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On the Analysis of a Complex Differential Game Using
Artificial Intelligence Techniques

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Abstract

The investigation is motivated by the dynamic conflict in an air-to-air combat between two aggressive aircraft, both equipped with medium-range guided missiles. This conflict can be viewed as an interaction of a two-target differential game (between the aircraft) with two independent missile-aircraft pursuit-evasion games. The information structure is, however, rather intricate: though perfect information can be assumed between the two aircraft, the missiles have a limited detection range, beyond which information has to be forwarded by the launching aircraft. Moreover, missile firing is assumed to be non-detectable. Problems of such complexity do not fit in the frame of classical differential game theory. The paper describes the analysis of the conflict by means of an Expert System with a "knowledge base" incorporating differential game solution elements. The system simultaneously evaluates potential success with the respective risks and advises the pilot when to fire his missile and when to start an evasive maneuver.

1. Introduction

Air-to-air combat is a most complex dynamic conflict situation depending on force ratios, aircraft performance and weapon system effectiveness. The inherent complexity of multiple aircraft scenarios can be relaxed by observing that the major events of an air-to-air combat (namely eventual aircraft destruction) are direct consequences of relatively brief duels between two opponents. This observation suggests considering the 1x1 air-to-air engagement as a building block of the more complex scenarios. Successful analysis of such an air combat duel is undoubtedly a prerequisite for further investigation. For these reasons the present paper concentrates on the analysis of air-to-air combat between two aggressively operating aircraft, both equipped with similar guided missiles. The objective of each pilot in such a duel is to shoot down the opponent aircraft without being hit.

The conflicting nature of the scenario calls for a non-cooperative differential game formulation [1]. Apparently such a dynamic conflict can be viewed as an interactive combination of a "two-target differential game" between the aircraft and two independent missile-aircraft "pursuit-evasion games". The target sets in the two-target

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game are the respective missile firing zones, each of which is the "capture zone" of a pursuit-evasion game of kind [2]. The encounter between the two aircraft (Blue and Red) exhibits a "threat reciprocity" [3] and it will terminate in one of the following outcomes:

a., Red alone is shot down = Blue wins
b., Blue alone is shot down = Red wins
c., Both are shot down = mutual kill
d., Both survive = draw

Following this description the solution of the relevant two-target differential game with given target sets is the decomposition of the set of admissible initial conditions into the respective zones of fixed outcome. Inside each of the two winning zones the termination of the game in some finite time is guaranteed.

If the engagement starts in the "mutual kill" zone both players have to be aggressive otherwise the state of the game may slip to the opponent's winning zone. However, cooperative strategies can drive the state of the game to the "draw" zone. In the "draw" zone each player can guarantee his own survival against any action of the opponent, but cooperative aggressive strategies can lead to "mutual kill".

In future air combat most engagements will start at rather long ranges, thus the initial conditions of the above described two-target game are in the "draw" zone. Therefore, the only guaranteed outcome of the non-cooperative game is a "draw". Such a result is unacceptable from an operational point of view, because it denies the very essence of air-to-air combat. At the other end, cooperative strategies are also inadmissible in a hostile environment.

The inherent non-cooperative nature of the scenario requires each player to determine his "preference ordering" [4] between "mutual kill" and "draw" and to act accordingly. Since such "preference ordering" is one of the elements of each player's strategy, it cannot be assumed to be known by the opponent. This inherent uncertainty implies, as pointed out recently [5], a major difficulty in the proper mathematical formulation and consequently in a meaningful analysis of future air combat.

Nevertheless, this challenging problem has attracted substantial research interest [6-9], without yielding yet any satisfying solution. Moreover, the interpretation of the results of [9] indicated a conceptual incompleteness of the two-target game formulation. Inside the Blue winning zone Red has no interest in playing defensively. Though he cannot force the game to his own target set (the no-escape envelope of his missile), he can still fire a missile within its classical (kinematic) firing envelope. Such a firing disrupts the original two-target game by starting an unexpected pursuit-evasion game between the Red missile and Blue aircraft. In order to survive, Blue

must take an evasive action. It leads to his successfull escape from the missile, but prevents him from reaching an effective firing opportunity, i.e., the win guaranteed by the two-target game solution. The above interpretation emphasizes the difficulty to model future air combat [5], within the classical differential game theory frame.

The objective of the present paper is to propose a new approach for the analysis of medium-range aggressive air-to-air engagements by using Artificial Intelligence (AI) techniques. The analysis is performed by an Expert System with a "knowledge base" composed of differential game solution elements.

In the next section the medium-range air combat duel with guided missiles is formulated, a simplified model for an exploratory study is outlined and the requirements to be satisfied by the Expert System are stated. It is followed by a description of the structure and algorithms used to satisfy the above requirements. Finally, some preliminary results are presented.

2. Problem Statement

The air combat duel of interest is between two supersonic fighters (Blue and Red) of similar performance and it is assumed to start at the limit of airborne radar detection range. Both aircraft are assumed to be equipped with a small number (2-4) of similar advanced medium-range air-to-air missiles (AMRAAM).

An AMRAAM type missile has two different guidance modes. After being fired it uses inertial midcourse guidance towards the predicted position of the target. For this purpose both the missile and its target have to be kept in the information cone of the launching aircraft. At some shorter range the missile's active radar seeker is turned on, it searches and "locks-on" the target. These missiles are fired beyond visual range and therefore the firing event is generally undetected by the target. Both aircraft may use radar and are also equipped with a passive warning system that detects missile "lock-on".

The entire duel engagement can be regarded as a set of missile-firing-exchange sequences composed of the following phases: target detection, pre-launch maneuver, missile deployment (launch), post-launch maneuver (until missile lock-on), eventual disengagement for missile avoidance, eventual reengagement.

The objective of each pilot (as stated in the Introduction) is to shoot down the opponent with certainty $(P_k=1)$ while guaranteeing own survival $(P_s=1)$. Generally this ideal objective cannot be achieved. The reason for this result is the very existence of a "draw" zone, where each player can guarantee his survival $(P_s=1)$. By using practical air combat terms one can say that unless a missile is fired inside its "no-escape" envelope it can be avoided if the target performs a proper disengagement maneuver. Generally, in order to reach a "no-escape" firing opportunity, own survival

has to be put at risk. The need for a post-launch maneuver (a consequence of AMRAAM properties) is an additional constraint.

For these reasons the objective is redefined using a more flexible formulation. It is stated (for Blue) as: "Maximize $P_k(R)$ - the probability of shooting down (Red) - while guaranteeing that the probability of own survival satisfies $P_s(B) \geq \bar{P}_s(B)$, where $\bar{P}_s(B) \leq 1$ is prescribed".

Note, that the value of $0 < \bar{P}_s < 1$ can be used as a measure of relative preference ordering. $\bar{P}_s = 1$ indicates a clear preference of "draw", while $\bar{P}_s = 0$ stands for preferring a "mutual kill".

Clearly, the optimization process yields

$$P_k^*(R) = \max_B P_k(R) = \Phi[\bar{P}_s(B)] \qquad (1)$$

where $\Phi[\cdot]$ is a monotonically decreasing function. Such formulation defines a game which (in game theory terms) is always "strongly playable".

The control set at the disposal of each pilot are of two types: (i) the actual control variables for aircraft maneuver; (ii) a set of decisions: when and whether to perform a transition from one phase to another. The most critical decision is the proper timing of missile firing.

In future air combat the above formulated complex differential game, has to be solved in real time. It is therefore required to design for the Blue aircraft a system which can aid the pilot in selecting the controls at his disposal in order to achieve the best outcome.

In this paper, a prototype "Pilot Advisory System" (PADS) is proposed for performing the task described above. PADS integrates conventional quantitative methods (such as solutions of differential sub-games) with rule based, AI type techniques of qualitative reasoning (e.g., selecting a differential sub-game, classifying opponents maneuver, etc.). For the sake of simplicity needed in an exploratory investigation, the prototype system uses a set of assumptions, which preserves the essential elements of the missile firing sequences, but decouples them from secondary effects of detailed aircraft dynamics. A recent simulation study [10] has demonstrated that such simplification is very important for gaining insight into the problem. The assumptions used in the prototype design are:

1. The engagement takes place in a horizontal plane of given altitude (h) (see Fig. 1).
2. Aircraft models are characterized by: constant speed (V), maximum load factor (n_{max}), maximum radar detection range(r_d), maximum radar look angle $(\Delta\lambda)$.
3. Both aircraft are equipped with identical missiles characterized by: thrust profile, aerodynamic drag model, PN guidance law, minimum turning radius, fixed range for active seeker "lock-on" (r_a).
4. A missile scores a hit if it can reach the vicinity of its target satisfying a set of inequalities concerning the time of flight, the final speed and the final closing rate.

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5. The information available for both aircraft is the following: perfect knowledge of own and opponent parameters, perfect measurements by radar, passive detection of radar illumination including missile lock-on.
6. The inherent uncertainties of the engagement are results of undetectable missile firings and unknown preference ordering of the opponent.
7. The Red aircraft has a conventional firing control system which computes own missile envelopes, but <u>does not have a PADS</u>. Consequently, Red will decide on missile firing by using some heuristic doctrine and will always start a disengagement maneuver if missile lock-on is detected.

Note, that based on assumptions 1-4 the optimal post-launch and disengagement (missile avoidance) maneuvers are uniquely determined (see Fig. 2).

In view of this last remark, the required functions of PADS, in a medium-range duel are to advise optimal prelaunch maneuver, missile firing, start of disengagement as well as to display estimated $P_k(R)$ for any given value of $\bar{P}_s(B) \leq 1$. The implementation of the design concept for PADS is based upon differential sub-games which are numerically solvable.

3. PADS - The Implementation Concept

3.1 The differential sub-games

A differential sub-game notion refers to a game of kind which is an element of the complete air combat game, as defined in section 2. Each sub-game is defined by a set of assumptions entered in the PADS. The barrier of such a sub-game defines a critical point on the trajectory. The most relevant subgames used in this work are described in the sequel.

The most important type refers to the NO-ESCAPE barrier denoted by NE¦X→Y. (The first argument indicates for whom the computation is carried out and the arrow indicates who fires on whom.) This is the barrier of the capture zone in a pursuit-evasion game between the <u>missile</u> fired by X and the <u>aircraft</u> Y.

Other subgames are defined by assumptions made on own and opponent behavior. The SHOOT NO-ESCAPE barrier SNE¦X→Y is based on the assumption of mutual (AMRAAM type) missile firing. It is further assumed that both aircraft perform a post-launch maneuver followed by an optimal evasion. In this sub-game X and Y do not behave similarly. X wishes to hit Y, ready to take any risk $[\bar{P}_s(X)=0)]$ and therefore guides his missile until lock-on. Y, on the other hand, wishes to survive $[\bar{P}_s(Y)=1]$, and starts an evasive maneuver when opponent missile lock-on is detected. Y's objective is to maximize the time of reaching the barrier. Since firing after this point provides a high probability to hit an aggressive opponent, it can be called the first "effective firing opportunity".

The NO-ESCAPE SHOOT barrier NES¦X←Y can be defined similarly to SNE¦X→Y, except that now both X and Y wish to destroy each other

and take the risk of $(\bar{P}_s(X)=\bar{P}_s(Y)=0)$. Thus Y also starts his evasive maneuver only after his missile lock-on. Moreover, his objective is to minimize the time of reaching the barrier. If Blue does not pass the point on the NES¦B←R barrier he can launch and guide his missile until it locks on without being hit. This point gives the last moment for a "safe and completed firing sequence".

On the LINE NO ESCAPE (LNE¦X→Y) sub-game barrier it is assumed that Y continues to fly in a straight line and will start his evasive maneuver only when X's missile lock-on is detected.

The last sub-game relates to the R-MAX barrier RMAX¦X→Y, which is defined by the assumption that Y flies in a straight line (does not perform an avoidance maneuver) even after missile lock-on.

All the above subgames are solved by simulations of the missile-aircraft encounter. The final miss distance r_f, satisfying the stopping conditions of the simulation, can be found from any initial condition and assumed aircraft behavior. The critical point on the barrier is that initial condition for which $r_f=d$, where "d" is the missile lethal range. Such a critical point can be found by searching for the zero of an implicit function while maximizing the required range. In the present work the "changing secant" method is used for finding the zero of the function and the maximization is done by a search around the expected solution. Only one iteration is performed each computer cycle, taking into account the real-time environment and assuming that the solution doesn't change much from cycle to cycle.

3.2 The target behavior model

In the introduction the difficulty to formulate (and solve) a complete air combat game was outlined. In order to allow solutions other than the "guaranteed draw" one has to assume non-optimal opponent behavior and play an appropriate reprisal strategy [11]. Since optimal post-launch and disengagement are uniquely defined, PADS needs a probabilistic model only for Red's launching moment. This behavior model eventually enables PADS to estimate $P_k(R)$ and $P_s(B)$. In generating such a behavior model it is assumed that Red can calculate the critical points of the following barriers: RMAX¦R→B, LNE¦R→B, NE¦R→B, NE¦R←B.

It can be safely assumed that Red will not fire before RMAX¦R→B is reached. Moreover, one can assume that firing after NE¦R→B is equally unreasonable, because it involves a trivially unnecessary risk. This is particularly true if [NE¦R←B]>[NE¦R→B]. Since missiles fired between RMAX and LNE are not effective if the opponent has a passive warning system, only a small percentage of the missiles are expected to be fired in this interval.

There may exist a set of several possible behavior models, from which one must be chosen. This choice can be based upon pilot intuition and previous knowledge about the opponent. Red's expected launching time

depends on several factors, some of which are deterministic and can be assumed known to Blue (e.g., aircraft and weapon characteristics), and some are unknown and must be "guessed" by Blue. The latter include: Red's preference ordering $(\bar{P}_s(R))$; Red's degree of sophistication (PADS-like equipment) and skill; Red's knowledge of Blue's parameters. These factors may be estimated to some degree by Blue observing Red's pre-launch behavior, and matching it to expected behavior patterns. Other information (such as intelligence from the ground) may be used as well. This estimation can be qualitatively performed using a ruled based Expert System.

3.3 PADS - Combat Analysis

PADS plans future actions based upon current data, including detected events concerning the opponent. PADS plans the prelaunch maneuver, the times of firing and of the eventual disengagement according to the prescribed minimal survival probability $(\bar{P}_s(B))$. In the post-launch phase PADS updates the required time for disengagement, according to concrete events like, missile detection, Red's change of strategy etc.

The first planning step is to assess the probability of a "safe winning" $[P_s(B)=1$ with $P_k(R)=1]$. The "safe winning" test is carried out by checking :

$$\text{time(SNE}|\text{B}{\rightarrow}\text{R)} < \text{time(NES}|\text{B}{\leftarrow}\text{R)} \quad (2)$$

If this outcome can be achieved then Blue is guided to carry out the appropriate planned sequence of actions. Otherwise relaxing assumptions have to be made by accepting less favorable results. Two assumptions are implemented: 1) Red launches a missile after SNE|B→R. 2) Blue starts to disengage before his missile locks on.

The first assumption enables evaluation of $P_k(R)$ based on the target behavior model. By the second one $P_s(B)$ increases but $P_k(R)$ decreases as a function of the probability that at lock-on-range Red will be inside the detection cone of the Blue missile. This probability depends again on Red's behavior. A rule guided search is made on the possible relaxations, maximizing $P_k(R)$ while satisfying $[P_s(B)>\bar{P}_s(B)]$. If such a solution with acceptable $P_k(R)$ exists, Blue is guided accordingly. Otherwise an escape maneuver or an early (thus ineffective) launch can be advised: such a launch may induce Red to disengage at missile lock-on detection and consequently prevent him from guiding his missile as required. Therefore, in spite of the initial disadvantage, Blue can survive without disengagement and reattack more efficiently.

Planning of the firing sequence is constantly updated until actual missile launch takes place. In the post-launch phase the situation is reassessed taking into account any new event. The last moment for safe disengagement and the required time for missile lock-on are again constantly updated

and the respective probability of kill and survival are displayed.

4. The PADS Structure

The schematic structure of PADS is presented in Fig. 3. One of PADS's major functions is to interface with the pilot. At pre-flight the pilot has to transfer, with help of the knowledge engineer, his subjective knowledge (experience, preference ordering etc.), general knowledge, and inference rules for solving the "subjective optimal" game to be played. During flight, PADS must display its advice, answer any questions and explain how different conclusions are derived. After the flight a debriefing may take place. For acting properly in flight, PADS needs to receive sensor-data and to communicate with the differential sub-game simulation module.

In the presently implemented scheme, the subjective knowledge takes the form of defining the opponents probabilistic behavior model and the prescribed limit of survival $(\bar{P}_s(B))$. General knowledge is the aerodynamic and other data on the aircraft and the missiles. The inference rules are the search rules for the different sub-games and the conclusions derived from them. For a given prelaunch trajectory the advice is the suggested time to launch a missile and the time to start the evasive maneuver. On each of the advised items, PADS can explain how the conclusions were derived.

The above tasks can be qualitatively performed using an Expert System. An Expert System is a rule based AI application program for doing a task which requires expertise. It generally consists of the following: user interface, data base, knowledge base and inference engine. The user interface allows interaction with the user such as asking quesitons and getting answers about the knowledge base, including the process of deduction; manually adding data to the system as a normal procedure, etc. The knowledge base which includes the knowledge about the specific problem domain, is organized in such a way as to enable the inference engine to search and find the proper path to the required goal.

Although in our prototype we implemented only a very small set of such rules, we laid the ground and prepared the tools and mechanism for a larger, more complex system that will be required for a more realistic duel, and much more so for a multi-aircraft situation, where the number of rules and the complexity of rule interaction will increase by orders of magnitude.

The rules and data in our system are organized in a so called "frame based system" [12] as shown in Fig. 4. A "frame" is a data structure that contains pointers to all information items relevant to the concept that is represented by the frame. For example, the Blue aircraft is such a concept. An aircraft in general is another. Relevant information can be the aircraft's maximal velocity, its weapon arsenal, its pilot's identity, etc. The frame bears the name of its concept, and the pointers inhabit "slots" of different types (called "facets") in the frame (e.g., explicitly given values, default

values, procedures that are invoked "if need" to compute a value, etc.). Frames are arranged in a hierarchy of class inclusions, linked by so called "IS-A" pointers. This hierarchy enables, for example, the Red aircraft to "inherit" the fact that it has wings from the data associated with the general aircraft frame, and thus reduces duplication of data.

The use of some of the rules implemented in the PADS is illustrated by the example given in the next section.

5. Illustrative Example

As a first example a case of equal velocity aircraft (350m/s) flying at the altitude of 8 km is selected. The engagement starts (t=0) at the range of 48 km (outside the RMAX barrier) with head-on initial conditions. Both aircraft turning rates are limited by a load factor of $n_{max}=9$.

At the initial planning phase the PADS computes three critical times:

(i) The latest time for starting a "safe disengagement" from all possible missiles, assuming straight flying trajectories of both aircraft. This "last escape" time serves merely as a reference value for the cases where the pilot decides not to deploy his own weapon. In the present example this occurs at $t_s=45.2$ sec and at the range of 16.3 km coinciding with the "no-escape" range.

(ii) The latest time for a "safe and completed firing sequence" against an aggressively acting intelligent opponent. This time is computed using the NES¦B←R barrier. In the present example this firing sequence starts at $t_\ell=32.3$ sec (at a range of 25.4 km) and allows missile updating until $t_g=49.5$ sec on a post-launch trajectory. Missiles fired earlier can be updated without any risk [$\bar{P}_s(B)=1$].

(iii) The first "effective firing opportunity" is computed by using the SNE¦B→R barrier. In the present case the firing sequence starts at $t_\ell=34.5$ sec (r=23.9 km) and missile lock-on is achieved at $t_g=48.3$ sec. Any missile fired later will hit the Red aircraft if it behaves as assumed.

The above results [time(NES)<time(SNE)] show that in the present case a "safe winning" cannot be guaranteed.

At that point, PADS evaluates a set of "compromise" solutions such as:

1. Planned firing prior to $t_\ell=32.3$ sec and compute probability of kill $P_k(R)$ based on target behavior model.

2. Planned firing at or slightly after $t_\ell=34.5$ sec evaluates two alternatives:

a) update missile until it locks on and assess risk [$P_s(B)$] based on opponent behavior model.

b) compute time for safe disengagement on the post-launch trajectory (for the worst case) and assess probability of missile lock-on based, again, on Red's behavior model.

All 3 cases (1,2a,2b) serve only as extreme points, because in each either

$P_s(B)=1$ or $P_k(R)=1$. For any prescribed value of $\bar{P}_s(B)<1$ an optimal firing sequence that maximizes $P_k(R)$ is found by a rule based search and displayed to the pilot as satisfactory advice.

Note that all of these assessments, carried out in the planning phase, assume straight trajectories until planned firing.

During the engagement actual opponent behavior and own trajectory are monitored and planning is constantly repeated and updated. Based on results obtained earlier [10] the optimal pre-launch trajectory is a lead-collision course for missile firing towards the future point of impact.

In our example we "detect" that at the range of 25.6 km (LNE¦R→B) at t=32 sec Red starts an apparent post-launch maneuver to the right towards a maximum look angle of $\psi_R=60°$. At the same time Blue reacts by turning to the respective optimal pre-launch direction (about 20°) and replans the optimal firing sequence leading to time(NES)=35.2 sec and time(SNE)=34.4 sec.

Since the situation is symmetrical due to equal velocities, it is clear that if Red did fire at t=32 sec and disengages when his missile locks-on, he cannot be hit because the NES barrier guarantees this outcome. However, if he continues to fly in the post-launch direction until Blue missile locks-on, he will be hit as a consequence of the "effective" Blue firing.

6. Conclusions

In this paper an approach of combining differential game concepts and Artificial Intelligence techniques is outlined for the analysis of an aggressive air combat duel. A prototype pilot advisory system (PADS) generated by this approach is described. This system is to operate as a real-time Expert System with a "knowledge-base" composed of differential sub-games solutions. The key elements of the system are the accelerated simulations of various missile-aircraft engagements. Assessing the outcome of these engagements serves as the basis of the advisory system for optimal timing of missile deployment and disengagement maneuver. PADS provides a meaningful solution for one of the basic and probably the most challenging tasks of a future Pilot Associate System. It presents to the pilot vital information which can be used either to guarantee survival, or to maximize the probability of winning with an accepted level of risk. Moreover, the system allows to take advantage of any non-optimal action of the opponent.

The results shown in the paper are of exploratory nature, a part of an ongoing investigation with a very simple dynamic model. These results are, however, very encouraging. They show that the concept of combining AI techniques and differential game theory can be implemented. They also demonstrate the great tactical advantage that such a system can provide to its user. Extension of the approach to multiple aircraft scenarios and a three-dimensional variable speed aircraft model is planned.

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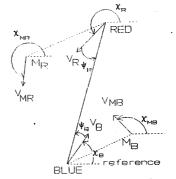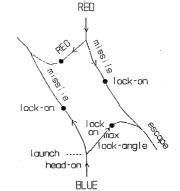
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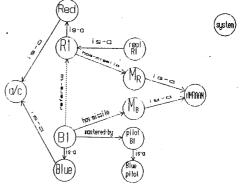

Fig. 2: Typical firing sequence

Fig. 3:Pilot Advisory System - general structure

Fig. 4: Upper level knowledge-base of BI