

请写出平面平行层构型下的恒星大气辐射转移方程和其通解（从恒星大气内部某层到恒星大气顶部出射的辐射强度通解）。

$$dI_\nu = -\kappa_\nu I_\nu ds + \eta_\nu ds$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \eta_\nu$$

平面平行层中， $ds = dz/\mu$

$$\begin{aligned}\mu \frac{\partial I_\nu(z, \mu)}{\partial z} &= -\kappa_\nu(z, \mu) I_\nu(z, \mu) + \eta_\nu(z, \mu) \\ &= \kappa_\nu(z, \mu) [S_\nu(z, \mu) - I_\nu(z, \mu)]\end{aligned}$$

$$\text{由 } d\tau_\nu = -\kappa_\nu dz, \quad \mu \frac{\partial I_\nu(\tau_\nu, \mu)}{\partial \tau_\nu} = I_\nu(\tau_\nu, \mu) - S_\nu(\tau_\nu, \mu)$$

$$\text{通解为 } I_\nu(\tau_{1\nu}, \mu) = I_\nu(\tau_{2\nu}, \mu) e^{-(\tau_{2\nu} - \tau_{1\nu})/\mu} + \int_{\tau_{1\nu}}^{\tau_{2\nu}} S_\nu(t_\nu, \mu) e^{-(t_\nu - \tau_{1\nu})/\mu} \frac{dt_\nu}{\mu}$$

$\tau_\nu$ 处出射强度 ( $\mu \geq 0$ ) :

$$I_\nu(\tau_\nu, \mu) = I_\nu(\tau_{2\nu}, \mu) e^{-(\tau_{2\nu} - \tau_\nu)/\mu} + \int_{\tau_\nu}^{\tau_{2\nu}} S_\nu(t_\nu, \mu) e^{-(t_\nu - \tau_\nu)/\mu} \frac{dt_\nu}{\mu}$$

由边界条件  $\lim_{\tau_{2\nu} \rightarrow \infty} I_\nu(\tau_{2\nu}, \mu) = 0, 0 \leq \mu \leq 1,$

$$I_\nu(\tau_\nu, \mu) = \int_{\tau_\nu}^{\infty} S_\nu(t_\nu) e^{-(t_\nu - \tau_\nu)/\mu} \frac{dt_\nu}{\mu}$$

从 $\tau_\nu$ 层到恒星大气顶部

$$\tau_{1\nu} = \tau_\nu = 0, \tau_{2\nu} = \tau_\nu$$

$$I_\nu(0, \mu) = I_\nu(\tau_\nu, \mu) e^{-\tau_\nu/\mu} + \int_0^{\tau_\nu} S_\nu(t_\nu, \mu) e^{-t_\nu/\mu} \frac{dt_\nu}{\mu} = \int_0^{\infty} S_\nu(t_\nu, \mu) e^{-t_\nu/\mu} \frac{dt_\nu}{\mu}$$