$$\min_{s_{ij}\geq 0, s_i \mathbf{1}_n = 1, rank(L_S)=n-c} \sum_{v=1}^{m} \left\| S - A^{(v)} \right\|_F. \qquad (4)$$

This equation looks pretty simplified and compact, but no weight factor is explicitly defined therein. We consider it as a normal issue and firstly write its Lagrange function

$$\min_{S} \sum_{v=1}^{m} \left\| S - A^{(v)} \right\|_F + \mathcal{G}(\Lambda, S), \qquad (5)$$

where Λ is the Lagrange multiplier, $\mathcal{G}(\Lambda, S)$ serves as a proxy for the constraints to S. Taking the derivative of Eq. (5) w.r.t S and setting the derivative to zero, we have

$$\sum_{v=1}^{m} w^{(v)} \frac{\partial \left\| S - A^{(v)} \right\|_F^2}{\partial S} + \frac{\partial G(\Lambda, S)}{\partial S} = 0, \qquad (6)$$

where $w^{(v)}$ is given as the following form[1]

$$w^{(v)} = 1 \Big/ \left(2 \left\| S - A^{(v)} \right\|_F \right). \qquad (7)$$

Obviously, from Eq. (7) we know that $w^{(v)}$ is dependent on S, which means the two factors of the first term in Eq. (6) are coupled with each other. But if we set $w^{(v)}$ stationary, Eq. (6) can be considered as the solution to the following problem

$$\min_{s_{ij}\geq 0, s_i \mathbf{1}_n = 1, rank(L_S)=n-c} \sum_{v=1}^{m} w^{(v)} \left\| S - A^{(v)} \right\|_F^2, \qquad (8)$$

公式 5 到公式 6 的求导我知道了， 就是把 $\left\| S - A^{(v)} \right\|_F$ 看成是 $\sqrt{x}$ ，那么他对 S(或者是 x)求导就是 $w^{(v)} = 1 \Big/ \left(2 \left\| S - A^{(v)} \right\|_F \right).$ (对应为 $\frac{1}{2\sqrt{x}}$)，再乘上内部对 S 的求导也就是 $\frac{\partial \left\| S - A^{(v)} \right\|_F^2}{\partial S}$ 。

但是要问为什么公式 4 和公式 8 有共同的解，我觉得就是把 $w^{(v)} = 1 \Big/ \left(2 \left\| S - A^{(v)} \right\|_F \right).$ 代入公式 8，这样不就把平方消掉了，和公式 4 一样了么(除了系数)？