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1. Solution:

$p(\text{open}) = p(\text{closed}) = 0.5$

$p(\text{open} \mid \text{open}, u) = 1$, $p(\text{closed} \mid \text{open}, u) = 0$

$p(\text{open} \mid \text{closed}, u) = 0.8$, $p(\text{closed} \mid \text{closed}, u) = 0.2$

$$p(\text{open} \mid u) = \sum_{x \in \{\text{open}, \text{closed}\}} p(\text{open} \mid x, u)\, p(x)$$

$$= p(\text{open} \mid \text{open}, u)\, p(\text{open}) + p(\text{open} \mid \text{closed}, u)\, p(\text{closed})$$

$$= 1 \times 0.5 + 0.8 \times 0.5 = 0.9$$

2. Solution:

$$p(x_t \mid u_{1:t}, z_{1:t}) = bel(x_t) = \eta \cdot p(z_t \mid x_t) \sum_{x_{t-1}} p(x_t \mid x_{t-1}, u_t)\, bel(x_{t-1})$$

① $$p(x_1 \mid u_1, z_1) = \eta \cdot p(z_1 \mid x_1) \cdot \sum_{x_0} p(x_1 \mid x_0, u_1)\, bel(x_0)$$

$$= \eta_1 \cdot \begin{bmatrix} p(z_1 = \text{open} \mid x_1 = \text{open}) \\ p(z_1 = \text{open} \mid x_1 = \text{close}) \end{bmatrix} \odot \begin{bmatrix} p(x_1 = \text{open} \mid x_0 = \text{open}, u_1)\, p(x_0 = \text{open}) + p(x_1 = \text{open} \mid x_0 = \text{close}, u_1)\, p(x_0 = \text{close}) \\ p(x_1 = \text{close} \mid x_0 = \text{open}, u_1)\, p(x_0 = \text{open}) + p(x_1 = \text{close} \mid x_0 = \text{close}, u_1)\, p(x_0 = \text{close}) \end{bmatrix}$$

$$= \eta_1 \begin{bmatrix} 0.8 \\ 0.3 \end{bmatrix} \odot \begin{bmatrix} 1 \times 0.5 + 0 \times 0.5 \\ 0 \times 0.5 + 1 \times 0.5 \end{bmatrix} = \begin{bmatrix} \frac{8}{11} \\ \frac{3}{11} \end{bmatrix}$$

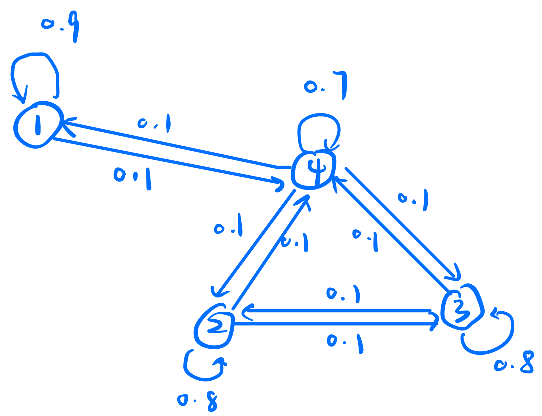
② $$p(x_2 \mid u_{1:2}, z_{1:2}) = \eta \cdot p(z_2 \mid x_2) \sum_{x_1} p(x_2 \mid x_1, u_2)\, bel(x_1)$$

$$= \eta_2 \begin{bmatrix} p(z_2 = \text{open} \mid x_2 = \text{open}) \\ p(z_2 = \text{open} \mid x_2 = \text{close}) \end{bmatrix} \odot \begin{bmatrix} p(x_2 = \text{open} \mid x_1 = \text{open}, u_2)\, bel(x_1 = \text{open}) + p(x_2 = \text{open} \mid x_1 = \text{close}, u_2)\, bel(x_1 = \text{close}) \\ p(x_2 = \text{close} \mid x_1 = \text{open}, u_2)\, bel(x_1 = \text{open}) + p(x_2 = \text{close} \mid x_1 = \text{close}, u_2)\, bel(x_1 = \text{close}) \end{bmatrix}$$

$$= \eta \cdot \begin{bmatrix} 0.8 \\ 0.3 \end{bmatrix} \odot \begin{bmatrix} 1 \times \frac{8}{11} + 0.9 \times \frac{3}{11} \\ 0 \times \frac{8}{11} + 0.1 \times \frac{3}{11} \end{bmatrix} \approx \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix}$$

3. Solution:

(1)



(2) $p(S_t=1 \mid S_{t-1}=1) = 0.9$

$p(S_t=2 \mid S_{t-1}=2) = p(S_t=3 \mid S_{t-1}=3) = 0.8$

$p(S_t=4 \mid S_{t-1}=4) = 0.7$

let $A = \begin{bmatrix} 0.9 & 0 & 0 & 0.1 \\ 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix} = V \Lambda V^{-1}$, $\Lambda = \mathrm{diag}(0.7, 0.9, 1, 0.6)$

when $k \to \infty$, $\Lambda^k \to \mathrm{diag}(0, 0, 1, 0)$

$$\begin{bmatrix} \bar{p}_1 \\ \bar{p}_2 \\ \bar{p}_3 \\ \bar{p}_4 \end{bmatrix} = \begin{bmatrix} 0.9 & 0 & 0 & 0.1 \\ 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix}^k \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = V \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 0 \end{bmatrix} V^{-1} \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

Therefore, the probs of the robot staying at each room is 0.25.

(3) From (2), the prob of the robot going through the door between ① and ④ is 0.25.

4. Solution:

$$p(x_t \mid z_{1:t}, u_{1:t}) = \frac{p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t})}{p(z_t \mid z_{1:t-1}, u_{1:t})}$$

$$= \eta \cdot p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t})$$

Since $p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$

then $p(x_t \mid z_{1:t}, u_{1:t}) = \eta \cdot p(z_t \mid x_t) \cdot p(x_t \mid z_{1:t-1}, u_{1:t})$

$\overline{bel}(x_t) = p(x_t \mid z_{1:t-1}, u_{1:t}) = \int p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t})\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1}$

Since $p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$

then $\overline{bel}(x_t) = \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1}$

then $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$

$bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$

5. Solution:

$bel(x_{0:t}) = p(x_{0:t} \mid z_{1:t}, u_{1:t})$

$= \eta \cdot p(z_t \mid x_{0:t-1}, x_t, z_{1:t-1}, u_{1:t})\, p(x_{0:t-1}, x_t \mid z_{1:t}, u_{1:t})$

$= \eta \cdot p(z_t \mid x_t)\, p(x_t \mid x_{0:t-1}, z_{1:t}, u_{1:t})\, p(x_{0:t-1} \mid z_{1:t}, u_{1:t})$

$= \eta \cdot p(z_t \mid x_t)\, p(x_t \mid x_{t-1}, u_t)\, bel(x_{0:t-1})$