Bayesian Insights into Aerial Bombing Strategies: An Ordered Logistic Regression Analysis of WWII Target Prioritization Against Germany*

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First sentence. Second sentence. Third sentence. Fourth sentence.

1 Introduction

You can and should cross-reference sections and sub-sections. We use R Core Team (2023) and Wickham et al. (2019).

The remainder of this paper is structured as follows. Section 2….

2 Data

Some of our data is of penguins (**?@fig-bills**), from Horst, Hill, and Gorman (2020).

Talk more about it.

And also planes (**?@fig-planes**). (You can change the height and width, but don't worry about doing that until you have finished every other aspect of the paper - Quarto will try to make it look nice and the defaults usually work well once you have enough text.)

*Code and data are available at: https://github.com/yunzhaol/aerial_bomb_priority.git.

Table 1: 2022 CES (Cooperative Election Study) Data (Cultural)

```
# analysis_data |>
#   ggplot(aes(x = width, y = length)) +
#   geom_point(alpha = 0.8) +
#   theme_minimal() +
#   labs(x = "Wing width (mm)",
#        y = "Wing length (mm)")
```

Talk way more about it.

3 Model

The goal of our modelling strategy is twofold. Firstly,...

Here we briefly describe the Bayesian analysis model used to investigate... Background details and diagnostics are included in Appendix B.

3.1 Model set-up

Define y_i as the number of seconds that the plane remained aloft. Then β_i is the wing width and γ_i is the wing length, both measured in millimeters.

$$y_i|\mu_i, \sigma \sim \text{Normal}(\mu_i, \sigma) \tag{1}$$
$$\mu_i = \alpha + \beta_i + \gamma_i \tag{2}$$
$$\alpha \sim \text{Normal}(0, 2.5) \tag{3}$$
$$\beta \sim \text{Normal}(0, 2.5) \tag{4}$$
$$\gamma \sim \text{Normal}(0, 2.5) \tag{5}$$
$$\sigma \sim \text{Exponential}(1) \tag{6}$$

We run the model in R (R Core Team 2023) using the `rstanarm` package of Goodrich et al. (2022). We use the default priors from `rstanarm`.

3.1.1 Model justification

We expect a positive relationship between the size of the wings and time spent aloft. In particular...

We can use maths by including latex between dollar signs, for instance θ.

4 Results

Our results are summarized in Table 2.

```
Model Info:
 function:     stan_polr
 family:       ordered [logistic]
 formula:      tgt_priority_explanation ~ tgt_industry + country_flying_mission +
       total_tons + ac_attacking
 algorithm:    sampling
 sample:       4000 (posterior sample size)
 priors:       see help('prior_summary')
 observations: 500

Estimates:
                                              mean   sd   10%   50%   90%
tgt_industryunidentified targets             -3.4    0.3 -3.9  -3.4  -3.0
tgt_industryurban areas                      -1.3    0.3 -1.6  -1.3  -1.0
country_flying_missionothers                  0.7    0.7 -0.1   0.8   1.6
country_flying_missionusa                     1.2    0.7  0.3   1.2   2.0
total_tons                                    0.0    0.0  0.0   0.0   0.0
ac_attacking                                  0.1    0.0  0.0   0.1   0.1
target of last resort|target of opportunity -4.8    0.8 -5.8  -4.7  -3.7
target of opportunity|secondary target      -0.4    0.8 -1.4  -0.4   0.6
secondary target|primary target              0.4    0.8 -0.6   0.4   1.3

Fit Diagnostics:
                                mean   sd   10%   50%   90%
mean_PPD:target of last resort 0.0    0.0  0.0   0.0   0.1
mean_PPD:target of opportunity 0.5    0.0  0.4   0.5   0.5
mean_PPD:secondary target      0.1    0.0  0.1   0.1   0.1
mean_PPD:primary target        0.4    0.0  0.3   0.4   0.4

The mean_ppd is the sample average posterior predictive distribution of the outcome variable

MCMC diagnostics
                                             mcse Rhat n_eff
tgt_industryunidentified targets             0.0  1.0  4885
tgt_industryurban areas                      0.0  1.0  4867
country_flying_missionothers                 0.0  1.0  4846
country_flying_missionusa                    0.0  1.0  4676
```

```
total_tons                                    0.0  1.0  4332
ac_attacking                                  0.0  1.0  3226
target of last resort|target of opportunity 0.0  1.0  5296
target of opportunity|secondary target      0.0  1.0  4979
secondary target|primary target             0.0  1.0  5012
mean_PPD:target of last resort              0.0  1.0  4230
mean_PPD:target of opportunity              0.0  1.0  3740
mean_PPD:secondary target                   0.0  1.0  3978
mean_PPD:primary target                     0.0  1.0  4404
log-posterior                               0.1  1.0   922

For each parameter, mcse is Monte Carlo standard error, n_eff is a crude measure of effective


Priors for model 'aerial_priority_model'
------

Coefficients
 ~ flat

Counts
 ~ dirichlet(concentration = [1,1,1,...])
------
See help('prior_summary.stanreg') for more details
```

5 Discussion

5.1 First discussion point

If my paper were 10 pages, then should be be at least 2.5 pages. The discussion is a chance to show off what you know and what you learnt from all this.

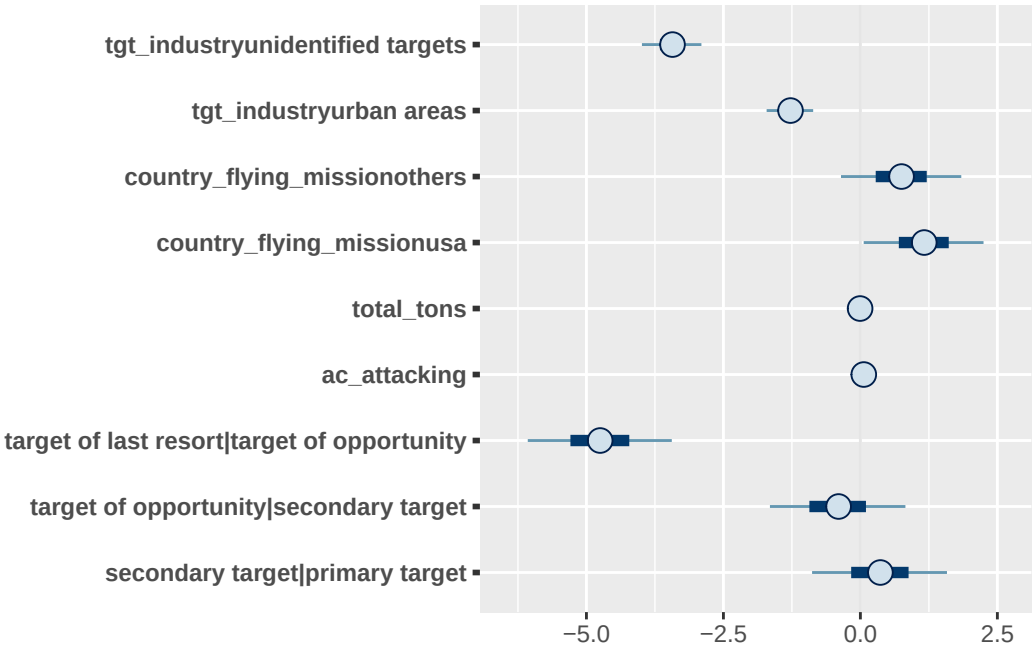
5.2 Second discussion point

5.3 Third discussion point

5.4 Weaknesses and next steps

Weaknesses and next steps should also be included.

Table 2: Explanatory models of flight time based on wing width and wing length



Appendix

A Additional data details

B Model details

B.1 Posterior predictive check

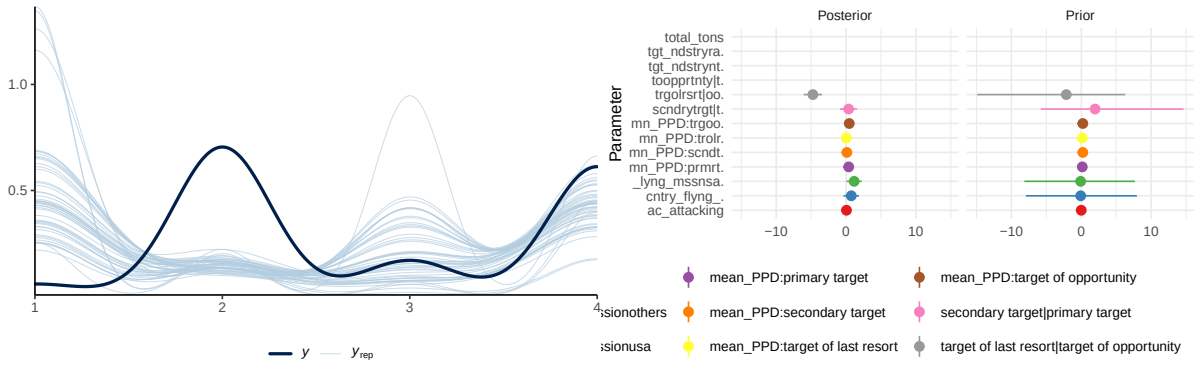
In Figure 1a we implement a posterior predictive check. This shows...

In Figure 1b we compare the posterior with the prior. This shows...

B.2 Diagnostics

Figure 2a is a trace plot. It shows... This suggests...

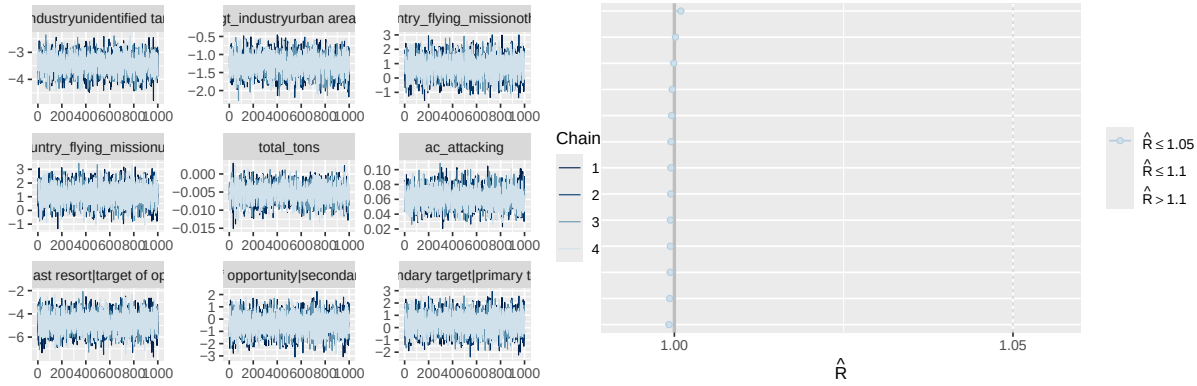
Figure 2b is a Rhat plot. It shows... This suggests...



(a) Posterior prediction check

(b) Comparing the posterior with the prior

Figure 1: Examining how the model fits, and is affected by, the data



(a) Trace plot

(b) Rhat plot

Figure 2: Checking the convergence of the MCMC algorithm

References

Goodrich, Ben, Jonah Gabry, Imad Ali, and Sam Brilleman. 2022. "Rstanarm: Bayesian Applied Regression Modeling via Stan." https://mc-stan.org/rstanarm/.

Horst, Allison Marie, Alison Presmanes Hill, and Kristen B Gorman. 2020. *Palmerpenguins: Palmer Archipelago (Antarctica) Penguin Data*. https://doi.org/10.5281/zenodo.3960218.

R Core Team. 2023. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. https://www.R-project.org/.

Wickham, Hadley, Mara Averick, Jennifer Bryan, Winston Chang, Lucy D'Agostino McGowan, Romain François, Garrett Grolemund, et al. 2019. "Welcome to the tidyverse." *Journal of Open Source Software* 4 (43): 1686. https://doi.org/10.21105/joss.01686.