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The Chinese University of Hong Kong, Shenzhen

# Introduction to Computer Science: Programming Methodology

## Lecture 8 Data Structure & Algorithm – Intro

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# Data structure and algorithm

- A **data structure** is a systematic way of organizing and accessing data
- An **algorithm** is a step-by-step procedure for performing some task in a finite amount of time.

# Why study data structure and algorithm?

- Important for **all other branches** of computer science
- Plays a **key role** in modern technological innovation
- **Moore's law** predicts that the density of transistors in integrated circuits would continue to double every 1 to 2 years
- However, in many areas, performance gains due to the **improvements in algorithms** have **greatly exceeded** even the dramatic performance gains due to increased processor speed

# Why study data structure and algorithm?

- Provide novel “lens” on processes outside of computer science and technology, such as quantum mechanics, economic markets, evolution
- Challenging (good for your brain!!) and fun

# Example: integer multiplication

- **Inputs:** two n-digits number  $x$  and  $y$
- **Output:** the product of  $x$  and  $y$
- **Primitive operations:** add or multiply  
2 single digit numbers

2	3	9	5	8	2	3	3
+-----+	+-----+	+-----+	+-----+	+-----+	+-----+	+-----+	+-----+
1 / 1 / 4 / 2 / 4 / 1 / 1 / 1 /	/   /   /   /   /   /   /   /   /   5						
01 / 0 / 5 / 5 / 5 / 0 / 0 / 5 / 5	+-----+	+-----+	+-----+	+-----+	+-----+	+-----+	+-----+
1 / 2 / 7 / 4 / 6 / 1 / 2 / 2 /	/   /   /   /   /   /   /   /   /   8						
02 / 6 / 4 / 2 / 0 / 4 / 6 / 4 / 4	+-----+	+-----+	+-----+	+-----+	+-----+	+-----+	+-----+
0 / 0 / 2 / 1 / 2 / 0 / 0 / 0 /	/   /   /   /   /   /   /   /   /   3						
17 / 6 / 9 / 7 / 5 / 4 / 6 / 9 / 9	+-----+	+-----+	+-----+	+-----+	+-----+	+-----+	+-----+
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01
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139676498390
= 139,676,498,390

$$\begin{array}{r}
 23958233 \\
 \times \quad \quad \quad 5830 \\
 \hline
 00000000 \text{ (} = 23,958,233 \times 0\text{)} \\
 71874699 \text{ (} = 23,958,233 \times 30\text{)} \\
 191665864 \text{ (} = 23,958,233 \times 800\text{)} \\
 + 119791165 \text{ (} = 23,958,233 \times 5,000\text{)} \\
 \hline
 139676498390 \text{ (} = 139,676,498,390\text{)}
 \end{array}$$

## Quarter square multiplication [edit]

This formula can in some cases be used, to make multiplication tasks easier to complete:

$$\frac{(x+y)^2}{4} - \frac{(x-y)^2}{4} = \frac{1}{4} ((x^2 + 2xy + y^2) - (x^2 - 2xy + y^2)) = \frac{1}{4} (4xy) = xy.$$

In the case where  $x$  and  $y$  are integers, we have that

$$(x+y)^2 \equiv (x-y)^2 \pmod{4}$$

because  $x+y$  and  $x-y$  are either both even or both odd. This means that

$$\begin{aligned}
 xy &= \frac{1}{4}(x+y)^2 - \frac{1}{4}(x-y)^2 \\
 &= ((x+y)^2 \text{ div } 4) - ((x-y)^2 \text{ div } 4)
 \end{aligned}$$

and it's sufficient to (pre-)compute the integral part of squares divided by 4 like in the following example.

[https://en.wikipedia.org/wiki/Multiplication\\_algorithm](https://en.wikipedia.org/wiki/Multiplication_algorithm)

## The algorithm designer's mantra

“Perhaps the most important principle for the good algorithm designer is to refuse to be content”

Aho, Hopcroft, and Ullman, *The Design and Analysis of Computer Algorithms*, 1974

## How do we define a “good” algorithm?

- The primary analysis of algorithms involves characterizing the **running times** and **space usage** of algorithms and data structure operations
- **Running time** is a natural measure of “goodness,” since time is a precious resource—computer solutions should run as fast as possible
- **Space usage** is another major issue to consider when we design an algorithm, since we only have limited storage spaces

# Measuring the running time experimentally

```
from time import time

startTime = time()

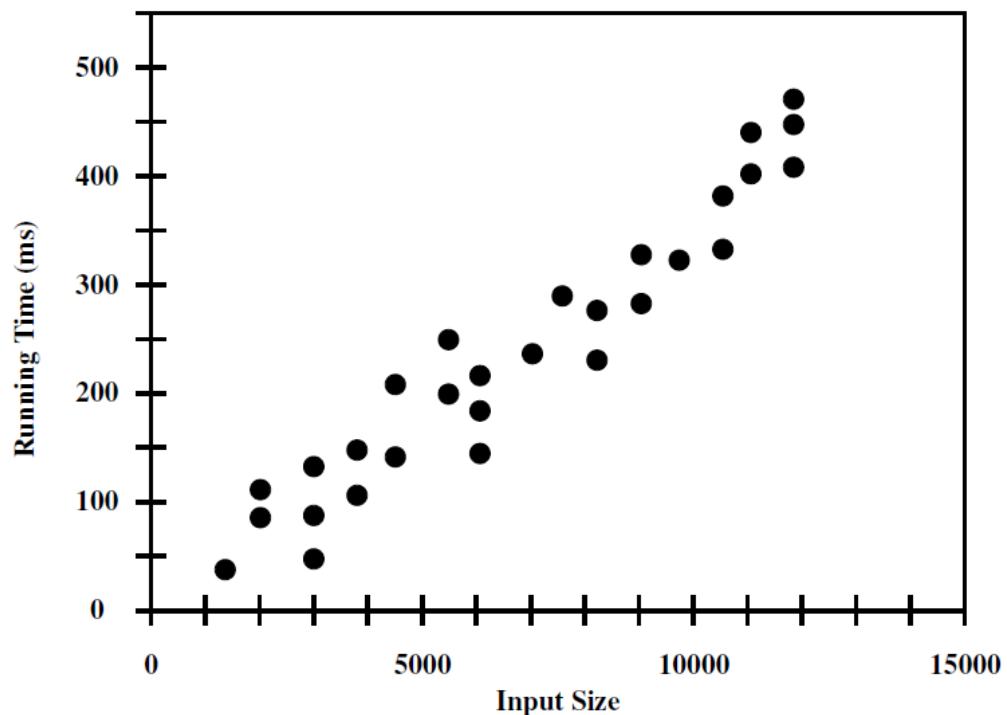
for i in range(1, 20000):
    if i%10 == 0:
        print(i)

endTime = time()

print('The time elapsed is:', endTime - startTime, 'seconds')
```

# Visualize the running time

- Running time and space usage are dependent on the **size of the input**
- Perform independent experiments on many different **test inputs of various sizes**
- Visualize the results by plotting the performance of each run of the algorithm as a point



## Challenges of experimental analysis

- Experimental running times of two algorithms are difficult to directly compare unless the experiments are performed in the same hardware and software environments
- Experiments can be done only on a limited set of test inputs; hence, they leave out the running times of inputs not included in the experiment (and these inputs may be important)
- An algorithm must be fully implemented in order to execute it to study its running time experimentally

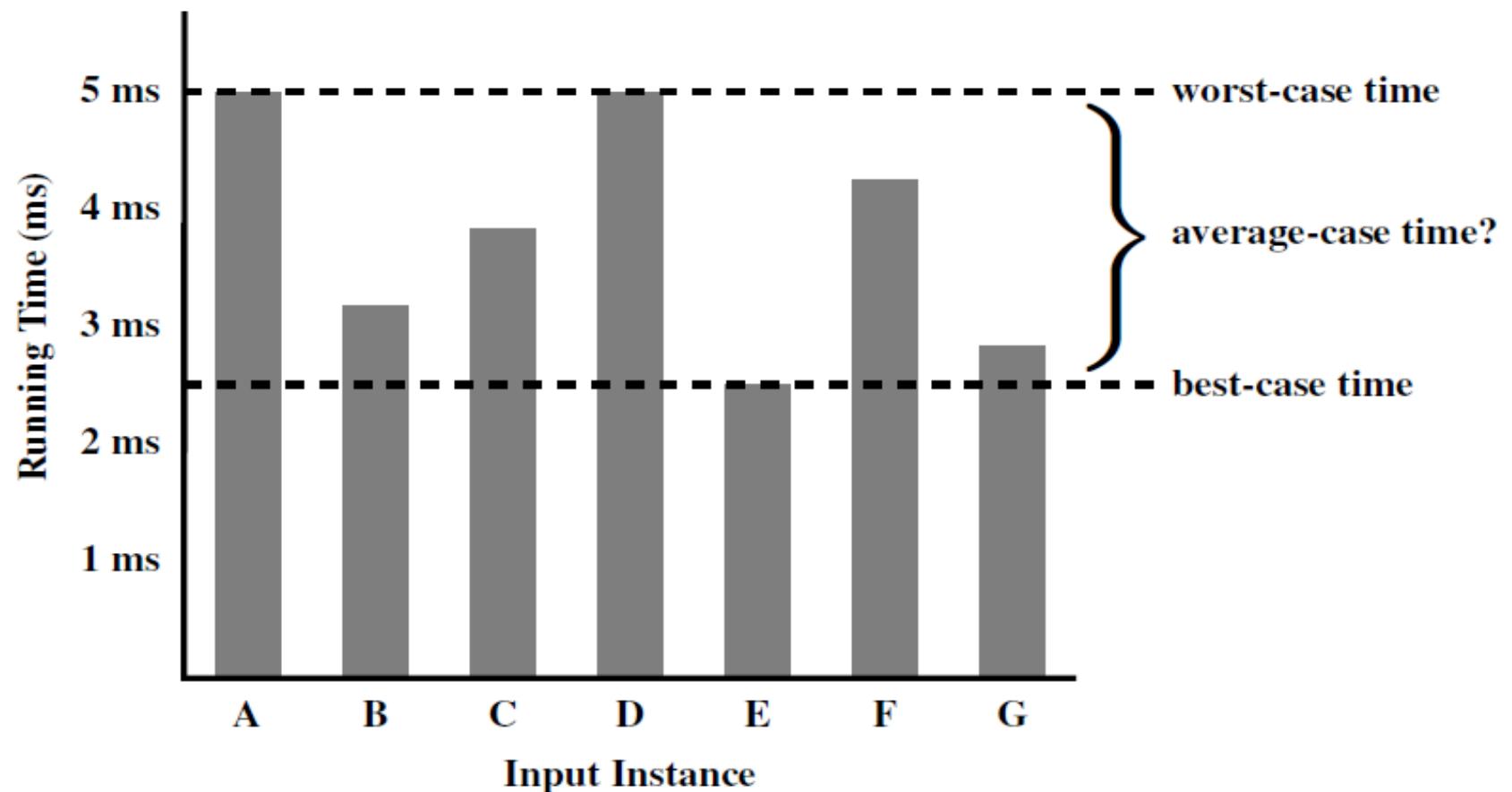
# Principle of algorithm analysis 1: counting primitive operations

- To analyse the running time of an algorithm without performing experiments, we perform an analysis directly on a high-level description of the algorithm
- We define a set of primitive operations such as the following:
  - ✓ Assigning an identifier to an object
  - ✓ Determining the object associated with an identifier
  - ✓ Performing an arithmetic operation (for example, adding two numbers)
  - ✓ Comparing two numbers
  - ✓ Accessing a single element of a Python list by index
  - ✓ Calling a function (excluding operations executed within the function)
  - ✓ Returning from a function

## Principle of algorithm analysis 2: measuring operations as a function of input size

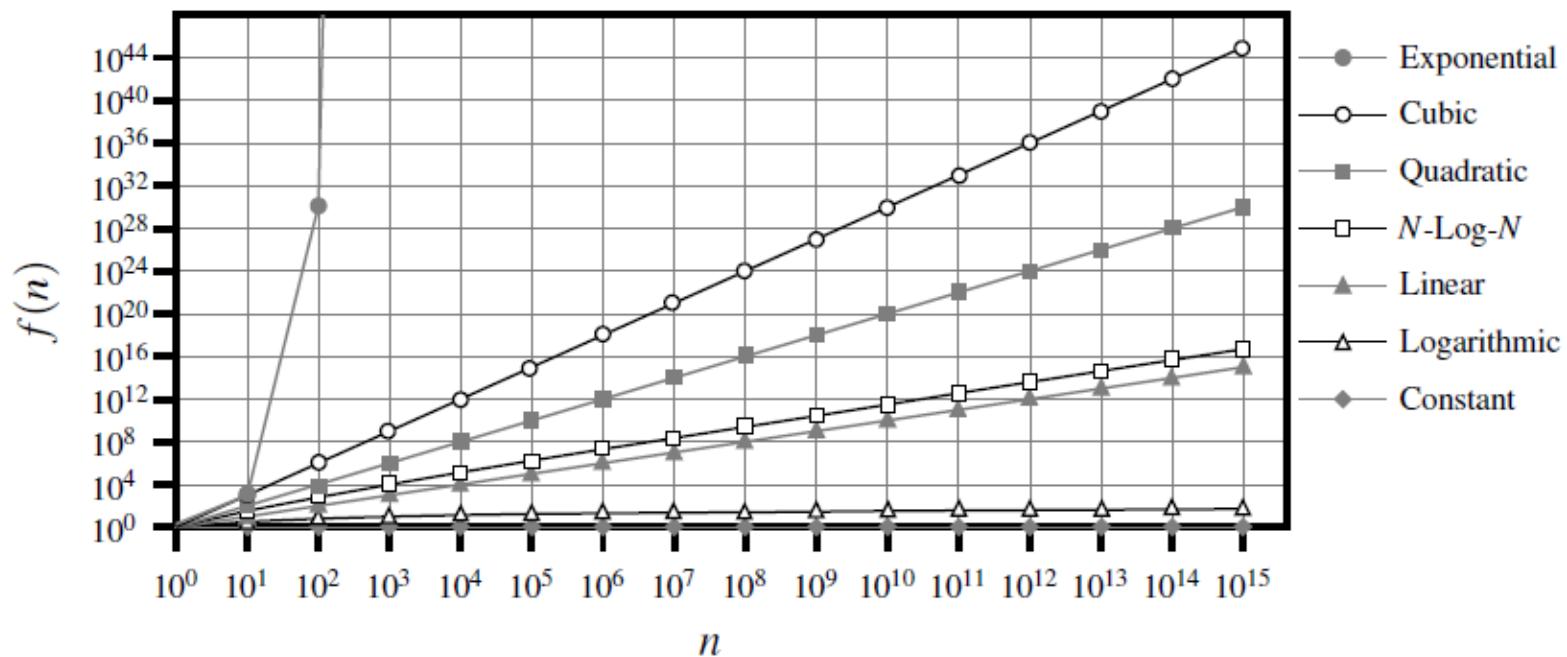
- To capture the order of growth of an algorithm's running time, we will associate, with each algorithm, a function  $f(n)$  that characterizes the number of primitive operations that are performed as a function of the input size  $n$

## Principle of algorithm analysis 3: focusing on the worst-case impact



# The 7 functions used in algorithm analysis

- We may use the following 7 functions to measure the time complexity of an algorithm: **constant, logarithm, linear, N-log-N, quadratic, cubic and other polynomials, exponential**



# Asymptotic analysis

- In algorithm analysis, we focus on the growth rate of the running time as a function of the input size  $n$ , taking a “big-picture” approach
- Vocabulary for the analysis and design of algorithms
- “Sweet spot” for high-level reasoning about algorithms
- Coarse enough to suppress unnecessary details, e.g. architecture/language/compiler...
- Sharp enough to make meaningful comparisons between algorithms

## The big Oh notation

- Let  $f(n)$  and  $g(n)$  be functions mapping positive integers to positive real numbers.
- We say that  $f(n)$  is  $O(g(n))$  if there is a real constant  $c > 0$  and an integer constant  $n_0 \geq 1$  such that

$$f(n) \leq cg(n), \text{ for } n \geq n_0$$

- This definition is often referred to as the “big-Oh” notation
- **Example:** The function  $8n+5$  is  $O(n)$ .

# The big Oh notation

- The big-Oh notation allows us to say that a function  $f(n)$  is “less than or equal to” another function  $g(n)$  up to a constant factor and **in the asymptotic sense** as  $n$  grows toward infinity
- The big-Oh notation is used widely to characterize **running times** and **space bounds** in terms of some parameter  $n$ , which varies from problem to problem, but is always defined as a chosen measure of the “**size**” of the problem

## Properties of the Big-Oh notation

- The big-Oh notation allows us to ignore constant factors and lower-order terms and focus on the main components of a function that affect its growth
- Example:  $5n^4 + 3n^3 + 2n^2 + 4n + 1$  is  $O(n^4)$
- Example:  $2^{n+2}$  is  $O(2^n)$
- Example:  $2n + 100\log n$  is  $O(n)$
- In general, we should use the big-Oh notation to characterize a function as closely as possible

## Comparative analysis

**Question:** Suppose two algorithms solving the same problem are available: an algorithm A, which has a running time of  $O(n)$ , and an algorithm B, which has a running time of  $O(n^2)$ . Which algorithm is better?

**Answer:** Algorithm A is **asymptotically better** than algorithm B

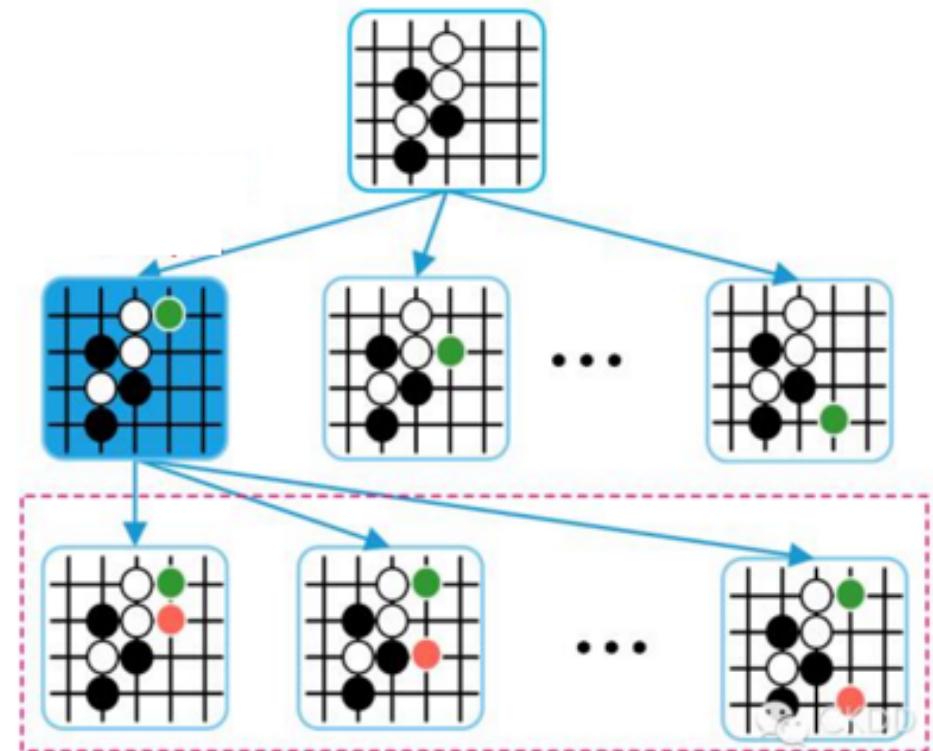
## Comparative analysis

- We can use the big-Oh notation to order classes of functions by asymptotic growth rate
- Our seven functions are ordered by increasing growth rate in the following sequence

$n$	$\log n$	$n$	$n \log n$	$n^2$	$n^3$	$2^n$
8	3	8	24	64	512	256
16	4	16	64	256	4,096	65,536
32	5	32	160	1,024	32,768	4,294,967,296
64	6	64	384	4,096	262,144	$1.84 \times 10^{19}$
128	7	128	896	16,384	2,097,152	$3.40 \times 10^{38}$
256	8	256	2,048	65,536	16,777,216	$1.15 \times 10^{77}$
512	9	512	4,608	262,144	134,217,728	$1.34 \times 10^{154}$

# Why AlphaGo is a remarkable achievement?

- If we use brutal-force to search the best move in Go, the time complexity is at the order of  $O(10^n)$
- The search space is even larger than the number of atoms in the universe!!!



## The line of tractability

- To differentiate **efficient** and **inefficient** algorithms, the general line is between **polynomial time algorithms** and **exponential time algorithms**
- The distinction between polynomial-time and exponential-time algorithms is considered a robust measure of **tractability**

## Example: finding the smallest number in a list

```
smallest_so_far = None
print('Before', smallest_so_far)

for num in [9, 39, 21, 98, 4, 5, 100, 65]:
    if smallest_so_far == None:
        smallest_so_far = num
    elif num < smallest_so_far:
        smallest_so_far = num
    print(smallest_so_far, num)

print('After', smallest_so_far)
```

- What is the time complexity of this algorithm?

# Recursion

- Recursion is a technique by which a function makes one or more calls to itself during execution
- Recursion provides an elegant and powerful alternative for performing repetitive tasks
- Recursion is an important technique in the study of data structures and algorithms

# Inception



## Example: the factorial function

- The **factorial** of a positive integer  $n$ , denoted  $n!$ , is defined as follows:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1 & \text{if } n \geq 1. \end{cases}$$

- The factorial function is important because it is known to equal the number of ways in which  $n$  distinct items can be arranged into a sequence, that is, the number of permutations of  $n$  items

## The recursive definition

- First, a recursive definition contains one or more **base cases**, which are defined **non-recursively** in terms of fixed quantities
- Second, it also contains one or more **recursive cases**, which are defined by appealing to the definition of the function being defined

## The recursive definition of factorial function

- The factorial function can be naturally defined in a recursive way, for example,  $5! = 5 \cdot (4 \cdot 3 \cdot 2 \cdot 1) = 5 \cdot 4!$
- More generally, for a positive integer  $n$ , we can define  $n!$  to be  $n \cdot (n-1)!$
- Therefore, the recursive definition of factorial function is:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{if } n \geq 1. \end{cases}$$

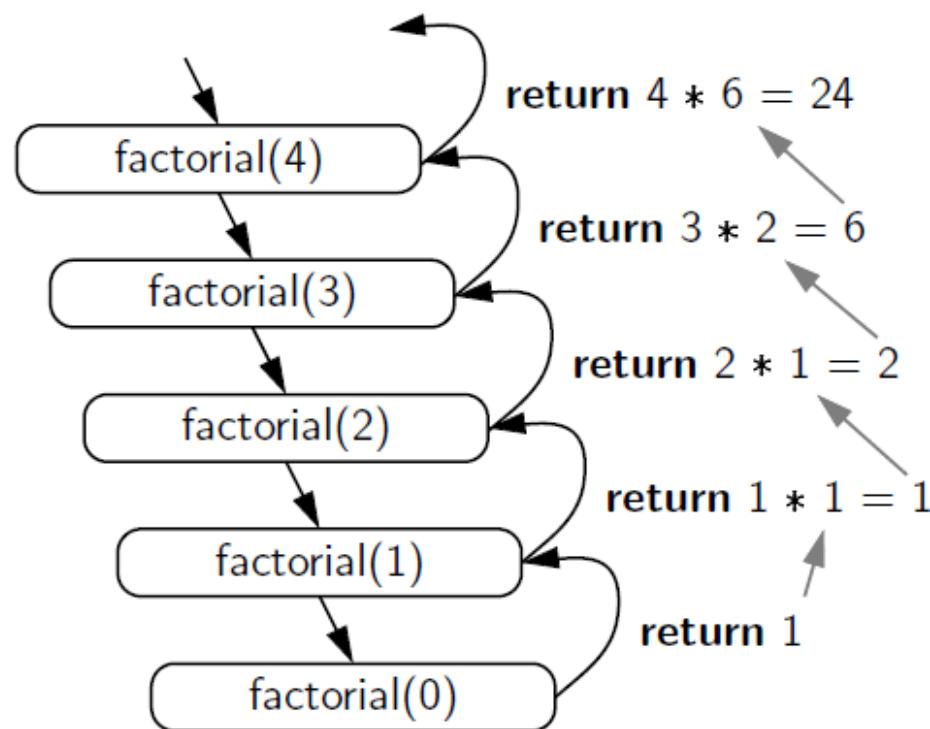
# Solution

```
def facFunc(n):  
  
    if n<0:  
        print('Invalid input.')  
        return None  
    elif n == 0:  
        return 1  
    else:  
        return n*facFunc(n-1)
```

# How Python implements recursion

- In Python, each time a function (recursive or otherwise) is called, a structure known as an **activation record** or **frame** is created to store information about the progress of that invocation of the function
- This activation record stores the function call's **parameters** and **local variables**
- When the execution of a function leads to a nested function call, the execution of the former call is suspended and its activation record stores the place in the source code at which the **flow of control should continue** upon return of the nested call

# The recursive trace

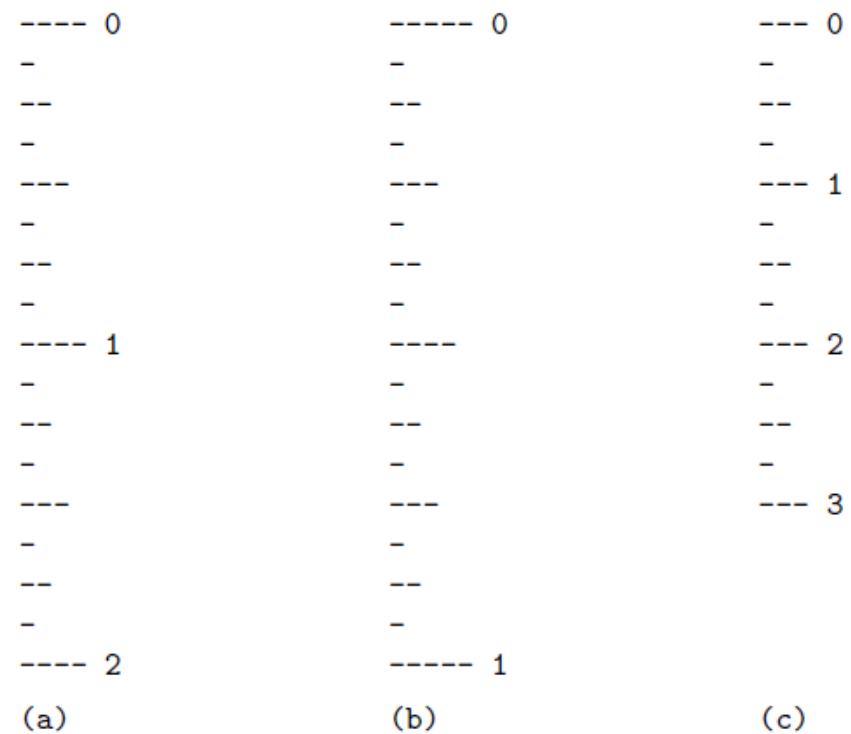


## Example: drawing an English ruler

- We denote the length of the tick designating a whole inch as the **major tick length**

- Between the marks for whole inches, the ruler contains a series of **minor ticks**, placed at intervals of  $1/2$  inch,  $1/4$  inch, and so on.

- As the size of the interval decreases by half, the tick length decreases by one



## Recursive implementation of English ruler

- An interval with a central tick length  $L \geq 1$  is composed of:
  - ✓ An interval with a central tick length  $L-1$
  - ✓ A single tick of length  $L$
  - ✓ An interval with a central tick length  $L-1$

# Solution

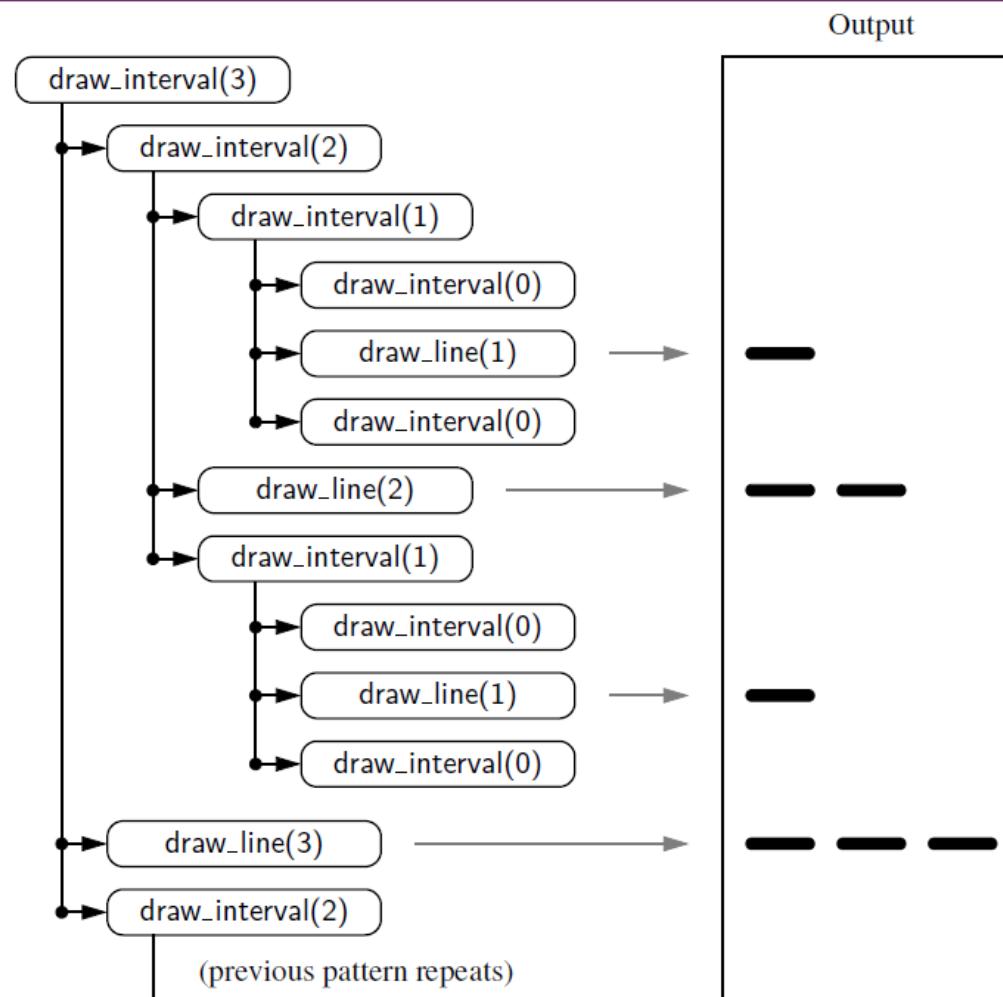
```
def draw_line(tickLen, tickLabel=' '):
    line = '-' * tickLen
    if tickLabel:
        line += ' ' + tickLabel
    print(line)

def draw_interval(centerLen):
    if centerLen > 0:
        draw_interval(centerLen - 1)
        draw_line(centerLen)
        draw_interval(centerLen - 1)

def draw_ruler(numInch, majorLen):
    draw_line(majorLen, '0')

    for j in range(1, 1 + numInch):
        draw_interval(majorLen - 1)
        draw_line(majorLen, str(j))
```

# The recursive trace for English ruler



## Example: binary search

- A classic and very useful recursive algorithm, **binary search**, can be used to efficiently locate a target value within a **sorted** sequence of **n** elements
- When the sequence is **unsorted**, the standard approach to search for a target value is to use a loop to examine every element, until either finding the target or exhausting the data set; This is known as the **sequential search** algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	5	7	8	9	12	14	17	19	22	25	27	28	33	37

# Binary search

- When the sequence is sorted and indexable, binary search is a much more efficient algorithm
- For any index  $j$ , we know that all the values stored at indices  $0, \dots, j-1$  are less than or equal to the value at index  $j$ , and all the values stored at indices  $j+1, \dots, n-1$  are greater than or equal to that at index  $j$

## The strategy of binary search

- We call an element of the sequence a **candidate** if, at the current stage of the search, we cannot rule out that this item matches the target
- The algorithm maintains two parameters, **low** and **high**, such that all the candidate entries have index at least **low** and at most **high**
- Initially, **low = 0** and **high = n-1**. We then compare the target value to the median candidate, that is, the item **data[mid]** with index

$$\text{mid} = \lfloor (\text{low}+\text{high})/2 \rfloor$$

## The strategy of binary search

- If the target equals `data[mid]`, then we have found the item we are looking for, and the search terminates successfully
- If `target < data[mid]`, then we recur on the first half of the sequence, that is, on the interval of indices from `low` to `mid-1`
- If `target > data[mid]`, then we recur on the second half of the sequence, that is, on the interval of indices from `mid+1` to `high`

# Solution

```
def binarySearch(data, target, low, high):
    if low>high:
        print('Cannot find the target number!')
        return False
    else:
        mid = (low+high)//2
        if target==data[mid]:
            print('The target number is at position', mid)
            return True
        elif target<data[mid]:
            return binarySearch(data, target, low, mid-1)
        else:
            return binarySearch(data, target, mid+1, high)

def main():
    data = [1, 3, 5, 6, 16, 78, 100, 135, 900]
    target = 16
    binarySearch(data, target, 0, len(data)-1)
```

## Time complexity of binary search

**Proposition:** The binary search algorithm runs in  $O(\log n)$  time for a sorted sequence with  $n$  elements

Why?

## Proof

**Justification:** To prove this claim, a crucial fact is that with each recursive call the number of candidate entries still to be searched is given by the value

$$\text{high} - \text{low} + 1.$$

Moreover, the number of remaining candidates is reduced by at least one half with each recursive call. Specifically, from the definition of  $\text{mid}$ , the number of remaining candidates is either

$$(\text{mid} - 1) - \text{low} + 1 = \left\lfloor \frac{\text{low} + \text{high}}{2} \right\rfloor - \text{low} \leq \frac{\text{high} - \text{low} + 1}{2}$$

or

$$\text{high} - (\text{mid} + 1) + 1 = \text{high} - \left\lfloor \frac{\text{low} + \text{high}}{2} \right\rfloor \leq \frac{\text{high} - \text{low} + 1}{2}.$$

Initially, the number of candidates is  $n$ ; after the first call in a binary search, it is at most  $n/2$ ; after the second call, it is at most  $n/4$ ; and so on. In general, after the  $j^{\text{th}}$  call in a binary search, the number of candidate entries remaining is at most  $n/2^j$ . In the worst case (an unsuccessful search), the recursive calls stop when there are no more candidate entries. Hence, the maximum number of recursive calls performed, is the smallest integer  $r$  such that

$$\frac{n}{2^r} < 1.$$

In other words (recalling that we omit a logarithm's base when it is 2),  $r > \log n$ .

Thus, we have

$$r = \lfloor \log n \rfloor + 1,$$

which implies that binary search runs in  $O(\log n)$  time. ■

Thanks