NUMERICAL ANALYSIS

MAT 214

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Part2:

So the one million dollar question is given F and B, how do we find the matrix that converts the image pixels of F into C (that is aligned with B)?

Fortunately for us, there are some reference points (shown with red on both F and B, provided by some expert who knows what she's doing). These are the 3 points for which we know their correspondences.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Coordinates in image **B(x,y)**	Coordinates in image **F(x',y')**
[1,2]	[2,2]
[2,1]	[-1,4]
[3,1]	[-4,4]

Now, if you can calculate the matrix A:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

that would mean you'd know how to transform B into F (we want the opposite F->B). Consequently, if you calculate the **inverse of A**, you would be able to transform F and align it with B! **Your goal is to calculate** A^{-1} .

Express the problem as a system of linear equations where the unknowns are the coefficients of A. Calculate A, and then take its inverse by solving *AZ=I* . You are free to use any of the methods presented in class for solving the two systems of linear equations. Do not code anything, but please send a detailed and formal report about how you solved the problem.

Hint:

$$xa_{11} + ya_{12} + 1a_{13} + 0a_{22} + 0a_{23} = x'$$
$$0a_{11} + 0a_{12} + 0a_{13} + xa_{21} + ya_{22} = y'$$

Answer:

Formula : $Ax = b$ The matrix A is multiplied with X vector and B vector is creatived.

Values of Coordinates in image **B(x,y)** are placed to X vector and values of Coordinates in image **F(x',y')** are placed to B vector. Thus, given formula is implemented and polynomial equations are generated.

[x,y]= [1,2] [x',y']= [2,2]	[x,y]= [2,1] [x',y']= [-1,4]	[x,y]= [3,1] [x',y']= [-4,4]
$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix}$
$E_1: a_{11} + 2a_{12} + a_{13} = 2$ $E_4: a_{21} + 2a_{22} + a_{23} = 2$	$E_2: 2a_{11} + a_{12} + a_{13} = -1$ $E_5: 2a_{21} + a_{22} + a_{23} = 4$	$E_3: 3a_{11} + a_{12} + a_{13} = -4$ $E_6: 3a_{21} + a_{22} + a_{23} = 4$

Sets of equation are generated from 1. and 2. rows in the matrices. Later, Gauss Elimination Method is implemented to these sets of equations, and 1. and 2. values of rows of A matrix is obtained.

Gauss Elimination Method

$E_1: a_{11} + 2a_{12} + a_{13} = 2$ $E_2: 2a_{11} + a_{12} + a_{13} = -1$ $E_3: 3a_{11} + a_{12} + a_{13} = -4$	$E_4: a_{21} + 2a_{22} + a_{23} = 2$ $E_5: 2a_{21} + a_{22} + a_{23} = 4$ $E_6: 3a_{21} + a_{22} + a_{23} = 4$
$\mathrm{E_2 - 2E_1 \rightarrow E_2}$ $\mathrm{E_3 - 3E_1 \rightarrow E_3}$	$\mathrm{E_5 - 2E_4 \rightarrow E_4}$ $\mathrm{E_6 - 3E_4 \rightarrow E_6}$
$E_1: a_{11} + 2a_{12} + a_{13} = 2$ $E_2: 0 - 3a_{12} - a_{13} = -5$ $E_3: 0 - 5a_{12} - 2a_{13} = -10$	$E_4: a_{21} + 2a_{22} + a_{23} = 2$ $E_5: 0 - 3a_{22} - a_{23} = 0$ $E_6: 0 - 5a_{22} - 2a_{23} = -2$
$\mathrm{E_3 - \frac{5}{3}E_1 \rightarrow E_3}$	$\mathrm{E_6 - \frac{5}{3}E_4 \rightarrow E_6}$
$E_1: a_{11} + 2a_{12} + a_{13} = 2$ $E_2: 0 - 3a_{12} - a_{13} = -5$ $E_3: 0 - 0 - \frac{1}{3}a_{13} = -\frac{5}{3}$	$E_4: a_{21} + 2a_{22} + a_{23} = 2$ $E_5: 0 - 3a_{22} - a_{23} = 0$ $E_6: 0 - 0 - \frac{1}{3}a_{23} = -2$
$a_{11} = -3$ $a_{12} = 0$ $a_{13} = 5$	$a_{21} = 0$ $a_{22} = -2$ $a_{23} = 6$

Inverse of the matrix A is calculated with determinant and cofactor methods.
The resulting inverse of matrix A helps us to transform F to align with B.

Values of obtained the matrix A	**Inverse of obtained the matrix A**
$A = \begin{bmatrix} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$	$A^{-1} \begin{bmatrix} -\frac{1}{3} & 0 & \frac{5}{3} \\ 0 & -\frac{1}{2} & 3 \\ 0 & 0 & 1 \end{bmatrix}$