linear algebra Gilbert Strang mit lecture 로 복습 ①
CS231n, python tutorial 참고

- Scaler : a single number $s \in \mathbb{R}$ (lower case) e.g 2.8
- Vector : an ordered list of number e.g $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$

$\boxed{\text{순서있음, 1-dim}}$ ⟷ ordered가 아닌 것은? ans) set

column vector, row vector 존재. e.g $x = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \in \mathbb{R}^3$ → column vector가 default.

- matrix : a two dimensional array of numbers.

e.g $A = \begin{bmatrix} 1 & 6 \\ 3 & 4 \\ 5 & 2 \end{bmatrix} \in \mathbb{R}^{3 \times 2}$

- Row vector : a horizontal vector
- Column vector : a vertical vector.

- A vector of n-dimensional is usually a column vector of size $n \times 1$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n = \mathbb{R}^{n \times 1}$$

$\mathbb{R}$: 실수

Thus, a row vector is usually written as its transpose.

$$x^T = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}^T = [x_1 \cdots x_n] \in \mathbb{R}^{1 \times n}$$

matrix notations

- $A \in \mathbb{R}^{n \times n}$: ~~a~~ square matrix (# rows = # columns)
- $A \in \mathbb{R}^{m \times n}$: Rectangular matrix ($m > n$, $m < n$)
- A^T : transpose of a matrix (mirroring across the main diagonal)

- $A_{ij} = (A)_{ij}$ – the component of A.
- $A_{i,:}$ = i-th row vector.
- $A_{:,j}$ = j-th column vector

사칙 연산

- $C = A+B$: element-wise addition i.e $C_{ij} = A_{ij} + B_{ij}$
  A, B, C should have the same size.
- ca, cA : scalar multiple of vector/matrix.
- $C = A \cdot B$: matrix-matrix multiplication, i.e $C_{ij} = \sum_k A_{ik} B_{kj}$

Matrix multiplication is not commutative

$\boxed{AB \neq BA}$

inner product $v = [x_1 \cdots x_n]$ $w = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$

$v \cdot w = x_1 y_1 + \cdots + x_n y_n$

$v \cdot w = [x_1 \cdots x_n] \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$

- $A(B+C) = AB + AC$: Distribution
- $A(BC) = (AB)C$
- $(AB)^T = B^T A^T$: property of transpose

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linear system.

- linear equation.

A linear equation in the variables $x_1 \cdots x_n$ is an equation that can be written in the form,

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b.$$

where b and the coefficients $a_1 \cdots a_n$ are real or complex numbers that are usually known in advance

The above equation can be written as

$a^T x = b$. $[a_1 \cdots a_n] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

where, $a = \begin{bmatrix} a_1 \\ a_2 \\ a_n \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_1 + 5x_2 + 3x_3 &= 3 \\ x_1 \qquad\quad + 8x_3 &= -1 \end{aligned}$$

은, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$.

이라 하자. $AX = B$로 나타낼 수 있다.

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$x_1 = -1$, $x_2 = 1$, $x_3 = 0$.

· Identity matrix.

(Def). An ~~identity~~ identity matrix is a square matrix whose diagonal entries are all 1's and all the other entries are zeros. Often, we denote it as $I_n \in \mathbb{R}^{n \times n}$

e.g. $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ...

· An identity matrix I_n preserves any vector $x \in \mathbb{R}^n$ after multiplying x by I_n. $\forall x \in \mathbb{R}^n$

- Inverse matrix

(Def) For a $\boxed{\text{square}}$ matrix $A \in \mathbb{R}^{n \times n}$, its inverse matrix A^{-1} is defined such that

$$A^{-1} \cdot A = A \cdot A^{-1} = I_n$$

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, its inverse matrix A^{-1} is defined as

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

ex) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A^{-1} = \dfrac{1}{1 \cdot 4 - 2 \cdot 3} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$$= -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & +1 \\ +\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

Futher case: Rectangular matrix.

$A^{-1} \cdot A = \cancel{AA^{-1}} =$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

~~A~~항상 역행렬이 존재하지 X.
$A \cdot A^{-1}$와 $A^{-1} \cdot A$ 가 Indentity matrix 가 아닐수도 있음.

$\exists$ inverse matrix

- Note that if A is invertiable, the solution is uniquely obtained as $X = A^{-1} b$

ex) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ ×2 ×2

- non-invertiable $\Rightarrow ad - bc = 0$.

For $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $ad-bc$ is called the determinant of A, or $\boxed{\det A}$.

$n \geq 3$ < Cramer rule / Reduce row echelon form.

- If A is non-invertible, $Ax=b$ will have either no solution or infinitely many solutions.

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \rightarrow \begin{aligned} x+2y&=3 \\ 2x+4y&=6 \end{aligned}$$: many solution

$$\begin{aligned} 2x+4y&=6 \\ \hline 0x+0y&=0 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \rightarrow \begin{aligned} x+2y&=3 \\ 2x+4y&=7 \\ 2x+4y&=6 \\ \hline 0x+0y&=1 \end{aligned}$$ ⇒ no solution.

m = # equations, n = # variable.

- $m<n$: more variables than equations.
  → usually infinitely many solutions exists (under-determined system)
- $m>n$: more equations than variables.
  → usually no solution exists (over-determined system)