Dynamics of Delayed-Feedback Opto-electronic Oscillators

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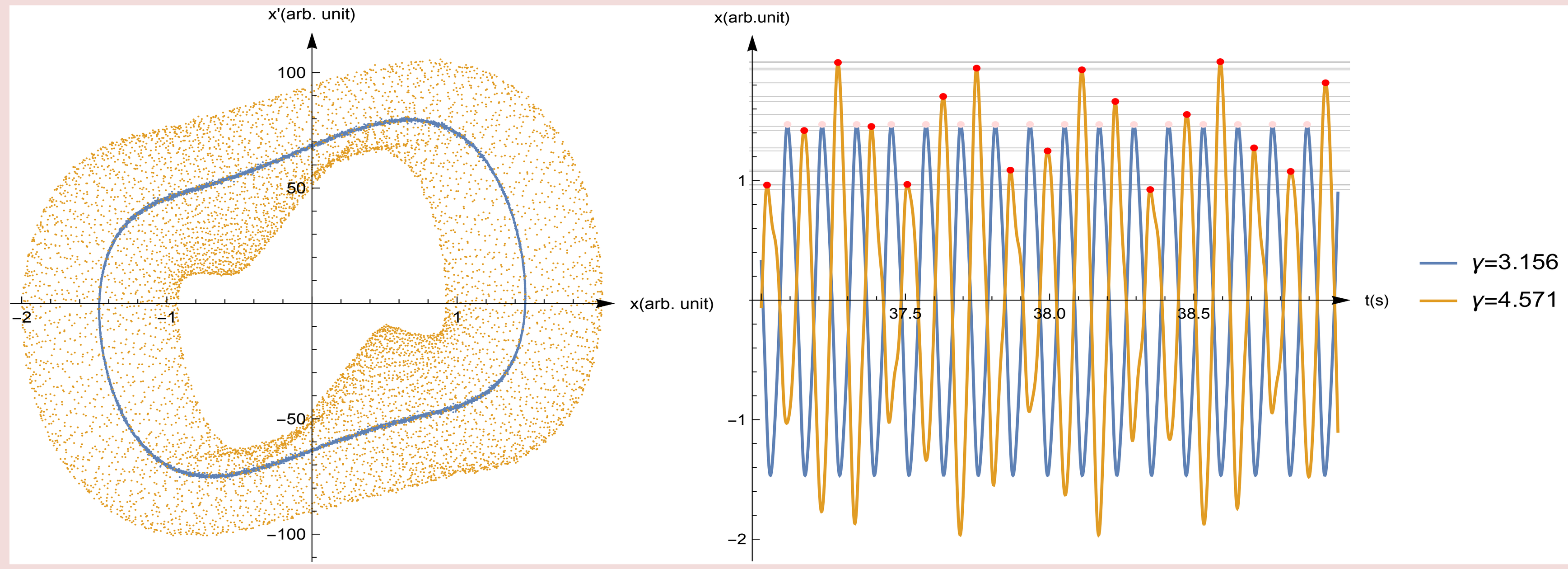
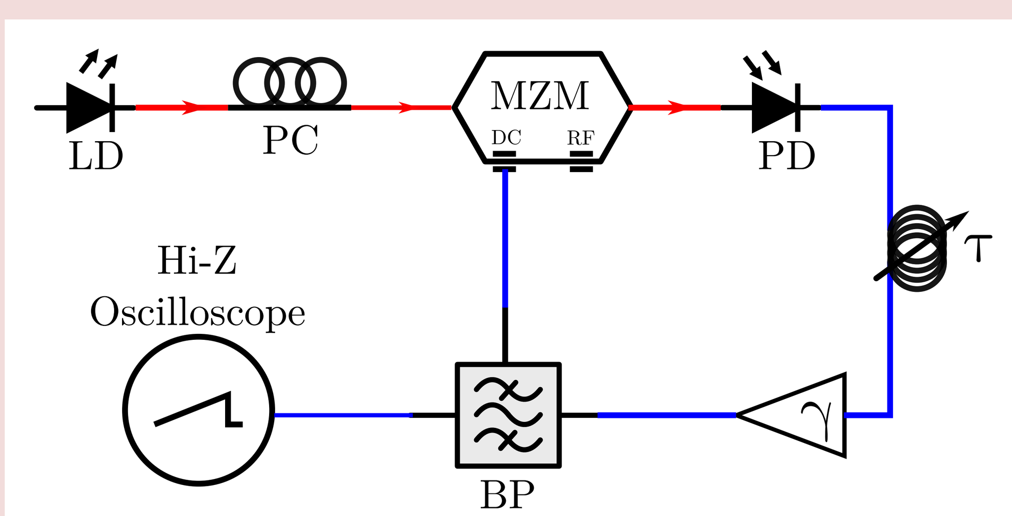
What is a Torus Bifurcation?



- Transition from a single-mode oscillation to a two-or-higher-mode oscillation
- Incommensurate frequencies resulting in quasi-periodic oscillations

Our Questions:
- Does the oscillating solution undergo a torus bifurcation and for what parameters?
- What are the frequencies and amplitudes of the signal envelopes after torus bifurcations?

Experiment Schematics and Model



- LD: Laser Diode
- PC: Polarization Controller
- MZM: Mach-Zehnder Modulator
- PD: Photodetector
- τ: Audio Delay
- γ: Amplification
- BP: Bandpass Filter

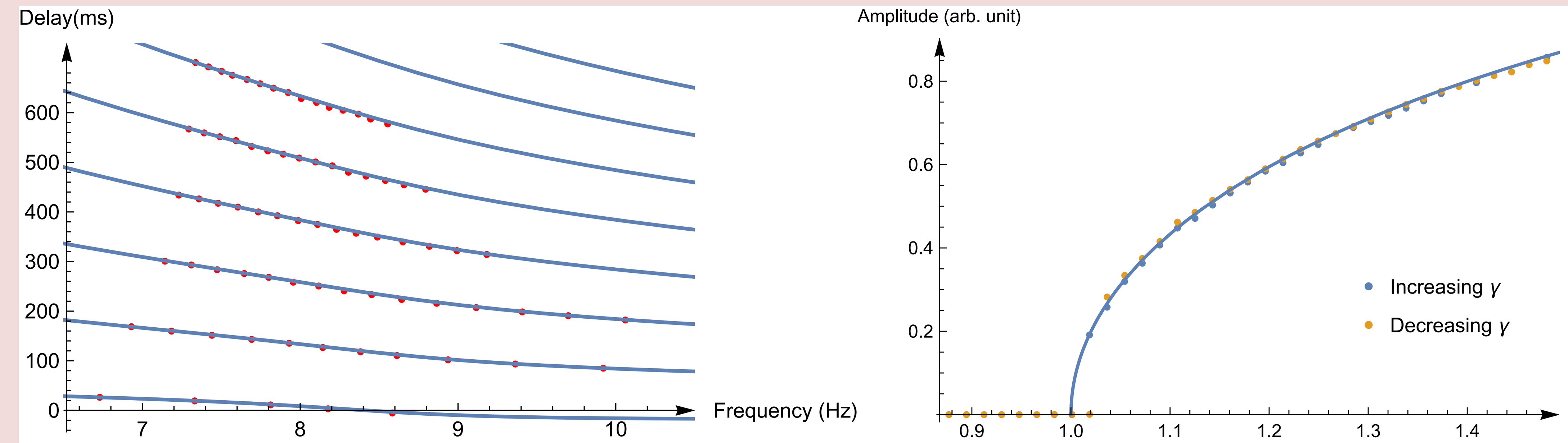
$$\underbrace{\dot{x} + \frac{x}{\Delta\omega} + \frac{\omega_0}{\Delta\omega}\int x\,dt}_{\text{Bandpass Filter}} = \underbrace{\gamma\cos^2[x(t-\tau)+\phi]}_{\text{Delayed Nonlinear Feedback}}$$

Fixed parameters: $\Delta\omega, \omega_0, \phi$
Controlled parameters: γ, τ
Measured variable: $x(t)$

Hopf Bifurcation: from Steady State to Sinusoidal Oscillation

As the feedback strength is increased, the zero solution loses stability, and a stable sinusoidal oscillatory solution starts to exist.

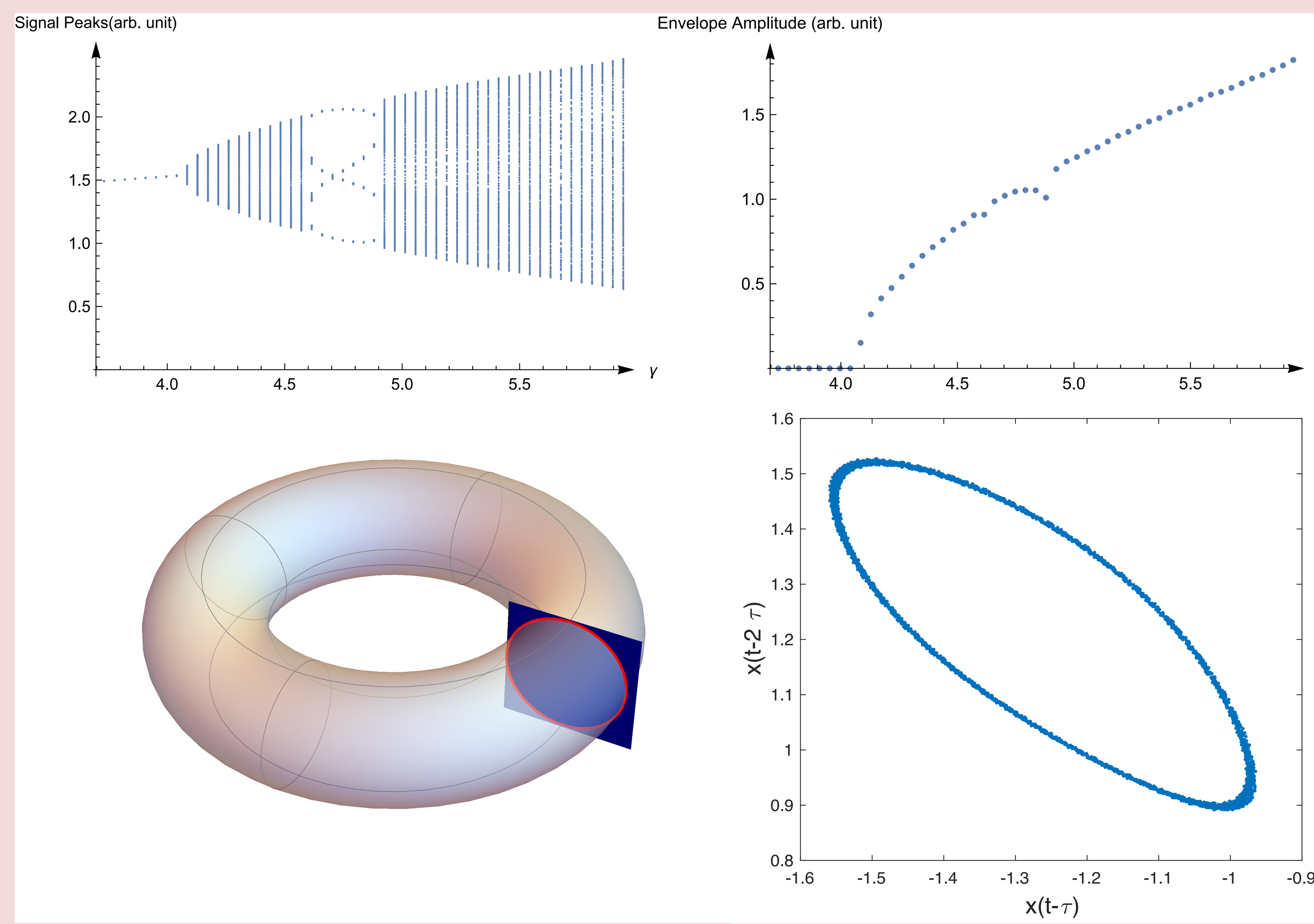
- Single-mode Ansatz: $x(t) = \mathcal{A}\cos(\omega t)$
- Jacobi-Anger expansion of the nonlinearity and neglect of higher order modes leads to self-consistent amplitude and frequency solutions



$$\frac{\omega_0}{\Delta\omega}\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right) = \tan(\omega\tau)$$

$$\mathcal{A} = -\gamma\sin(2\phi)\cos(\omega\tau)J_1(2\mathcal{A})$$

Torus Bifurcation



Further increasing the loop amplification, a torus bifurcation emerges, *i.e.*, a signal with periodically oscillating peaks.

- Torus bifurcations exist for short delays.
- Torus bifurcations for short delays require higher feedback gains than those for large delays, which may indicate a slightly different mechanism of bifurcation.

Conclusions

The system exhibits two main bifurcation types: Hopf bifurcations and torus bifurcations. Measurements of the amplitudes and frequencies of the oscillatory signals that are created through Hopf bifurcations of the steady state are in good agreement with analytic predictions. Torus bifurcations exist for all feedback delays that we measured. However, bifurcation points in short delay cases differ greatly from those in long delay cases.

Acknowledgments

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References

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[2] Banyas, E. & Illing, L., Modeling single and dual-delay optoelectronic oscillators, *Unpublished Research Note*, 2016