is applied, the ratio of mode 3 induced pore pressure over mode 1 and mode 2 induced pore pressure is around 0.01%-0.02%. This is also in line with the findings from the classical poroelastic model (Detournay et al. (1988).

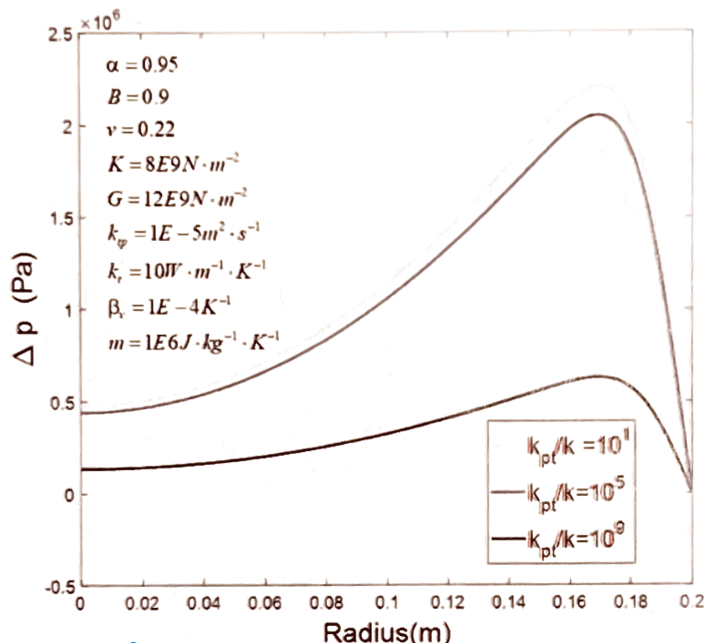

Figure 5. Various ratio s of k_{pt} / k (thermo-osmosis's influence, on impacting the mode 2 induced pore pressure($t = 10^{-4}$ days).

shown at

4.2 Thermo-osmosis and thermo-filtration effects

So far, all results have been presented with a single combination of parameters governing thermos-osmosis and thermo-filtration. To examine these effects, we will present the internal pressure profile at t=10 days after thermal (mode 2) loading. Three different ratios (1, 10^5, 10^9) of k_{pt} / k are selected to show the different developments of the temperature induced pore pressure (Figure 5), noting that k_{pt} / k =1 corresponds to the case presented in Fig. 4. All three cases show a peak pore pressure near the surface region; however, the magnitude of the peak induced pore pressure has considerable reduction when the thermo-osmotic coefficient is significantly larger than the hydraulic conductivity (i.e., $k_{pt} / k = 10^9$). While it is not clear if this ratio is realistic (it might be, but there has been little study), it is clear that thermos-osmosis does have potential to contribute a reduction in the induced pore pressure from temperature loading.

The role of thermo-filtration is make most apparent by observing the temperature profiles resulting from the mode 2 (temperature) loading. A base case is shown in Figure 6. As the time proceeds, temperature is increasing monotonically from surface towards the inner core of the plug, and finally arrives at equilibrium. ~~which is the equilibrium of the boundary conditions at the later time~~. Three different ratios of thermal conductivity over the thermal filtration coefficients are selected to show the impact of the thermal filtration effect on the temperature profile, as if ~~that it~~ changed by the pore pressure gradient from mode 1 loading. As shown in Figure 7, among the three selected ratios, the maximum temperature difference induced by the pore pressure gradient is within 1 Celsius. Furthermore, when the thermal conductivity is significantly larger than the thermal filtration coefficient ($k_t / k_{tp} = 10^8$), the induced temperature differences are negligible under the current model settings and inputs.

Commented [BAP39]: It looks like you might have used different parameters than you specified previously. You need to tell us what the parameters are that were used for this, including and importantly, the ratio of kT/ktp.

Commented [LY40R39]: Figure 6 is the temperature change due to mode 2 (temperature loading) and figure 7 is the temperature change due to mode 1 (pore pressure loading)

Commented [BAP41]: Check consistency, kT, or kt.

Commented [LY42R41]: Updated through the manuscript.

Commented [BAP43]: I don't understand. Shouldn't the change be around 40 deg C because of the boundary condition? So Fig 7 is like one of the curves in Fig 6, and then showing varying kT/ktp?

Commented [LY44R43]: As same above, Figure 6 is the temperature change due to mode 2 (temperature loading) and figure 7 is the temperature change due to mode 1 (pore pressure loading)

keep 1K if you are using K everywhere else. I prefer °C everywhere, but OK as long as you are consistent

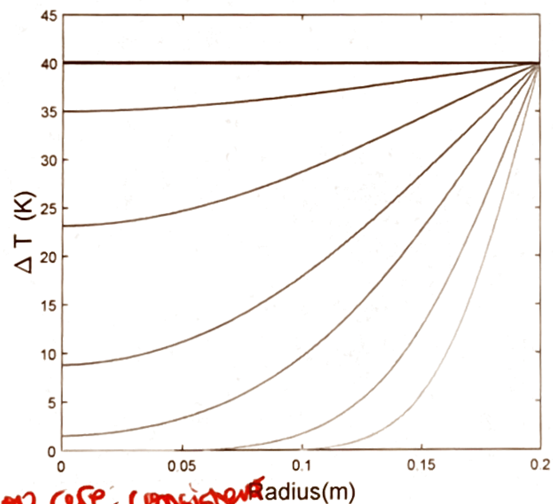

Figure 6. The development of temperature change (ΔT) under the loading mode 2, where the curves grade from gray to black as time increases.

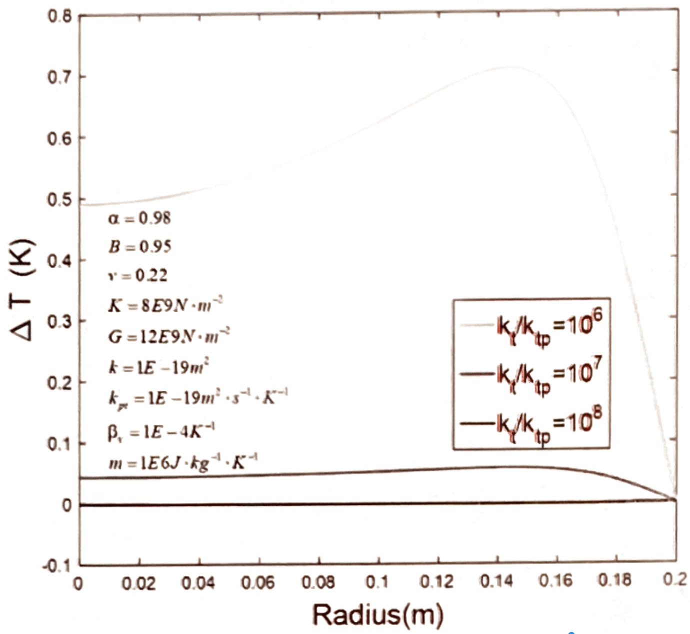

Figure 7. Various ratios of k_t / k_{tp} (thermal filtration's influence) ~~on~~ impacting the temperature change (ΔT) due to loading mode 1.

4.3 Effective Stress

There is a vast literature pointing to the impact of the so-called effective stress on behavior of porous media, where effective stress is defined as the difference between the total stress and the pore pressure (Biot, 1941; Terzaghi, 1925). Its significance lies in that it quantifies the total stress carried by the solid skeleton. Hence, when it becomes tensile, it indicates that the skeleton is subjected to tension. In materials like cement, rock, and soil, the tensile strength is low and so generation of tension indicates risk of tensile failure. Here, the effective radial stress $\sigma_r^{'}$ is obtained as the superposition of the effective stress $\sigma_{rr}^{i'}$ from three loading modes as

again, I think here is the first point that the initial values matter, which is why you need to write your equations explicitly. Not because people don't know what is effective stress, but to be clear how to compute it from quantities in your work.

$$\sigma_r^{'} = \sigma_{rr}^{1'} + \sigma_{rr}^{2'} + \sigma_{rr}^{3'} \qquad (26)$$

Commented [BAP45]: This is a confusing notation. You should not put a mode of loading in the subscript as it looks like it is a component of the tensor. I suggest

\sigma_{rr}^{\prime (1)} for mode 1 loading, etc.

Commented [LY46R45]: Updated.

The result is plotted in Fig. 8, ~~Recall~~ recalling that tension is positive. It is observed that after the cement is placed under the pore pressure, temperature, and stress loading, the induced pore pressure will increase and will therefore reduce the radial effective stress near the boundary, ~~and~~ the results show this can create a region where effective stress is tensile. However, outside of this region, and at early times, the rest of the material will still be subjected to compressive effective stresses. ~~However,~~ But then, because the pore pressure diffusion process is very slow compared to thermal diffusion, the pore pressure continues to increase through to the center and eventually a tensile region will be created throughout the central region. Finally, at a later time when induced pore pressure is fully dissipated, the radial effective stress will return to be compressive again. This demonstration of the whole diffusion process indicates that cementing under the HTHP has potential to induce a high value of pore pressure, which will lower the effective stress and can even generate a tensile region. This coupled behavior can therefore lead to cracking of the cement, jeopardizing the integrity of the cementing system with the potential to trigger unwanted consequences.

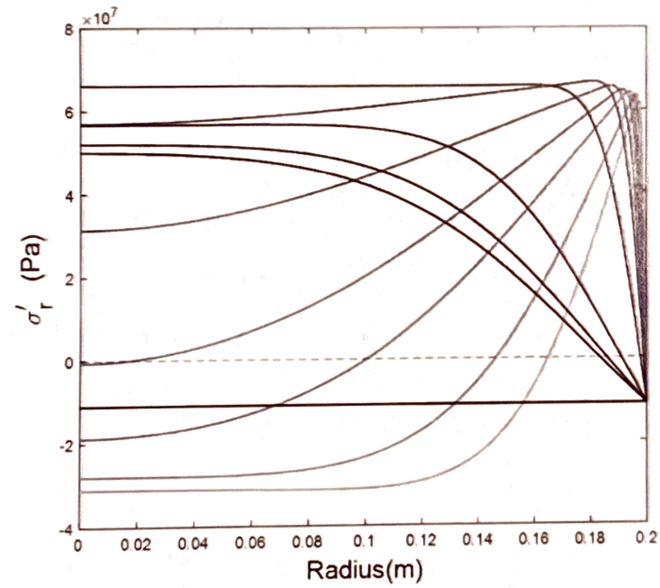

Figure 8. Effective radial stress of cement, with tension positive, where the curves grade from gray to black as time increases (boundary conditions and input values of each parameter are stated at Eq. 25).

Commented [BAP47]: Need to be more specific. Give the parameters an equation number. And a little confusing because we just went through the thermo-filtration part which seams to have had a different set of parameter values (though not explained why this choice was made).

Commented [LY48R47]: Updated, please check.

5. Pairwise Bivariate Analysis and Dimensional Analysis

The system behaviors discussed so far depend on the different values of input parameters and the mutual interactions of the physical processes they determine. If some of these parameters take on different values, the results can be much different. Thus, it is important to investigate the potential prevalence of the high pore pressure generation and its associated generation of tensile effective stress. The analysis starts with assigning values at random to each parameter within a certain range, as summarized in Table 2 ~~in Appendix A~~. Once these variables have been given values according to this Monte Carlo approach, they are substituted into the semi-analytical solution and a new solution is thus computed. For each solution, the most tensile effective radial stress at any location and any time is extracted from the data and then used to classify the case as "tensile" or "compressive". The details of the above-mentioned procedures are summarized in the flowchart in Figure 9.

Move to main body. No point to put a table in an appendix with no text.

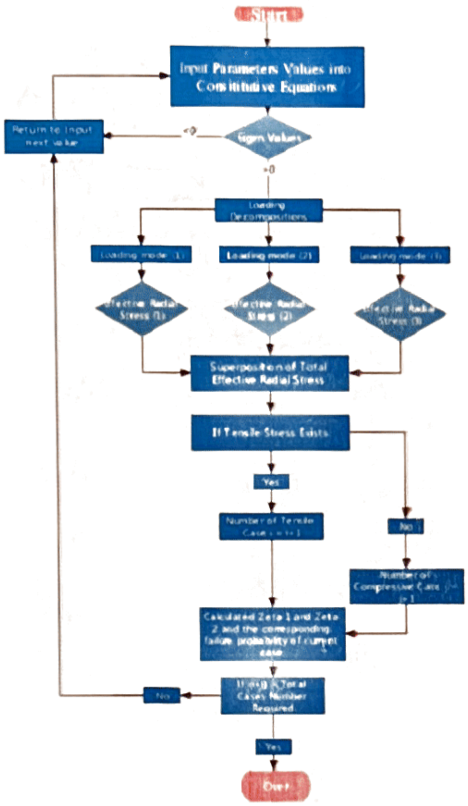

Figure 9. The ~~A~~ algorithm that is used to construct the fully parametric studies.

Commented [BAP49]: I think somewhere in the text we need a symbol defined for the probability you are computing here. And I don't think you are after a "fully parametric study" (not sure what that is). The caption should reflect more the objective of the exercise.

Commented [LY50R49]: Have updated, please check

I still see no definition of the probability in the text, and still not sure what are "fully parametric studies" compared to just "parametric studies".

5.1 Pairwise Bivariate Analysis for Each Variable

Following the procedures detailed in the flowchart described in Figure 9, a total of 3000 cases are calculated and then categorized into two groups: tensile (1914 cases) and compressive (1217 cases). Next, the relationship between each variable and the outcome of "tensile" is examined by the pairwise bivariate distributions that are shown in Figure 10. The non-diagonal elements are scatter plots which display the correlation between two variables and give insight on the distribution features of these variables. The matrix of the results is symmetric about its diagonal. The diagonal elements are univariate distribution plots which are drawn to show the probability density function of each variable. Based on the information provided by the diagonal components, it can be found that in the events of a tensile case, the specific heat and permeability are more concentrated at their lower range, and the thermal conductivity and the thermal expansion coefficient differences are more concentrated at their higher range, whereas the rest of the parameters are almost evenly distributed along their whole range. This indicates a general trend that, within the framework of PTEOF, at higher values of β_v, k_t and lower values of k and m will increase the propensity for generating tensile radial effective stress increases.

Commented [BAP51]: Is this insight coming just from the diagonal components? Is there anything to be gained from the off-diagonal?

Commented [LY52R51]: Yes, it is just from the diagonal parts, I addressed it in the context. I think for the off-diagonal parts, we may need more statistical analysis?

Figure 10. Pairwise bivariate distributions for eleven variables. Each orange color dot represents a tensile case and each blue color dot represents a compressive case. The blue distributions on diagonals indicate distributions of the parameter for compressive cases, while orange shows distributions for tensile cases. Differences between these distributions indicates impact the parameter has on the "tensile" versus "compressive" outcomes.

The impact of permeability on tendency to generate tension is a point that bears further discussion. One common cement design principle is that lower hydraulic conductivity is synonymous with a better cement barrier. This cement designing philosophy is possibly effective under many working conditions (i.e., ambient temperature and pressure) for which ~~that cement~~ is originally designed ~~built for~~. However, when the cement is placed under ~~the~~ HTHP conditions, and poromechanics are taken into consideration, our results show that the lower permeability will actually increase the probability of the cement experiencing tensile effective stress, which can be detrimental to the integrity of the cementing system. This is mainly due to the fact that under the framework of porous media, the lower permeability will greatly slow down the pore fluid diffusion process when the pore pressure is rapidly built up within the cement due to the HTHP boundary conditions. Hence, the pursuit of low permeability regardless of the intrinsic porous properties of the cement itself and without guidance from poromechanical models could instead increase likelihood of degradation of the cement and hence reduce its effectiveness at providing mechanical support and zonal isolation. In other words, there is a negative aspect to having permeability that is too low, which we will henceforth call a "permeability penalty". To avoid the permeability penalty for cementing design under HTHP, perhaps the most favorable solution is to keep the permeability at certain ranges which can achieve the sealing function, but at the same time, allowing the diffusion of pore pressure that is built up by the HTHP conditions and therefore not cause unnecessary damage induced by the excess pore pressure. This permeability-forgiveness design is actually very popular in pavement design of permeable porous systems (PPS) (Scholz et al., 2007) where one or two special drainage layers with relative higher permeability are designed ~~in a more effective and efficient way that~~ to reduces the runoff rates during ~~in the~~ storm while providing a hard surface for the traffic flow. The PPS design has been successfully turned into wide variety of residential, commercial, and industrial applications in the last two decades (Drake et al., 2013). Furthermore, finding a suitable permeability window should in principle be possible for wellbore cementing owing to the vast difference between the length scale associated with drainage of pore pressure to a radial boundary and the length scale associated with fluid diffusion through the length of the barrier. A permeability that is high enough to allow the former while preventing the latter should be attainable.

It also worth mentioning that the specific heat capacity has received little attention in previous research of HTHP cementing design. However, based on our results, it does play a very important role, because ~~similar to the role of thermal conductivity,~~ as a large specific heat capacity will effectively slow down the heat transfer process and hence reduce the tendency to generate ~~the~~ thermally-induced pore pressure. This will give the pore-pressure more time to dissipate compared to the rate of its build-up. This observation suggests a new direction for creating cement with high specific heat capacity and low thermal conductivity which would comprise suitable design for HTHP conditions.