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1 INTRODUCTION

Using the features of the quantum computer, we will learn to perform operations with qubits, albeit in limited numbers, and with this we will better understand some of the interesting features of quantum mechanics. We will do this by first designing a coin toss game with one quantum bit, then perform an experiment with another qubit on quantum numbers and finally entangling 2 different qubits and super positioning them.

2 QUANTUM COIN GAME

According to the rules of the quantum coin game, a computer and a human make moves, respectively, on a coin whose initial condition is heads. The computer makes the first move, then the human and finally the computer, and after these moves, the state of the "coin" is measured. The computer wins in the case of heads and human wins in case of tails.

What makes this coin toss game different from the classical coin toss game is that the state of the coin can enter superposition as well as being just heads or tails.

While the human can only invert the state of the cain or leave it alone, the computer can put the coin in a superposition state with Hadamard product. This makes it such that the computer has an overall advantage over humans and the game generally results with the computer winning.

We can express some concepts of the game mathematically as follows.

$$|1\rangle = \begin{bmatrix}1\\0\end{bmatrix} \; and \; |0\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

where the states 1 and 0 represent heads and tails, respectively.

$$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \; (Invert\ Product)$$
$$I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \; (Identity\ Product)$$
$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} \; (Hadamard\ Product)$$

where X stands for flipping the coin, I for skipping turn, and H for putting the coin in a superposition with equal probability.

The analysis of the state of the coin as a result of the moves made by the computer and the human is as follows, respectively.

1) The computer puts the coin in a superposition on its move with Hadamard product.

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$$

2) The person flips the coin or skips his move, but in both cases since the both states of the coin have the same probility, moves made by human don't really affect the coin in superposition.

$$\frac{1}{\sqrt{2}}X(|1\rangle + |0\rangle) = \frac{1}{\sqrt{2}}I(|1\rangle + |0\rangle) = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$$

3) The computer uses the Hadamard product again, causing the coin's state to collapse into being heads.

$$\frac{1}{\sqrt{2}}H(|1\rangle + |0\rangle) = |1\rangle$$

When the final result is measured, it will be seen that the coin is heads in every theoretical case. However, this result may not be absolute in practise, since the quantum computer isn't ideal, the coin may be more prone to a state when it is in its superposition. Although this is a small possibility, it allows a person to win.

3 Stern Gerlach Experiment

The spin quantum number, which is one of the four quantum numbers, was not known to be a discrete number at first, and it was assumed that a continuous value could be obtained by a measurement. However, the idea proposed by the quantum theory that some properties of particles are quantized has aroused the thought that this spin number may be discrete instead of continuous. The Stern Gerlach experiment shows that electrons' spin value can only take 2 values, either up or down.

The aim of the experiment is that as a result of unpolarized neutral silver particles passing through a certain magnetic field, the electrons in last orbitals obtain a certain spin, change their path accordingly and create a general pattern on the screens they reach.

An image of two straight lines that are separated from each other forms as a result of this experiment, and it shows that there are only 2 states of electron spin either up or down, nothing in between.

Although the experiment itself is carried out with the help of neutral silver particles and a certain magnetic field, the experiment itself can be simulated with a quantum computer since the working mechanism of the quantum computer also makes use of magnetic fields to measure the state of a qubit.

By super positioning a qubit, we can assign a random value to the electron's direction of spin. In this state, the qubit's spin may have any value and we can't really make an absolute prediction of its state. Then, if we measure the state of this qubit with quantum computer, that is, if we apply a magnetic field to qubit like in the experiment, we can reach a similar conclusion as told above, depending on whether the measured state of the qubit is discrete or continuous.

As a result of measuring 8192 particles sequentially, it is seen that 4052 of them are in the upward direction and 4150 of them are in the downward direction. It is noticed from this that the spin number of the electron can only be up or down. Of course, in this experiment, it is seen that the electron can be up or down with equal probability, but if we change the phase of this electron in superposition, we can change these probabilities. For instance, if we pass the sample qubit through the T gate, the probability changes to 85% and 15%.

4 BELL STATE

Bell State refers to the situation when two or more qubits are placed in a quantum entanglement state. In this case, the quantum state of the qubits can be expressed as a superposition, meaning linear combination of all states that can occur with n classical bits.

But it is important to realize that when there are n bits, there are 2^n possible outcomes, and all of these outcomes form the basis of the quantum state in the superposition. In order to express this quantum state, it is necessary to know as many coefficients as the number of basis vectors.

$$|v\rangle = a_1|b_1\rangle + a_2|b_2\rangle + \cdots + a_m|b_m\rangle, \qquad m = 2^n$$

It can be understood that while classical n bits can store n bits of information, n qubit can store up to 2^n bits of information.

For the case when we have 2 qubits, if we superposition one of the qubits and pass the other one through the controlled not gate, a quantum state with the superposition of $|00\rangle$ and $|01\rangle$ forms. When we replace the controlled not gate with the swap gate, a quantum state with the superposition of $|00\rangle$ and $|10\rangle$ occurs.

As a result of the experiment with the control gate, 4101 out of 8192 cases resulted in 00, 4091 out of 8192 resulted in 11. As a result, qubits can take advantage of their superposition ability to store more information than is possible from normal bits as the number of qubits increase.

Although these properties of the qubit make it seem more effective than the classical bits, it becomes more difficult to entangle the qubits with the increasing number, and operations with qubits may take longer time. However, the ability to superposition multiple operations and solve them at the same time and return only the important results, makes the qubit interesting.