

Problem 6.1: The man in the painting

(The following puzzle appears in **russel** question 9.11.) A man stands in front of a painting and says the following:

Brothers and sisters have I none, but that man's father is my father's son.

What is the relationship between the man in the painting and the speaker? Use the predicates

$Male(x) : x \text{ is male.}$
 $Father(x, y) : x \text{ is the father of } y.$
 $Son(x, y) : x \text{ is a son of } y.$
 $Parent(x, y) : x \text{ is a parent of } y.$
 $Child(x, y) : x \text{ is a child of } y.$
 $Sibling(x, y) : x \text{ is a sibling of } y$

and the knowledge

- A sibling is another child of one's parents.

$$\forall x, y \quad Sibling(x, y) \Leftrightarrow x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)$$

- Parent and child are inverse relations.

$$\forall p, c \quad Parent(p, c) \Leftrightarrow Child(c, p)$$

to solve the riddle with first-order logic.

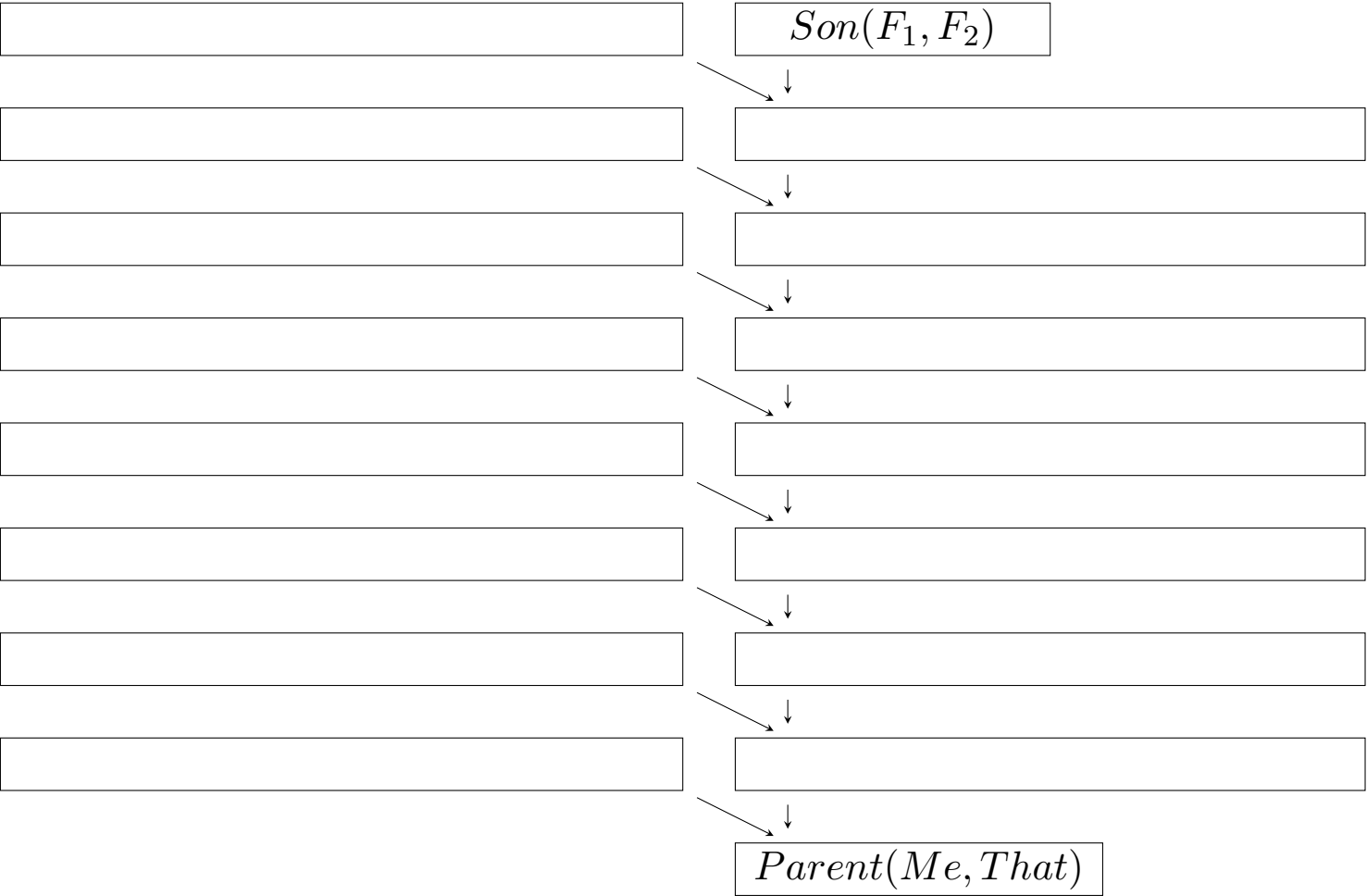
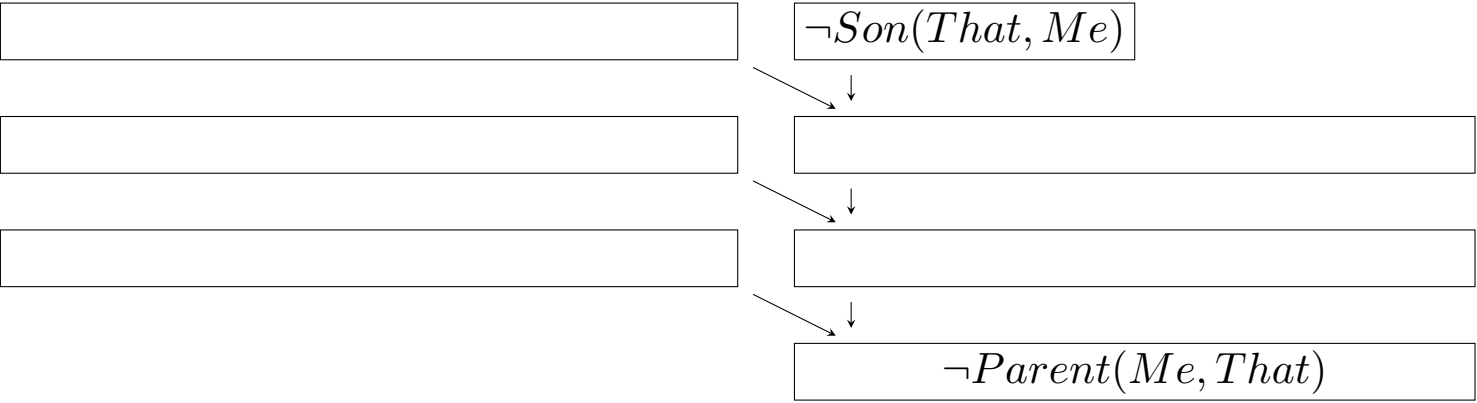
Problem 6.1.1: Define the rule which relates *son*, *child*, and *male*. Define also the rule which relates *father* and *parent*.

Problem 6.1.2: Using the constants *Me* for the speaker and *That* for the person depicted in the painting, formalise the sentences regarding the sexes of the people in the puzzle.

Problem 6.1.3: Formalise the sentences “Brothers and sisters have I none” and “That man's father is my father's son” in first-order logic.

Problem 6.1.4: Solve this puzzle informally and decide what is the relation between the man in the painting and the speaker.

Problem 6.1.5: Using the resolution technique for first-order logic, prove your answer. You can use the two diagrams on the next page to structure your proof.



Problem 6.2: Backward chaining

(The following exercise is taken from **russel** Question 9.10.) Suppose you are given the following axioms:

1. $0 \leq 3$
2. $7 \leq 9$
3. $\forall x \quad x \leq x$
4. $\forall x \quad x \leq x + 0$
5. $\forall x \quad x + 0 \leq x$
6. $\forall x, y \quad x + y \leq y + x$
7. $\forall w, x, y, z \quad w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$
8. $\forall x, y, z \quad x \leq y \wedge y \leq z \Rightarrow x \leq z.$

Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that lead to success, not the irrelevant steps.