NUMERICAL ANALYSIS

MAT 214

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QUESTION 2) Solve exercise 5 of section 2.2 and calculate the theoretical number of iterations required according to Corollary 2.5.

Section 2.2 / 5. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

ANSWER 2)

$x^4 - 3x^2 - 3 = 0$ on $[1, 2]$ $\quad p_0 = 1 \quad \varepsilon = 10^{-2}$

Formula: $|g'(x)| \le k$, for all (a, b)

$p_n = g(p_{n-1}) \quad n \ge 1$

$|p_n - p| \le \frac{k^n}{1-k} |p_0 - p_1|$

$f(x) = x^4 - 3x^2 - 3 = 0$

$x^4 = 3x^2 + 3$

$g(x) = x = \sqrt[4]{3x^2 + 3} \qquad g'(x) = \frac{1}{4} * (3x^2 + 3)^{-\frac{3}{4}} * 6x$

$g'(1) = \frac{1}{4} * (3 * 1 + 3)^{-\frac{3}{4}} * 6 \qquad \Leftrightarrow \qquad g'(2) = \frac{1}{4} * (3 * 4 + 3)^{-\frac{3}{4}} * 12$

$g'(1) = 0.3912 \qquad g'(2) = 0.3936 \cong 0.394$

$p_0 = 1$

$g(1) = \sqrt[4]{3 * 1 + 3} \Rightarrow p_1 = 1.565$

$k = 0.394$

$|p_n - p| = 10^{-2} \quad \Rightarrow \quad 10^{-2} \le \frac{(0.394)^n}{1-0.394} |1 - 1.565|$

$\Rightarrow \frac{10^{-2} * 0.606}{0.565} < (0.394)^n$

$\frac{10^{-2} * 0.606}{0.565} \cong 0.01073$

$\Rightarrow \log(0.01073) < \log(0.394)^n$

$\Rightarrow \log(0.01073) < n * \log(0.394)$

$$=> \frac{\log(0.01073)}{\log(0.394)} < n$$

$$4.868 < n$$

$n = 5$ It is theoretically expected to have a root in step 5.

Pratically :

$$g(x) = \sqrt[4]{3x^2 + 3} \quad => \quad p_n = g(p_{n-1}) \quad n \geq 1 \quad p_0 = 1 \quad \varepsilon = 10^{-2}$$

$$\boldsymbol{p_1} = g(p_0) = \sqrt[4]{3 * (p_0)^2 + 3} \quad => \quad p_1 = 1.56508458 \cong 1.57$$

$$\boldsymbol{p_2} = g(p_1) = \sqrt[4]{3 * (p_1)^2 + 3} \quad => \quad p_2 = 1.79357288 \cong 1.79$$

$$\boldsymbol{p_3} = g(p_2) = \sqrt[4]{3 * (p_2)^2 + 3} \quad => \quad p_3 = 1.88594374 \cong 1.89$$

$$\boldsymbol{p_4} = g(p_3) = \sqrt[4]{3 * (p_3)^2 + 3} \quad => \quad p_4 = 1.92284784 \cong 1.92$$

$$\boldsymbol{p_5} = g(p_4) = \sqrt[4]{3 * (p_4)^2 + 3} \quad => \quad p_5 = 1.93750754 \cong 1.94$$

$$\boldsymbol{p_6} = g(p_5) = \sqrt[4]{3 * (p_5)^2 + 3} \quad => \quad p_6 = 1.94331693 \cong 1.94$$

$$\boldsymbol{p_7} = g(p_6) = \sqrt[4]{3 * (p_6)^2 + 3} \quad => \quad p_7 = 1.94561686 \cong 1.95$$

$|x_6 - x_5| < \varepsilon$ => There is root

$|1.94 - 1.94| < \varepsilon$

$= 0.00 < 0.01$ There is a root in step 6 and it is 1.9433.

QUESTION 3) Solve exercises 4 and 5 of section 2.3.

Section 2.3 / 4. Let $f(x) = -x^3 - \cos x$. With $p_0 = -1$ and $p_1 = 0$, find p_3.

a. Use the Secant method. **b.** Use the method of False Position.

Section 2.3 / 5. Use Newton's method to find solutions accurate to within 10^{-4} for the following problems.

a. $x^3 - 2x^2 - 5 = 0$, $[1, 4]$ **b.** $x^3 + 3x^2 - 1 = 0$, $[-3, -2]$

c. $x - \cos x = 0$, $[0, \pi/2]$ **d.** $x - 0.8 - 0.2 \sin x = 0$, $[0, \pi/2]$

ANSWER 3)

$f(x) = -x^3 - \cos x \qquad p_0 = -1 \qquad p_1 = 0 \quad p_3 = ?$

4.a.) The secant method

Formula: $$p_n = p_{n-1} - \frac{f(p_{n-1}) * (p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}$$

$f(x) = -x^3 - cos(x)$ $\qquad \frac{\pi}{2} \quad 90^0 \qquad \underline{-1 \quad \theta} \qquad \underline{\theta = -57.30}$

$f(p_0) = -(-1)^3 - cos(-57.30) = 0.459759$

$f(p_1) = -(-0)^3 - \cos(0) = 0 - 1 = -1$

$$p_2 = p_1 - \frac{f(p_1) * (p_1 - p_0)}{f(p_1) - f(p_0)} = 0 - \frac{(-1) * (0 - (-1))}{-1 - 0.459759} = -0.685044$$

$\frac{\pi}{2} \quad 90^0 \qquad \underline{-0.685044 \quad \theta} \qquad \underline{\theta = -39.25}$

$f(p_2) = -(-0.685044)^3 - \cos(-39.25) = -0.452911$

$$p_3 = p_2 - \frac{f(p_2) * (p_2 - p_1)}{f(p_2) - f(p_1)} = -0.685044 - \frac{(-0.452911) * (-0.685044 - 0)}{-0.452911 - (-1)} =$$

$p_3 = -1.252161$

4.b.) The method of False Position

Formula: $$p_n = p_{n-1} - \frac{f(p_{n-1}) * (p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}$$

if $(f(p_n) * f(p_{n-2}) < 0) \qquad p_{n-1} = p_n$, else $p_{n-2} = p_n$

$f(x) = -x^3 - cos(x)$ $\qquad \frac{\pi}{2} \quad 90^0 \qquad \underline{-1 \quad \theta} \qquad \underline{\theta = -57.30}$

$f(p_0) = -(-1)^3 - cos(-57.30) = 0.459759$

$f(p_1) = -(-0)^3 - \cos(0) = 0 - 1 = -1$

$$p_2 = p_1 - \frac{f(p_1) * (p_1 - p_0)}{f(p_1) - f(p_0)} = 0 - \frac{(-1) * (0-(-1))}{-1 - 0.459759} = -0.685044$$

$$\frac{\pi}{2} \qquad 90^0 \qquad \underline{\theta = -39.25}$$

$$\underline{-0.685044 \qquad \theta}$$

$$f(p_2) = -(-0.685044)^3 - \cos(-39.25) = -0.452911$$

⇨ $\big(f(p_0) * f(p_2)\big) = \big(0.459759 * (-0.452911)\big) < 0 \quad Thus, \qquad p_1 = p_2$

$$p_3 = p_2 - \frac{f(p_2) * (p_2 - p_0)}{f(p_2) - f(p_0)} = -0.685044 - \frac{(-0.452911) * (-0.685044 - (-1)}{-0.452911 - 0.459759} =$$

$$p_3 = -0.841342$$

5. a.) $x^3 - 2x^2 - 5 = 0$, [1, 4] $\quad p_0 = 2 \quad \varepsilon = 10^{-4}$

Formula : $$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

$$f'(x) = 3x^2 - 4x$$

$$f(x) = x^3 - 2x^2 - 5$$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} => 2 - \frac{(-5)}{4} = 2 + \frac{5}{4} = \frac{13}{4} => p_1 = 3.25$$

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} => 3.25 - \frac{(3.25)^3 - 2(3.25)^2 - 5}{3(3.25)^2 - 4(3.25)} => p_2 = 2.8110$$

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} => 2.8110 - \frac{(2.811)^3 - 2(2.811)^2 - 5}{3(2.811)^2 - 4(2.811)} => p_3 = 2.6980$$

$$p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} => 2.6980 - \frac{(2.698)^3 - 2(2.698)^2 - 5}{3(2.698)^2 - 4(2.698)} => p_4 = 2.6907$$

$$p_5 = p_4 - \frac{f(p_4)}{f'(p_4)} => 2.6907 - \frac{(2.6907)^3 - 2(2.6907)^2 - 5}{3(2.6907)^2 - 4(2.6907)} => p_5 = 2.69065$$

$|p_n - p_{n-1}| < \varepsilon \rightarrow$ There is root

$|p_5 - p_4| < \varepsilon \rightarrow$ ROOT $\quad 2.69065 \approx 2.6907$

$0.0000 < 0.0001$

5. b.) $x^3 + 3x^2 - 1 = 0, [-3,-2]$ $p_0 = -3$ $\varepsilon = 10^{-4}$

Formula : $$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

$f'(x) = 3x^2 + 6x$

$f(x) = x^3 + 3x^2 - 1$

$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} \Rightarrow -3 - \frac{(-1)}{9} = -3 + \frac{1}{9} = -\frac{26}{9} \Rightarrow p_1 = -2.8889$

$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \Rightarrow -2.8889 - \frac{(-2.8889)^3 + 3(-2.8889)^2 - 1}{3(-2.8889)^2 + 6(-2.8889)} \Rightarrow p_2 = 2.8140$

$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \Rightarrow 2.8140 - \frac{(2.8140)^3 + 3(2.8140)^2 - 1}{3(2.8140)^2 + 6(2.8140)} \Rightarrow p_3 = -2.8794$

$p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} \Rightarrow -2.8794 - \frac{(-2.8794)^3 + 3(-2.8794)^2 - 1}{3(-2.8794)^2 + 6(-2.8794)} \Rightarrow p_4 = -2.8794$

$|p_n - p_{n-1}| < \varepsilon \rightarrow$ There is root

$|p_4 - p_3| < \varepsilon \rightarrow$ ROOT $-2.87939 \approx -2.8794$

$0.0000 < 0.0001$

5.c.) $x - \cos x = 0, [0, \pi/2]$ $p_0 = 0$ $\varepsilon = 10^{-4}$

Formula : $$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

$f(x) = x - \cos(x)$

$f'(x) = 1 + \sin(x)$

$f(p_0) = 0 - \cos(0) = -1$

$f'(p_0) = 1 + \sin(0) = 1$

$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} \Rightarrow 0 - \frac{(-1)}{1} \Rightarrow p_1 = 1$

$\frac{\pi}{2}$ 90^0 $\underline{\theta = 57.29}$

$\underline{1 \qquad \theta}$

$f(p_1) = 1 - cos(57.29) = 0.45961$

$f'(\mathrm{p}_1) = 1 + \sin(57.29) = 1.84141$

$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} => 1 - \frac{0.45961}{1.84141} => p_2 = 0.75040$

$\frac{\pi}{2}$ 90^0 $\underline{\theta = 42.99}$

$\underline{0.75040 \quad \theta}$

$f(p_2) = 0.75040 - cos(42.99) = 0.01892$

$f'(p_2) = 1 + sin(42.99) = 1.68187$

$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} => 0.75040 - \frac{0.01892}{1.68187} => p_3 = 0.73915$

$\frac{\pi}{2}$ 90^0 $\underline{\theta = 42.35}$

0.73915 θ

$f(p_3) = 0.73915 - cos(42.35) = 0.00011$

$f'(p_3) = 1 + sin(42.35) = 1.67366$

$p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} => 0.73915 - \frac{0.00011}{1.67366} => p_4 = 0.73908$

$|p_n - p_{n-1}| < \varepsilon \rightarrow$ There is root
$|p_4 - p_3| < \varepsilon \rightarrow$ ROOT 0.73908
0.00007 $<$ 0.0001

5. d.) $x - 0.8 - 0.2 \sin x = 0$, $[0, \pi/2]$ $p_0 = 0$ $\varepsilon = 10^{-4}$

Formula : $$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

$f(x) = x - 0.8 - 0.2\, sin(x)$

$f'(x) = 1 - 0.2\, cos(x)$

$f(p_0) = 0 - 0.8 - 0.2\, sin(0) = -0.8$

$f'(p_0) = 1 - 0.2\,cos(0) = 0.8$

$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} => 0 - \frac{(-0.8)}{0.8} => p_1 = 1$

$$\begin{array}{ll} \frac{\pi}{2} & 90^0 \\ \underline{1} & \underline{\theta} \end{array} \qquad \underline{\theta = 57.29}$$

$f(p_1) = 1 - 0.8 - 0.2\,sin(57.29) = 0.03172$

$f'(p_1) = 1 - 0.2\,cos(57.29) = 0.89192$

$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} => 1 - \frac{0.03172}{0.89192} => p_2 = 0.96443$

$$\begin{array}{ll} \frac{\pi}{2} & 90^0 \\ \underline{0.96443} & \underline{\theta} \end{array} \qquad \underline{\theta = 56.26}$$

$f(p_2) = 0.96443 - 0.8 - 0.2\,sin(56.26) = 0.00008$

$f'(p_2) = 1 - 0.2\,cos(56.26) = 0.88602$

$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} => 0.96443 - \frac{0.00008}{0.88602} => p_3 = 0.96434$

$|p_n - p_{n-1}| < \varepsilon \rightarrow$ There is root
$|p_3 - p_2| < \varepsilon \rightarrow$ ROOT 0.96434
$0.00009 < 0.0001$