Constants:
$\quad b_i$:seats of booking i
$\quad v_j$:vehicle capacity of vehicle assigned to trip j

Known:
$\quad \mathcal{T}(\mathcal{L})$:a set of trips $T(L)$, L is a combination of locations $\{l_{i_1^L}, l_{i_j}$

Variables:
$\quad f_j$:operation fee of vehicle assigned to trip j
$\quad c_j$:distance cost of trip j

$$y_j : y_j = \begin{cases} 1 & \text{if trip } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{jl} : Y_{jl} = \begin{cases} 1 & \text{if booking } j \text{ has } l \text{ as destination} \\ 0 & \text{otherwise} \end{cases}$$

$$\chi_{ij}^{bT} : \chi_{ij}^{bT} = \begin{cases} 1 & \text{if booking } i \text{ is assigned to trip } j \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_{il}^{bL} : \alpha_{il}^{bL} = \begin{cases} 1 & \text{if booking } i \text{ has location } l \text{ as destination} \\ 0 & \text{otherwise} \end{cases}$$

$$\chi_{jl}^{TL} : \chi_{jl}^{TL} = \begin{cases} 1 & \text{if trip } j \text{ covers location } l \\ 0 & \text{otherwise} \end{cases}$$

Given b as booking b's seat, v as vehicle v's' capacity, f operation fee of each vechicle , α requirement of bookings, $C(TSP)$ the outcome of TSP. the goal is to minimize the total cost

$$\text{COST} = \sum_j (f_j + c_j) \cdot y_j$$

when satisfies

$$\chi_{ij}^{bT} + \chi_{i'j}^{bT} = 1 \qquad \text{if } i \text{ and } i' \text{ are incompatible} \qquad (1)$$

$$\sum_j \chi_{ij}^{bT} = 1 \qquad (2)$$

$$\sum_i b_i \cdot \chi_{ij}^{bT} \leq v_j \qquad v_j \in V \qquad (3)$$

$$\sum_{l \in \chi_{C(TSP)}} \chi_{C(TSP)} \leq \chi_{ik}^{L} \qquad (4)$$

The constraints are based on the following consideration:

1. Incompatible bookings constraint, the more stricter version is

   $$\chi_{ij}^{T} \cdot \chi_{i'j}^{T} = 0$$

   . It introduces nonlinearity, so in the formalization we adopt the linear one.
2. Each booking is required to be picked up by a vehicle.
3. Each booking's destination (location) request should be met by any route
4. Each vehicle's capacity should not be exceeded.

Concerns:

The number of the third constraints could be exponential due to the exponetial combination of different locations into different route for a vehicle. (There are ways to handle this).