Periodic and Quasiperiodic Dynamics of Optoelectronic Oscillators

Yunjia Bao & L. Illing, Reed College, Portland, Oregon, USA

Abstract

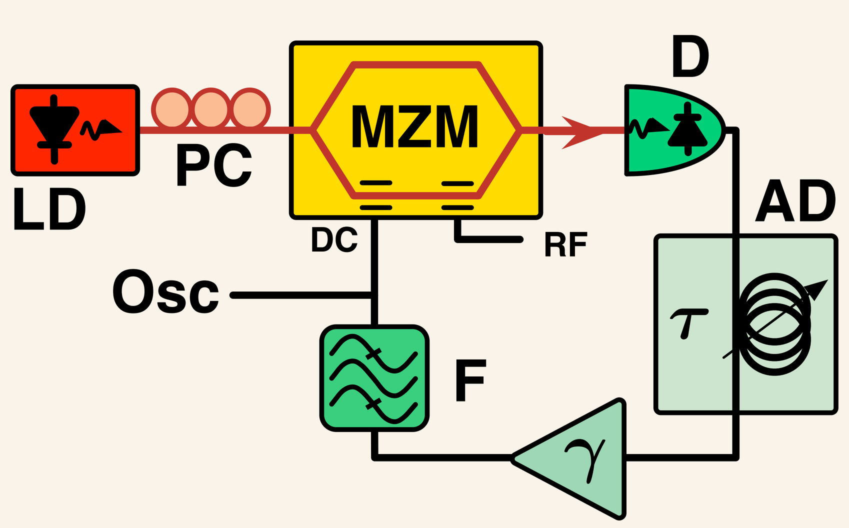
Optoelectronic oscillators with narrowband time-delayed feedback produce periodic solutions and are useful as high-purity signal generators, but their nonlinear dynamics sets limits on available signal amplitudes. Starting from an integro-differential model, we utilize approximate analytic solutions to find the stability boundaries of the periodic solutions and to quantify signal distortions. Our analytical predictions are confirmed by numerical simulations and experiments.

Background and Motivation

Narrowband optoelectronic oscillators are used as high spectral-purity and low phase-noise signal generators [1], with application that include sensors and detectors [2]. The system nonlinearity, however, limits the ability to produce stable periodic signals that also have large amplitude, due to bifurcations that lead to instabilities and quasiperiodic behavior [3]. Signal distortions also occur. Numerical simulations show that distortions are most pronounced in the short delay limit. Thus, we extend previous approaches [3] to the short delay case. Experiments are conducted at audio frequencies for ease of implementation.

Experimental Setup & Model

LD: Laser Diode
PC: Polarization Control
MZM: Mach-Zehnder Modulator
AD: Audio delay
D: Photodetector
F: Bandpass Filter
Osc: Oscilloscope



Adjustable Parameters: τ — Feedback delay (adjustable audio delay)
γ — Feedback gain (adjustable amplifier)

Two-pole Bandpass Filter Model

$$\underbrace{x + \frac{1}{\Delta\Omega}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\Omega_0^2}{\Delta\Omega}\int_{t_0}^{t} x(s)\,\mathrm{d}s}_{\text{Bandpass Filter}} = \gamma\,\underbrace{\cos^2\left[x(t-\tau_D)+\phi\right]}_{\text{Nonlinearity } F[\cdot]}$$

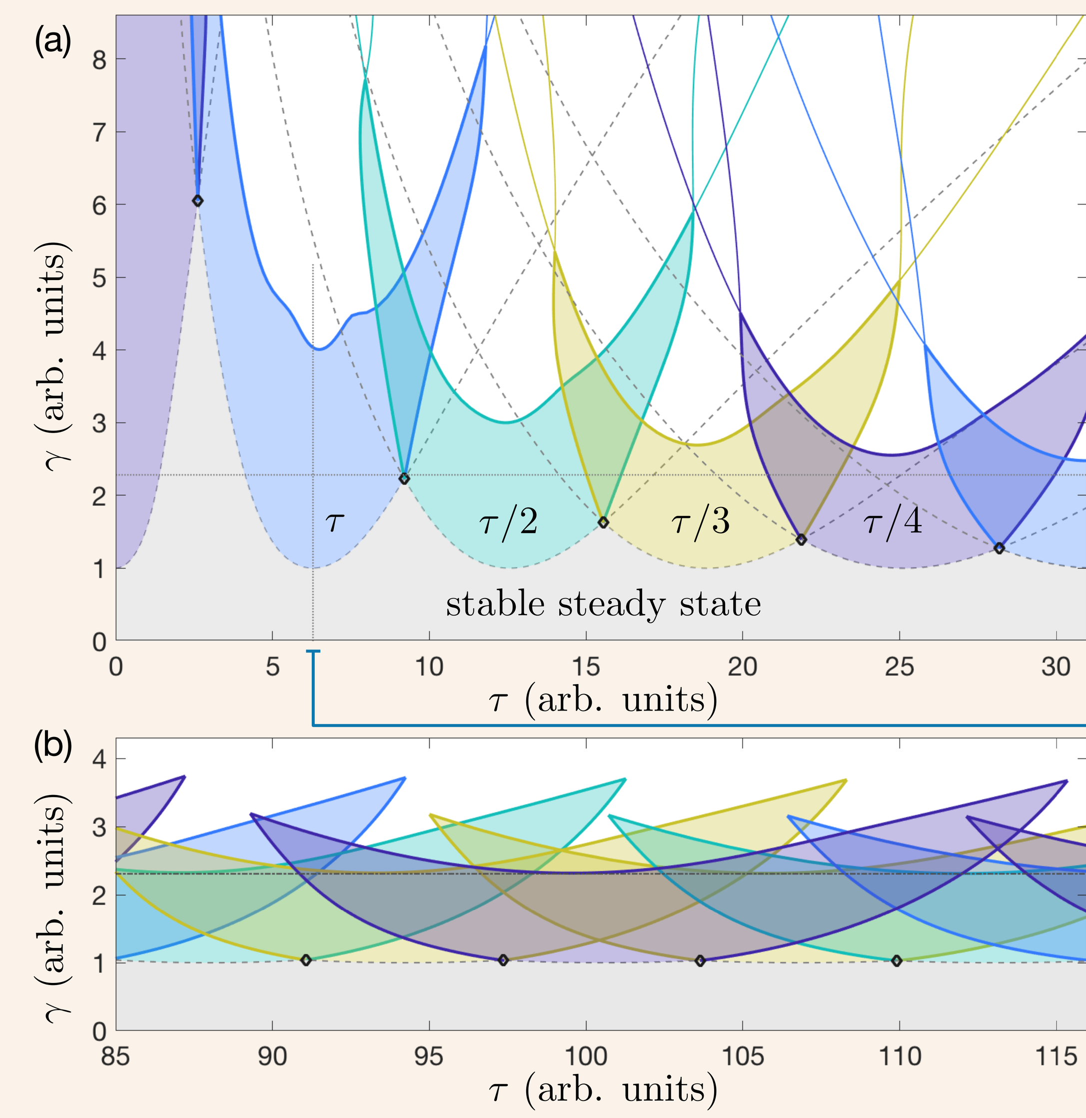
With small parameter $\mu = \dfrac{\Delta\Omega_0}{\Omega_0}$ and $t \to \Omega_0 t$ one obtains

2nd order nonlinear DDE: $\boxed{\ddot{x} + \mu\,\dot{x} + x = \mu\,\gamma\,\frac{d}{dt}F\left[x^{\tau}\right]}$

References : [1] X. S. Yao and L. Maleki, J. Opt. Soc. Am. B **13**, 8 (1996).
[2] X. Zou *et al.*, IEEE J. Quantum Electron. **52**, 1 (2016).
[3] Y. Chembo *et al.*, Opt. Lett. **32**, 17 (2007).

This study was supported in part by Galakatos Funds and the Reed College Science Research Fellowship.

Numerical Bifurcation Analysis



DDE Biftool
quasiperidicity/chaos
coexisting periodic solutions
period two solution
period one solution
stable steady state
--- Hopf Bifurcation
◇ Hopf-Hopf Bifurcation
— Torus Bifurcation

Theory

Perturbative solution based on smallness of narrowband filter parameter μ and corresponding smallness of higher harmonic terms.

$$x = x_0 + \hat{x}_1 + x_1 + \ldots \qquad x_0 = \mathcal{A}\cos(\omega t)$$

and

$$F[x^{\tau}] \approx F[x_0^{\tau}] + F'[x_0^{\tau}]\,x_1^{\tau} + \ldots \qquad F[x_0^{\tau}] = \sum_{n=0}^{\infty} C_n \cos(n\omega[t-\tau])$$

Dominant periodic single mode

$$\ddot{x}_0 + \mu\,\dot{x}_0 + x_0 = \mu\,\gamma\,\frac{C_1}{\mathcal{A}}\dot{x}_0^{\tau}$$

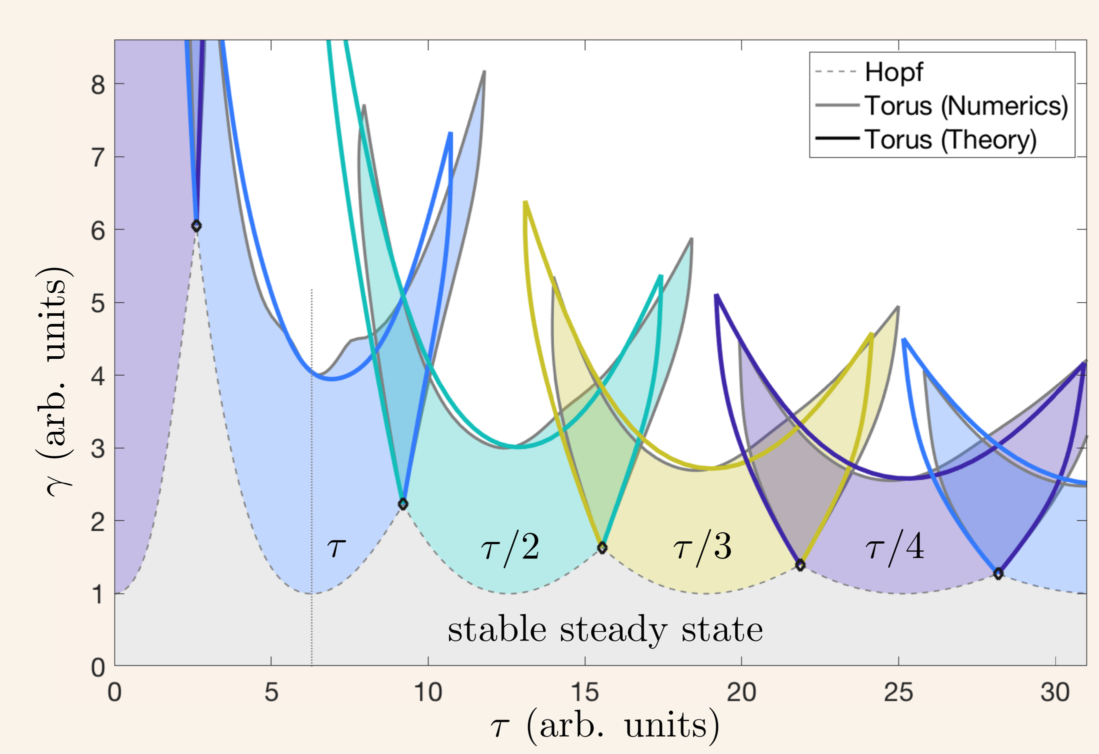
Distortions due to higher harmonics

$$\ddot{\hat{x}}_1 + \mu\,\dot{\hat{x}}_1 + \hat{x}_1 = \mu\,\gamma\,\frac{d}{dt}\sum_{n=2}^{\infty} C_n\cos(n\omega[t-\tau])$$

Stability equation of periodic solution

$$\ddot{x}_1 + \mu\,\dot{x}_1 + x_1 = \mu\,\gamma\,\frac{d}{dt}\left\{F'[x_0^{\tau}]\,x_1^{\tau}\right\}$$

Result

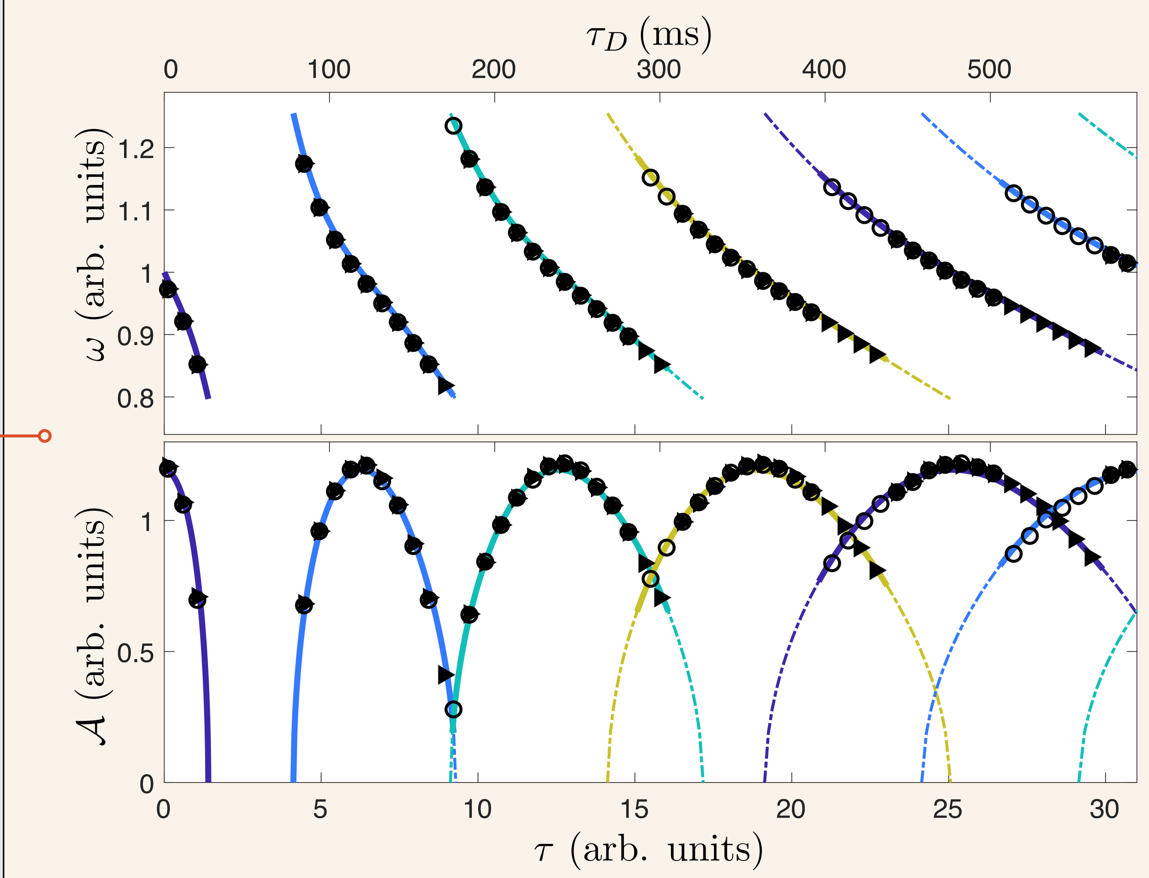


First order perturbation theory correctly predicts "upper" torus bifurcation curve as well as torus bifurcation curves emanating from Hopf-Hopf bifurcation points.

Behavior close to the intersection of torus bifurcation curves remains to be explored.

Periodic Solutions & Torus Bifurcations

Amplitude and frequency of periodic solutions for $\gamma = 2.28$

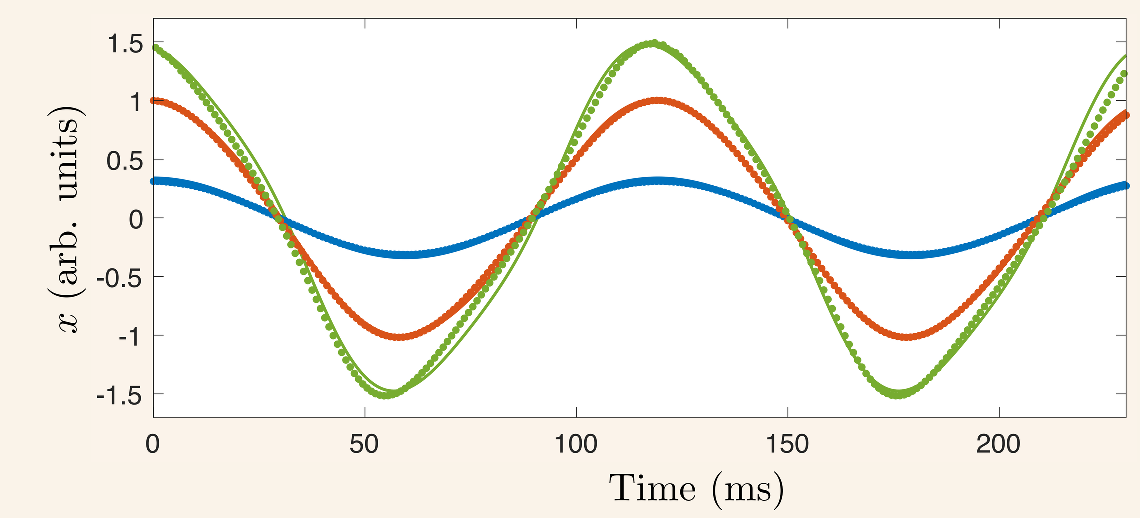


Experiment : delay was
▶ increased
○ decreased

Theory : periodic soln.
— stable
--- unstable

Distortion due to higher harmonics at large feedback gain

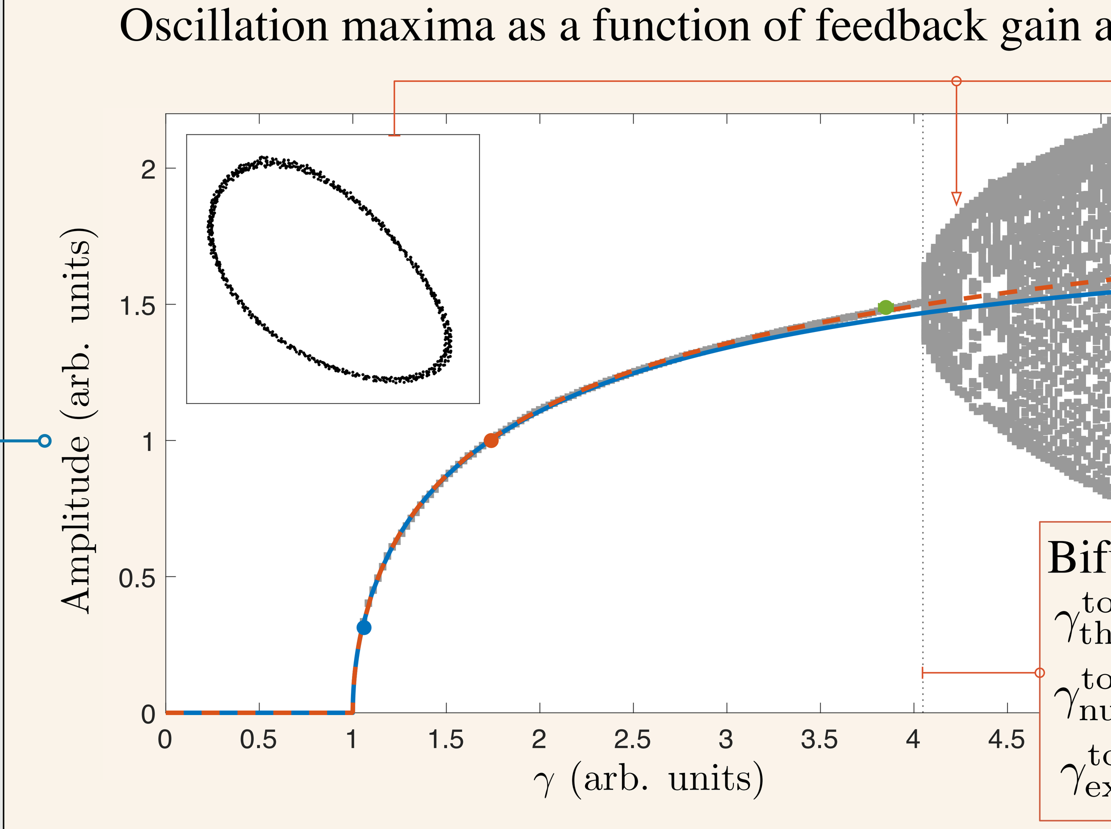
The higher harmonics correction $\hat{x}_1$ has only a minor effect on amplitude $\mathcal{A}$ but changes the waveform shape ("shark fin shape" instead of sinusoidal).



····· Experiment
— Theory

Torus bifurcations lead to stable quasiperiodic oscillations.

Oscillation maxima as a function of feedback gain at $\gamma = 2.28$



Poincaré section of a 3D delay embedding of a time series ($\gamma = 4.24$)

Bifurcation threshold
$\gamma^{\mathrm{tor}}_{\mathrm{theo.}} = 4.05$
$\gamma^{\mathrm{tor}}_{\mathrm{num.}} = 4.04$
$\gamma^{\mathrm{tor}}_{\mathrm{exp.}} = 4.05 \pm 0.03$

Conclusion

Periodic oscillations change stability due to torus bifurcations. The range of feedback gain that results in stable periodic oscillations strongly depends on the delay considered. Distortions of large amplitude signals are unavoidable and regions of multi-stability exist.