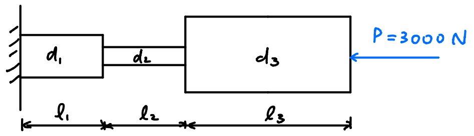
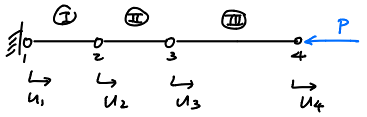
PROBLEM 1





	d (m)	A (m²)	l (m)	E (Pa)
1	4×10^{-2}	$4\times10^{-4}\pi$	0.1	80×10^{9}
2	2×10^{-2}	$1\times10^{-4}\pi$	0.1	
3	6×10^{-2}	$9\times10^{-4}\pi$	0.2	

1.1. elementary stiffness matrices

$$\boxed{\begin{aligned} &\text{I}: \quad [k]^{\text{I}} = \frac{EA_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_1\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}, \qquad k_1 = 3.2\times10^{8}\pi \ \text{N/m} \\ &\text{II}: \quad [k]^{\text{II}} = \frac{EA_2}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_2\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}, \qquad k_2 = 8.0\times10^{7}\pi \ \text{N/m} \\ &\text{III}: \quad [k]^{\text{III}} = \frac{EA_3}{l_3}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}, \qquad k_3 = 3.6\times10^{8}\pi \ \text{N/m} \end{aligned}}$$

1.2.

$$[K] = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 3.2\times10^{8} & -3.2\times10^{8} & 0 & 0 \\ -3.2\times10^{8} & 4.0\times10^{8} & -8.0\times10^{7} & 0 \\ 0 & -8.0\times10^{7} & 4.4\times10^{8} & -3.6\times10^{8} \\ 0 & 0 & -3.6\times10^{8} & 3.6\times10^{8} \end{bmatrix} \cdot \pi \ \text{N/m}$$

$$\{F\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -P \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -3000 \end{Bmatrix} \text{N}$$

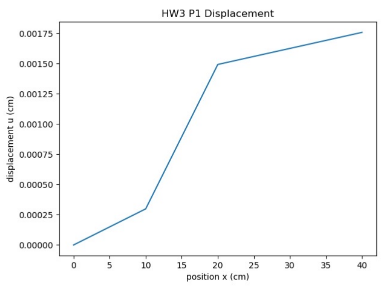
1.3. $[K]\{u\} = \{F\} \Rightarrow \{u\} = [K]^{-1}\{F\}$

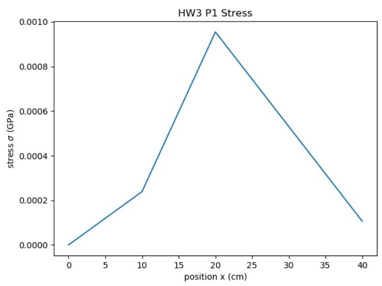
enforce displacement B.C. at node 1 : $u_1 = 0$

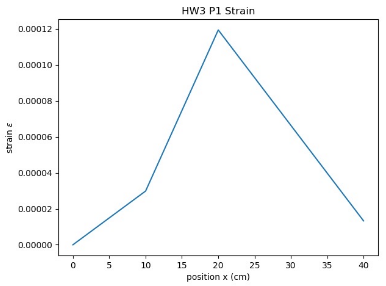
$\Rightarrow$ row 1 & col 1 in $[K]$ and $F[1]$ all go to zero for consistency

$$\begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -P \end{Bmatrix} \Rightarrow \boxed{\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2.98\times10^{-8} \\ 1.49\times10^{-7} \\ 1.76\times10^{-7} \end{Bmatrix} \text{m} = \begin{Bmatrix} 0 \\ 0.000298 \\ 0.00149 \\ 0.00176 \end{Bmatrix} \text{cm}}$$

1.4. $\varepsilon = \dfrac{\partial u}{\partial x} = \dfrac{\text{diff}(u)}{\text{diff}(x)}$ $\qquad \sigma = E\varepsilon$

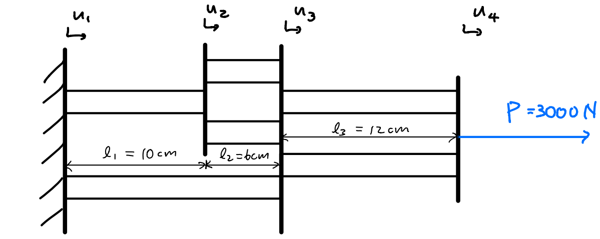


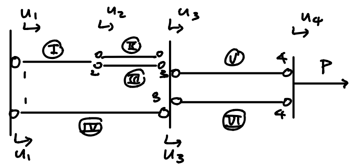




Stress & strain plots look the same except for stress (σ) is scaled by E.

PROBLEM 2





$$\left.\begin{array}{l} E = 70\ GPa = 7\times10^{10}\ Pa \\ A = 0.1\ cm^2 = 1\times10^{-5}\ m^2 \end{array}\right\}\ \text{for all elements}$$

2.1.

elementary stiffness matrices:

$$\text{I}:\ [k]^{\text{I}} = \frac{EA}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_1 \begin{array}{c} \begin{array}{cc} ① & ② \end{array} \\ \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \end{array}\begin{array}{c} ① \\ ② \end{array}$$

$$\text{II}:\ [k]^{\text{II}} = \frac{EA}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_2 \begin{array}{c} \begin{array}{cc} ② & ③ \end{array} \\ \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \end{array}\begin{array}{c} ② \\ ③ \end{array}$$

$$\text{III}:\ [k]^{\text{III}} = \frac{EA}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_3 \begin{array}{c} \begin{array}{cc} ② & ③ \end{array} \\ \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \end{array}\begin{array}{c} ② \\ ③ \end{array}$$

$$\text{IV}:\ [k]^{\text{IV}} = \frac{EA}{l_1+l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_4 \begin{array}{c} \begin{array}{cc} ① & ③ \end{array} \\ \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \end{array}\begin{array}{c} ① \\ ③ \end{array}$$

$$\text{V}:\ [k]^{\text{V}} = \frac{EA}{l_3}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_5 \begin{array}{c} \begin{array}{cc} ③ & ④ \end{array} \\ \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \end{array}\begin{array}{c} ③ \\ ④ \end{array}$$

$$\text{VI}:\ [k]^{\text{VI}} = \frac{EA}{l_3}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = k_6 \begin{array}{c} \begin{array}{cc} ③ & ④ \end{array} \\ \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \end{array}\begin{array}{c} ③ \\ ④ \end{array}$$

i	l_i (m)	k_i (10^6 N/m)
1	0.10	7
2	0.06	11.667
3	0.06	11.667
4	0.16	4.375
5	0.12	5.833
6	0.12	5.833

assemble global [K]

$$[K] = \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \begin{bmatrix} k_1+k_4 & -k_1 & -k_4 & 0 \\ -k_1 & k_1+k_2+k_3 & -k_2-k_3 & 0 \\ -k_4 & -k_2-k_3 & k_2+k_3+k_4+k_5+k_6 & -k_5-k_6 \\ 0 & 0 & -k_5-k_6 & k_5+k_6 \end{bmatrix} \end{array} \begin{array}{c} ① \\ ② \\ ③ \\ ④ \end{array}$$

force vector

$$\{F\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ P \end{Bmatrix}$$

apply displacement B.C. @ node 1 : $u_1 = 0$

$\Rightarrow$ row 1 & col 1 in [k] go to 0

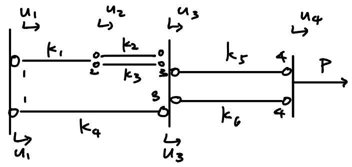
$$\begin{bmatrix} k_1+k_2+k_3 & -k_2-k_3 & 0 \\ -k_2-k_3 & k_2+k_3+k_4+k_5+k_6 & -k_5-k_6 \\ 0 & -k_5-k_6 & k_5+k_6 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P \end{bmatrix}$$

$$\begin{bmatrix} k_1+k_2+k_3 & -k_2-k_3 & 0 \\ -k_2-k_3 & k_2+k_3+k_4+k_5+k_6 & -k_5-k_6 \\ 0 & -k_5-k_6 & k_5+k_6 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 30.333 & -23.333 & 0 \\ -23.333 & 39.375 & -11.667 \\ 0 & -11.667 & 11.667 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 3000 \end{Bmatrix} \Rightarrow \boxed{\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.000236 \\ 0.000307 \\ 0.000565 \end{Bmatrix} m = \begin{Bmatrix} 0 \\ 0.0236 \\ 0.0307 \\ 0.0565 \end{Bmatrix} cm}$$

PROBLEM 2 CONT'D

2.2.



$\Rightarrow$

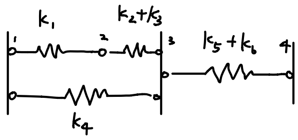
parallel : $k_{eq} = k_i + k_j \qquad u_{eq} = u_1 = u_2 \qquad F_{eq} = F_1 + F_2$

series : $k_{eq} = \dfrac{k_i k_j}{k_i + k_j} \qquad u_{eq} = u_1 + u_2 \qquad F_{eq} = F_1 = F_2$

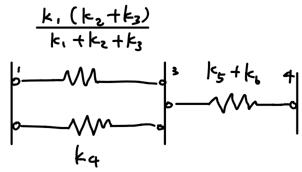
$k_2 \,//\, k_3$

$k_5 \,//\, k_6$

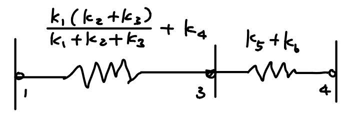
$\Rightarrow$



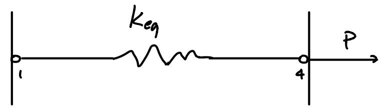
$\Rightarrow$



$\Rightarrow$



$\Rightarrow$



$$k_{eq} = \frac{\left(\dfrac{k_1(k_2+k_3) + k_4(k_1+k_2+k_3)}{k_1+k_2+k_3}\right)\cdot\left(k_5+k_6\right)}{\left(\dfrac{k_1(k_2+k_3) + k_4(k_1+k_2+k_3)}{k_1+k_2+k_3}\right)+\left(k_5+k_6\right)}$$

$k_{eq} = 5.314 \times 10^6$ N/m

Hook's law : $u_4 = \dfrac{P}{k_{eq}}$

$u_3 = \dfrac{P}{k_{1234}}$

$u_2 = \dfrac{k_2+k_3}{k_1+k_2+k_3}\, u_3$

$k_1 u_2 = (k_2+k_3)(u_3 - u_2)$

$(k_1+k_2+k_3)\, u_2 = (k_2+k_3)\, u_3$

$$\boxed{\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.000236 \\ 0.000307 \\ 0.000565 \end{Bmatrix} m = \begin{Bmatrix} 0 \\ 0.0236 \\ 0.0307 \\ 0.0565 \end{Bmatrix} cm}$$

* same as FEM