FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE
Exercise 7: Probability theory and Bayesian networks – Problems   Winter Semester 2022/23
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1 Presented Problems

Problem 7.1:

Consider the following probabilistic inference problem with random variables for driver expertise, road conditions and accidents. The joint probability distribution is provided in the table below (values are guessed). The first variable $R \in \{dry, wet, snow/ice\}$ is a discrete random variable and represents the considered road conditions. The rest of the variables are Boolean: E is associated to the event that the driver is experienced or not and A to the event that an accident happens or not. The joint probability distribution is given by the table below.

E	A	$\mathbf{P}(R = dry, E, A)$	$\mathbf{P}(R = wet, E, A)$	$\mathbf{P}(R = snow/ice, E, A)$
t	t	0.0607	0.0449	0.0084
t	f	0.3605	?	0.0240
f	t	0.0851	0.0654	0.0152
f	f	0.1435	0.0400	0.0022

a. Calculate the value of the missing probability in the table.
b. What is the prior probability distribution of the random variables R and E?
c. What is the probability that the driver is not experienced given that there is an accident and the road is wet?
d. Construct a corresponding Bayesian network with the conditional probability tables (calculate the required table entries).

Problem 7.2:

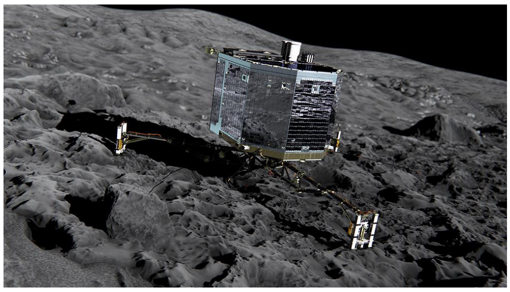
(Taken from [1] Ex. 13.21) Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable.

a. Is it possible to calculate the most likely color for the taxi?
b. What if you know that 9 out of 10 Athenian taxis are green?

2 Additional Problems

Problem 7.3:

Suppose that we send a space robot on a comet to collect and analyze samples from the terrain. Due to the uncertain environment we know that the probability that the robot finds a correct pose to extract useful samples is 45%. Fortunately, the robot has a measurement system that senses a correct pose for taking a sample with 80% of reliability, performing one measurement. This sensing system has the same reliability for the detection of an unacceptable pose (the probability values are guessed).



Artistic impression of a comet lander: the Rosetta's Philae lander. Source: ESA.

a. Consider that the robot is in some pose and takes a measurement with positive result. What is the probability that the robot is in a correct pose for collecting a sample?

b. Consider that the robot should take a sample if the probability that it is useful is at least 90%. Can the robot collect a sample if it takes another measurement and the result is again positive? Calculate the corresponding posterior probability.

Problem 7.4:

(Taken from [1] Ex. 13.3) For each of the following statements, either prove it is true or give a counterexample.

a. If $P(a|b,c) = P(b|a,c)$, then $P(a|c) = P(b|c)$.

b. If $P(a|b,c) = P(a)$, then $P(b|c) = P(b)$.

c. If $P(a|b) = P(a)$, then $P(a|b,c) = P(a|c)$.

References

[1] S. Russell and P. Norvig, *Artificial Intelligence: A Modern Approach*. Prentice Hall, 2010.