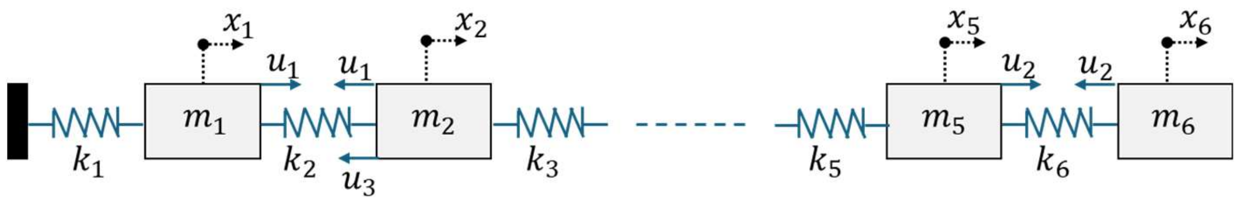
# FALL 2025 - EE 571 FINAL EXAM

Consider the spring-mass system below. The system involves a total of 6 individual rigid bodies connected with linear springs. The system has three inputs: two of them are related to tension on the first and last springs, and another one is a force applied on the second body. We will design a regulator for this system.




The equations of motion for this system would be as follows:

$$
\begin{aligned}
m_1\ddot{x}_1 &= -2x_1 + x_2 + u_1 \\
m_1\ddot{x}_2 &= x_1 - 2x_2 + x_3 - u_1 - u_3 \\
&\vdots \\
m_1\ddot{x}_5 &= x_4 - 2x_5 + x_6 + u_2 \\
m_1\ddot{x}_6 &= -x_5 - x_6 - u_2
\end{aligned}
$$

One state-space realization for this system is given in the preparation code “prep_final.m”. This realization assumes that only a single output is measured (the displacement of the first body). Suppose that the controller will be implemented on the discrete model of the system with T=0.01 sec. The prep code has the discrete model and a simulation of the system.

Please answer the following questions:

1) Note that the discrete system is not observable. Please obtain the Kalman decomposition of this and find the observable/unobservable modes.
2) Adding another sensor at the 6th body will fix the observation problem. Suppose the system now has the C matrix below accordingly. Please design an observer for this system and simulate the comparison of the observer's estimations against the real states. You can augment the simulation code snippet in the prep code. The initial conditions for the states in the estimated and actual systems are given below.

$$C = \begin{bmatrix} 1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0 \end{bmatrix}$$
$$x_0 = [0\,0\,0\,1\,1\,1\,0\,0\,0\,0\,0\,0]$$
$$\hat{x}_0 = [0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0]$$

3) Consider the observable system in (2). Please design an LQR based on the nominal cost function given below and simulate its performance. Note that this regulator will use the estimated states.

$$J = \sum u^T u + y_1^2 + y_6^2, \text{ where } u = [u_1 \;\; u_2 \;\; u_3]$$

4) Suppose the third input is gone. Please design a new LQR for the system in (3) with the same nominal cost function and simulate its performance. Compare the performance of this controller against the one in (3) in terms of the total cost incurred and the maximum input required.

5) Consider the system with three inputs. This time suppose each of the actuators and the sensors has an uncertainty as described below. Please design a linear quadratic estimator (Kalman filter) to estimate the states. Simulate the observer using the actual and estimated initial conditions given in (2). Compare the real and estimated states.

$$x_{k+1} = A_d x_k + B_d u_k + B_d w_k$$
$$y_k = C_d x_k + v_k$$

where $v \sim N(0, 0.1 I_p)$ and $w \sim N(0, 0.05 I_m)$
where p is the number of outputs and m is the number of inputs

6) Use the LQR design in (3) on the system developed in (5). This operation combines LQR with the estimated states obtained via Kalman filter. Simulate the system and compare the outputs against those obtained in (3).

7) Consider the closed-loop system in (6). Suppose we added more sensors to the setup. Please simulate the performance of the system when the C matrix becomes as the ones below. Note that these additional sensors will have the same uncertainty level as others. Does having more sensors help with the estimation and/or regulation?

Case 1:

$$C = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix}$$

Case 2:

$$C = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix}$$

<u>Grading Policy:</u>

*Part 1 to 6*: 15 points

*Part 7*: 10 points

**Submission Instructions:**

- Please explain your solutions step-by-step and submit your report in pdf format. The report should include all the necessary plots and should be self-sufficient (I should not need to run your code to understand the results).
- Please submit your codes packed in a zipped folder. The code for each part should have its own file and should be named clearly. Your code may use some external code files. Please include them in the submission folder. Your code should be able to run when called within that folder.
- In summary, we expect one report (in pdf) and a zipped folder that contains the code files.