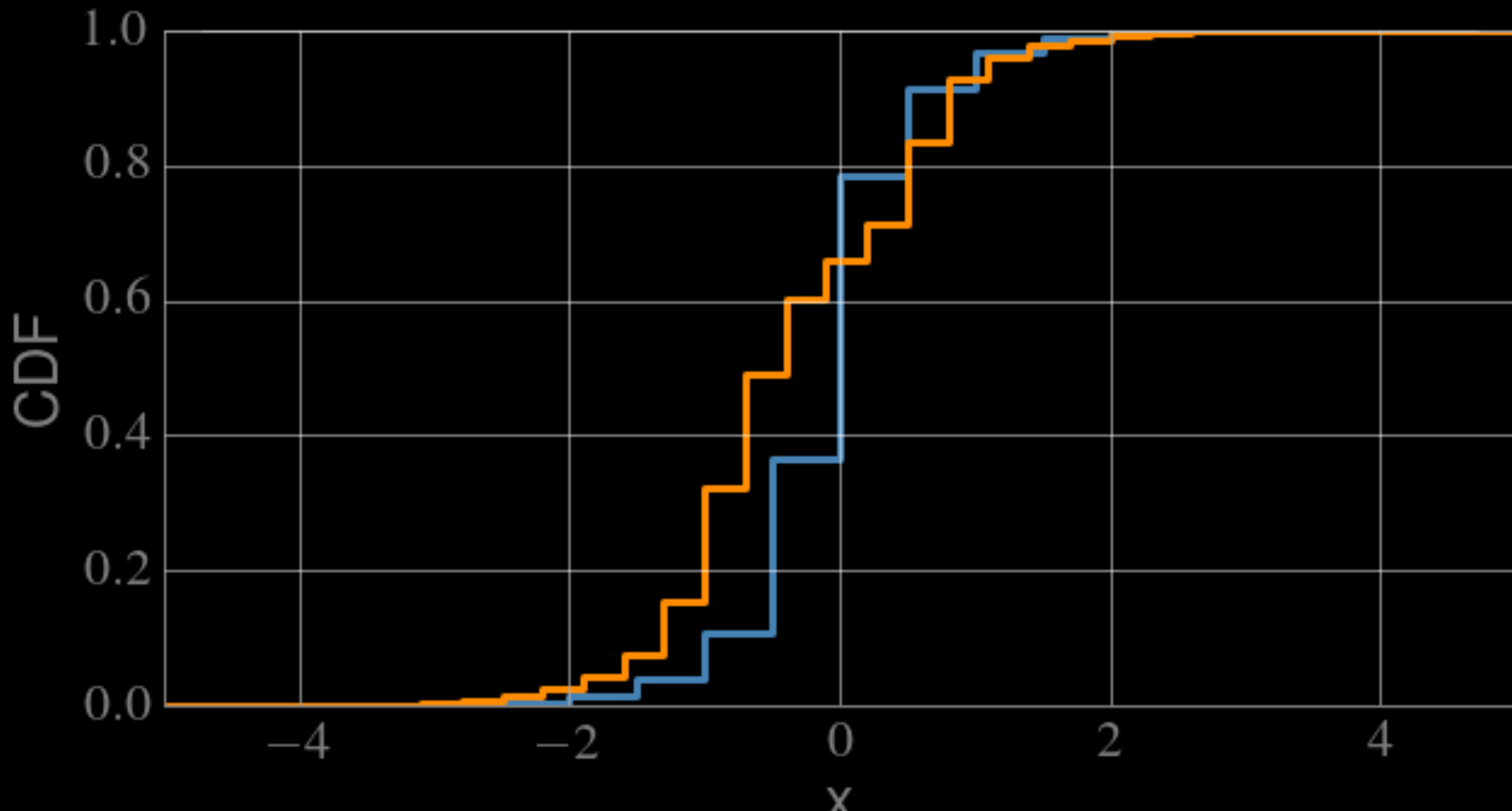
−2
0
2
4


CDF
x
0.0
0.2
0.4
0.6
0.8
1.0
−4
−2
0
2
4

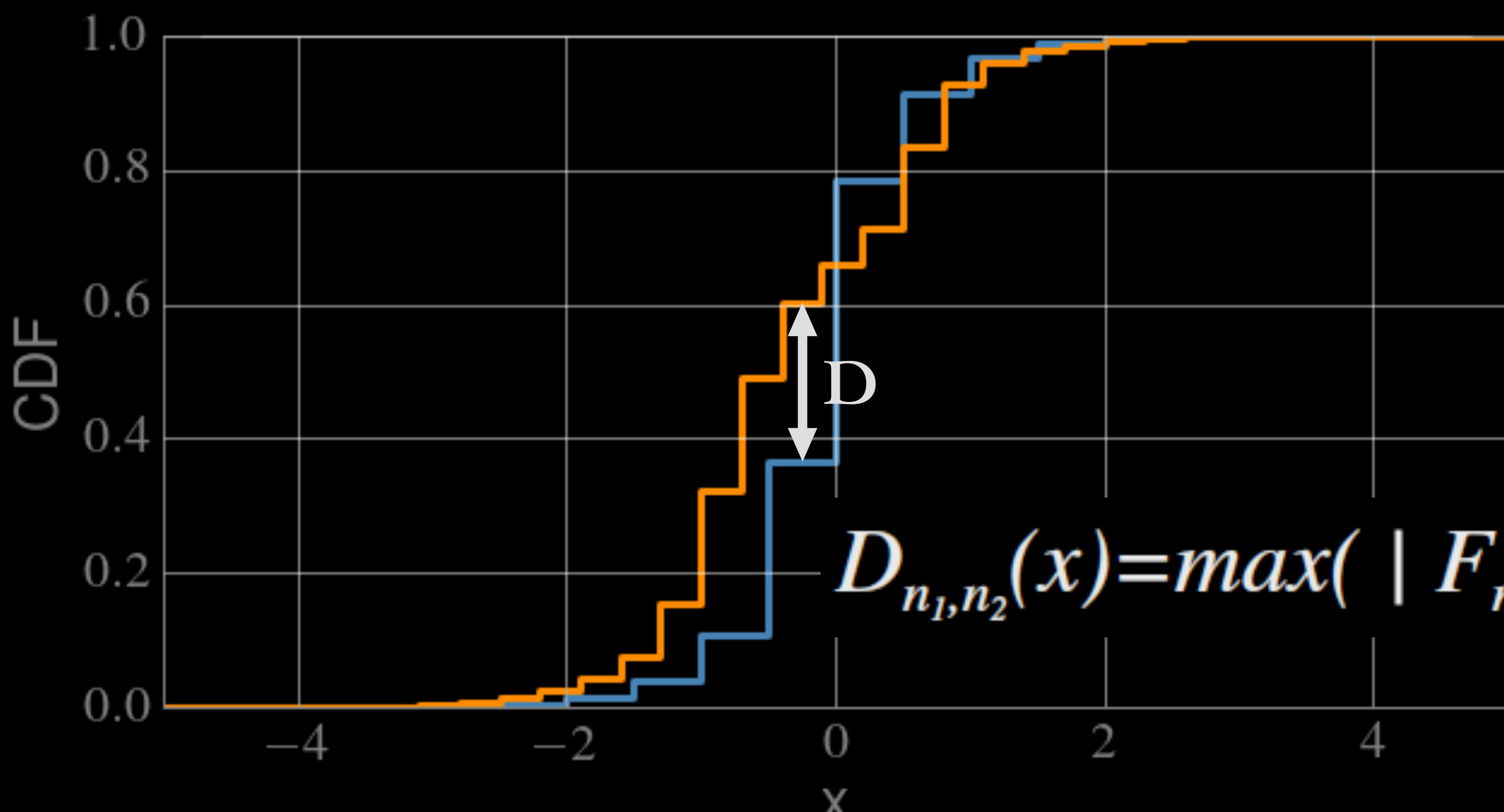
Two sample Kolmogorov Smirnoff test:

null hypothesis H_0: the samples come from the same parent distribution

H_0 is rejected at level α if $$D(n_1,n_2) > c(\alpha)\sqrt{\frac{n_1+n_2}{n_1 n_2}}$$

with $c(\alpha)$ given by a table

NOTE: it ONLY woks in 2D where the Euclidian distance is uniquely defined!

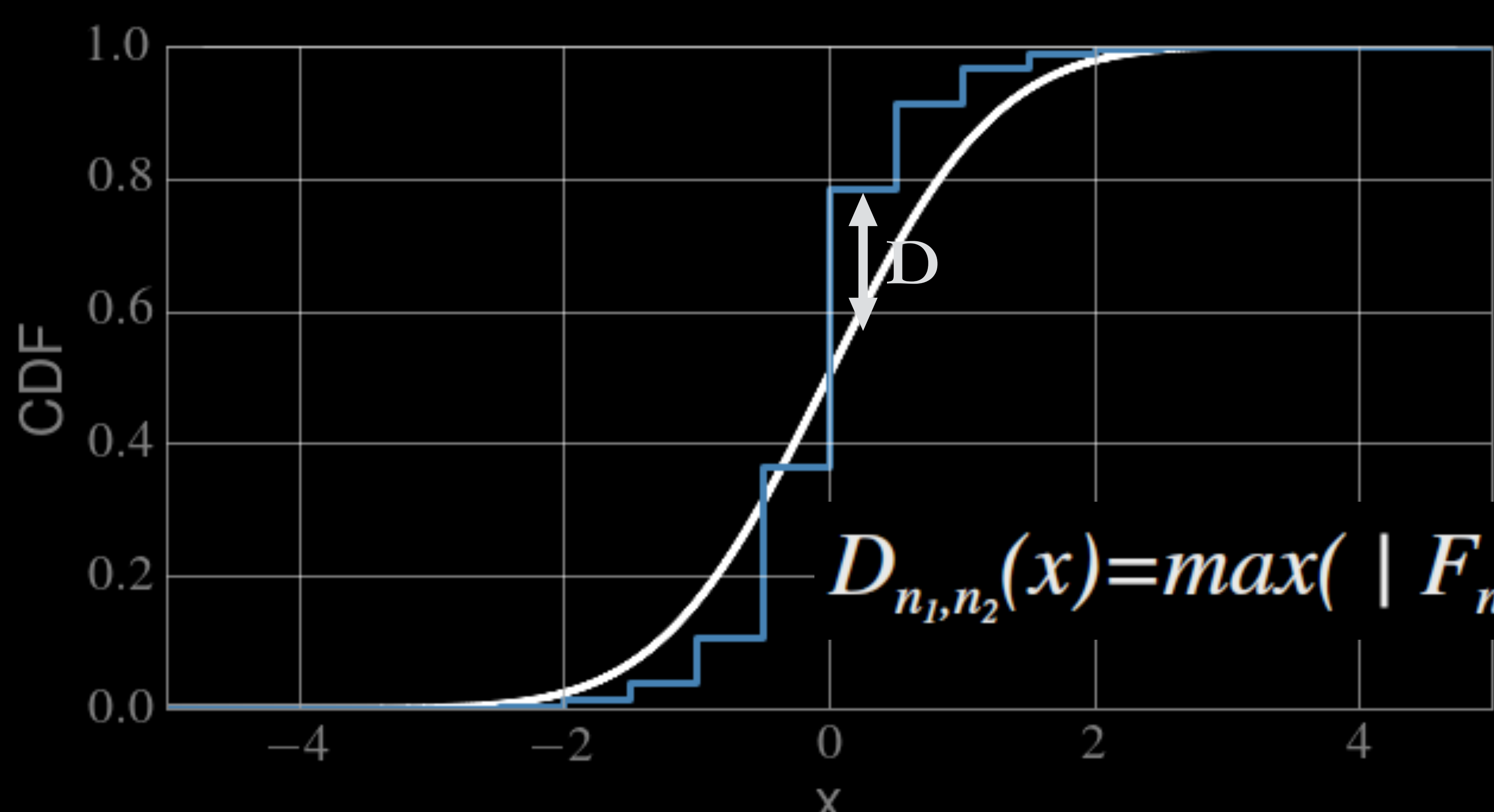


$$D_{n_1,n_2}(x)=max(\,|\,F_{n_1}(x)-F_{n_2}(x)\,|\,)$$

Goodness-of-fit Kolmogorov Smirnoff test:

null hypothesis H_0: the sample does comes from the *model* distribution

H_0 is rejected at level α if $\sqrt{n}\, D_n > K_\alpha$ where $P(K \leq K_\alpha) = 1 - \alpha$



$$D_{n_1,n_2}(x) = max(\,|\,F_{n_1}(x) - F_{n_2}(x)\,|\,)$$

Tests Cheat Sheet:
2 (+) samples comparison

	metric (statistic)	compare to	
KS	$D_{n_1,n_2}(x)=max(\mid F_{n_1}(x)-F_{n_2}(x)\mid)$	$c(\alpha)\sqrt{\frac{n_1+n_2}{n_1 n_2}}$	*Non parametric 2 samples only*
K-sample Anderson-Darling	$ADK=\frac{n-1}{n^2(k-1)}\sum_{i=1}^{k}\frac{1}{n(i)}\left(\sum_{j=1}^{L}h_j\frac{(nF_{ij}-n_iH_j)^2}{H_j(n-H_j)-nh_j/4}\right)$	•AK table	*Non parametric, N samples*
Pearson's	$r_{xy}=\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$	The interpretation of a correlation coefficient depends on the context and purpose	-1 anticorrelated 0 uncorrelated 1 correlated .
Spearman's	$\rho=1-\frac{6\sum(x_i-y_i)^2}{n(n^2-1)}$	t test $t=r\sqrt{\frac{n-2}{1-r^2}}$	*ranked data only* p-value from t-test, Fisher's transformation +z score, permutation test

LAB: Compare Tests for Goodness of fit

(synthetic data)

The following are 5 tests that can be used to assess the goodness of fit of a model

- **K-S**
- **Pearson's Chi squared**
- **Anderson-Darling**
- **K-L Divergence**
- **(Likelihood ratio, you do not need to do this yet!)**

Use KS, K-L divergence, and one more test (AD or Chisq) to quantify the difference between a binomial & Gaussian distribution and a Poisson & Gaussian distribution as a function of the parameters of the first distribution (np for binomial, λ for poisson)

For each test plot the relevant parameter (the K-L parameter, Anderson-Darling statistics, p-value for KS, Chi-sq parameter), against the distribution parameter (np, λ)

Compare Tests for Correlation and Goodness of fit:

The following are 3 tests that assess correlation between 2 samples:

- Pearson's test
- Spearman's test
- K-S test

e.g. Estimate number of MTA Bus passengers at different hours (morning, afternoon, or in time chunks as 7:30-10:30, 10:30-1:30, 1:30-3, 3:6, 6:9, you can do it per bus line, per origin or destination neighborhood…)

The following are 5 tests that can be used to assess the goodness of fit of a model

- K-S
- Pearson's Chi squared
- Anderson-Darling
- K-L Divergence
- Likelihood ratio

e.g. Estimate number of MTA Bus passengers per bus line within an interval of time: are the passengers randomly distributing on busses.

In the lab/homework you will 2 out of these tests to assess if 2 samples are related (measure their correlation, or decide if they come from the same parent distributions) and 2 out of the goodness of fit tests to see if a dataset comes from a normal distribution, or from another distribution (where possible) of your choice.

Compare Tests for Correlation and Goodness of fit:

The following are 3 tests that assess correlation between 2 samples:

- Pearson's test
- Spearman's test
- K-S test

e.g. Age distribution of male vs female Citibikes riders. Age distribution in different seasons. Age distribution for long/short trips

The following are 5 tests that can be used to assess the goodness of fit of a model

- K-S
- Pearson's Chi squared
- Anderson-Darling
- K-L Divergence
- Likelihood ratio

e.g. Estimate Age of riders: could be Gaussian, could be lognormal, power law, bimodal

In the lab/homework you will 2 out of these tests to assess if 2 samples are related (measure their correlation, or decide if they come from the same parent distributions) and 2 out of the goodness of fit tests to see if a dataset comes from a normal distribution, or from another distribution (where possible) of your choice.

Tests Cheat Sheet: 2 (+) samples comparison

	metric (statistic)	compare to	
KS	$D_{n_1,n_2}(x)=max(\mid F_{n_1}(x)-F_{n_2}(x)\mid)$	$c(\alpha)\sqrt{\frac{n_1+n_2}{n_1 n_2}}$	*Non parametric 2 samples only*
K-sample Anderson-Darling	$ADK=\frac{n-1}{n^2(k-1)}\sum_{i=1}^{k}\frac{1}{n(i)}\left(\sum_{j=1}^{L}h_j\frac{(nF_{ij}-n_iH_j)^2}{H_j(n-H_j)-nh_j/4}\right)$	•AK table	*Non parametric, N samples*
Pearson's	$r_{xy}=\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$	The interpretation of a correlation coefficient depends on the context and purpose	-1 anticorrelated 0 uncorrelated 1 correlated .
Spearman's	$\rho=1-\frac{6\sum(x_i-y_i)^2}{n(n^2-1)}$	t test $t=r\sqrt{\frac{n-2}{1-r^2}}$	*ranked data only* p-value from t-test, Fisher's transformation +z score, permutation test

Tests Cheat Sheet: goodness of fit

	metric (statistic)	compare to	
KS	$D_{n_1,n_2}(x)=max(\lvert F_n(x)-F(x) \rvert)$	$\frac{K_\alpha}{\sqrt{n}}$	power in the core only
Pearson's chi square	$\chi^2_{red}=\frac{\chi^2}{df}=\frac{1}{df}\sum\frac{(O-E)^2}{\sigma^2}$	scipy.stats.chisquare(f_obs, f_exp=None, ddof=0, axis=0)[	
Anderson-Darling	$A=n\int_{-\infty}^{\infty}\frac{(F_n(x)-F(x))^2}{F(x)\ (1-F(x))}dF(x)$	scipy.stats.anderson(x, dist='norm')	power in the tails
K-L divergence	$D_{KL} = -\int_x p(x)\ log(q(x)) + p(x)\ log(p(x))$	scipy.stats.entropy(pk. qk=<not None>)	relates to information entropy
Likelihood ratio	$\frac{L\ (model\ 1 \mid\ data)}{L\ (model\ 2 \mid\ data)}$		suitable to bayesian analysis

assignment: Z-test and chi sq test

- Reproduce the analysis of the Hard to Employ program. Reproduce the results in cell 2 and 10. Follow the notebook in the HW directory (turn in the python notebook in the HW GitHub folder

assignment. Compare Tests for Correlation

The following are 3 tests that assess correlation between 2 samples of citibike data:

- **Pearson's test** (answer: are the 2 samples correlated?)
- **Spearman's test** (answer: are the 2 samples correlated?)
- **K-S test** (answer: are the 2 samples likely to come from the same parent distribution?)

Use:

-trip duration for day vs night. State your result in words in terms of the Null Hypothesis

-Extra Credit : age of bikers in BK vs Man and assess the correlation/independence of the 2 samples in each case..

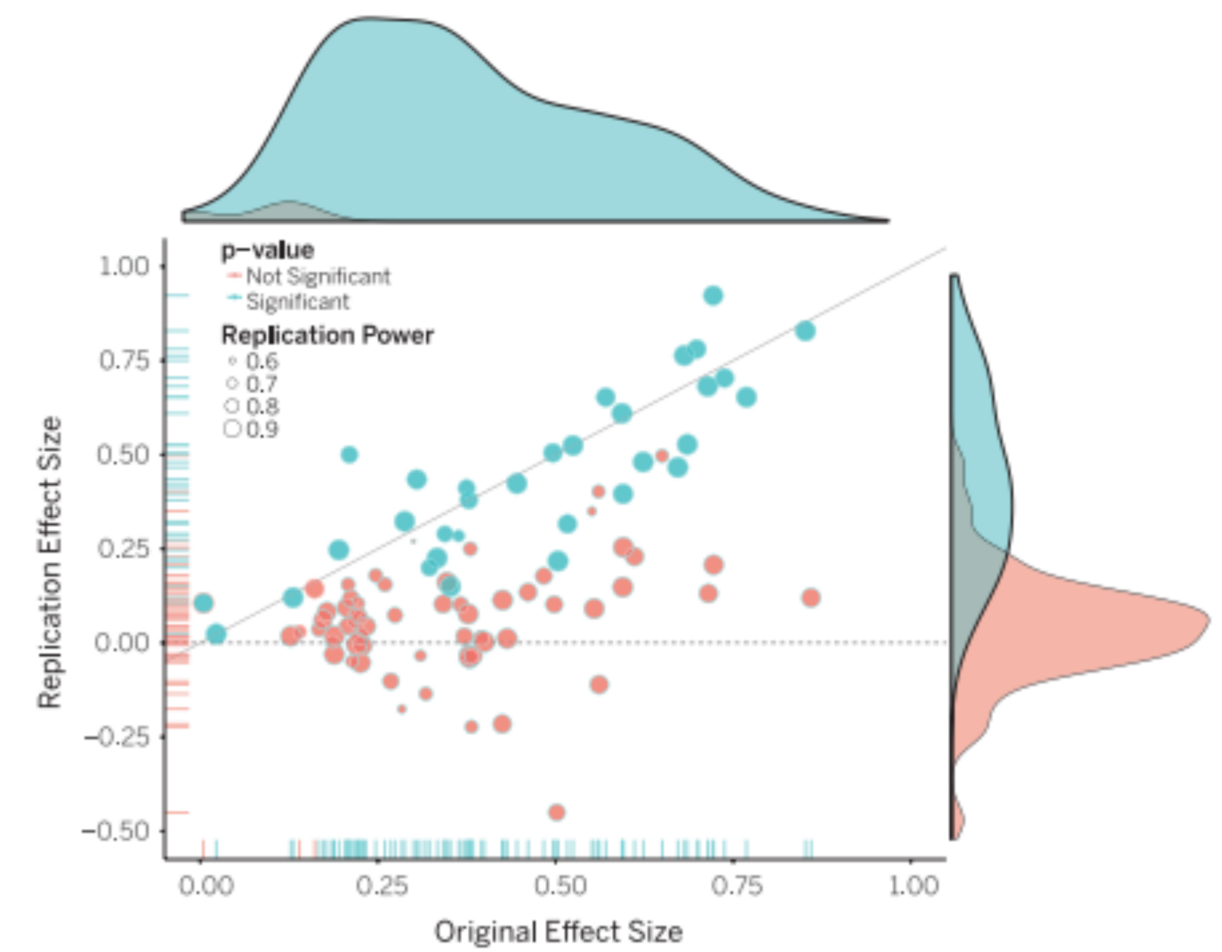
Homework: READING

RESEARCH ARTICLE SUMMARY

PSYCHOLOGY

Estimating the reproducibility of psychological science

Open Science Collaboration*



Original study effect size versus replication effect size (correlation coefficients). Diagonal line represents replication effect size equal to original effect size. Dotted line represents replication effect size of 0. Points below the dotted line were effects in the opposite direction of the original. Density plots are separated by significant (blue) and nonsignificant (red) effects.

http://www.sciencemag.org/content/349/6251/aac4716.full.pdf

MUST KNOWS:

- How to choose (and perform) a statistical test
- Statistical errors
- how to perform Z and chisel test
- PDF vs CDF
- correlation vs causation
- KS test for 2 samples, Pearson's, Spareman

Resources:

Sarah Boslaugh, Dr. Paul Andrew Watters, 2008
Statistics in a Nutshell (Chapters 3,4,5)
https://books.google.com/books/about/Statistics_in_a_Nutshell.html?id=ZnhgO65Pyl4C

David M. Lane et al.
Introduction to Statistics (XVIII)
http://onlinestatbook.com/Online_Statistics_Education.epub
http://onlinestatbook.com/2/index.html

Reckova & Irsova
Publication Bias in Measuring Climate Sensitivity
IES Working Paper: 14/2015
http://salserver.org.aalto.fi/vanhat_sivut/Opinnot/Mat-2.4108/pdf-files/emet03.pdf