FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE
Exercise 6: Inference in First Order Logic – Problems Winter Semester 2022/23
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Problem 6.1: The man in the painting

(*The following puzzle appears in* **russel** *question 9.11.)* A man stands in front of a painting and says the following:

> Brothers and sisters have I none, but that man's father is my father's son.

What is the relationship between the man in the painting and the speaker? Use the predicates

$$\begin{array}{rcl} Male(x) & : & x \text{ is male.} \\ Father(x,y) & : & x \text{ is the father of } y. \\ Son(x,y) & : & x \text{ is a son of } y. \\ Parent(x,y) & : & x \text{ is a parent of } y. \\ Child(x,y) & : & x \text{ is a child of } y. \\ Sibling(x,y) & : & x \text{ is a sibling of } y \end{array}$$

and the knowledge

- A sibling is another child of one's parents.

$$\forall x, y \quad Sibling(x,y) \Leftrightarrow x \neq y \wedge \exists p \quad Parent(p,x) \wedge Parent(p,y)$$

- Parent and child are inverse relations.

$$\forall p, c \quad Parent(p,c) \Leftrightarrow Child(c,p)$$

to solve the riddle with first-order logic.

Problem 6.1.1: Define the rule which relates *son*, *child*, and *male*. Define also the rule which relates *father* and *parent*.

Problem 6.1.2: Using the constants Me for the speaker and $That$ for the person depicted in the painting, formalise the sentences regarding the sexes of the people in the puzzle.

Problem 6.1.3: Formalise the sentences "Brothers and sisters have I none" and "That man's father is my father's son" in first-order logic.

Problem 6.1.4: Solve this puzzle informally and decide what is the relation between the man in the painting and the speaker.

Problem 6.1.5: Using the resolution technique for first-order logic, prove your answer. You can use the two diagrams on the next page to structure your proof.

$\neg Son(That, Me)$

$\neg Parent(Me, That)$

$Son(F_1, F_2)$

$Parent(Me, That)$

Problem 6.2: Backward chaining

(*The following exercise is taken from* **russel** *Question 9.10.*) Suppose you are given the following axioms:

1. $0 \leq 3$
2. $7 \leq 9$
3. $\forall x \quad x \leq x$
4. $\forall x \quad x \leq x + 0$
5. $\forall x \quad x + 0 \leq x$
6. $\forall x, y \quad x + y \leq y + x$
7. $\forall w, x, y, z \quad w \leq y \land x \leq z \;\Rightarrow\; w + x \leq y + z$
8. $\forall x, y, z \quad x \leq y \land y \leq z \;\Rightarrow\; x \leq z.$

Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that lead to success, not the irrelevant steps.