ECE 9203/9023 – Random Signals, Adaptive and Kalman Filtering
Winter 2021, MATLAB Assignment #1
Due date & time: Friday, February 19, 2021, 11 PM EDT.
Upload to "Assignments" section. Email or DropBox submissions not accepted.

1. (10 marks) Generate the following random signals in MATLAB. Plot the estimated probability density function (PDF) and power spectral density (PSD) for each. Comment on how close the estimated PDF and PSD are to their theoretical values. (see "wgn.mlx" for guidance).

   a. White Gaussian Noise, mean = 0, variance = 1
   b. White Uniform Noise, mean = 0, variance = 1
   c. Pink Gaussian Noise, mean = 0, variance = 1

2. (10 marks) In this exercise, we will apply AR modeling to speech samples. Download "m01ae.wav", "w01ae.wav", "w01ih.wav", and "w01uw.wav" from "Resources -> MATLAB" directory. Complete the following:
   a. For each speech sample, plot the estimated variance of the white noise input against the model order, with the model order ranging from 1 to 25. See the documentation for "aryule" command for accessing the estimated variance. Comment on the results. What would be a good model order for modelling these waveforms?
   b. For each speech sample and the chosen model order, compute and plot the periodogram and AR spectral estimates. See "LinearPredictionExample.mlx" for guidance. Comment on the results. In particular, what is the AR spectral estimate trying to model? Are the AR spectral estimates the same across the four speech samples?

3. (5 marks) The input to a Wiener filter of length two is described by the difference equation, $u(n) = x(n) + v_2(n)$, where $x(n) = 0.3x(n-1) + 0.64\,x(n-2) + v_1(n)$, and $v_1(n)$ and $v_2(n)$ are zero-mean white noise processes of variances 0.4 and 0.2 respectively. The desired input is given by the difference equation, $d(n) = 0.1x(n) + 0.52\,x(n-1)$. We derived the equations for the error performance surface and the Wiener filter for this example in class. Do the following in MATLAB (see "WienerFilterExample1.m" for guidance):
   a. Plot the error performance surface as function of the weights.
   b. Plot the contours of the error performance surface and indicate the Wiener solution on this plot.
   c. Plot the gradient vectors and comment on their orientation.

4. (15 marks) Consider a one-step adaptive predictor for a generic second order real AR process defined by the difference equation $u(n) + a_1\,u(n-1) + a_2\,u(n-2) = v(n)$, where $v(n)$ is a zero-mean white noise process with variance, σ_v^2.

   a. Derive the equations for r(0), r(1), and r(2).
   b. If $a_1 = -1.9$, $a_2 = 0.95$, and $\sigma_v^2=1$, compute the eigenvalues, eigenvectors, and the eigenvalue spread.
   c. Define $e(n) = u(n) - \hat{u}(n)$, $\varepsilon_1(n) = 1.9 - w_1(n)$, and $\varepsilon_2(n) = -0.95 - w_2(n)$. Using the LMS algorithm with the convergence parameter, $\mu = 0.003$, plot the power spectral densities for $e(n)$, $\varepsilon_1(n)$, and $\varepsilon_2(n)$. Include these plots in your assignment, clearly labeling the axes. What observations can you make from these plots?
   d. Compute the ensemble-average learning curve of the LMS process by averaging the squared values of the prediction error, $e(n)$. Similarly, compute the ensemble-average learning curves for the NLMS algorithm using $\delta = 0.05$. Include these plots with your assignment, along with your observations comparing the NLMS and LMS results.