Semi-Analytical Solution for Thermo-Poro-Elastic Stresses in a Wellbore Cement Plug and Implications for Cement Properties that Minimize Risk of Failure

Commented [LY1]: May I know do we have to emphasis the HTHP in the title or just put them in the keywords would also work. Temporarily put them in the keywords now.

Commented [BAP2R1]: Up to you, but my opinion is that HTHP have kind of unclear definitions and it is not clear HTHP is a necessary condition for your results to be relevant. Maybe just HT or just HP would suffice, for example. So I think reducing that stipulation in the title would be better.

Commented [LY3R1]: Agree.

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Abstract

Cementing materials used for plugging wellbores are subjected to evolving temperature, stress, and pore pressure conditions during their service lives. The induced pore pressure changes can be ~~particularly~~ problematic, especially in high temperature and high pressure (HTHP) environments and especially in low permeability materials. However, a design goal of most cement plugs is to achieve very low permeability, with the idea that lower permeability leads to better isolation. Here, with aid of a new semi-analytical solution for thermo-poro-elastic (TPE) stresses in a cylindrical cement plug that includes consideration of full coupling between hydraulic and thermal transport models (so-called "porothermoelastic-osmosis-filtration", or "PTEOF" model), this work shows that lower permeability is not always better. Specifically, the solution shows that materials that are unable to drain excess pore pressure quickly enough compared to the rate at which these pressures build due to thermal changes are more prone to generate regions of internal tensile effective stress and hence are more likely to be damaged. The specific parameter groups associated with this ~~newly identified~~ "permeability penalty" are obtained through a combination of ~~dimensional analysis~~ and pairwise bivariate analysis. These approaches give rise to two dimensionless groups of parameters that are mainly associated with propensity to generate TPE tensile effective stresses. The parametric space defined by these two groups is shown to have three distinct regions based on the probability of generating tensile effective stresses in a plug with a given set of material properties. By shifting the focus of material design from achieving the lower possible permeability to instead achieving the lowest permeability that will not incur increase likelihood of failure due to pore pressure buildup, this work provides a new design concept for wellbore cement. Furthermore, this work highlights for the first time the important role of specific heat of the cement in preventing pore pressure buildup, thereby showing a new way forward for cement design to increase this quantity.

Keywords: wellbore Cementing Plug and Abandonment (P&A) ; High-temperature High pressure (HTHP); Thermo-poro-elastic; Thermal Osmosis; Thermal filtration; ~~Permeability penalty~~; ~~Phase change cement;~~

Commented [LY4]: Since we remove the phase change cement from the discussions and conclusions, should we remove it from here as well?

1. Introduction

Properly designed and executed wellbore cementing and plugging operations are important for various ~~Earth science-related~~ geotechnical applications. These include radioactive waste disposal, deep-well plug and abandonment (P&A), drilling and completion in unconventional reservoirs, Enhanced Geothermal System (EGS), and carbon capture utilization and storage (CCUS) (Gruber et al., 2021; Hargis et al., 2021; Koťátková et al., 2017; Olson et al., 2015; Vrålstad et al., 2019). Despite the advancements in ~~technological development of~~ cementing materials over the last several decades, the quality of cementing is still often associated with some deficiencies, mainly due to the harsh environments where cement is placed (Ahmed et al., 2020; Allahvirdizadeh, 2020; Kiran et al., 2017). Taking wellbore P&A as an example, over the years and across companies, the upper range of the reservoirs' pressure and temperature have been pushing up to 275 MPa and 315 Celsius (DeBruijn et al., 2008; Khalifeh et al., 2020). However, the cement is originally designed for low-temperature and low-pressure conditions. Under harsh wellbore conditions, its stability over an extended period of time is unknown. To mitigate this problem, extensive research has been focusing on reinforcing the cement by inclusion of various additives aiming to provide better mechanical and hydraulic properties, with the goal of maintaining the system integrity under the extreme conditions (Cai et al., 2022; Ge et al., 2018; Katende et al., 2020; Krakowiak et al., 2018; Massion et al., 2021; Massion et al., 2022; Qin et al., 2021; Samarakoon et al., 2022).

Commented [BAP5]: Suggest to write in C and MPa as well

Commented [LY6R5]: Updated

While much effort has been focused on development of materials and additives, the identification of what comprises "better" mechanical and hydraulic properties of cement are still unclear. This is especially true for high temperature and high pressure (HTHP) environments.

Wellbore cement can be classified as cementitious saturated porous material with permeability ranges from milli- to nano-Darcies (Banthia et al., 1989; Goto et al., 1981; Meng et al., 2021; Picandet et al., 2011). Such a material can be heavily influenced by thermo-hydraulic-mechanical (THM) coupling in the pore space, especially when it is experiencing large temperature and pressure variations. Within the permeability range mentioned above, a very large pore pressure could be induced by the THM coupling ~~and the pore pressure would be progressively reestablished over the time,~~ which will also lead to ~~the~~ changing of the effective stress and increase the possibility of ~~shear failure, hydraulic fracturing, or even tensile~~ failure (Ghabezloo et al., 2010). While THM coupling phenomena in porous media has been studied extensively, the wellbore-related applications have been mainly focusing on wellbore stability during drilling and fluid injection into borehole (Gao et al., 2017; Song et al., 2019; Tao et al., 2010; Zhou et al., 2009). The THM coupling effect in the cementing designs and ~~operations~~ Performance has rarely been considered and fully investigated under the HTHP conditions. This knowledge gap means that conditions that will generate stresses that are able to damage the material are not well understood, and therefore it has potentially serious consequences.

To include these mutual interactions between thermal, hydraulic, and mechanical systems in the non-isothermal conditions, Biot (Biot, 1977) extended the traditional theory of poromechanics (Biot, 1941) to include the uncoupled thermal effects by incorporating the thermo-molecular diffusion and dynamic forces using a variational Lagrangian thermodynamics approach. Later on, the thermal diffusion process was coupled in solid and fluid deformation by Derski (1979), as well as others (Bear et al., 1981; Kurashige, 1989; Smith et al., 1993). The abovementioned porothermoelastic formulations include an assumption to neglect the non-linear term associated with connective heat transfer, which is thought to be most appropriate for low permeability materials (Chen et al., 2005; Delaney, 1982; Gomar et al., 2014; Wang et al., 2003). Within the framework of linear porothermoelasticity, ~~substantial~~ Past studies have been performed for coupled THM behavior of isotropic porous media, but most of the works are assuming the fluid flux and heat flux are dominated by the pore pressure gradient and thermal gradient, respectively (Ghassemi et al., 2002; Ghassemi et al., 2009; Valov et al., 2022). That is to say, the thermo-osmosis denoted by k_{pt} (fluid flux generated by thermal gradient) and mechano-caloric effects denoted by k_{tp} (heat flux generated by pore pressure gradient) are neglected in the transport equations

$$\vec{q} = -{k}/{u}\,\nabla p + k_{pt}\nabla T \,, \tag{1}$$

$$\vec{h} = k_{tp}\nabla p - k_T\nabla T \,. \tag{2}$$

Here $\vec{q}$ is ~~denoting~~ the fluid flux, ~~and~~ $\vec{h}$ is the heat flux, ~~while~~ p is the pore pressure, and T is the temperature field. Note~~d~~ that the mechano-caloric coefficient is also known as thermal filtration coefficient (Cheng, 2016).

Although thermo-osmosis and mechano-caloric effects are often neglected (if for no other reason, this assumption greatly simplifies solution methods), for porous material with low permeability, these two effects can play important roles (Gonçalvès et al., 2010; Roshan et al., 2015; Trémosa et al., 2010). For example, Carnahan (1983) has shown that the thermo-~~osmosis~~ Osmotic flow through kaolinite can be two orders of magnitude higher than Darcy's flow (that is, the fluid flux driven by the pressure gradient term in Eq. (1)) near a nuclear waste repository. Thus, when designing the cementing under the HTHP conditions, both the thermo-osmosis and mechano-caloric effects should be taken into consideration and should not be dismissed at the outset of the solution. To the best of our knowledge, under the HTHP conditions, their influences on the cement integrity are still unclear.

Commented [BAP7]: I'm sure that somehow, maybe indirectly, one could propose additives to help address these issues. Suggest to leave out the line I just deleted so it doesn't raise reviewer concerns.

Commented [LY8R7]: Agree.

Commented [BAP9]: Are you sure this is the first? Is that what is referenced in Wang's book?

Commented [LY10R9]: Sorry – the firstly word here is indeed confusing. I was originally wanting to emphasis the variational Lagrangian thermodynamics approach that he used. I deleted it here. And added the Biot (1941) to the Endnote Library and reference as well.

Commented [BAP11R9]: OK - but here it still reads that his 1977 paper was first to put thermal part into poroelastic framework. Is this correct?

Commented [LY12R9]: Yes.

By introducing a fully-coupled porothermoelastic model, which incorporates both of the thermo-osmosis and the mechano-caloric (thermal filtration) effects, dubbed here as "porothermoelastic-osmosis-filtration" (PTEOF), the present work uses a cylindrical geometry and boundary conditions inspired by cement plugs for P&A as an example to highlight the cementing challenges that are associated with HTHP conditions. The motivation of creating the PTEOF model is to have a comprehensive understanding of the cement behaviors under the HTHP conditions. and to build up a general framework and solutions for future cementing studies ~~and analysis~~. Specifically, we derive a semi-analytical solution (analytical up to a numerical inversion of a Laplace transport) that draws inspiration from the method of Sarout et al. (2011) and therefore leverages the mathematical similarity between PTEOF and linear chemo-poroelasticity. After presenting the governing equations, solution method, and examples of the behaviors predicted by the model, the key parameters associated with preventing tensile effective stresses from developing in a cement plug are identified. These are identified through a combination of dimensional analysis and pairwise bivariate analysis, leading to dimensionless groups that define a parametric space with regions that are "safe" and regions that are associated with material parameter combinations more likely to sustain damage. The work concludes with a discussion of implications for design of cement materials that do not just pursue the lowest possible permeability, but rather pursue a combination of material properties that will provide the necessary isolation without incurring elevated risk of damage from PTEOF phenomena under HTHP conditions.

2. Governing Equations

The formulation begins with classical tensorial strain-stress constitutive relation that expatiates upon the coupled thermo-hydro-mechanical behaviors of fluid saturated porous medium could be rewritten as (Cheng, 2016)

Should also write this in coordinate free notation to match other equations.

$$\sigma_{ij} = 2G\varepsilon_{ij} + \frac{2vG}{1-2v}\delta_{ij}\varepsilon - \alpha p - \alpha_d T, \tag{3}$$

where σ_{ij} and ε_{ij} is stress tensor components and strain tensor components, respectively; ε is the volumetric strain (the trace of the strain tensor); δ_{ij} is the Kronecker delta; p is pore pressure change from virgin pore pressure and T is temperature change from the reference temperature T_0; G is shear modulus and v is Poisson's ratio; α is the Biot effective stress coefficient and α_d is the thermoelastic effective stress coefficient. Note that, following the sign convention in Detournay et al. (1988), positive stress is considered to be tensile within the present work.

Furthermore, based on the fundamental work of Cheng (2016) and Wang (2017), within the framework of linear thermoporoelasticity, the coupled volumetric response relations are

$$\begin{pmatrix} \mathcal{E} \\ \zeta \\ \mathcal{S} \end{pmatrix} = \mathcal{N} \begin{pmatrix} \sigma \\ p \\ T \end{pmatrix}, \quad \text{where } \mathcal{N} = \begin{pmatrix} -\dfrac{1}{K} & \dfrac{\alpha}{K} & \beta_d \\ -\dfrac{\alpha}{K} & \dfrac{\alpha}{BK} & -\beta_v \\ \beta_d & -\beta_v & m \end{pmatrix}. \tag{4}$$

Note that one of these three equations can be obtained from contraction on Eq. (3). In Eq. (4), $\mathcal{E}$ and σ are volumetric strain and stress, respectively, given as the traces of their respective tensors; ζ is the variation of fluid content per unit volume; $\mathcal{S}$ is entropy density. The material constants include the drained bulk modulus tensor K, Biot effective stress coefficient α, Skempton pore pressure coefficient B, coefficient of volumetric thermal expansion of porous media frame β_d, coefficient of volumetric thermal expansion of variations in fluid content in the solid-fluid system β_v (Cheng, 2016), and m represents the specific heat of the porous medium at the reference temperature.

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Before moving on to the solution method, it is intuitive to reflect that from Eq. (4), deformation of the solid frame is caused by changes in stress, pore pressure, and/or temperature. The fluid phase in the porous medium is not only deforming with the solid frame, but at the same time, driven by pore pressure gradient and thermal forces, causing the pore fluid to be entering or leaving the solid frame of unit volume. Similarly, the stress and temperature change will cause the change of the entropy of the porous system based on the generalized-energy relation. The entropy density is therefore a function of volumetric strain of the solid frame, fluid content, and the change of temperature. Thus, the constitutive equations relate and couple volumetric strain, fluid content, and energy variables $\{\mathcal{E}, \zeta, \mathcal{S}\}$ with total stress, pore stress, and temperature variables $\{\sigma, p, T\}$ with the material constants $\{\alpha, B, K, \beta_d, \beta_v, m\}$. The notations ~~that are~~ used within this work are summarized in Table 1 ~~in Appendix~~ A.

I think this can also come to main body

Next, based on quasi-static equilibrium, the divergence of the stress tensor is taken to be zero, that is

$$\nabla \cdot \sigma = 0. \tag{5}$$

Furthermore, if the displacement is $\mathcal{U}$, then the classical small strain assumption is adopted whereby the strain-displacement relations are

probably want to somehow denote a tensor as distinct from σ

maybe $\bar{\bar{\sigma}}$ and $\bar{\bar{\varepsilon}}$ so that $\mathrm{tr}(\bar{\bar{\sigma}}) = \sigma$ (etc)

Commented [BAP13]: I like the rearranged section, but I see now that Eq 10 should come right after Eq 3. Then you can define some of the quantities in Eq 3 right there and use them in Eq 10. You could also move the blue-highlighted discussion to that point. Then you switch gears and get into the rest of the model, concluding the section with the yellow highlighted sentence.

Commented [LY14R13]: Rearranged accordingly, please check.

Commented [BAP15]: Write this in coordinate free notation, not indicial notation. This is actually correct only for rectangular coordinate system as you have written here. The version with \nabla \cdot \sigma is correct for any coordinate system.

Commented [LY16R15]: Updated, please check

From memory, I think this should be $\underline{\underline{\varepsilon}} = \frac{1}{2}(\nabla \bar{u} + \nabla^T \bar{u})$ Please check

$$\varepsilon = \frac{1}{2}(\nabla \mathcal{U} + \nabla \mathcal{U}^T) \tag{6}$$

I suggest vector notation

Commented [BAP17]: This also needs to be written in coordinate free notation for reasons mentioned on previous equation.

Commented [LY18R17]: Updated, please check

Commented [BAP19]: Define U

Commented [LY20R19]: Defined in line 173. Updated, please check

Next, we consider the fluid in the pore spaces to be incompressible so that the divergence of the fluid flux (q) is directly balanced by rate of change of the variation of fluid content (ζ), hence (Cheng, 2016)

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot \vec{q} = 0. \tag{7}$$

make sure consistent w/ this vector notation.

A similar conservation law assumes the heat flux only through conduction and relates the divergence of the heat flux (h) to the rate of change of the entropy density ($\mathcal{S}$) according to (Cheng, 2016)

$$\frac{\partial \mathcal{S}}{\partial t} + \nabla \cdot \vec{h} = 0. \tag{8}$$

By substituting both transport laws (Eq. (1) and Eq. (2)) into the conservation laws, the fully coupled diffusion equations are obtained as

$$\frac{\partial \zeta}{\partial t} = k \nabla^2 p - k_{pT} \nabla^2 T, \tag{9}$$

and

$$\frac{\partial \mathcal{S}}{\partial t} = -k_{Tp} \nabla^2 p + k_T \nabla^2 T. \tag{10}$$

In these equations, the balancing of the rate of change of fluid content and entropy density with the Laplacian of the fluid pressure and temperature, respectively, comprise the classical uncoupled diffusion equations. However, these equations also have "off-diagonal" terms that relate rate of change of fluid content to the Laplacian of the temperature field, as well as the rate of change of entropy density to the Laplacian of the fluid pressure. These impacts are known as the thermal osmosis effect and thermal filtration effect, respectively. The model fully coupling all of these terms will henceforth be referred as the porothermoelastic-osmosis-filtration (PTEOF) model. While it may indeed be valid at times to neglect thermal osmosis and/or thermal filtration, here we retain the full coupling in order to elucidate conditions in which they may have an important impact.

With the addition of boundary and initial conditions (to be discussed later), Eqs. (1-10) comprise a complete model, sufficient to solve for all unknown quantities.

Commented [BAP21]: I'm not seeing where you state the initial conditions.

Commented [LY22R21]: Addressed in the line between #317-#323

capitalize
S

3. PTEOF ~~solution~~ Solution for Constrained Cylinder

3.1 Problem description ~~and~~ boundary conditions

Consider a thermoporoelastic cylinder of radius R and length 2L (Figure 1a). Both of its ends are jacketed which is fixed and impermeable for fluid and thermal isolation from heat. The radial boundary conditions are unjacketed and can have fluid and heat exchange at the boundary. This constrained cylinder geometry is inspired by the geometry and boundary conditions of the primary plug in P&A (Figure 1a) where the length of the primary plug is usually 50 to 100 times larger than its diameter that often ranges from 5 to 20 inches and its loadings mainly co~~mes~~ from the high temperature ~~the~~ pore pressure from the formation and the far-field in-situ stress. Thus, it is appropriate to apply the generalized plane-strain assumption where the pore pressure and thermal diffusio~~n~~ only appear in the isotropic plane that is perpendicular to the length axis of the plug which is fully saturated. U~~nder the plane-strain conditions,~~ the ra~~dial stress can be expressed as (Cheng~~, 2016)

$$\sigma_{rr} \cancel{= 2Gv/(1-2v)\varepsilon + 2Ge_{rr} - \alpha p} - \alpha_d T . \quad (11)$$

this is just Eq3 – can it be cut?

Is Eq3 already plane strain?
If so, need to mention that when introducing Eq 3.

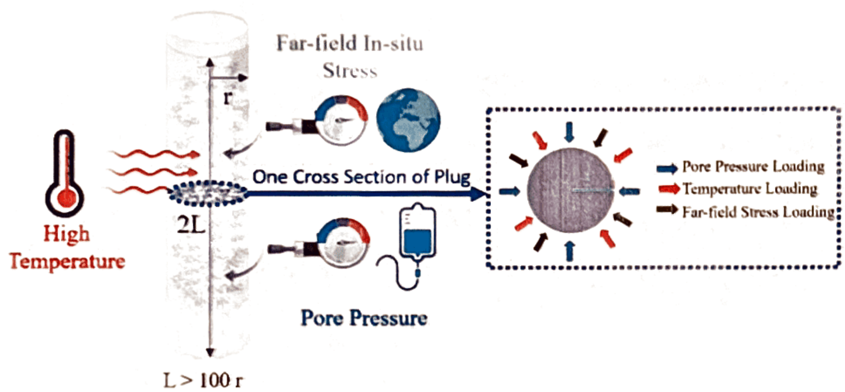

Figure 1(a) Sketch showing a primary plug in P&A and its boundary conditions; (b) A ~~zoom-in sketch showing a~~ cross-section of the plug in plane-strain conditions and its boundary conditions.

In line with the loading decomposition scheme proposed by Detournay et al. (1988) in the context of poroelasticity, the PTEOF model can be decomposed into three sub-loading cases to simplify the analysis. These are: 1) pore pressure loading ($\cancel{p^{(1)}}$), 2) temperature loading ($\cancel{T^{(2)}}$), and 3) isotropic far-field stress loading ($\cancel{\sigma^{(3)}}$), w~~here the superscript i is denoting by the stress field that induced by the loading mode j~~

Need page #s!

I don't understand the notation. See below for my suggestion