# Single Qubit Clifford Gates

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| name | Unitary matrix | Primary gate decomposition |
|---|---|---|
| C1 | $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ | $I$ |
| C2 | $\exp\left(-i\frac{\pi}{4}\sigma_x\right) = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$ | $X_{\pi/2}$ |
| C3 | $\exp\left(-i\frac{\pi}{2}\sigma_x\right) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$ | $X_{\pi}$ |
| C4 | $\exp\left(-i\frac{3\pi}{4}\sigma_y\right) = \frac{\sqrt{2}}{2}\begin{pmatrix} -1 & -i \\ -i & -1 \end{pmatrix}$ | $X_{-\pi/2}$ |
| C5 | $\exp\left(-i\frac{\pi}{4}\sigma_y\right) = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ | $Y_{\pi/2}$ |
| C6 | $\exp\left(-i\frac{\pi}{2}\sigma_y\right) = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$ | $Y_{\pi}$ |
| C7 | $\exp\left(-i\frac{3\pi}{4}\sigma_x\right) = \frac{\sqrt{2}}{2}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$ | $Y_{-\pi/2}$ |
| C8 | $\exp\left(-i\frac{\pi}{4}\sigma_z\right) = \frac{\sqrt{2}}{2}\begin{pmatrix} 1-i & 0 \\ 0 & 1+i \end{pmatrix}$ | $X_{\pi/2}Y_{\pi/2}X_{-\pi/2}$<br>$X_{-\pi/2}Y_{-\pi/2}X_{\pi/2}$<br>$Y_{\pi/2}X_{-\pi/2}Y_{-\pi/2}$<br>$Y_{-\pi/2}X_{\pi/2}Y_{\pi/2}$ |
| C9 | $\exp\left(-i\frac{\pi}{2}\sigma_z\right) = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$ | $X_{\pi}Y_{\pi}$<br>$Y_{\pi}X_{\pi}$ |
| C10 | $\exp\left(-i\frac{3\pi}{4}\sigma_z\right) = \frac{\sqrt{2}}{2}\begin{pmatrix} -1-i & 0 \\ 0 & -1+i \end{pmatrix}$ | $X_{\pi/2}Y_{-\pi/2}X_{-\pi/2}$<br>$X_{-\pi/2}Y_{\pi/2}X_{\pi/2}$<br>$Y_{\pi/2}X_{\pi/2}Y_{-\pi/2}$<br>$Y_{-\pi/2}X_{-\pi/2}Y_{\pi/2}$ |
| C11 | $\exp\left(-i\frac{\pi}{2\sqrt{2}}(\sigma_x+\sigma_y)\right) = \frac{\sqrt{2}}{2}\begin{pmatrix} 0 & -1-i \\ 1-i & 0 \end{pmatrix}$ | $X_{\pi/2}Y_{\pi/2}X_{\pi/2}$<br>$X_{-\pi/2}Y_{-\pi/2}X_{-\pi/2}$<br>$Y_{\pi/2}X_{\pi/2}Y_{\pi/2}$<br>$Y_{-\pi/2}X_{-\pi/2}Y_{-\pi/2}$ |
| C12 | $\exp\left(-i\frac{\pi}{2\sqrt{2}}(\sigma_x-\sigma_y)\right) = \frac{\sqrt{2}}{2}\begin{pmatrix} 0 & 1-i \\ -1-i & 0 \end{pmatrix}$ | $X_{\pi/2}Y_{-\pi/2}X_{\pi/2}$<br>$X_{-\pi/2}Y_{\pi/2}X_{-\pi/2}$<br>$Y_{\pi/2}X_{-\pi/2}Y_{\pi/2}$<br>$Y_{-\pi/2}X_{\pi/2}Y_{-\pi/2}$ |

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|---|---|---|
| C13 | $\exp\left(-i\frac{\pi}{2\sqrt{2}}(\sigma_x+\sigma_z)\right)=\frac{\sqrt{2}}{2}\begin{pmatrix}-i & -i\\ -i & i\end{pmatrix}$ | $Y_{-\pi/2}X_\pi$<br>$X_\pi Y_{\pi/2}$ |
| C14 | $\exp\left(-i\frac{\pi}{2\sqrt{2}}(\sigma_x-\sigma_z)\right)=\frac{\sqrt{2}}{2}\begin{pmatrix}i & -i\\ -i & -i\end{pmatrix}$ | $Y_{\pi/2}X_\pi$<br>$X_\pi Y_{-\pi/2}$ |
| C15 | $\exp\left(-i\frac{\pi}{2\sqrt{2}}(\sigma_y+\sigma_z)\right)=\frac{\sqrt{2}}{2}\begin{pmatrix}-i & -1\\ 1 & i\end{pmatrix}$ | $X_{\pi/2}Y_\pi$<br>$Y_\pi X_{-\pi/2}$ |
| C16 | $\exp\left(-i\frac{\pi}{2\sqrt{2}}(\sigma_y-\sigma_z)\right)=\frac{\sqrt{2}}{2}\begin{pmatrix}i & -1\\ 1 & -i\end{pmatrix}$ | $X_{-\pi/2}Y_\pi$<br>$Y_\pi X_{\pi/2}$ |
| C17 | $\exp\left(-i\frac{\pi}{3\sqrt{3}}(\sigma_x+\sigma_y+\sigma_z)\right)=\frac{1}{2}\begin{pmatrix}1-i & -1-i\\ 1-i & 1+i\end{pmatrix}$ | $X_{\pi/2}Y_{\pi/2}$ |
| C18 | $\exp\left(-i\frac{2\pi}{3\sqrt{3}}(\sigma_x+\sigma_y+\sigma_z)\right)=\frac{1}{2}\begin{pmatrix}-1-i & -1-i\\ 1-i & -1+i\end{pmatrix}$ | $Y_{-\pi/2}X_{-\pi/2}$ |
| C19 | $\exp\left(-i\frac{\pi}{3\sqrt{3}}(\sigma_x-\sigma_y+\sigma_z)\right)=\frac{1}{2}\begin{pmatrix}1-i & 1-i\\ -1-i & 1+i\end{pmatrix}$ | $Y_{-\pi/2}X_{\pi/2}$ |
| C20 | $\exp\left(-i\frac{2\pi}{3\sqrt{3}}(\sigma_x-\sigma_y+\sigma_z)\right)=\frac{1}{2}\begin{pmatrix}-1-i & 1-i\\ -1-i & -1+i\end{pmatrix}$ | $X_{-\pi/2}Y_{\pi/2}$ |
| C21 | $\exp\left(-i\frac{\pi}{3\sqrt{3}}(\sigma_x+\sigma_y-\sigma_z)\right)=\frac{1}{2}\begin{pmatrix}1+i & -1-i\\ 1-i & 1-i\end{pmatrix}$ | $Y_{\pi/2}X_{\pi/2}$ |
| C22 | $\exp\left(-i\frac{2\pi}{3\sqrt{3}}(\sigma_x+\sigma_y-\sigma_z)\right)=\frac{1}{2}\begin{pmatrix}-1+i & -1-i\\ 1-i & -1-i\end{pmatrix}$ | $X_{-\pi/2}Y_{-\pi/2}$ |
| C23 | $\exp\left(-i\frac{\pi}{3\sqrt{3}}(-\sigma_x+\sigma_y+\sigma_z)\right)=\frac{1}{2}\begin{pmatrix}1-i & -1+i\\ 1+i & 1+i\end{pmatrix}$ | $Y_{\pi/2}X_{-\pi/2}$ |
| C24 | $\exp\left(-i\frac{2\pi}{3\sqrt{3}}(-\sigma_x+\sigma_y+\sigma_z)\right)=\frac{1}{2}\begin{pmatrix}-1-i & -1+i\\ 1+i & -1+i\end{pmatrix}$ | $X_{\pi/2}Y_{-\pi/2}$ |

**Comments**

- The primary gates are chosen as $\{I, X_{\pm\pi/2}, Y_{\pm\pi/2}, X_\pi, Y_\pi\}$. $X_\pi$ and $Y_\pi$ can be replaced by $X_{-\pi}$ and $Y_{-\pi}$.
- The primary gates are applied **from right to left**, i.e., in the pulse sequence the rightmost gate is the earliest one.
- The primary decomposition might differ from the Clifford gate by a factor of $-1$.
- For each Clifford gate there might be multiple primary gate decompositions. The above table lists all possible decompositions with the shortest length. In a randomized benchmarking experiment, one can randomly choose one of them.