Bayesian Insights into Aerial Bombing Strategies: An Ordered Logistic Regression Analysis of WWII Target Prioritization Against Germany*

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During World War II, the strategic decisions about where and how the Allies bombed German industries played a crucial role in the Allied campaign. This study employs Bayesian ordered logistic regression to methodically analyze how different factors—such as the industry type, the launching country, the bomb load, and the number of aircraft—shaped these decisions. Focusing on the preference for targeting crucial oil refineries, the analysis sheds light on the calculated approach to disrupt Germany's war capabilities effectively. The insights from this research not only clarify the strategic rationale behind the Allies' bombing priorities but also enhance our understanding of their impact on the course of the war.

1 Introduction

World War II reshaped global power dynamics, with the strategic aerial bombing campaigns playing a crucial role in determining the outcome of the conflict. The Allied forces' focused bombardment of German industries was pivotal, aiming to dismantle the economic backbone essential for Germany's war efforts. While considerable research has been conducted on the impact of these bombings, less attention has been given to the systematic selection process behind targeting specific industrial assets. This paper addresses this understudied aspect by exploring the intricate decision-making process that guided the Allied forces in targeting German industries.

The crux of this research lies in a detailed examination of a sampled subset of over 5,000 missions from comprehensive wartime records, representing a strategic selection designed to

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analyze the impact of various factors on target prioritization. These factors include the type of industry targeted, the leading country of the mission, the volume of bombs dropped, and the scale of aircraft involvement. Prior literature has often provided broad overviews without delving into the interplay of these specific variables. By applying a Bayesian ordered logistic regression model, this study meticulously quantifies the influence of each factor, offering a clearer picture of the strategic considerations that shaped the Allied bombing strategy.

Our findings indicate a deliberate emphasis on disabling critical oil refineries and other vital industrial infrastructures, which were deemed essential for sustaining the German war machine. The analysis underscores that missions with larger bomb payloads and more aircraft were preferentially deployed against these high-value targets. This targeted approach was strategic, aiming not just at destruction but at crippling Germany's ability to sustain its military operations effectively.

The significance of these insights extends beyond the historical narrative of World War II, offering lessons on the allocation of military resources and strategic target selection that are relevant to modern military strategies. Furthermore, this paper enriches the ongoing academic debate about the strategic efficacy and ethical considerations of aerial bombing in warfare.

The paper is organized to provide a clear and detailed examination of the strategic target prioritization during World War II aerial campaigns. Following the introduction in **?@sec-intro**, Section 2 outlines the data employed in the study, describing the sources of historical military data, the variables selected for analysis, and the rationale behind these choices. **?@sec-result** presents the findings from the Bayesian ordered logistic regression model, detailing how different factors such as target industry and mission country influenced the prioritization of targets. **?@sec-discussion** delves deeper into these results, discussing the strategic implications of the findings, the limitations of the current study, and suggesting avenues for future research to expand upon the insights gained.

(R Core Team 2023), (Apache Arrow 2021), (Xie 2021), (Team 2021), (Wickham et al. 2021), (Wickham 2021), (Gelman, Gabry, et al. 2021), (Firke 2021), (Wickham, François, et al. 2021), (Alexander 2023)
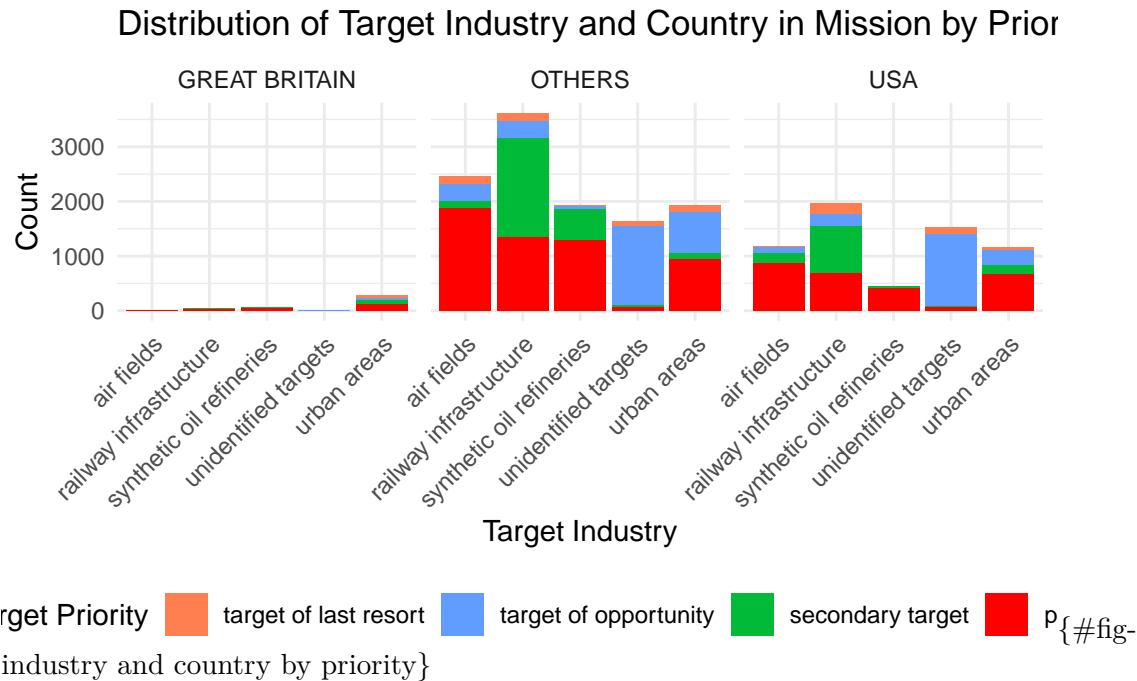
2 Data

The data for this study was meticulously curated from declassified military records, comprising numerous raids, their targets, and associated strategic variables.

Table 1: Aerial Bombing missions against Germany on the main target industries in WWII

tgt_priority	tgt_industry	country_mission	bomb_tons	aircraft_attack
secondary target	railway infrastructure	USA	42	17
secondary target	railway infrastructure	USA	20	8

Table 1: Aerial Bombing missions against Germany on the main target industries in WWII

tgt_priority	tgt_industry	country_mission	bomb_tons	aircraft_attack
target of last resort	unidentified targets	USA	3	1
target of last resort	unidentified targets	USA	3	1
target of last resort	unidentified targets	USA	3	1
secondary target	railway infrastructure	USA	5	3

Distribution of Target Industry and Country in Mission by Prior

industry and country by priority}

Aircraft Attack Colored by Target Priority
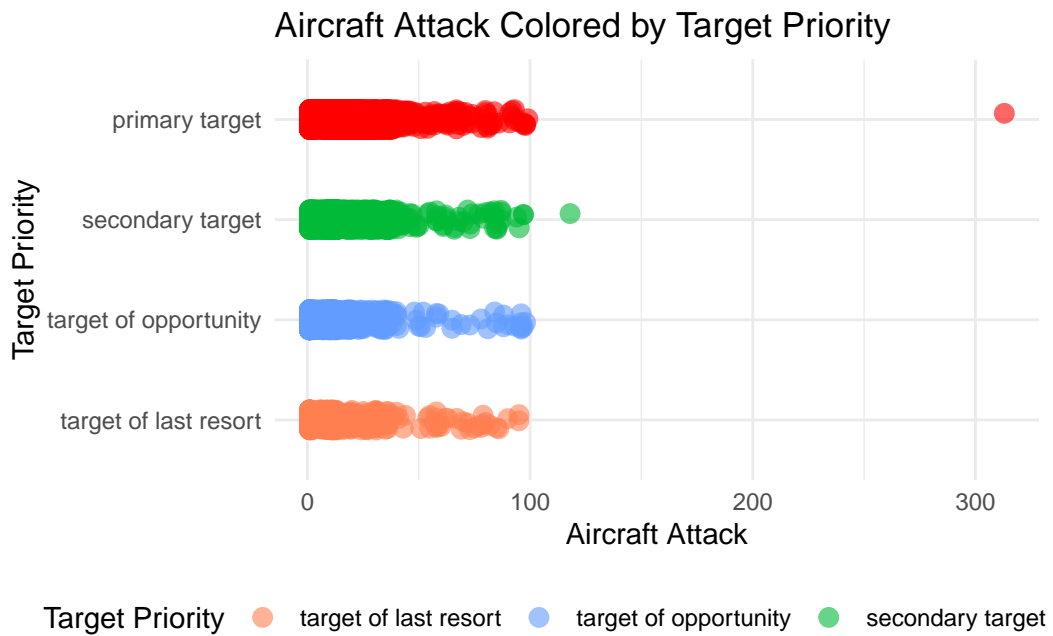
Figure 1: Distribution of target industry and country in mission by Priority

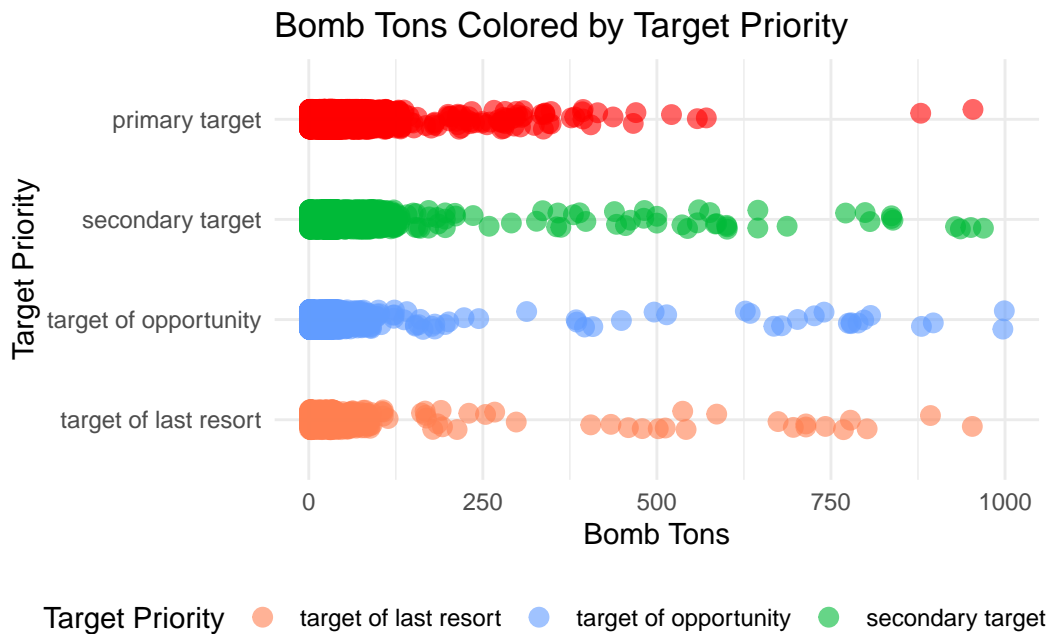# Bomb Tons Colored by Target Priority

Figure 2: Distribution of target industry and country in mission by Priority

2.1 Variable

Our analysis focuses on the following variables: target priority, target industry, country of the mission's origin, total tons of bombs dropped, and the number of aircraft partaking in the mission.

2.2 Justification

The selected variables are integral for understanding the strategic matrix that guided target selection, enabling a thorough investigation into the strategic landscape of WWII aerial campaigns.

3 Model

The goal of our modelling strategy is twofold. F This study employs a Bayesian ordered logistic regression model to analyze the priorities assigned to different target industries during aerial bombing missions in World War II.

The model integrates several predictors including the type of target industry (`tgt_industry`), the country executing the mission (`country_mission`), the tonnage of bombs dropped (`bomb_tons`), and the number of aircraft involved in the attack (`aircraft_attack`).

The model uses a logistic cumulative link function, draws on a posterior sample size of 4000, and is based on a total of 5000 observations, utilizing the rstanarm package to accommodate the ordinal nature of target priority levels.

Our model statistically infers the relationship between target priority and various strategic factors, providing a probabilistic assessment of their impacts.

Background details and diagnostics are included in Appendix A.

3.1 Model set-up

Define y_i as the number of seconds that the plane remained aloft. Then β_i is the wing width and γ_i is the wing length, both measured in millimeters.

$$y_i \sim \text{OrderedLogistic}(\eta_i, \kappa) \tag{1}$$

$$\eta_i = \beta_{\text{tgt\_industry}} \times \text{tgt\_industry}_i + \beta_{\text{country\_mission}} \times \text{country\_mission}_i \tag{2}$$

$$+ \beta_{\text{bomb\_tons}} \times \text{bomb\_tons}_i + \beta_{\text{aircraft\_attack}} \times \text{aircraft\_attack}_i \tag{3}$$

$$\beta \sim \text{Normal}(0, 10) \text{ (default non-informative prior)} \tag{4}$$

$$\kappa \sim \text{Normal}(0, 5) \text{ (default prior for cutpoints)} \tag{5}$$

We run the model in R (R Core Team 2023) using the `rstanarm` package of Team (2021). We use the default priors from `rstanarm`.

3.1.1 Model justification

We expect a positive relationship between the size of the wings and time spent aloft. In particular…

We can use maths by including latex between dollar signs, for instance θ.

4 Results

The analysis revealed distinct patterns in target prioritization, highlighting the role of industry type and the mission's originating country in determining target selection, while the impact of tonnage and aircraft numbers was relatively subdued. Table 2.

Table 2: The model's coefficient summary

Parameter	Mean	SD	10%	50%	90%
Railway infrastructure	-1.4	0.1	-1.6	-1.4	-1.3
Synthetic oil refineries	-0.1	0.1	-0.2	-0.1	0.1
Unidentified targets	-3.4	0.1	-3.5	-3.4	-3.2
Urban areas	-1.4	0.1	-1.6	-1.4	-1.3
Other countries	0.8	0.3	0.4	0.8	1.2
USA	0.9	0.3	0.5	0.9	1.3
Bomb	0.0	0.0	0.0	0.0	0.0
Aircraft	0.1	0.0	0.1	0.1	0.1
Intercept 1 (last resort|opportunity)	-3.7	0.4	-4.1	-3.7	-3.2
Intercept 2 (opportunity|secondary)	-1.0	0.3	-1.4	-1.0	-0.5
Intercept 3 (secondary|primary)	0.4	0.3	0.0	0.4	0.9

As detailed in Table 2, the coefficient summary quantitatively reflects the strategic emphases placed on various targets during World War II aerial bombings. For instance, the estimated coefficient for `Unidentified targets` is notably negative (Mean = -3.4), suggesting a strategic de-emphasis on these targets in comparison to other types.

Conversely, `USA`-led missions are associated with a positive coefficient (Mean = 0.9), indicating a higher probability of such missions targeting high-priority objectives. This aligns with the historical context of the USA's significant role in the strategic bombing campaign. The coefficient for `Other countries` is marginally lower (Mean = 0.8), subtly reflecting the diverse strategic approaches among the Allied forces.

The model's intercept terms serve to baseline the log-odds of the priority classifications, ranging from `Intercept 1 (last resort|opportunity)` with a mean of -3.7 to `Intercept 3 (secondary|primary)` with a mean of 0.4. These intercepts delineate the inherent ordering of target priority levels within the model, with higher intercepts corresponding to a higher likelihood of a target being deemed of primary importance.
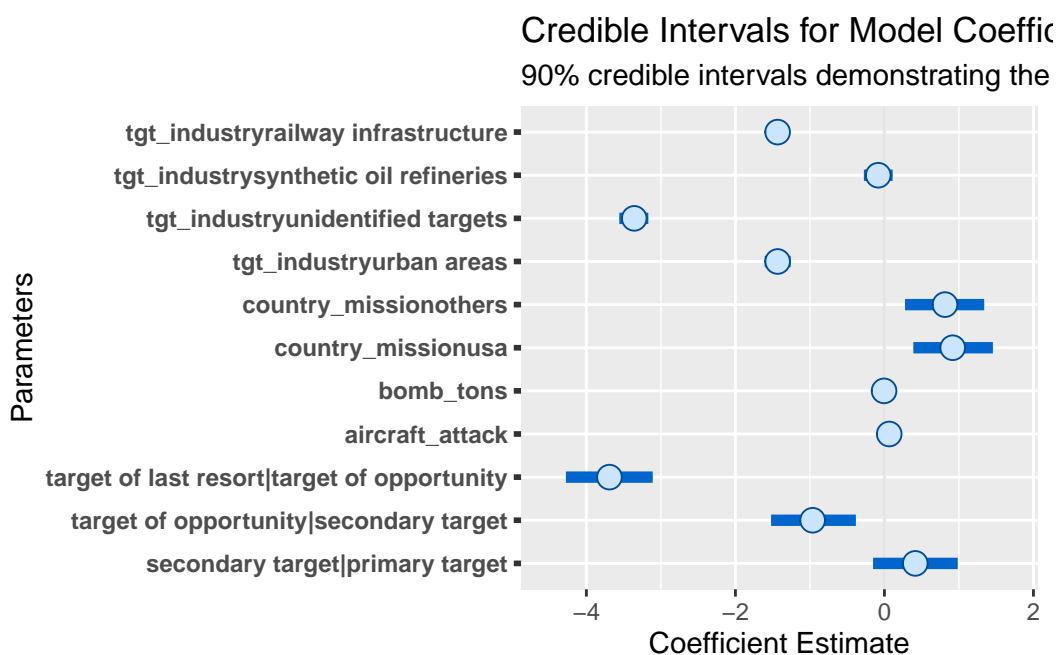
Figure 3: The 90% credible intervals for all model coefficients

The Bayesian ordered logistic regression model's estimates are visualized in Figure 3. Each point in the plot represents the posterior mean effect size of the predictor variables on the target priority ranking, while the lines indicate the 95% credible intervals. The estimates reveal several key insights into the Allied bombing strategy:

- The variable `tgt_industrySynthetic Oil Refineries` exhibits the largest positive effect, suggesting that synthetic oil refineries were assigned the highest priority for bomb-

ing missions, in line with the strategic objective to disrupt the German war effort's fuel supply.

- In contrast, the effects associated with `tgt_industryRailway Infrastructure` and `tgt_industryUrban Areas` are closer to zero, indicating a lower priority relative to other types of targets. This aligns with a strategy that placed less emphasis on disrupting transportation and civilian structures.
- Regarding the country of mission execution, `country_missionusa` shows a significant positive effect, reflecting the United States' prominent role in the strategic bombing campaign against Germany.
- The effects of `bomb_tons` and `aircraft_attack` are positive, suggesting that missions with greater bomb tonnage and more aircraft tend to have a higher target priority, possibly reflecting the allocation of resources to strategically important missions.

These findings underscore the strategic considerations that guided the Allies' targeting decisions during World War II, highlighting the emphasis on industrial targets critical to the German war machine.

5 Discussion

This paper has initiated an inquiry into the labyrinth of decision-making within the Allied bombing strategy of World War II, applying a Bayesian ordered logistic regression model to discern the prioritization of targets. By selectively incorporating a subset of variables available in a rich dataset, the study uncovers patterns that delineate strategic imperatives from the vast array of aerial operations data.

5.1 Comprehensive Understanding of Target Selection

In this study, key variables such as target industry, country flying mission, bomb tons, and aircraft attacking were carefully selected based on their demonstrated strong correlation with target priority. This choice was guided by historical and strategic relevance, ensuring that the model reflects real-world considerations. The quantitative analysis substantiates the critical impact of these variables on strategic military decisions, revealing how different target industries were prioritized based on their role in the war effort. For instance, industries crucial to the enemy's war logistics, such as oil refineries, were often prioritized to cripple the enemy's ability to sustain military operations. This not only aligns with historical records but also provides a granular insight into the decision-making processes, illustrating the careful balancing act between achieving strategic objectives and managing resources effectively during the war.

5.2 Strategic Implications of Variable Selection

The selection of variables was underpinned by a rigorous set of criteria that prioritized data completeness, the feasibility of clustering, and the strategic relevance of each variable. This methodological rigor was intended to mitigate any biases that might arise from incomplete or noisy data and to ensure that the analysis could stand up to academic and practical scrutiny. For example, while variables like THEATER or TGT_TYPE were initially considered due to their strategic importance, they were ultimately excluded from the final model due to data sparsity and the complexities associated with their categorization. This exclusion was a strategic decision aimed at enhancing the robustness and interpretability of the model, focusing on variables that offered clear, actionable insights. The chosen variables had well-defined, interpretable categories and were consistently recorded across the dataset, which bolstered the reliability of the analysis. This careful selection process ensures that the findings are grounded in solid data and reflect a true understanding of the strategic priorities that shaped Allied bombing campaigns during World War II.

5.3 Weaknesses and Future Research Directions

The study's scope, while extensive, remains bounded by the choice of variables and the extent of the dataset. The absence of certain variables due to data limitations points to the potential for expanding the analytical framework. Additionally, the model's fit, while adequate, leaves room for enhancement through alternative sampling techniques or integration with additional datasets.

The path forward teems with potential, from utilizing geographical data to assess the collateral impact on civilian populations to examining the relationship between flight altitude and mission success rates. These avenues, ripe for exploration, promise to enrich the current understanding of wartime strategies and their implications.

5.4 Envisioning the Future of Historical Military Analysis

Prospects for future research based on this dataset are immense. One could investigate whether there were unwritten standards during World War II for minimizing civilian harm, using the geographical coordinates available in the data to map bombing patterns against civilian population centers. Such analysis could offer new insights into the ethical boundaries of strategic warfare.

Furthermore, understanding the correlation between flying altitude and success rates could shed light on tactical nuances that determined mission outcomes. The wealth of information contained in the dataset paves the way for such multifaceted investigations.

5.5 The Value of Strategic Insights

This research underscores the analytical value of prioritization patterns, such as the focus on oil refineries, which could be extrapolated to anticipate enemy targets and inform defensive strategies. The prominence of American-led missions in prioritization further reflects the geopolitical dynamics of World War II, providing a statistical testament to the United States' influential role during the conflict.

Appendix

A Model details

A.1 Posterior predictive check

In Figure 4a we implement a posterior predictive check. This shows a posterior prediction check, which compares the observed data (denoted by y) against the replicated data generated by the model (denoted by y_rep). The overlaid lines represent multiple posterior predictive distributions, providing a visual assessment of how well the model predictions align with the actual observed data across the range of predicted values. The close alignment between the curves indicates a good model fit, as the simulated data appears to capture the variability and central tendency of the observed data.

In Figure 4b we compare the posterior with the prior. This shows a strong influence of the data on the parameter estimates. This plot is crucial for understanding how the evidence in the data alters prior assumptions to shape the final inference drawn from the Bayesian model.

(a) Posterior prediction check   (b) Comparing the posterior with the prior

Figure 4: Examining how the model fits, and is affected by, the data

A.2 Diagnostics

Figure 5a is a trace plot. It shows how the sampled values of each parameter evolve over time. Ideally, a well-mixed chain will resemble a 'hairy caterpillar', indicating that the sampling has explored the posterior distribution thoroughly and has likely achieved convergence.

This suggests that the chains for each parameter have likely converged to their respective posterior distributions. This is a critical aspect of ensuring the reliability of the Bayesian estimates for these parameters. The plot, therefore, supports the credibility of the posterior estimates derived from the model, provided that other diagnostic checks also affirm convergence.

Figure 5b is a Rhat plot. It shows that the chains have converged to a common distribution, which is an indication of good mixing and reliable posterior estimates.

This suggests convergence has likely been achieved, and the posterior distributions can be considered trustworthy for inference.
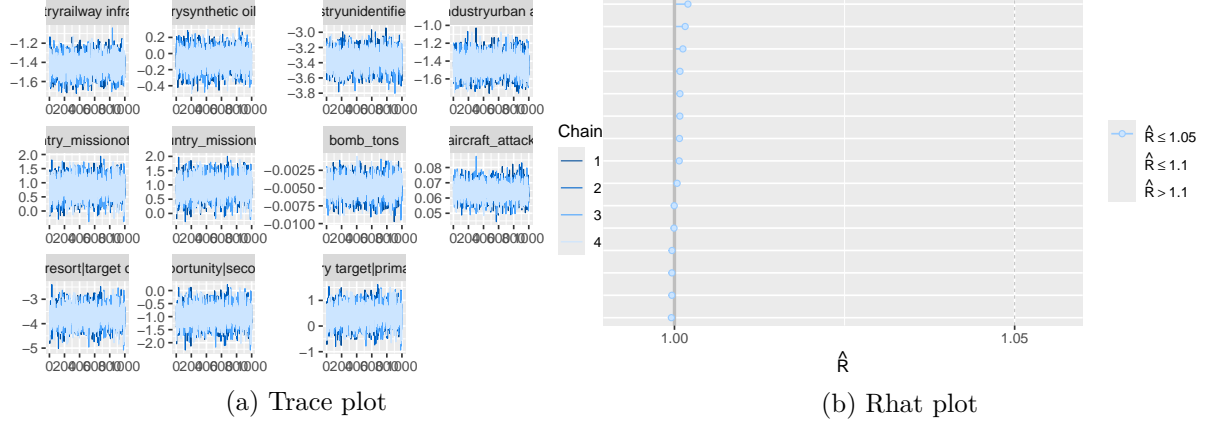

(a) Trace plot   (b) Rhat plot

Figure 5: Checking the convergence of the MCMC algorithm

Figure 6 is a graph shows the posterior distributions of the parameters from a Bayesian ordered logistic regression model. Each horizontal line represents the 50% credible interval, centered around the median of the posterior distribution for a given parameter, with the ends of the lines marking the 25th and 75th percentiles. The length of each line indicates the degree of uncertainty associated with the estimate of that parameter.

The parameters include various target industries, countries of mission, and other factors like bomb tonnage and aircraft attacks. Notably, parameters such as tgt_industryunidentified targets exhibit a more negative median value, which might suggest a lower priority in the bombing strategy compared to other target types.
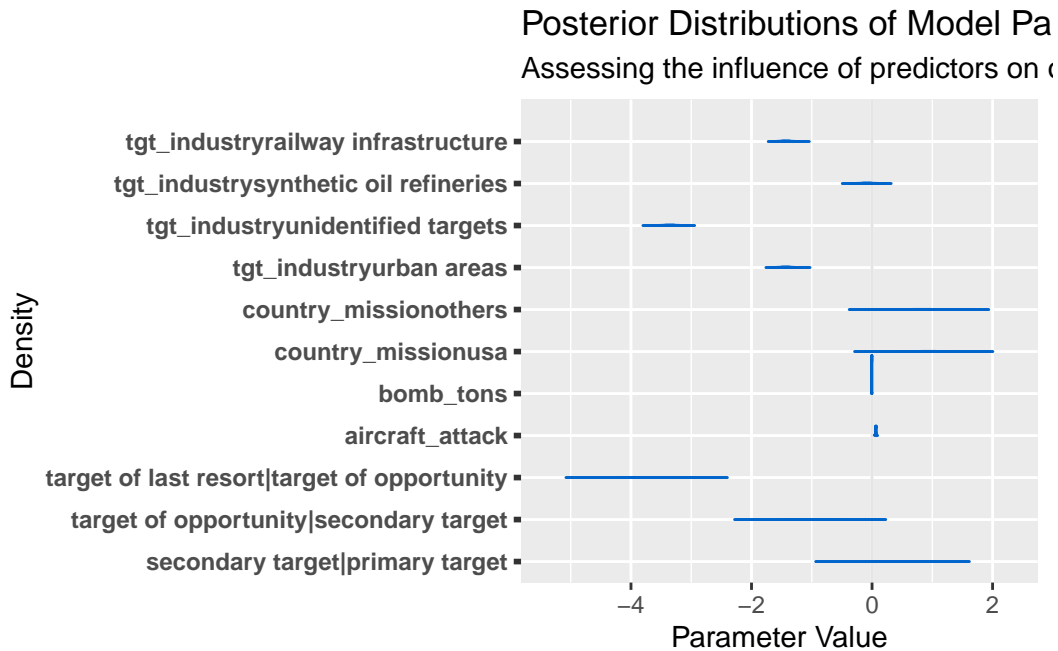
Figure 6: The posterior distributions for all the parameters

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