3. This problem relates to the QDA model, in which the observations within each class are drawn from a normal distribution with a class-specific mean vector and a class specific covariance matrix. We consider the simple case where $p = 1$; i.e. there is only one feature.

Suppose that we have K classes, and that if an observation belongs to the kth class then X comes from a one-dimensional normal distribution, $X \sim N(\mu_k, \sigma_k^2)$. Recall that the density function for the one-dimensional normal distribution is given in (4.16). Prove that in this case, the Bayes classifier is *not* linear. Argue that it is in fact quadratic.

Hint: For this problem, you should follow the arguments laid out in Section 4.4.1, but without making the assumption that $\sigma_1^2 = \ldots = \sigma_K^2$.

$$P_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_l)^2\right)}$$

$$\Rightarrow \delta_k(x) = \log(\pi_k) - \frac{1}{2}\log(2\pi\sigma_k) - \frac{1}{2\sigma_k^2}(x-\mu_k)^2$$

$\because$ σ_k can't be ignored in QDA

$\therefore$ We should keep $(x-\mu_k)^2$, which is quadratic

The boundary of QDA is:

$$\delta_{k_1}(x) = \delta_{k_2}(x)$$

$\Rightarrow$ The boundary should be quadratic.

Suppose that we wish to predict whether a given stock will issue a dividend this year ("Yes" or "No") based on X, last year's percent profit. We examine a large number of companies and discover that the mean value of X for companies that issued a dividend was $\bar{X} = 10$, while the mean for those that didn't was $\bar{X} = 0$. In addition, the variance of X for these two sets of companies was $\hat{\sigma}^2 = 36$. Finally, 80 % of companies issued dividends. Assuming that X follows a normal distribution, predict the probability that a company will issue a dividend this year given that its percentage profit was $X = 4$ last year.

Hint: Recall that the density function for a normal random variable is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$. *You will need to use Bayes' theorem.*

$$p_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)}$$

$\sigma = 6 \quad \mu_{Yes} = 10 \quad \mu_{NO} = 0 \quad \pi_{Yes} = 0.8 \quad \pi_N = 0.2$

$$\Rightarrow P(Y = Yes \mid X = 4) = \frac{0.13 \times 0.6}{0.13 \times 0.6 + 0.03 \times 0.8}$$

$$= 0.76$$