Bayesian Estimation and Inference in R

Yun-Xiao Li

Fri 13th Jun, 2025

What?

What are Bayesian estimation and Bayesian inference?

What?

WARWICK

What are Bayesian estimation and Bayesian inference?

- **Bayesian estimation** is a way of using data and prior knowledge to estimate unknown values, like the mean or proportion in a population;

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What are Bayesian estimation and Bayesian inference?

- **Bayesian estimation** is a way of using data and prior knowledge to estimate unknown values, like the mean or proportion in a population;
- **Bayesian inference** is a broader term. It's the whole approach of using Bayes' Theorem to update beliefs and make decisions in the face of uncertainty.

Why?

Why should I learn Bayesian inference?

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Why should I learn Bayesian inference?

- Mirrors real-life thinking and is easy to understand (to some extent);
- Makes use of prior knowledge;
- Gives direct answers to real questions (not like p-values);

Outline

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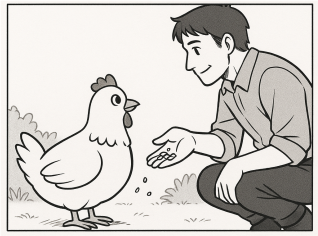
1. Bayes Rule

2. Bayesian Estimation: Examples of Coins

3. Bayesian one-sample t-test

Bayes Rule

Bayes Rule
Example



Bayes Rule
Example

Bayes Rule
Example

- Two fundamental perceptions of Bayesian inference:
  - You have a prior belief.
  - New observations update this belief.

Definition (Conditional probability)

Let A stands for an event, and D stands for the observation, then the probability of event A occurring given that B is observed is

$$p(A|D) = \frac{p(AD)}{p(D)}.$$

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In the example above:

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- $P(AD)$: The chance of observing the man with an axe and the chicken being killed

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- $P(AD)$: The chance of observing the man with an axe and the chicken being killed
- $p(A|D)$: The chance of the man killing the chicken today if the chicken sees him with an axe;
- What is the value of $P(AD)$ and $P(D)$?

Let's warm up the brain!

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$$p(A|D)p(D) = p(AD)$$

$$p(D|A)p(A) = p(DA)$$

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Theorem (Bayes' Theorem)

$$p(A|D) = \frac{p(D|A)p(A)}{p(D)}.$$

- $P(A)$ is the **prior** belief of the chance that the man will kill the chicken;
- $P(D|A)$ is the **likelihood** of observing the man with an axe if the man decides to kill the chicken.

Let's decompose $P(D)$!

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p(D|A) & & p(D|\neg A)
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$$p(D) = p(D|A)p(A) + p(D|\neg A)p(\neg A)$$

Bayes Rule

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Example

Assume the incidence rate of a disease is 1/1000. A particular test for whether someone has this disease is 95% sensitive, meaning that if the patient has the disease, this test has a 95% probability of being positive. This test, however, has a 1% probability of being positive even though the patient does not suffer from this disease.

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- *Bob got a positive result for this test.*

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Hints:

- $P(A)$ is the chance of having the disease;
- $p(D|A)$ is the chance of having a positive result if the person has the disease;

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- *Bob got a positive result for this test.*

- $P(A) = \frac{1}{1000}$;
- $P(D|A) = 0.95$;
- $P(D|\neg A) = 0.01$.

$$p(A|D) = \frac{p(D|A)p(A)}{p(D|A)p(A) + p(D|\neg A)p(\neg A)}.$$

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- *Bob got a positive result for this test.*

- $P(A) = \frac{1}{1000}$;
- $P(D|A) = 0.95$;
- $P(D|\neg A) = 0.01$.

$$P(A|D) = \frac{0.95 \times 0.001}{0.95 \times 0.001 + 0.01 \times 0.999} \approx 0.087$$

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- *Bob got a positive result for this test.*
- *Bob got a negative result in the second test.*

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$$P(A|D) = \frac{0.05 \times 0.087}{0.05 \times 0.087 + 0.99 \times 0.913} \approx 0.0048$$

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An alternative solution:

$$P(A|D) = \frac{0.95 \times 0.05 \times 0.001}{0.95 \times 0.05 \times 0.001 + 0.01 \times 0.99 \times 0.999} \approx 0.0048$$

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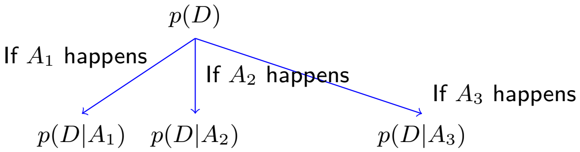

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Theorem (Bayes' Theorem)

If $A = \{A_1, A_2, \cdots, A_N\}$ is an event set, then the probability of event $A_i \in A$ occurring given that D is observed is

$$p(A_i|D) = \frac{p(D|A_i)p(A_i)}{\sum_{i=1}^{N} p(D|A_i)p(A_i)}.$$

- The main idea of the Bayes rule is: Prior $+$ Observation (Likelihood) $=$ Posterior

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- The mathematical form of this formula is

$$p(A_i|D) = \frac{p(D|A_i)p(A_i)}{\sum_{i=1}^{N} p(D|A_i)p(A_i)}$$

Bayesian Estimation: Examples of Coins

Bayesian Estimation: Examples of Coins

A Discrete Distribution

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Example

We flipped a coin six times and got the observation $\{1, 0, 1, 1, 0, 1\}$*, where 1 represents heads, while 0 means tails. What is the probability that the coin will come up heads?*

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- Find a probabilistic description of the observation. — Likelihood function

Bayesian Estimation: Examples of Coins

A Discrete Prior

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Denote The underlying probability of heads as θ. This idea can be written formally as

$$p(y = 1|\theta) = \theta.$$

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Therefore,

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The probability of the observation $Y = \{1, 0, 1, 1, 0, 1\}$ (i.e., the likelihood function) is

$$p(Y|\theta) = \theta^4(1 - \theta)^2.$$

Bayesian Estimation: Examples of Coins

A Discrete Prior

- Find a probability distribution of the parameter(s). — Prior distribution

Bayesian Estimation: Examples of Coins

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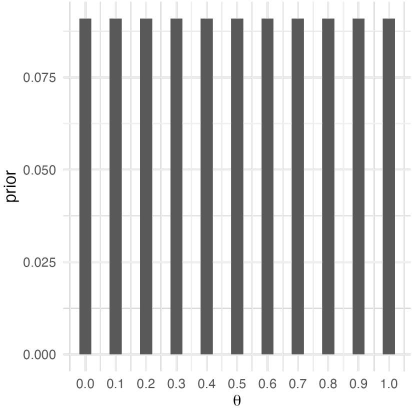
We start with a prior that θ is equally likely to be 0, 0.1, 0.2, ..., 0.9, 1.



Bayesian Estimation: Examples of Coins

A Discrete Prior

- Following Bayes' theorem, we can reallocate credibility across the values of the parameter(s). — Posterior Distribution

Bayesian Estimation: Examples of Coins

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$$p(\theta_i|Y) = \frac{p(Y|\theta_i)p(\theta_i)}{\sum_i^N p(Y|\theta_i)p(\theta_i)}$$

Bayesian Estimation: Examples of Coins

A Discrete Prior

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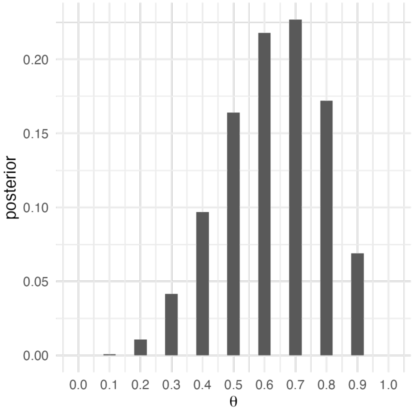

Bayesian Estimation: Examples of Coins

A Discrete Prior

Bayesian estimation follows three steps:

1. Find a probabilistic description of the observation. — Likelihood function;
2. Find a probability distribution of the parameter(s). — Prior distribution;
3. Following Bayes' theorem, we can reallocate credibility across the values of the parameter(s). — Posterior Distribution.

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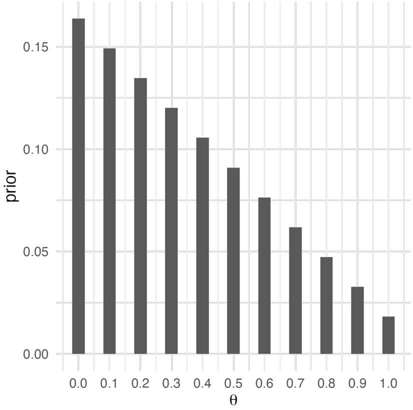
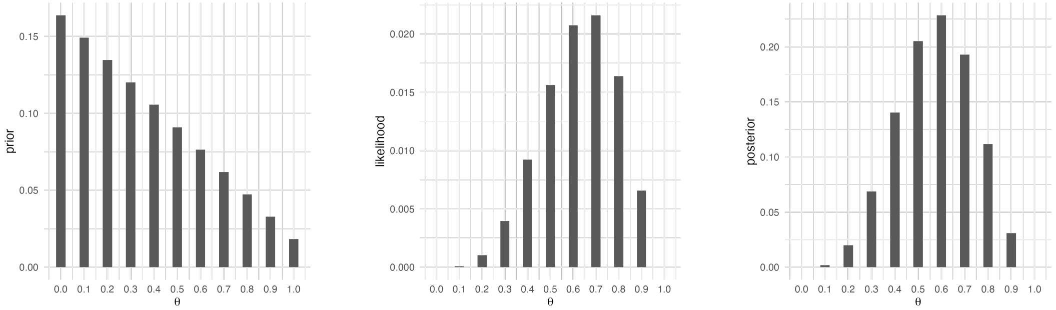
Let's try a new discrete prior!



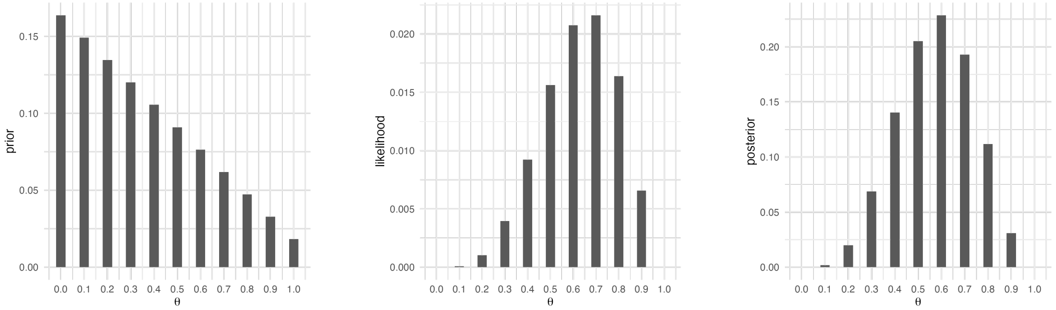
Bayesian Estimation: Examples of Coins

A Discrete Prior



Bayesian Estimation: Examples of Coins

A Discrete Prior



- The shape of the posterior is the combination of the shape of the prior and the likelihood.

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Let's think about a more reasonable prior: a Beta distribution.

Bayesian Estimation: Examples of Coins

A Continuous Prior

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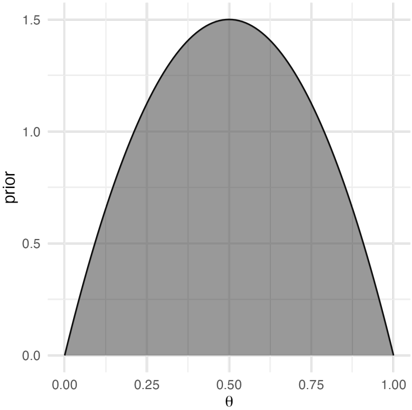

Fig. 1: The density function of $Beta(2, 2)$.

Bayesian Estimation: Examples of Coins

A Continuous Prior

- Prior: $\theta \sim Beta(2, 2)$;
- Likelihood:

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Bayesian Estimation: Examples of Coins

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- To "calculate" the posterior distribution, we used the Markov Chain Monte Carlo (MCMC) method.

Bayesian Estimation: Examples of Coins

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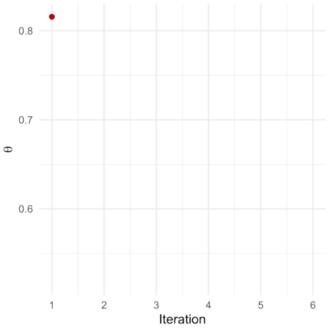

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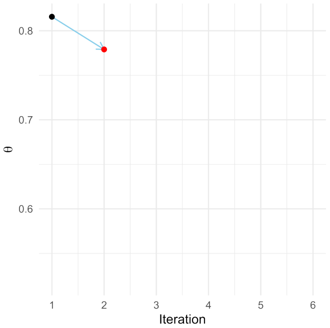

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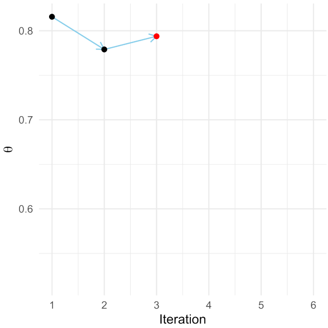

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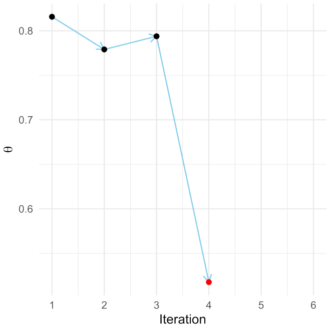

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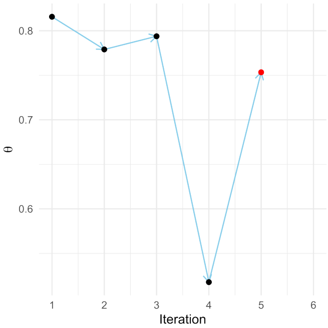

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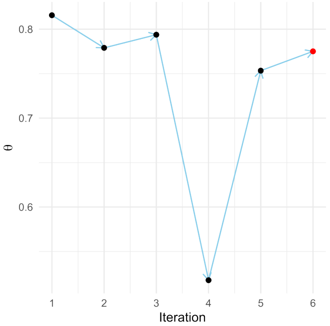

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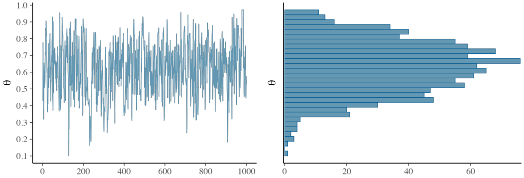

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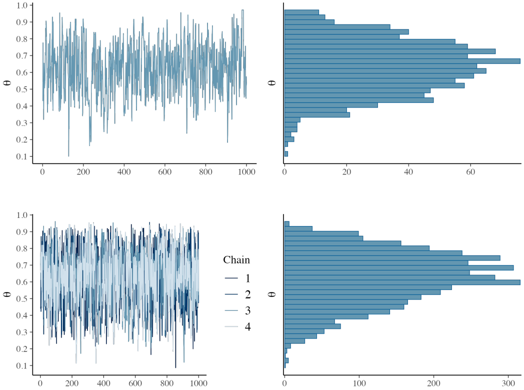

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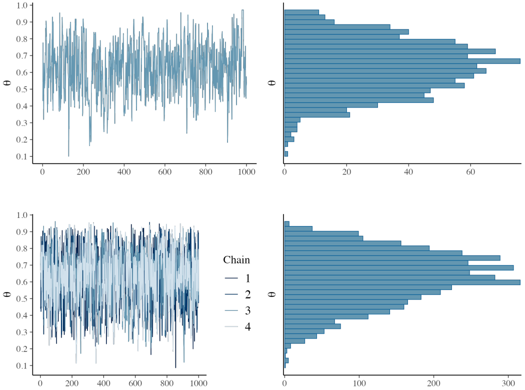

- The MCMC is implemented in a `.stan` file.

What is a `.stan` file?

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What is a `.stan` file?

```
data {
 // The observations
}

parameters {
 // The parameters
}

model {
 // The prior and likelihood function
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```

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Bayesian Estimation: Examples of Coins

A Continuous Prior

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We will write two `.stan` files:

- A “Hello world” example (See *hello_world.stan*);
- The Bayesian estimation of the coin example (See *coin.stan*).

Bayesian Estimation: Examples of Coins

Summary

- The full version of Bayes' Theorem:

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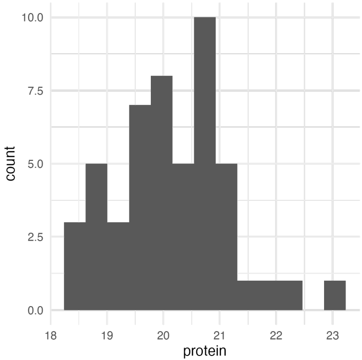
- The posterior distribution $p(\theta|D)$ can be approximated by MCMC, which is implemented in Stan;

Bayesian one-sample t-test

WARWICK

Example

There is a type of energy bar that claims each bar contains 20 grams of protein. We collected 50 samples and tested the protein content of this product. We want to know whether the label is accurate.



Bayesian one-sample t-test

The Frequentist t-test

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WARWICK

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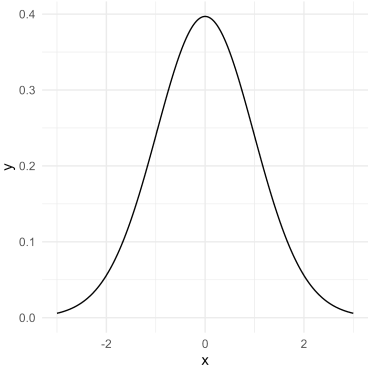

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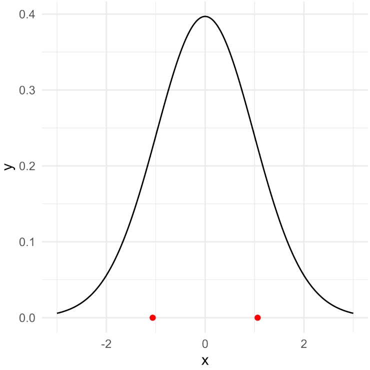

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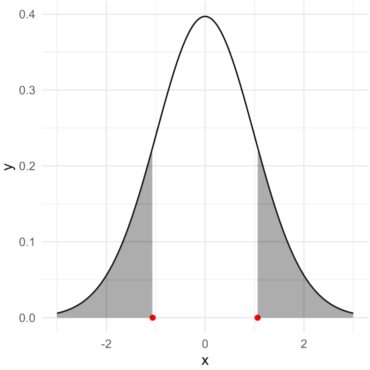

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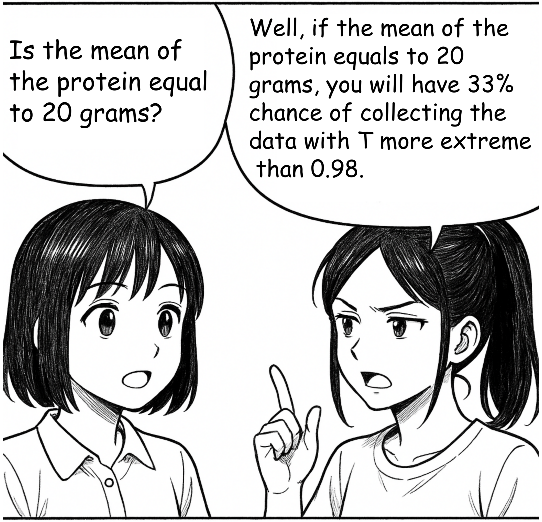
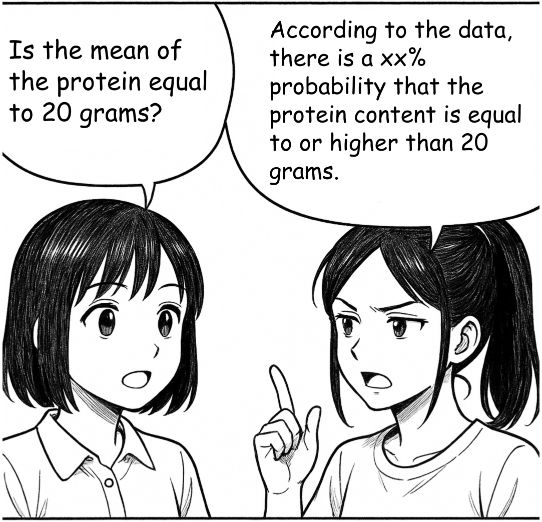
$$p = P(t < -|T| \; |\mu = 20) + P(t > |T| \; |\mu = 20)$$

Bayesian one-sample t-test
The Frequentist t-test



Bayesian one-sample t-test
The Bayesian Estimation



- Likelihood function:
- Prior:

Bayesian one-sample t-test

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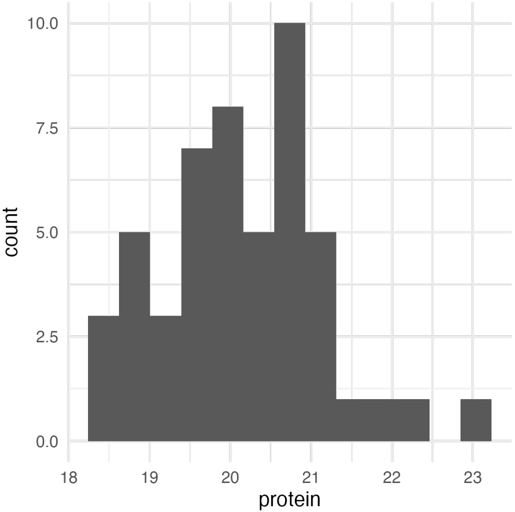

Bayesian one-sample t-test

The Bayesian Estimation

WARWICK

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$$L(D|\mu, \sigma) \sim N(\mu, \sigma^2)$$

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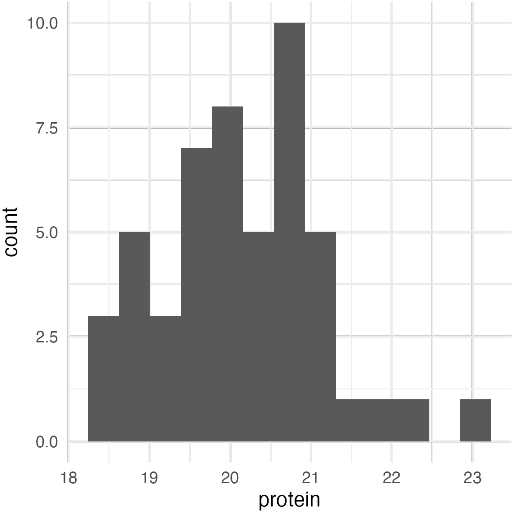

Bayesian one-sample t-test

The Bayesian Estimation

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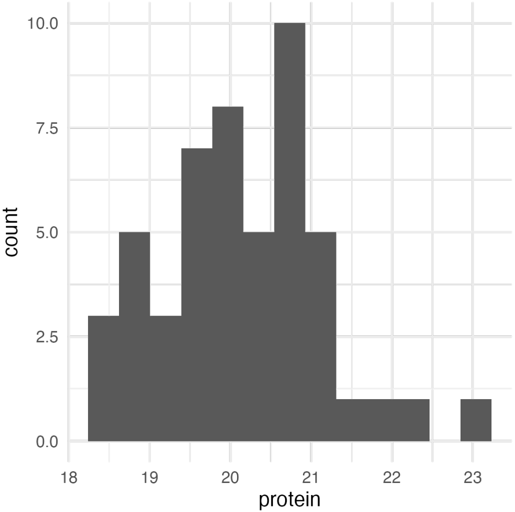
$$\mu \sim N(20,10)$$
$$\sigma \sim N(0,10)T[0,\infty]$$



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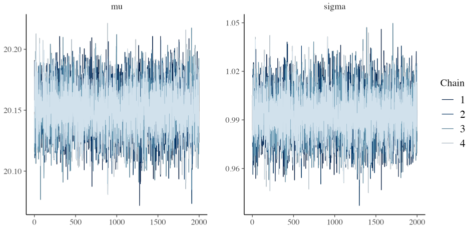

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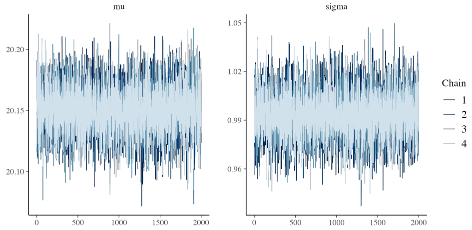
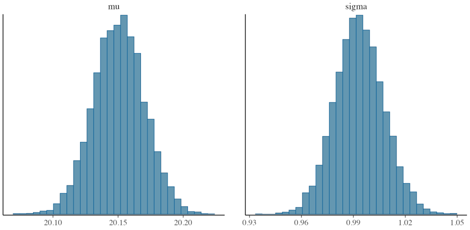
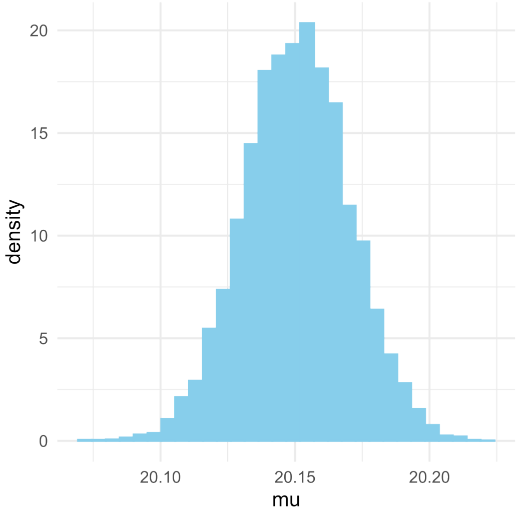


- The posterior distribution:

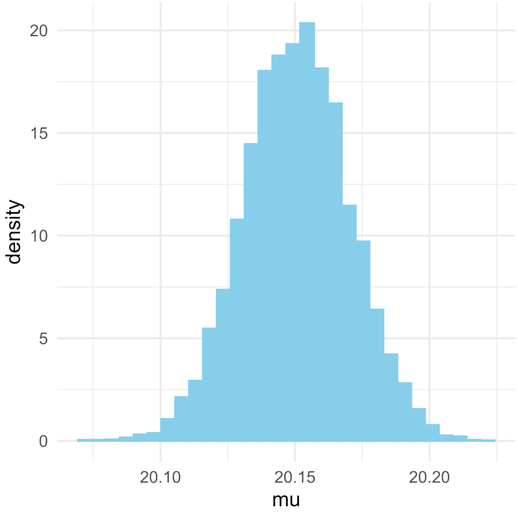


Bayesian one-sample t-test

The Bayesian Estimation

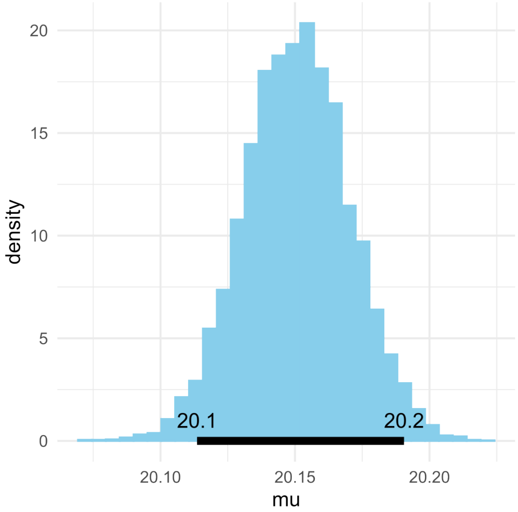


How do we summarise the posterior distribution?



How do we summarise the posterior distribution?

- HDI (highest density interval): A 95% HDI is the shortest interval that covers the 95% mass of the distribution.



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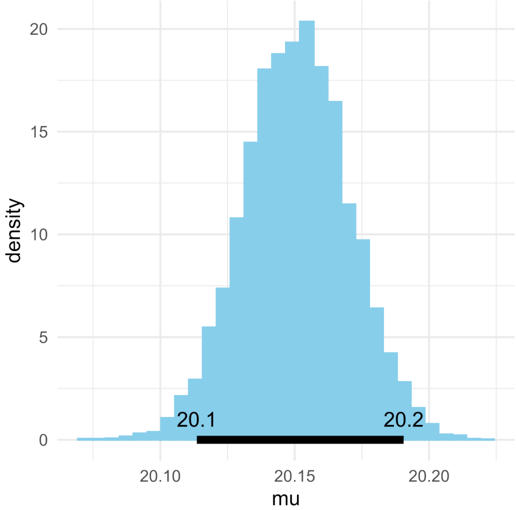
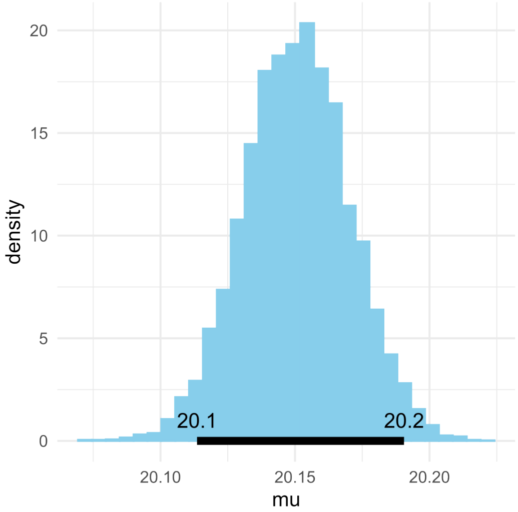
- The 95% CI means that if we did this experiment over and over, 95% of those intervals would include the true mean.
- The 95% HDI means that there is a 95% probability that the true mean lies in this interval, given our data and assumptions.

Bayesian one-sample t-test

The Bayesian Inference



How do we conclude?



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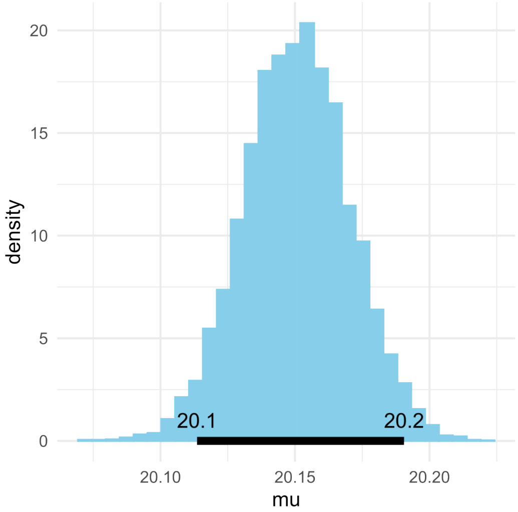
The value of 20 grams is excluded from the 95% HDI.

Bayesian one-sample t-test

The Bayesian Inference



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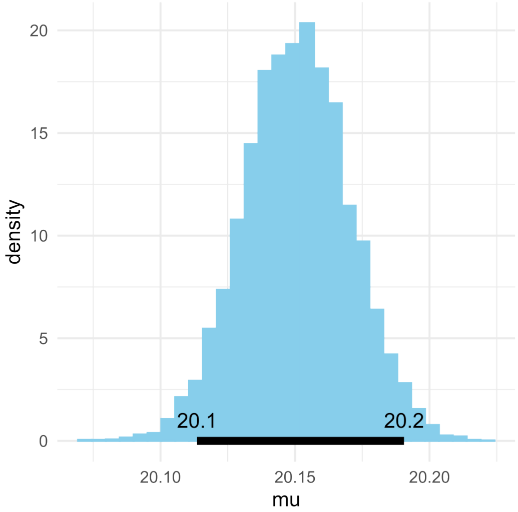

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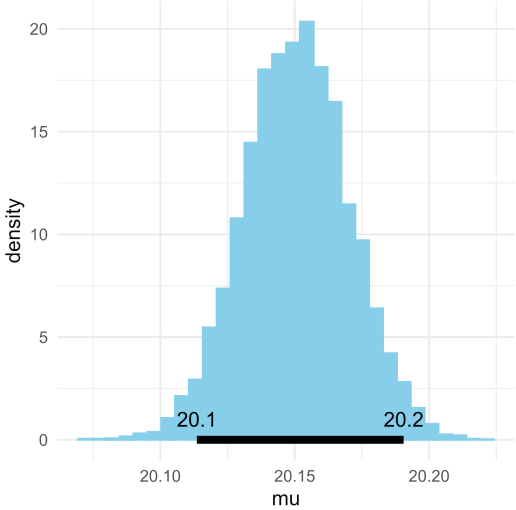

How do we conclude?

The value of 20 grams is excluded from the 95% HDI. Can we reject the null hypothesis now?

Not yet. We also want to know whether the difference between the posterior distribution and the null hypothesis is big *enough*.

One more thing...
Adam Tow | AllThingsD.com

ROPE (region of practical equivalence):

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$$d = \frac{|\mu - \mu_0|}{\sigma}.$$

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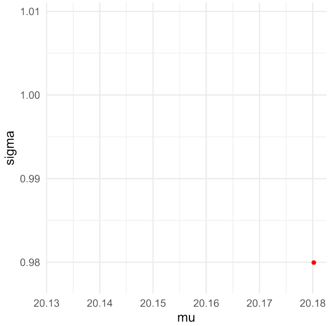
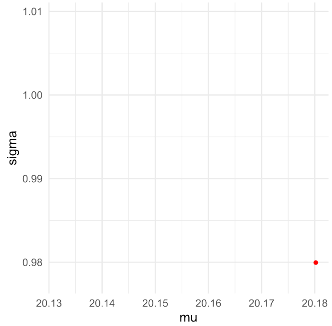
$$d = \frac{|\mu - \mu_0|}{\sigma}.$$

- Cohen (1988) noted that the effect is "small" when $d = 0.2$. In other words, when $d \leq 0.2$, the estimate of μ is practically equivalent to the null value μ_0.

Bayesian one-sample t-test

The Bayesian Inference





$$\text{Effect size} = \frac{|\mu - 20|}{\sigma}$$

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Bayesian one-sample t-test

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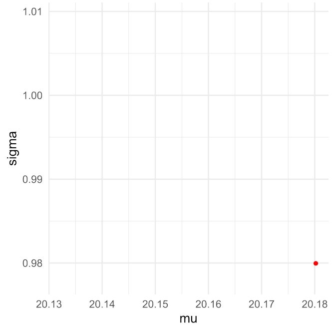

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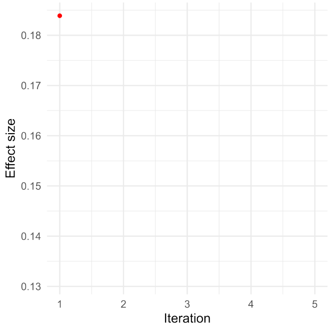

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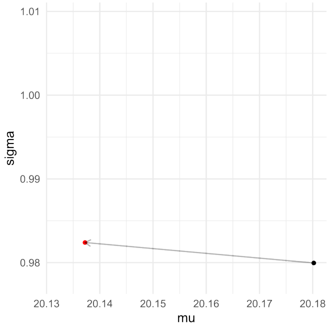

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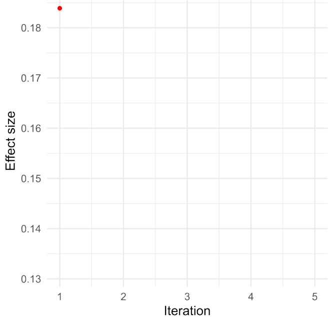

Bayesian one-sample t-test
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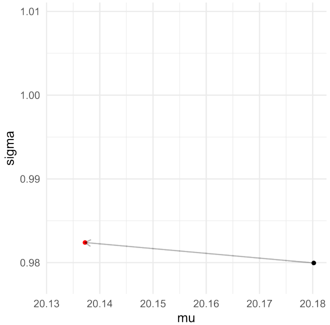

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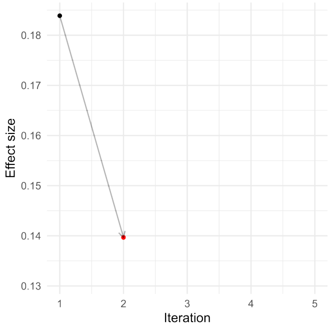

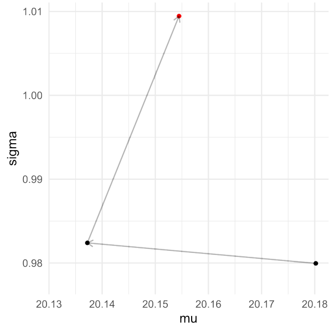

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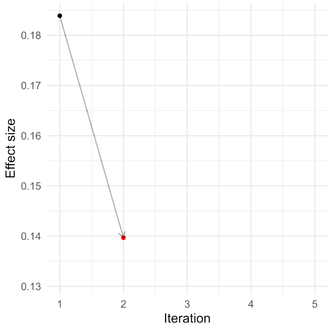

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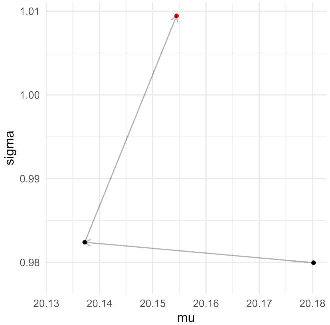

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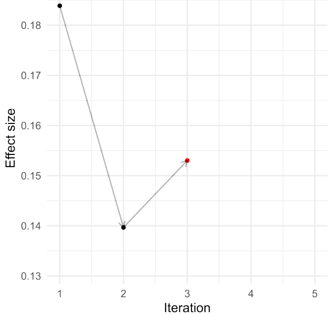

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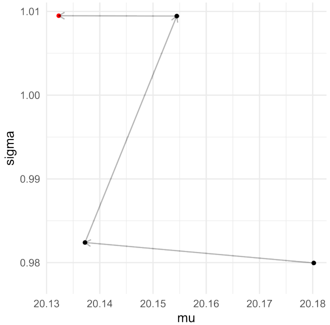

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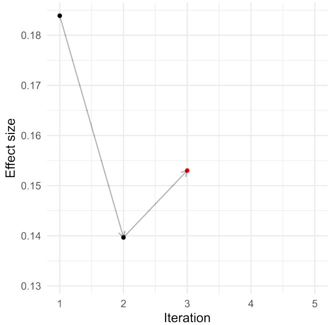

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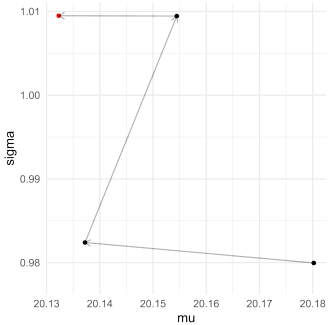

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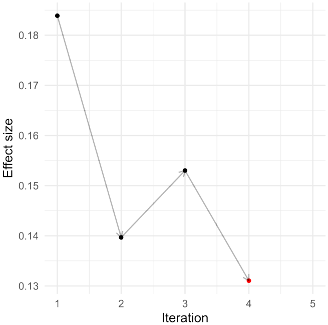

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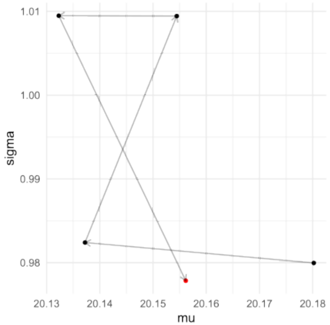

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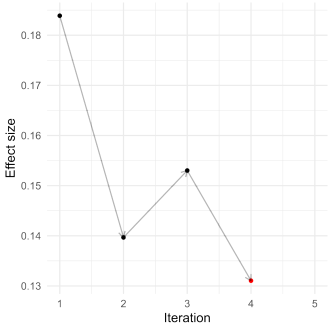

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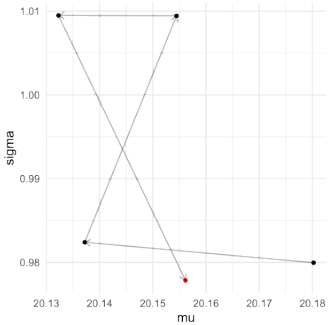

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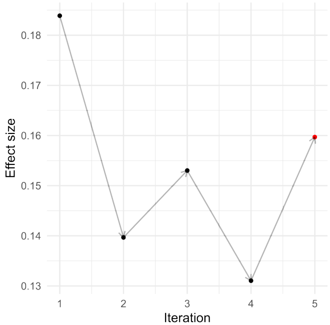

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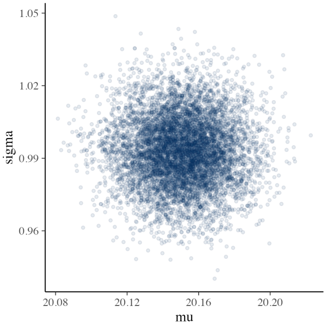

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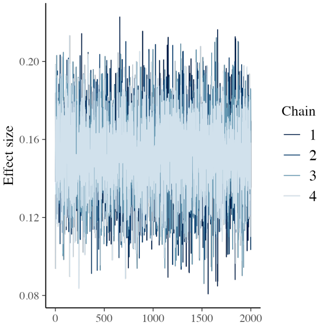

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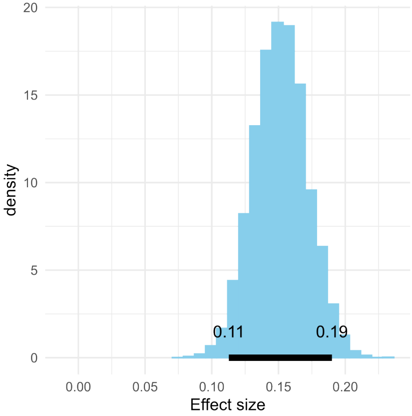

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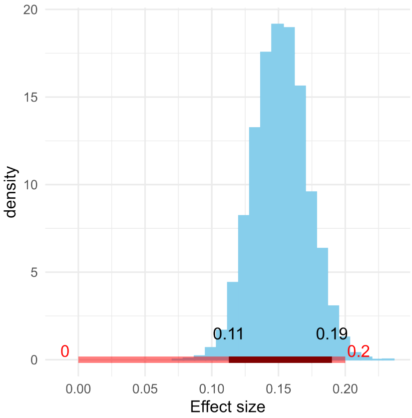

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- 95% HDI and ROPE overlap: current data are insufficient to yield a clear conclusion.

Bayesian one-sample t-test
Summary

- The idea of the p-values does not answer the question we are interested in;

Bayesian one-sample t-test

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- Bayesian estimation can provide a more direct answer to our question;
- (Advanced topic) Practically, we need 95% HDI and ROPE to test the hypotheses.

Thanks!

References I

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Lawrence Erlbaum Associates.

Kruschke, J. K. (2012).Bayesian estimation supersedes the t test.. *Journal of Experimental Psychology: General, 142*(2), 573–603. https://doi.org/10.1037/a0029146