MATH 132A

Project 1 Linear Programming

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Description of the project: Auto Assembly

A large company manufacturing automobiles produces trucks, small cars, and midsized luxury cars. One plant close to Detroit, MI, assembles two models of midsized luxury cars. The first model, the Family Adventurer, is a four-door sedan with vinyl seats, plastic interior, standard features, and excellent gas mileage. It is marketed as a smart buy for middle-class families with tight budgets, and each Family Adventurer sold generates a modest profit of $3,700 for the company. The second model, the Classic Transporter, is a two-door luxury sedan with leather seats, wooden interior, custom features, and navigational capabilities. It is marketed as a privilege of affluence for upper-middle-class families, and each Classic Transporter sold generates a profit of $5, 300 for the company.

William Smith, the manager of the plant, is currently deciding the production schedule for the next month. Specifically, he must decide how many Family Adventurers and how many Classic Transporters to assemble in the plant to maximize the profit for the company. He knows that the plant possesses a capacity of 48,500 labor-hours during the month. He also knows that it takes 6 labor-hours to assemble one Family Adventurer and 10.5 labor-hours to assemble one Classic Transporter. Because the plant is simply an assembly plant, the parts required to assemble the two models are not produced at the plant. They are instead shipped from other plants around the Michigan area to the assembly plant. For example, tires, steering wheels, windows, seats and doors all arrive from various supplier plants. For the next month, William knows that he will be able to obtain only 20,000 doors (10,000 left-hand doors and 10,000 right-hand doors) from the door supplier. A recent labor strike forced the shutdown of that particular supplier plant for several days, and that plant will not be able to meet its production schedule for the next month. Both the Family Adventurer and the Classic Transporter use the same door part.

In addition, a recent company forecast of the monthly demands for different automobile models suggests that the demand for the Classic Transporter is limited to 3,500 cars. There is no limit on the demand for the Family Adventurer within the capacity limits of the assembly plant.

Problem 1. Formulate a linear programming model for this problem to determine the number of Family Adventurers and Classic Transporters that should be assembled. Explain if a linear model is appropriate for this problem.

Based on the problem description, we can list all the information in the following table:

	Number of doors	Profit	Time
Family Adventurer	4	$3,700	6h
Classic Transporter	2	$5,300	10.5h

We know that the plant possesses a capacity of 48,500 labor-hours during the month, and

William can only obtain 20,000 doors (10,000 left-hand doors and 10,000 right-hand doors) from the door supplier. What's more, the demand for the Classic Transporter is limited to 3,500 cars. Suppose the decision variables x_1 represents the number of Family Adventurer (first model) sold in the month, and x_2 represents the number of Classic Transporter (second model) sold in the month. In this problem, we want to maximize the profit. Therefore, our objective function is $z(x_1, x_2) = 3700x_1 + 5300x_2$. We can formulate the linear programming model as the following:

$$\begin{array}{ll} \textbf{Maximize} & 3700x_1 + 5300x_2 \\ \textbf{Subject to:} & 4x_1 + 2x_2 \leq 20000 \\ & 6x_1 + 10.5x_2 \leq 48500 \\ & x_2 \leq 3500 \\ & x_1 \geq 0, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{array}$$

Now we check the four assumptions of linear programming.
1. **Proportionality**: The problem satisfies proportionality since the coefficients of the decision variables are 3700, 5300, 4, 2, 6, 10.5, and 1, which are all constants.
2. **Additivity**: The problem satisfies additiviy because all variables are added or subtracted together.
3. **Divisibility**: The problem does not satisfy divisibility because we can only sell an entire car but not part of a car at a time. Thus, the solution to the problem must be integers. This is not a serious concern since we can use integer programming.
4. **Certainty**: In the short term, we can assume that the problem satisfies the certainty.

Problem 2. Solve the problem using Lingo or Excel and interpret the result. Determine if this problem has a unique solution.

We input our equations into Lingo and solve them.

Lindo Model – Lingo1.ltx*

```
max 3700x1+5300x2
subject to
4x1+2x2<20000
6x1+10.5x2<48500
x2<3500
x1>0
x2>0
end

GIN x1
GIN x2
```

Figure 1: Code in Lingo to solve the problem.

```
Solution Report – Lingo1.ltx
LINGO/OSX64 20.0.8 (17 Oct 2022), LINDO API 14.0.5099.166

Licensee info: Eval Use Only
License expires: 26 JUL 2023

Global optimal solution found.
Objective value:                              0.2700930E+08
Objective bound:                              0.2700930E+08
Infeasibilities:                               0.000000
Extended solver steps:                                0
Total solver iterations:                              3
Elapsed runtime seconds:                           0.13

Model Class:                                       PILP

Total variables:                    2
Nonlinear variables:                0
Integer variables:                  2

Total constraints:                  6
Nonlinear constraints:              0

Total nonzeros:                     9
Nonlinear nonzeros:                 0


                              Variable           Value        Reduced Cost
                                    X1        3766.000           -3700.000
                                    X2        2467.000           -5300.000

                                   Row    Slack or Surplus      Dual Price
                                     1       0.2700930E+08        1.000000
                                     2        2.000000            0.000000
                                     3       0.5000000            0.000000
                                     4        1033.000            0.000000
                                     5        3766.000            0.000000
                                     6        2467.000            0.000000
```

Figure 2: Report from Lingo.

From Figure 2, we can see that when $x_1 = 3766$ and $x_2 = 2467$, the profit is \$27,009,300, which is the maximum. This implies that when the plant assembles 3766 Family Adventurers and 2467 Classic Transporters, the profit can be maximized and it equals to \$27,009,300.

This problem has a unique solution. We draw the feasible region of this linear programming model (Figure 3). We can notice that when we move the function $3700x_1 + 5300x_2$ in the direction to maximize it, the last intersection between the function and the feasible region is at the point (3766.667, 2466.667) (the black line). Since the results of this problem must be integers, we now check all the integer points around (3766.667, 2466.667).

(1) $x_1 = 3766, x_2 = 2467$: This is the result solved by Lingo. From the above report, we know that the profit is \$27,009,300.
(2) $x_1 = 3767, x_2 = 2466$: We get $3767 \times 3700 + 2466 \times 5300 = 27007700$. Since \$27,007,700 $<$ \$27,009,300, this point is not the optimal solution.
(3) $x_1 = 3766, x_2 = 2466$: We get $3766 \times 3700 + 2466 \times 5300 = 27004000$. Since \$27,004,000 $<$ \$27,009,300, this point is not the optimal solution.
(4) $x_1 = 3767, x_2 = 2467$: Since $3767 > 3766.667$ and $2467 > 2466.667$ at the same time, it is not possible for this integer point to be in the feasible region. Thus, we don't consider this point.

Therefore, the optimal solution $x_1 = 3766, x_2 = 2467$ is unique.

Figure 3: Feasible Region of the problem.

Problem 3. The marketing department knows that it can pursue a targeted $500,000 advertising campaign that will raise the demand for the Classic Transporter next month by 20 percent. Should the campaign be undertaken?

Since the demand for the Classic Transporter will be raised by 10 percent next month, we update the restriction to $x_2 \leq 3500 \times 1.2 = 4200$. We can formulate the following linear programming model.

$$\begin{array}{ll} \textbf{Maximize} & 3700x_1 + 5300x_2 - 500000 \\ \textbf{Subject to:} & 4x_1 + 2x_2 \leq 20000 \\ & 6x_1 + 10.5x_2 \leq 48500 \\ & x_2 \leq 4200 \\ & x_1 \geq 0, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{array}$$

We input our equations into Lingo and solve them.

Lindo Model – Lingo1.ltx

```
max 3700x1+5300x2
subject to
4x1+2x2<20000
6x1+10.5x2<48500
x2<4200
x1>0
x2>0
end

GIN x1
GIN x2
```

Figure 4: Code in Lingo to solve the problem.

Solution Report – /Lingo1.ltx

```
LINGO/OSX64 20.0.8 (17 Oct 2022), LINDO API 14.0.5099.166

Licensee info: Eval Use Only
License expires: 26 JUL 2023

Global optimal solution found.
Objective value:                              0.2700930E+08
Objective bound:                              0.2700930E+08
Infeasibilities:                               0.000000
Extended solver steps:                                0
Total solver iterations:                              3
Elapsed runtime seconds:                           2.55

Model Class:                                       PILP

Total variables:                      2
Nonlinear variables:                  0
Integer variables:                    2

Total constraints:                    6
Nonlinear constraints:                0

Total nonzeros:                       9
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                                    X1        3766.000           -3700.000
                                    X2        2467.000           -5300.000

                                   Row    Slack or Surplus      Dual Price
                                     1       0.2700930E+08        1.000000
                                     2        2.000000            0.000000
                                     3        0.5000000           0.000000
                                     4        1733.000            0.000000
                                     5        3766.000            0.000000
                                     6        2467.000            0.000000
```

Figure 5: Code in Lingo to solve the problem.

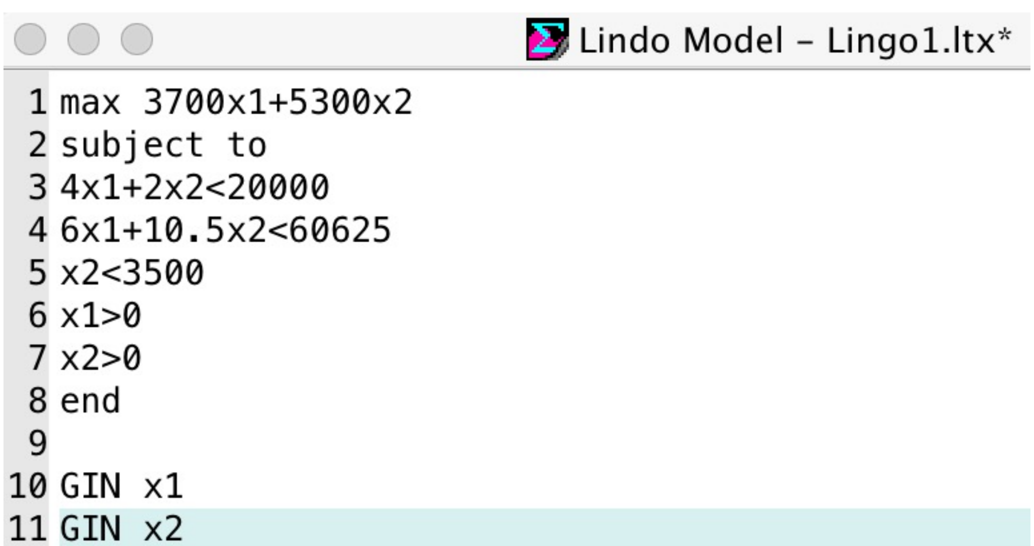
From Figure 5, we can see that when $x_1 = 3766$ and $x_2 = 2467$, the profit is \$27,009,300 − \$500,000 = \$26,509,300, which is the maximum. This implies that when the plant assembles 3766 Family Adventurers and 2467 Classic Transporters, the profit can be maximized and it equals to \$26,509,300. The company should not undertake the campaign since the profit \$26,509,300 < \$27,009,300.

Problem 4. William knows that he can increase next month's plant capacity by using overtime labor. He can increase the plants labor-hour capacity by 25 percent. With the new assembly plant capacity, how many Family Adventurers and how many Classic Transporters should be assembled?

Since the plants labor-hour capacity can be increased by 25 percent, we update the restriction to $6x_1 + 10.5x_2 \leq 48500 \times 1.25 = 60625$. We can formulate the following linear programming model.

$$
\begin{array}{ll}
\textbf{Maximize} & 3700x_1 + 5300x_2 \\
\textbf{Subject to:} & 4x_1 + 2x_2 \leq 20000 \\
 & 6x_1 + 10.5x_2 \leq 60625 \\
 & x_2 \leq 3500 \\
 & x_1 \geq 0, x_2 \geq 0 \\
 & x_1, x_2 \in \mathbb{Z}
\end{array}
$$

We input our equations into Lingo and solve them.



```
Lindo Model – Lingo1.ltx*
max 3700x1+5300x2
subject to
4x1+2x2<20000
6x1+10.5x2<60625
x2<3500
x1>0
x2>0
end

GIN x1
GIN x2
```

Figure 6: Code in Lingo to solve the problem.

Solution Report – Lingo1.ltx

```
LINGO/OSX64 20.0.8 (17 Oct 2022), LINDO API 14.0.5099.166

Licensee info: Eval Use Only
License expires: 26 JUL 2023

Global optimal solution found.
Objective value:                              0.3057500E+08
Objective bound:                              0.3057500E+08
Infeasibilities:                               0.000000
Extended solver steps:                                0
Total solver iterations:                              0
Elapsed runtime seconds:                           0.08

Model Class:                                       PILP

Total variables:                      2
Nonlinear variables:                  0
Integer variables:                    2

Total constraints:                    6
Nonlinear constraints:                0

Total nonzeros:                       9
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                                    X1        3250.000           -3700.000
                                    X2        3500.000           -5300.000

                                   Row    Slack or Surplus      Dual Price
                                     1       0.3057500E+08        1.000000
                                     2        0.000000            0.000000
                                     3        4375.000            0.000000
                                     4        0.000000            0.000000
                                     5        3250.000            0.000000
                                     6        3500.000            0.000000
```

Figure 7: Code in Lingo to solve the problem.

From Figure 7, we can see that when $x_1 = 3250$ and $x_2 = 3500$, the profit is \$30,575,000, which is the maximum. This implies that when the plant assembles 3250 Family Adventurers and 3500 Classic Transporters, the profit can be maximized and it equals to \$30,575,000.

Problem 5. William knows that overtime labor does not come without an extra cost. What is the maximum amount he should be willing to pay for all overtime labor beyond the cost of this labor at regular time rates?

As long as the total profit with overtime labor is still higher than the profit without overtime labor, William should be willing to use overtime labor. Thus, the maximum amount he would pay for all overtime labor should be \$30,575,000 − \$27,009,300 = \$3,565,700.

Problem 6. William explores the option of using both the targeted advertising campaign and the overtime labor-hours. The advertising campaign raises the demand for the Classic Transporter by 20 percent, and the overtime labor increases the plants labor-hour capacity by 25 percent. How many Family Adventurers and how many Classic Transporters should be assembled using the advertising campaign and overtime labor-hours if the profit from each Classic Transporter sold and each Family Adventurer sold continue to be the same?

Since the demand for the Classic Transporter raises by 20 percent when using the advertising campaign and the labor-hour capacity increases by 25 percent when using overtime labor-hours, we update the restriction to $6x_1 + 10.5x_2 \leq 48500 \times 1.25 = 60625$ and $x_2 \leq 3500 \times 1.2 = 4200$. Since the profit from each Classic Transporter sold and each Family Adventurer sold continue to be the same, we can formulate the following linear programming model.

$$
\begin{array}{ll}
\textbf{Maximize} & 3700x_1 + 5300x_2 \\
\textbf{Subject to:} & 6x_1 + 10.5x_2 \leq 60625 \\
& 4x_1 + 2x_2 \leq 20000 \\
& x_2 \leq 4200 \\
& x_1 \geq 0, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{array}
$$

We input our equations into Lingo and solve them.

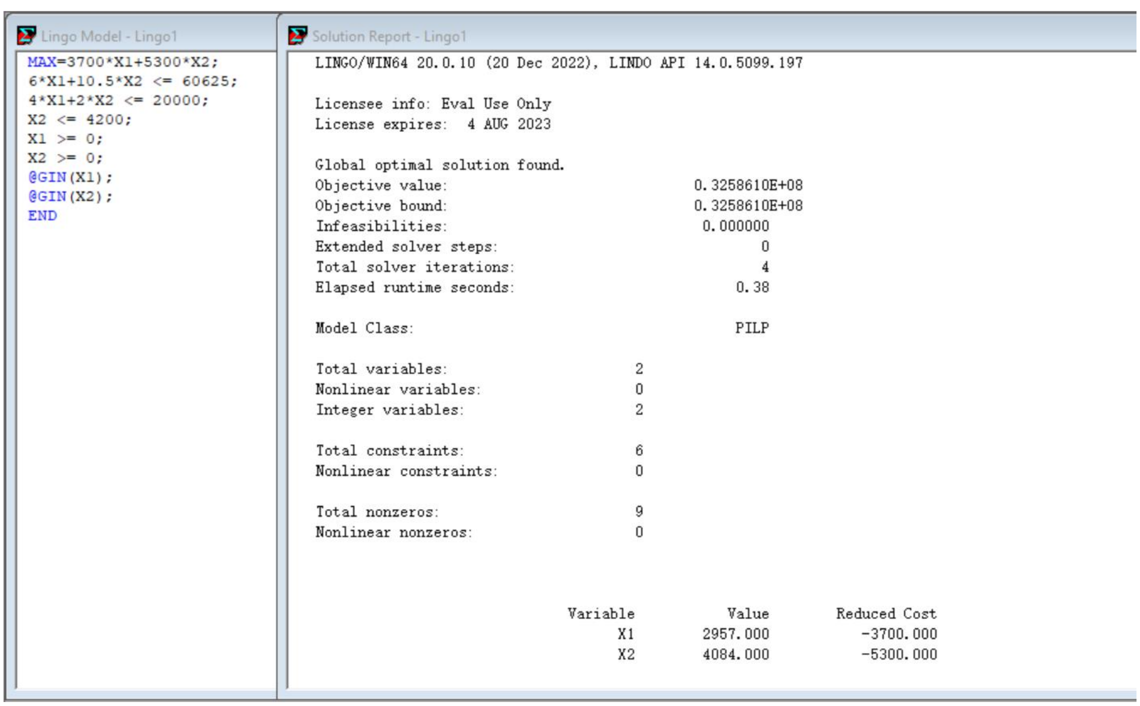


Figure 8: Code in Lingo to solve the problem.

From Figure 8, we can see that when $x_1 = 2957$ and $x_2 = 4084$, the profit is \$32,586,100, which is the maximum. Therefore, 2957 Family Adventurers and 4084 Classic Transporters should be assembled using the advertising campaign and overtime labor-hours if the profit from each Classic Transporter sold and each Family Adventurer sold continue to be the same.

Problem 7. Knowing that the advertising campaign costs \$500,000 and the maximum usage of overtime labor-hours cost \$1,600,000, is the solution found in part (6) a wise decision compared to the solution found in part (1)?

Since the advertising campaign costs \$500,000 and the maximum usage of overtime labor-hours cost \$1,600,000, the total cost is $\$500,000 + \$1,600,000 = \$2,100,000$. We update the objective function to **Maximize** $3700x_1 + 5300x_2 - 2100000$. Now we can formulate the following linear programming model.

$$
\begin{array}{ll}
\textbf{Maximize} & 3700x_1 + 5300x_2 - 2100000 \\
\textbf{Subject to:} & 6x_1 + 10.5x_2 \leq 60625 \\
& 4x_1 + 2x_2 \leq 20000 \\
& x_2 \leq 4200 \\
& x_1 \geq 0, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{array}
$$

We input our equations into Lingo and solve them.

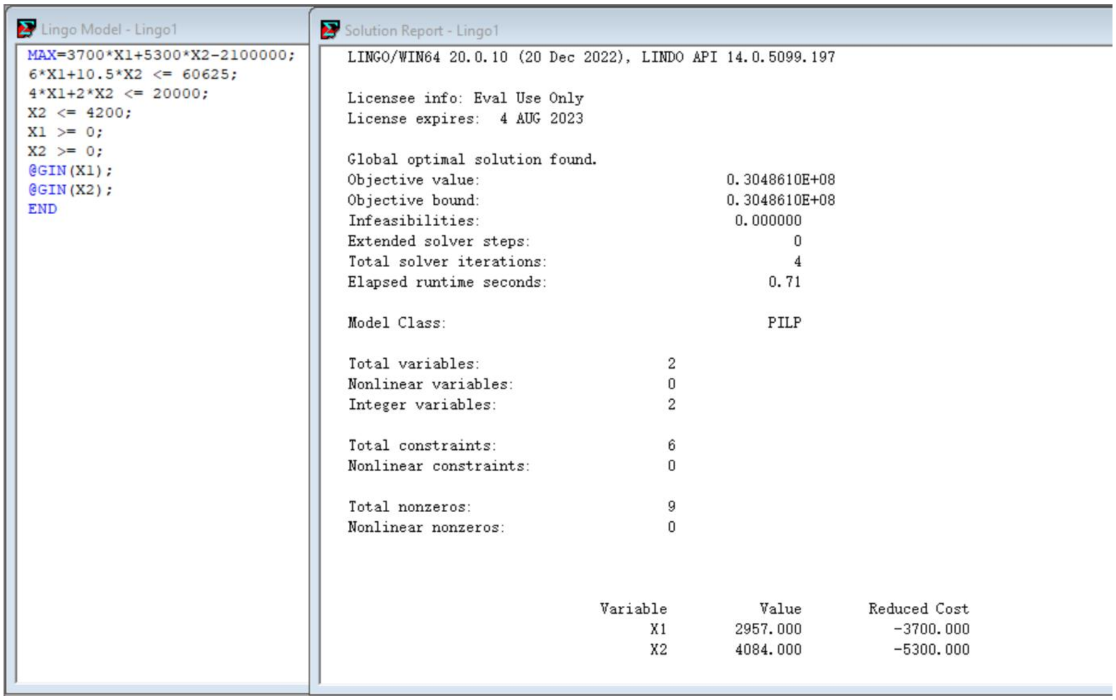


Figure 9: Code in Lingo to solve the problem.

From Figure 9, we can see that when $x_1 = 2957$ and $x_2 = 4084$, the profit is \$30,486,100, which is the maximum. The solution found in part (1) is \$ 27,009,300. Since $\$30,486,100 > \$27,009,300$, the solution found in part (6) is a wise decision compared to the solution found in part (1).

Problem 8. The company has determined that dealerships are actually heavily discounting the price of the Family Adventurers to move them off the lot. Because of a profit-sharing agreement with its dealers, the company is therefore not making a profit of \$3,700 on the Family Adventurer but is instead making a profit of \$2,800. Determine the number of Family Adventurers and the number of Classic Transporters that should be assembled given this new discounted price. Consider all the possible scenarios: using/not using advertising campaign and using/not using overtime labor. What is the most profitable this time?

Since the company is making a profit of \$2,800 on the Family Adventurer, we change the coefficient of x1 in the object function from \$3,700 to \$2,800. To find the most profitable possibility, we need to consider 4 cases: (i) not using advertising campaign and not using overtime labor (ii) using advertising campaign and not using overtime labor (iii) not using advertising campaign and using overtime labor (iv) using advertising campaign and using overtime labor.

(i) not using advertising campaign and not using overtime labor:

We update the objective function to **Maximize** $2800x_1 + 5300x_2$ and the restrictions to

$6x_1 + 10.5x_2 \leq 48500$ and $x_2 \leq 3500$. Now we can formulate the following linear programming model.

$$\begin{array}{ll} \textbf{Maximize} & 2800x_1 + 5300x_2 \\ \textbf{Subject to:} & 6x_1 + 10.5x_2 \leq 48500 \\ & 4x_1 + 2x_2 \leq 20000 \\ & x_2 \leq 3500 \\ & x_1 \geq 0, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{array}$$

We input our equations into Lingo and solve them.

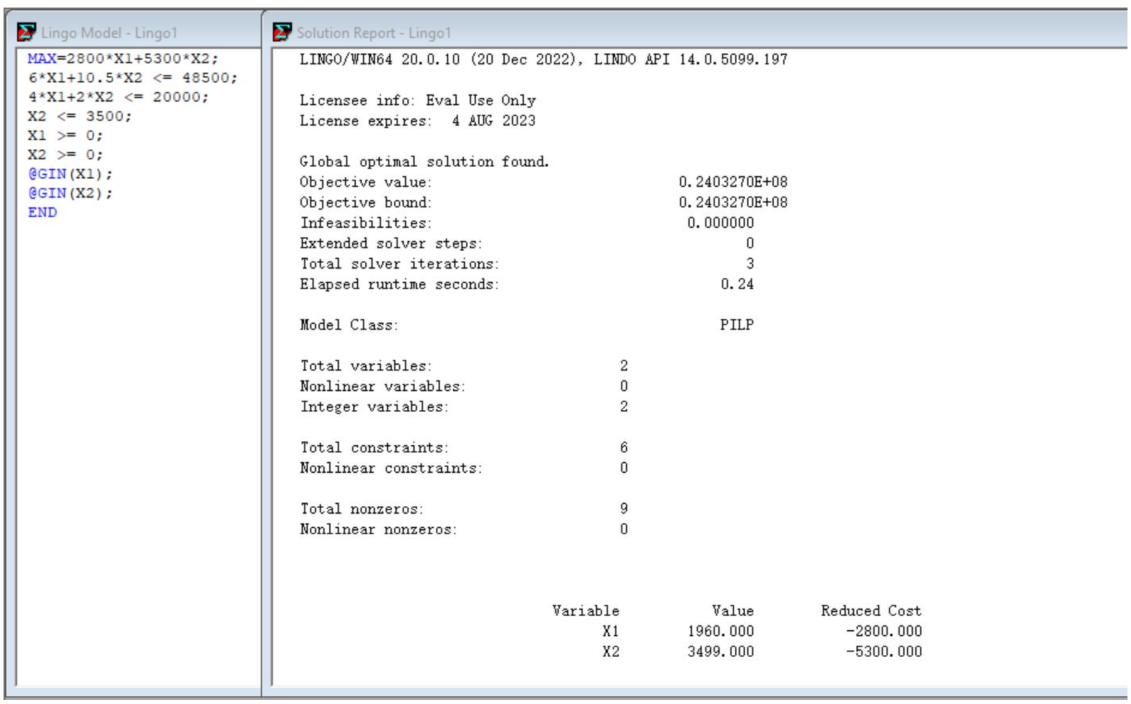


Figure 10: Code in Lingo to solve the problem.

From Figure 10, we can see that when $x_1 = 1960$ and $x_2 = 3499$, the profit is \$24,032,700, which is the maximum.

(ii) using advertising campaign and not using overtime labor:
We update the objective function to **Maximize** $2800x_1 + 5300x_2 - 500000$ and the restrictions to $6x_1 + 10.5x_2 \leq 48500$ and $x_2 \leq 4200$. Now we can formulate the following linear

programming model.

$$
\begin{aligned}
\textbf{Maximize} \quad & 2800x_1 + 5300x_2 - 500000 \\
\textbf{Subject to:} \quad & 6x_1 + 10.5x_2 \leq 48500 \\
& 4x_1 + 2x_2 \leq 20000 \\
& x_2 \leq 4200 \\
& x_1 \geq 0, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
$$

We input our equations into Lingo and solve them.

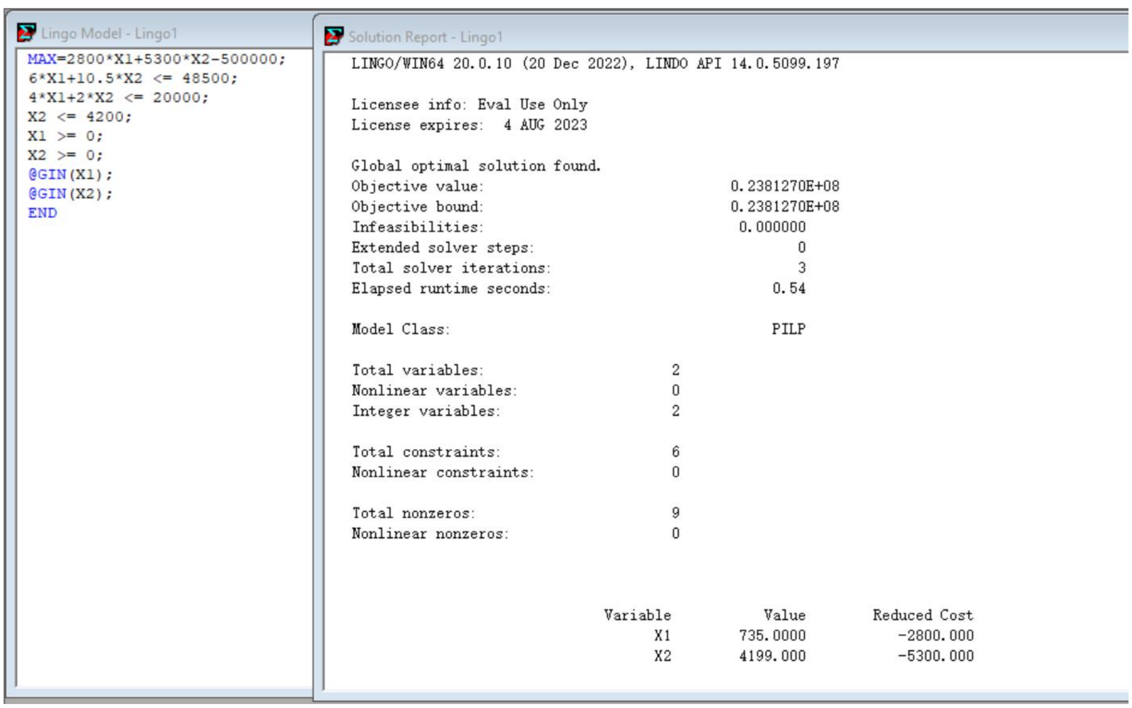


Figure 11: Code in Lingo to solve the problem.

From Figure 11, we can see that when $x_1 = 735$ and $x_2 = 4199$, the profit is \$23,812,700, which is the maximum.

(iii) not using advertising campaign and using overtime labor:

We update the objective function to **Maximize** $2800x_1 + 5300x_2 - 1600000$ and the restrictions to $6x_1 + 10.5x_2 \leq 60625$ and $x_2 \leq 3500$. Now we can formulate the following linear

programming model.

$$\begin{array}{ll} \textbf{Maximize} & 2800x_1 + 5300x_2 - 1600000 \\ \textbf{Subject to:} & 6x_1 + 10.5x_2 \leq 60625 \\ & 4x_1 + 2x_2 \leq 20000 \\ & x_2 \leq 3500 \\ & x_1 \geq 0, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{array}$$

We input our equations into Lingo and solve them.

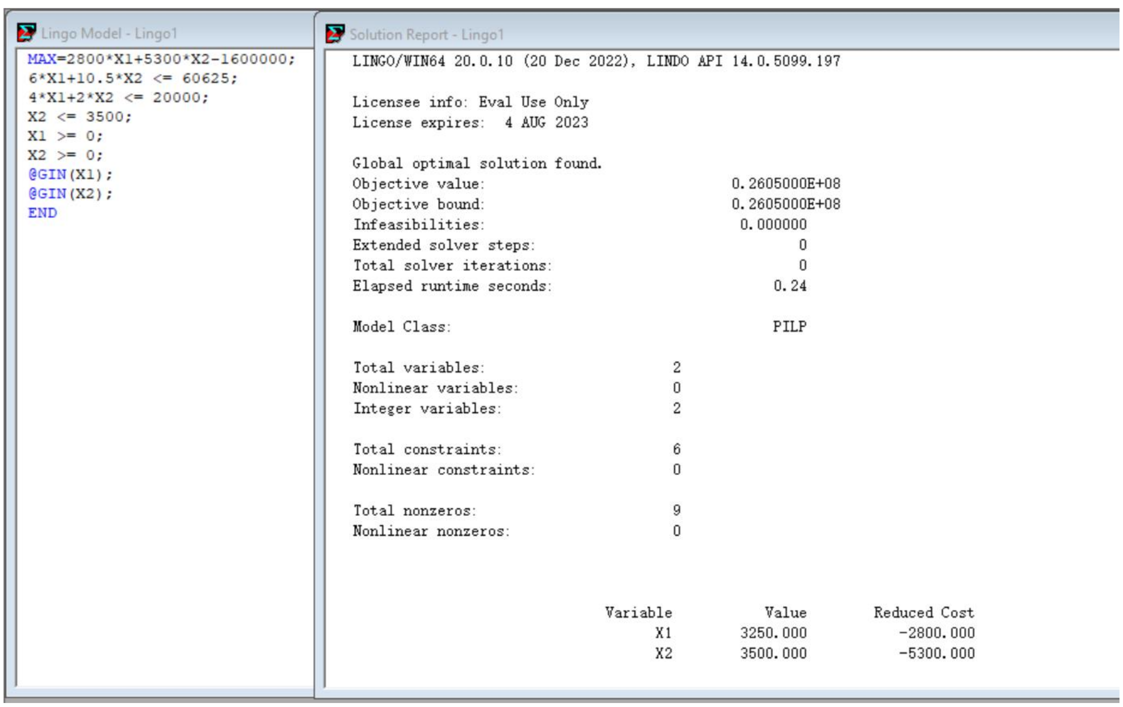


Figure 12: Code in Lingo to solve the problem.

From Figure 12, we can see that when $x_1 = 3250$ and $x_2 = 3500$, the profit is \$26,050,000, which is the maximum.

(iv) using advertising campaign and using overtime labor:

We update the objective function to **Maximize** $2800x_1 + 5300x_2 - 2100000$ and the restrictions to $6x_1 + 10.5x_2 \leq 60625$ and $x_2 \leq 4200$. Now we can formulate the following linear

programming model.

$$
\begin{aligned}
\textbf{Maximize} \quad & 2800x_1 + 5300x_2 - 2100000 \\
\textbf{Subject to:} \quad & 6x_1 + 10.5x_2 \leq 60625 \\
& 4x_1 + 2x_2 \leq 20000 \\
& x_2 \leq 4200 \\
& x_1 \geq 0, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
$$

We input our equations into Lingo and solve them.

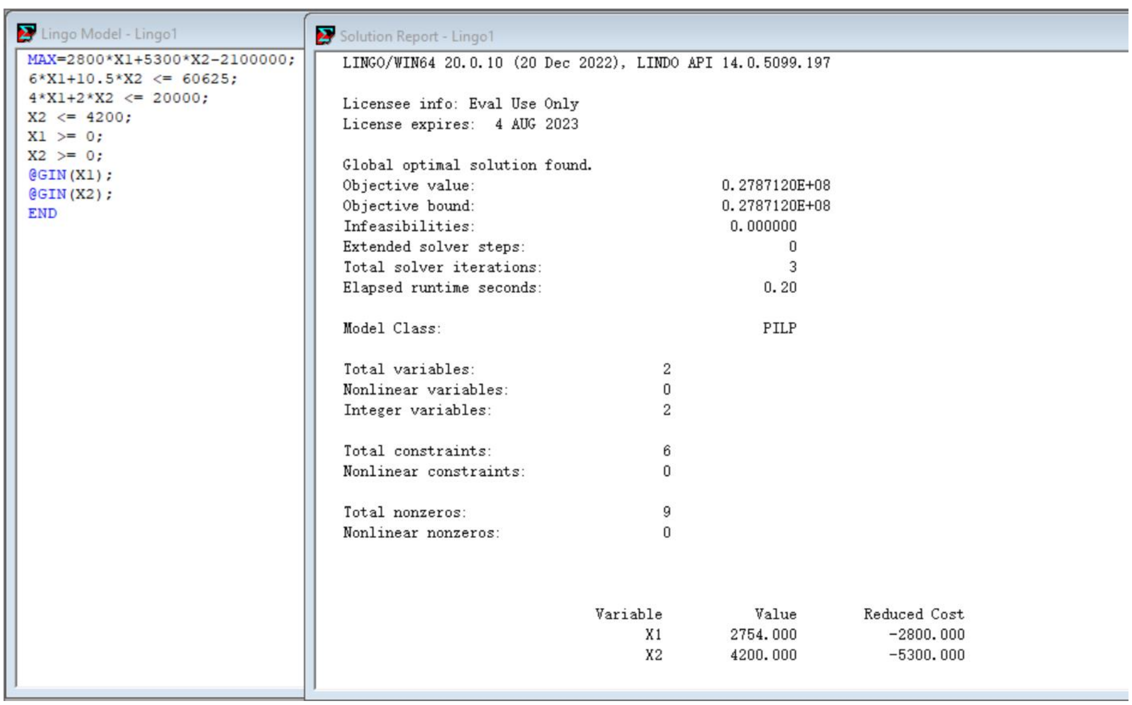


Figure 13: Code in Lingo to solve the problem.

From Figure 13, we can see that when $x_1 = 2754$ and $x_2 = 4200$, the profit is \$27,871,200, which is the maximum.

Therefore, according to these 4 cases, we see that advertising campaign and overtime labor should be used to get the maximum profit, which is \$27,871,200. In this case, 2754 Family Adventurers and 4200 Classic Transporters should be assembled given this new discounted price.

Problem 9. The company has discovered quality problems with the Family Adventurer by randomly testing Adventurers at the end of the assembly line. Inspectors have discovered that in over 60 percent of the cases, two of the four doors on an Adventurer do not seal properly. Because the percentage of defective Adventurers determined by the random testing is so high, the floor supervisor has decided to perform quality control tests on every Adventurer at the end of the line. Because of the added tests, the time it takes to assemble one Family Adventurer has increased from 6 to 7.5 hours. Determine the number of units of each model that should be assembled given the new assembly time for the Family Adventurer.

Since the time it takes to assemble one Family Adventurer has increased from 6 to 7.5 hours, We update the objective function to **Maximize** $3700x_1 + 5300x_2$ and the restriction to $7.5x_1 + 10.5x_2 \leq 48500$. Now we can formulate the following linear programming model.

$$
\begin{aligned}
\textbf{Maximize} \quad & 3700x_1 + 5300x_2 \\
\textbf{Subject to:} \quad & 7.5x_1 + 10.5x_2 \leq 48500 \\
& 4x_1 + 2x_2 \leq 20000 \\
& x_2 \leq 3500 \\
& x_1 \geq 0, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
$$

We input our equations into Lingo and solve them.

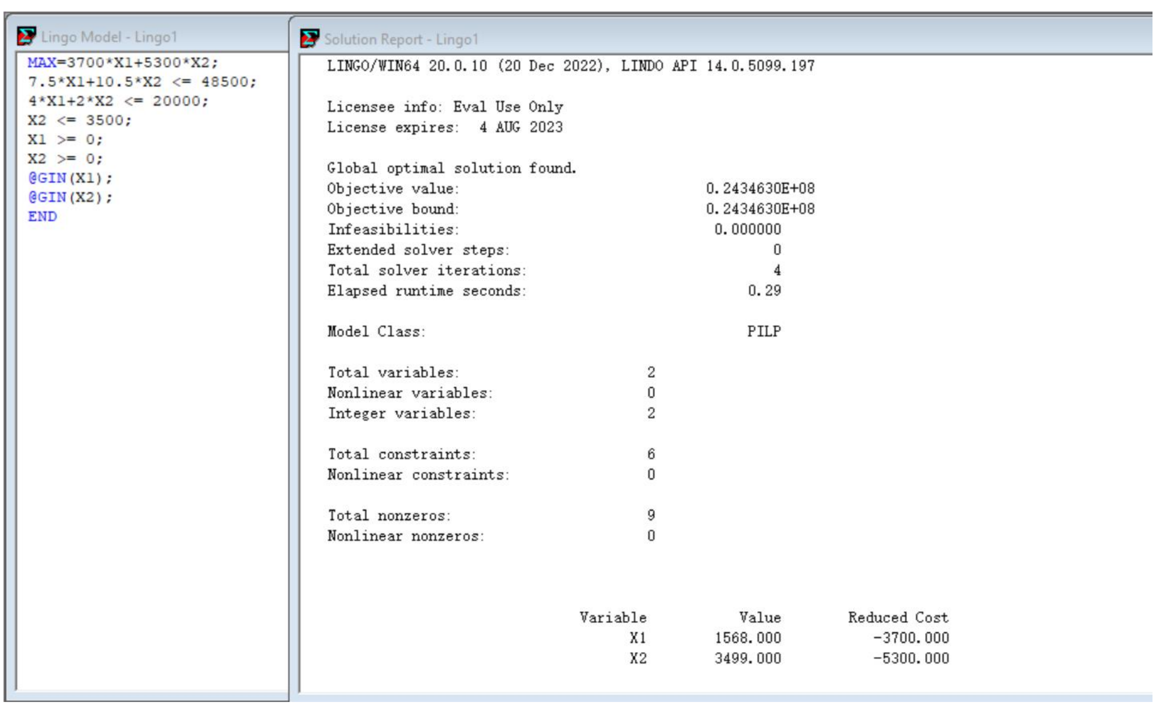


Figure 14: Code in Lingo to solve the problem.

From Figure 14, we can see that when $x_1 = 1568$ and $x_2 = 3499$, the profit is \$24,346,300, which is the maximum. Therefore, 1568 Family Adventurers and 3499 Classic Transporters should be assembled given the new assembly time for the Family Adventurer.

Problem 10. The board of directors of the automobile company wishes to capture a larger share of the luxury sedan market and therefore would like to meet the full demand for Classic Transporters. They ask William to determine by how much the profit of his assembly plant would decrease as compared to the profit found in part (1). They then ask him to meet the full demand for Classic Transporters if the decrease in profit is not more than \$2,000,000. Should he meet the full demand?

Since the board of directors of the automobile company would like to meet the full demand for Classic Transporters, $x_2 = 3500$. We update the objective function to **Maximize** $3700x_1 + 5300 \times 3500$ and the restriction to $6x_1 + 10.5 \times 3500 \leq 48500$ and $4x_1 + 7000 \leq 20000$. Now we can formulate the following linear programming model.

$$\begin{array}{ll}
\textbf{Maximize} & 3700x_1 + 5300 \times 3500 \\
\textbf{Subject to:} & 6x_1 + 10.5 \times 3500 \leq 48500 \\
& 4x_1 + 7000 \leq 20000 \\
& x_1 \geq 0, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{array}$$

We input our equations into Lingo and solve them.

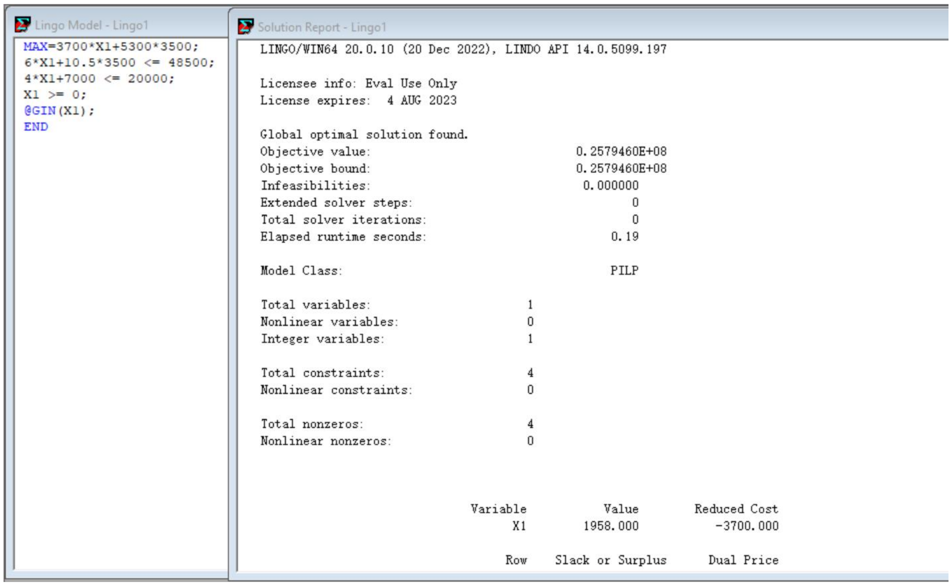


Figure 15: Code in Lingo to solve the problem.

From Figure 15, we can see that when $x_1 = 1958$, the profit is \$25,794,600, which is the maximum. We see that the maximum profit when the full demand is met is \$25,794,600, and the profit found in (1) is \$27,009,300. Therefore, William would decrease $\$27,009,300 - \$25,794,600 = \$1,214,700$ profit compared to the profit found in (1). Since $\$1,214,700 < \$2,000,000$, he should meet the full demand.

Problem 11. William now makes his final decision by combining all the new considerations described in parts (6), (7), and (8). What are his final decisions on whether to undertake the advertising campaign, whether to use overtime labor, the number of Family Adventurers to assemble, and the number of Classic Transporters to assemble?

From parts (6), (7), and (8), we know that William can use overtime labor which increases labor capacity by 25 percent with \$1,600,000 extra cost, use a campaign which increases the demand for Classic Transporter by 20 percent with \$500,000 extra cost, and the profit for Family Adventurer can be either \$3,700 or \$2,800. We will discuss all the situations in the following.

Case 1: The profit of Family Adventures is \$2,800.
In this case, the problem would be the same as Problem 8. We will have the following results:
(1) **Not using advertising campaign, not using overtime labor**: The company should assemble 1960 Family Adventurers and 3499 Classic Transporters, and the profit will be \$24,032,700, which is the maximum.
(2) **Not using advertising campaign, using overtime labor**: The company should assemble 3250 Family Adventurers and 3500 Classic Transporters, and the profit will be \$26,050,000, which is the maximum.

(3) **Using advertising campaign, not using overtime labor**: The company should assemble 735 Family Adventurers and 4199 Classic Transporters, and the profit will be \$23,812,700, which is the maximum.
(4) **Using advertising campaign, using overtime labor**: The company should assemble 2754 Family Adventurers and 4200 Classic Transporters, and the profit will be \$27,871,200, which is the maximum.

Therefore, according to these 4 cases, we see that advertising campaign and overtime labor should be used to get the maximum profit, which is \$27,871,200. In this case, 2754 Family Adventurers and 4200 Classic Transporters should be assembled given this new discounted price.

Case 2: The profit of Classic Transporter is \$3,700.
(1) **Not using advertising campaign, not using overtime labor**: When the company doesn't use advertising campaign and overtime labor, the linear programming model should be the same as that in problem 1. Thus we have the solution that the company should assemble 1960 Family Adventurers and 3499 Classic Transporters, and the profit will be \$27,009,300, which is the maximum.
(2) **Not using advertising campaign, using overtime labor**: When the company doesn't use advertising campaign but uses overtime labor, the linear programming model should be the same as that in problem 4, and we just need to subtract the additional cost for the overtime labor. Thus the company should assemble 3250 Family Adventurers and 3500 Classic Transporters, and the profit will be \$30,575,000-\$1,600,000 = \$28,975,000, which is the maximum.
(3) **Using advertising campaign, not using overtime labor**: When the company uses advertising campaign but doesn't use overtime labor, the linear programming model will be the same as that in problem 3. Thus the company should assemble 735 Family Adventurers and 4199 Classic Transporters, and the profit will be \$26,509,300, which is the maximum.
(4) **Using advertising campaign, using overtime labor**: When the company uses both advertising campaign and overtime labor, we can formulate the following linear programming model.

$$
\begin{array}{ll}
\textbf{Maximize} & 3700x_1 + 5300x_2 - 2100000 \\
\textbf{Subject to:} & 6x_1 + 10.5x_2 \leq 60625 \\
& 4x_1 + 2x_2 \leq 20000 \\
& x_2 \leq 4200 \\
& x_1 \geq 0, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{array}
$$

We input our equations into Lingo and solve them.

Lindo Model – Lingo1.ltx*

```
max 3700x1+5300x2
subject to
4x1+2x2<20000
6x1+10.5x2<60625
x2<4200
x1>0
x2>0
end

GIN x1
GIN x2
```

Figure 16: Code in Lingo to solve the problem.

Solution Report – Lingo1.ltx

```
LINGO/OSX64 20.0.8 (17 Oct 2022), LINDO API 14.0.5099.166

Licensee info: Eval Use Only
License expires: 26 JUL 2023

Global optimal solution found.
Objective value:                              0.3258610E+08
Objective bound:                              0.3258610E+08
Infeasibilities:                               0.000000
Extended solver steps:                                0
Total solver iterations:                              4
Elapsed runtime seconds:                           0.10

Model Class:                                       PILP

Total variables:                      2
Nonlinear variables:                  0
Integer variables:                    2

Total constraints:                    6
Nonlinear constraints:                0

Total nonzeros:                       9
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                                    X1        2957.000           -3700.000
                                    X2        4084.000           -5300.000

                                   Row    Slack or Surplus      Dual Price
                                     1       0.3258610E+08        1.000000
                                     2        4.000000            0.000000
                                     3        1.000000            0.000000
                                     4        116.0000            0.000000
                                     5        2957.000            0.000000
                                     6        4084.000            0.000000
```

Figure 17: Code in Lingo to solve the problem.

From Figure 17, we can see that when $x_1 = 2957$ and $x_2 = 4084$, the profit is \$32,586,100 $-$ \$500,000 $-$ \$1,600,000 $=$ \$30,486,100, which is the maximum. This implies that when the plant assembles 2975 Family Adventurers and 4084 Classic Transporters, the profit can be maximized and it equals to \$30,486,100.

After comparing all the results above, we can notice that when the profit of Family Adventurer is \$3,700 and the company uses both advertising campaign and overtime labor, the total profit will be maximized. The company should assemble 2957 Family Adventurers and 4084 Classic Transporters, and the profit will be \$30,486,100.