

Mathematics Solution

Yunying Liang

9/17/2020

Let X_1, X_2 be the start time of 2 computer jobs.

X_1, X_2 are random variables that have distribution $U(0, 5)$.

The joint pdf of X_1, X_2 is

$$f_{X_1, X_2}(x_1, x_2) = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}, 0 \leq x_1 \leq 5, 0 \leq x_2 \leq 5$$

Let $Y = X_1 - X_2$

The cdf of Y is

$$F_Y(y) = P(y \leq Y) = P(x_1 - x_2 \leq Y)$$

$$F_Y(y) = \begin{cases} \int_0^{y+5} \int_{x_1-y}^5 \frac{1}{25} dx_1 dx_2, & -5 \leq y < 0 \\ 1 - \int_y^5 \int_0^{x_2-y} \frac{1}{25} dx_2 dx_1, & 0 \leq y \leq 5 \end{cases}$$

$$F_Y(y) = \begin{cases} \frac{y^2}{50} + \frac{y}{5} + \frac{1}{2}, & -5 \leq y < 0 \\ -\frac{y^2}{50} + \frac{y}{5} + \frac{1}{2}, & 0 \leq y \leq 5 \end{cases}$$

$$P(\text{the system is down}) = P(|X_1 - X_2| \leq 1) = P(-1 \leq X_1 - X_2 \leq 1) = P(-1 \leq Y \leq 1) = F_Y(1) - F_Y(-1) = \frac{9}{25}$$

The expected value of annual lost is: $365 \cdot \frac{9}{25} \cdot 1000 = \131400