DS-GA 3001.008 Modelling time series data

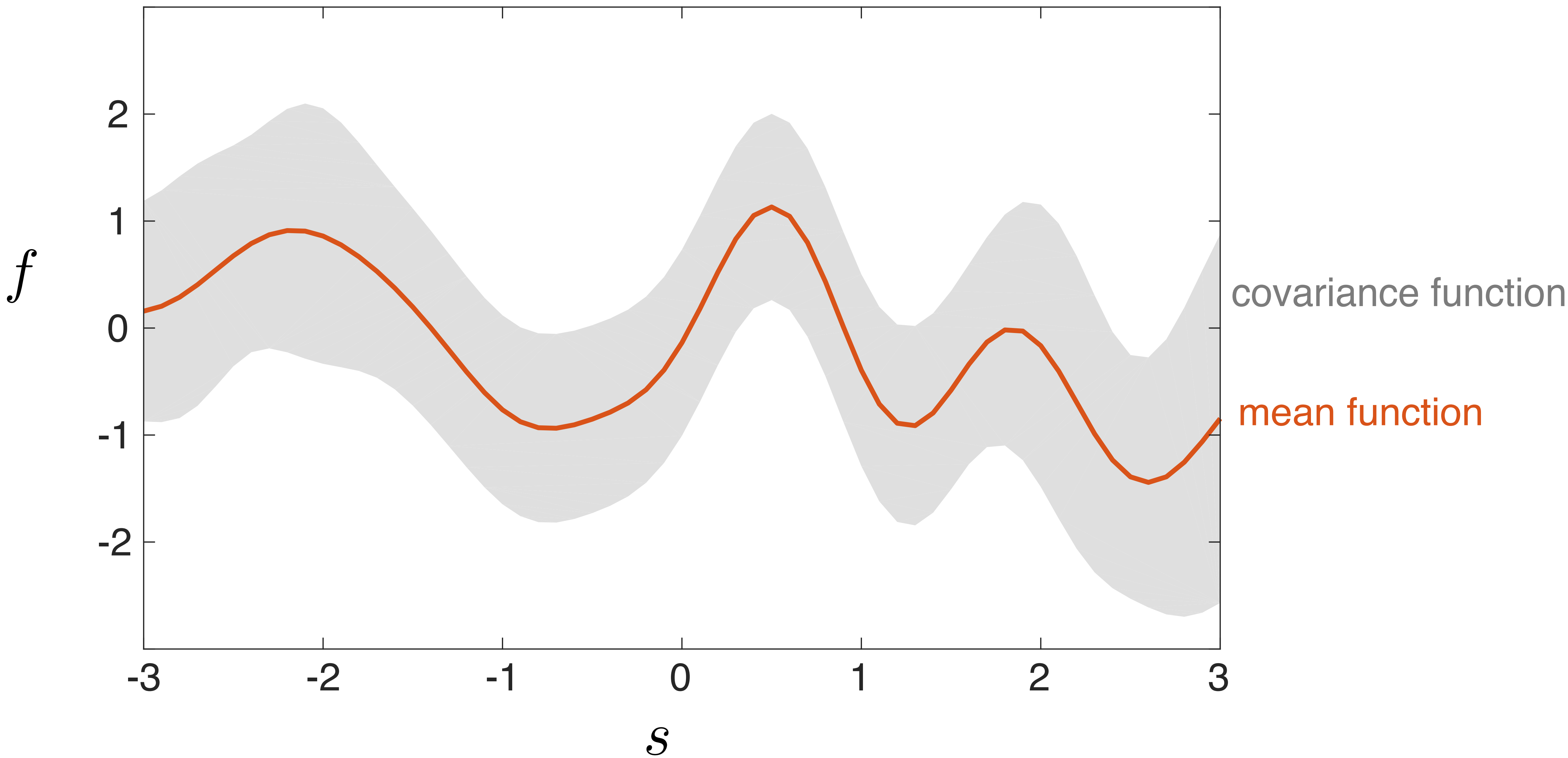
L9. An intro to Gaussian Processes

Instructor: Cristina Savin

NYU, CNS & CDS

Probabilistic inference with functions

$$f \sim \mathrm{GP}\left(f; \mu(s), k(s_1, s_2)\right)$$



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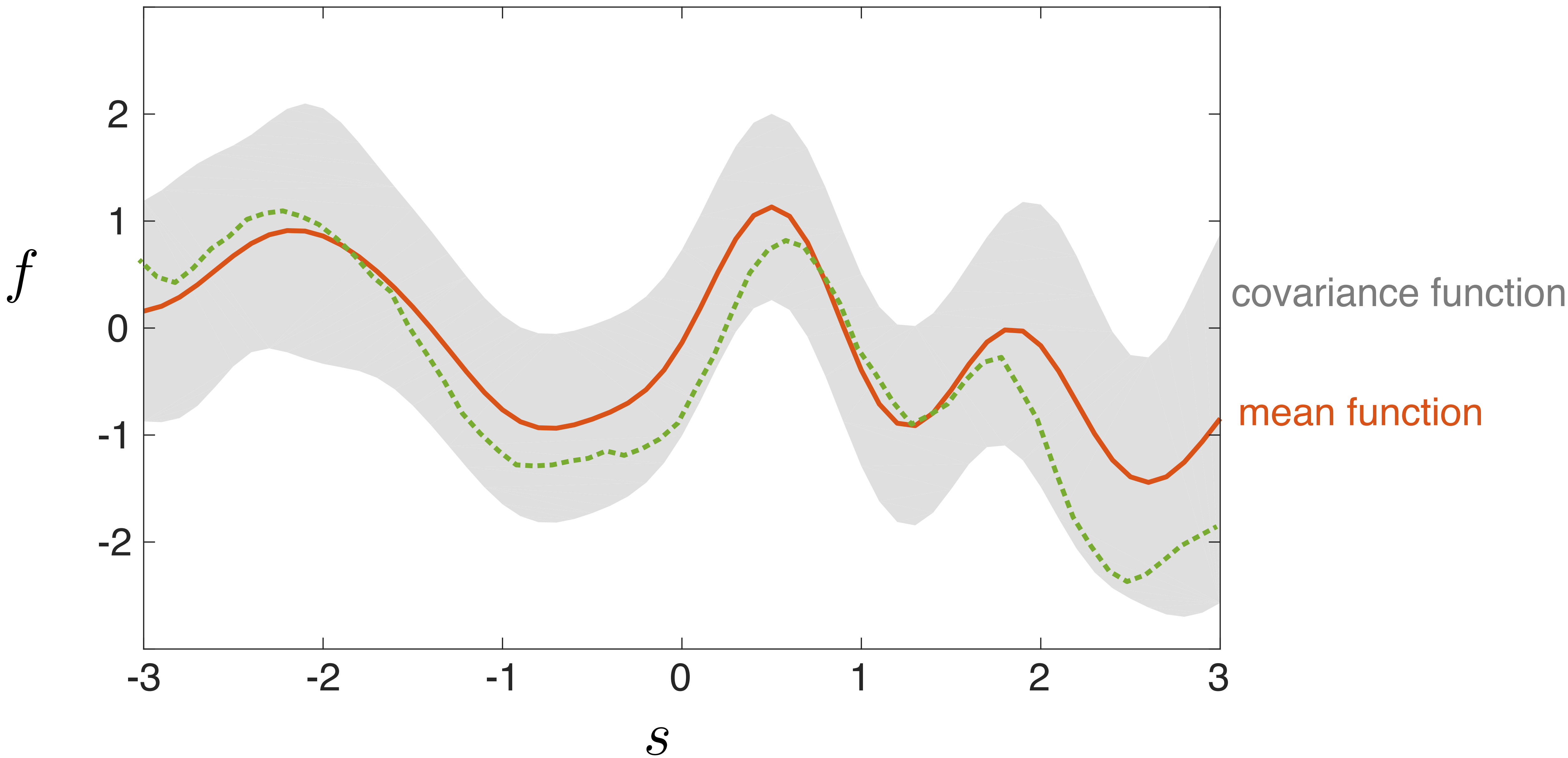

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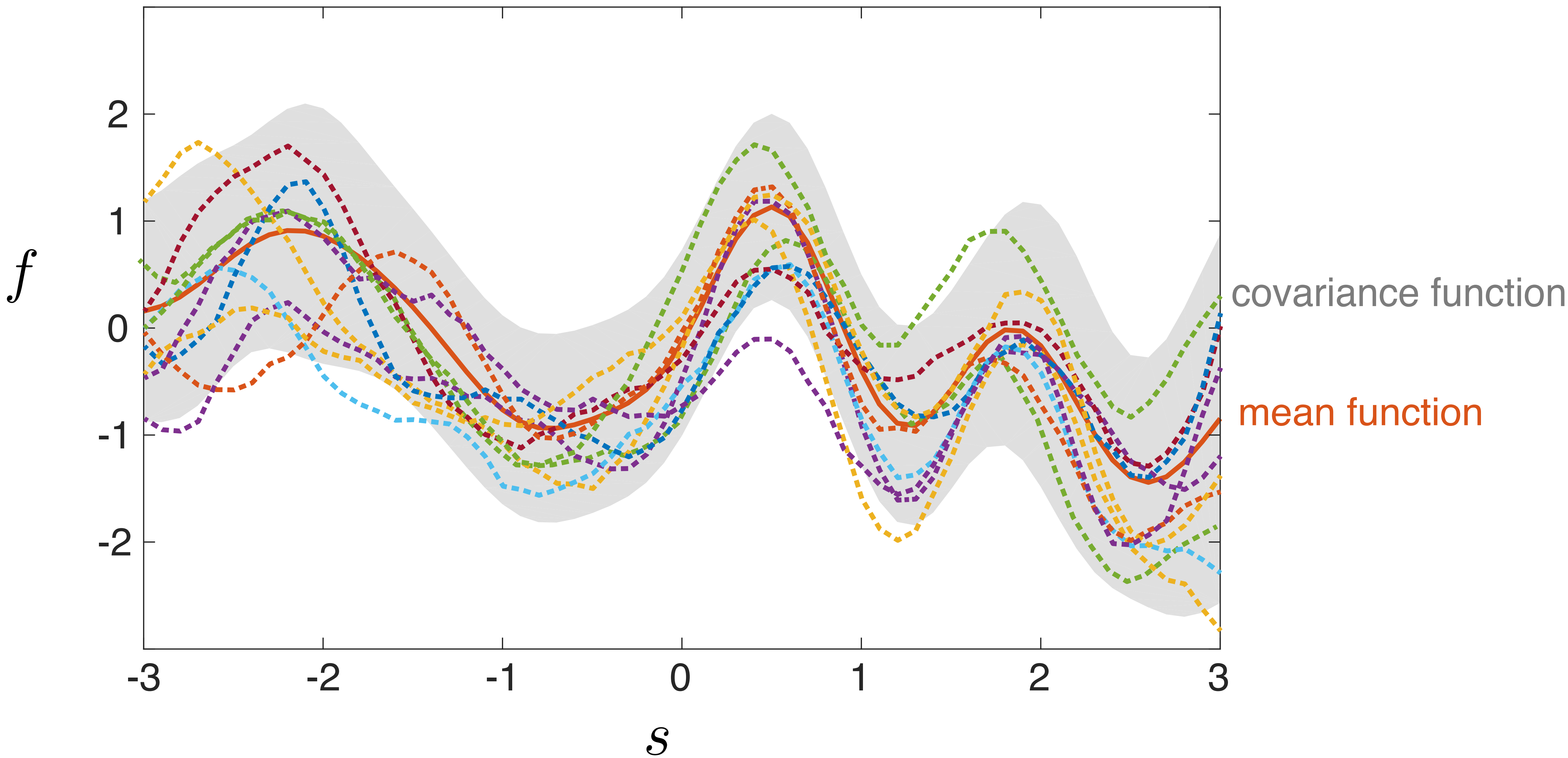
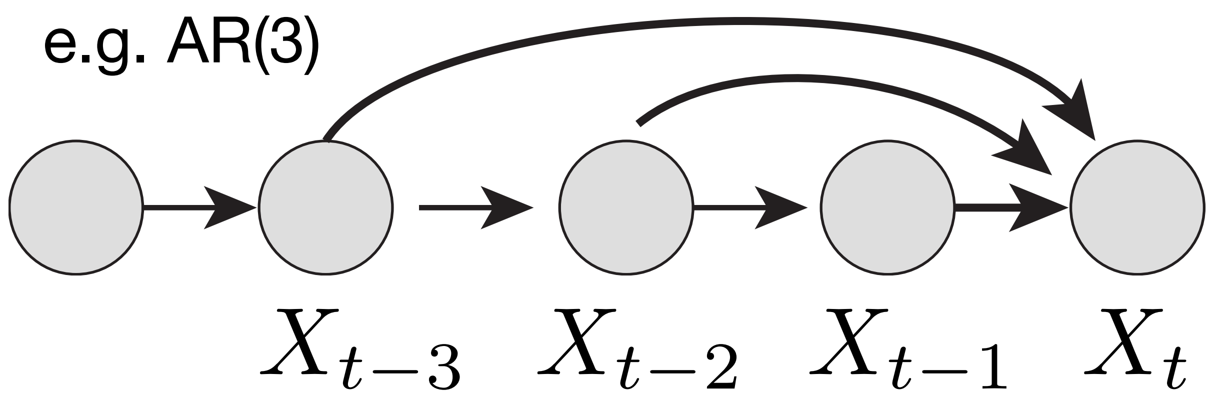


Big picture

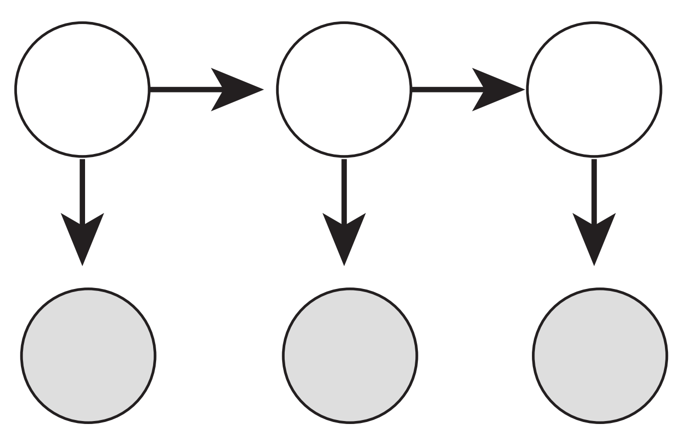
ARIMA: directly model data statistics



simple dependencies

linear prediction

Latent state models: introduce latent variables



more complex dependencies

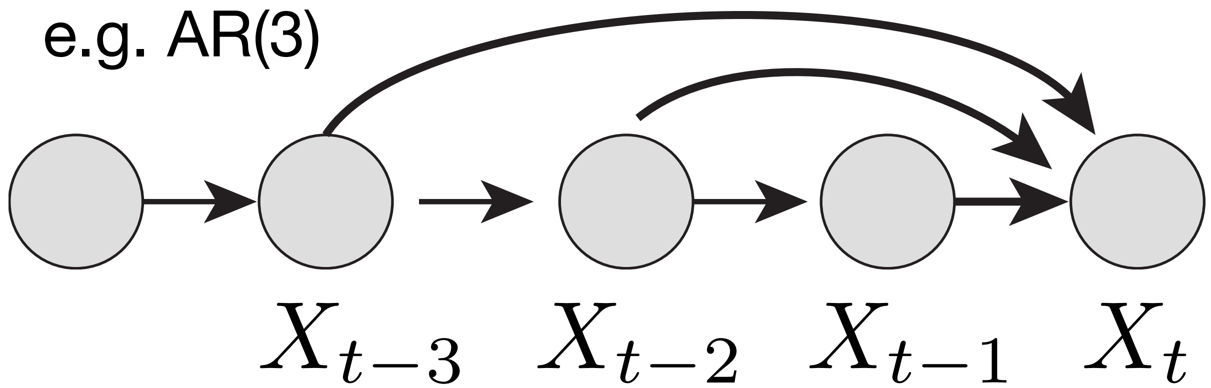
Markov dynamics in latent space

more complicated prediction

Big picture

ARIMA: directly model data statistics

e.g. AR(3)



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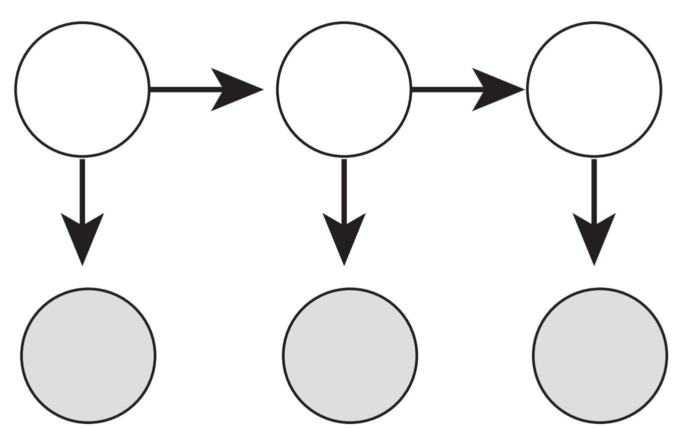
GP: complex functional dependencies

Latent state models: introduce latent variables



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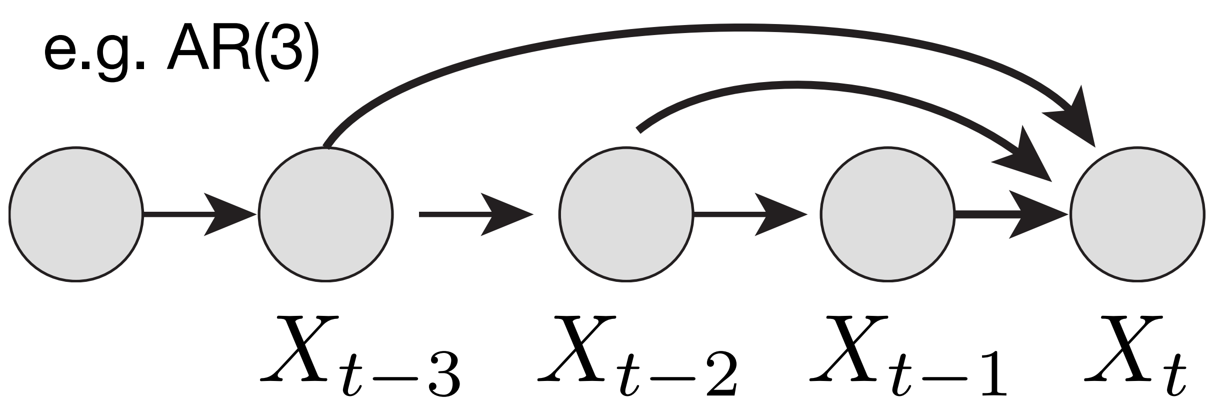

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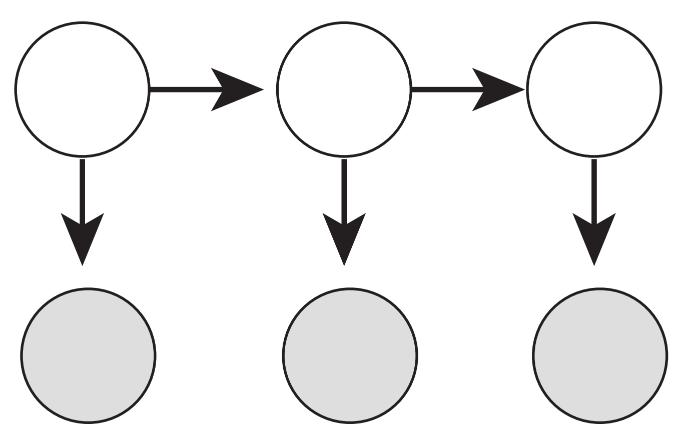

more complex dependencies

Markov dynamics in latent space

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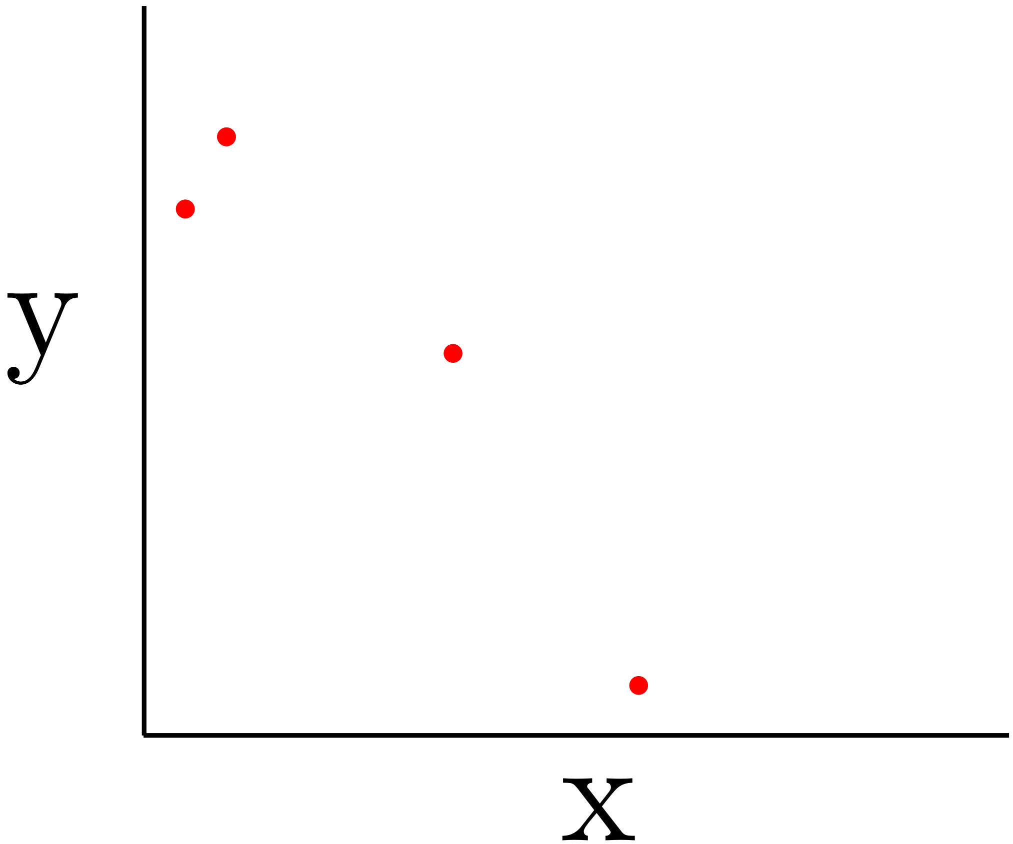
GP: complex latent dynamics

Motivation: non-linear regression



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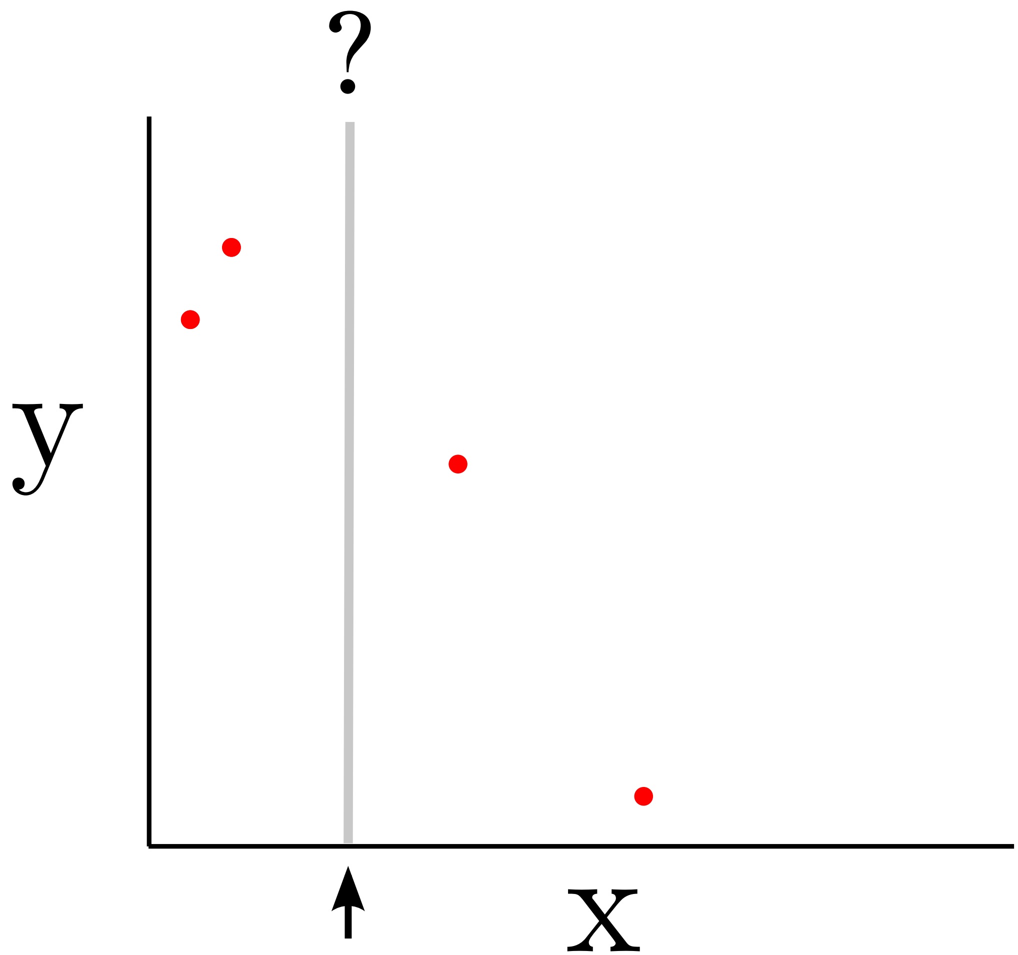

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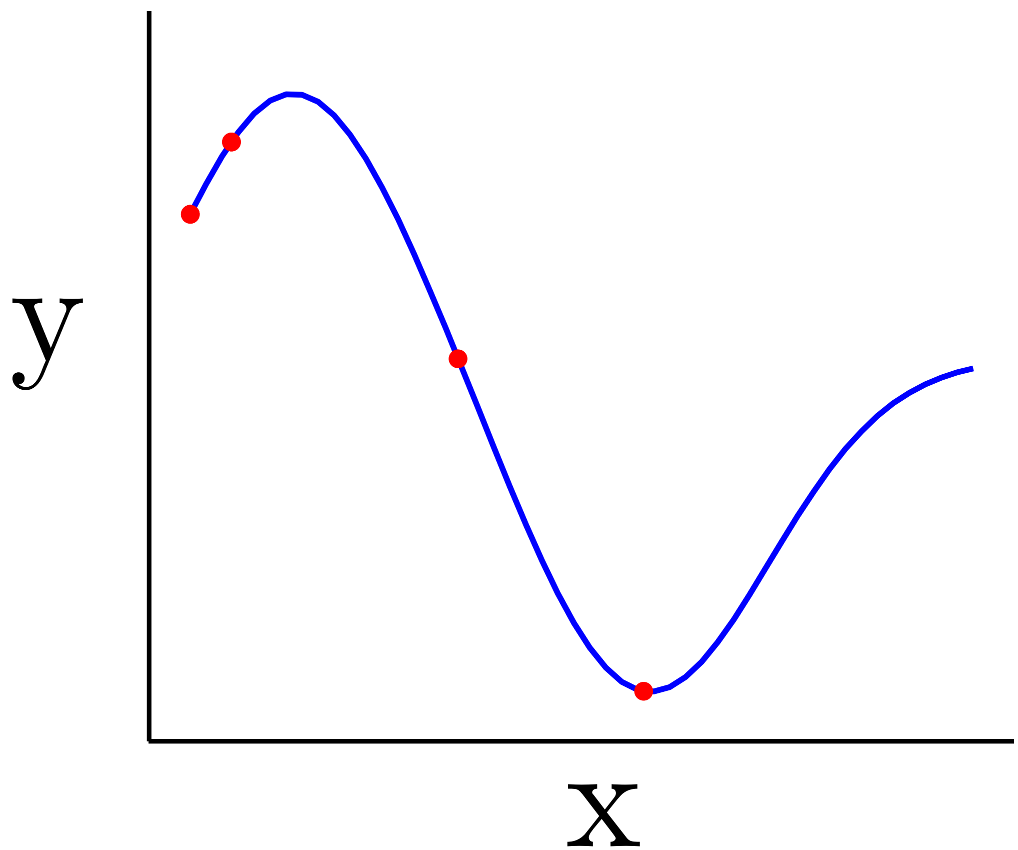

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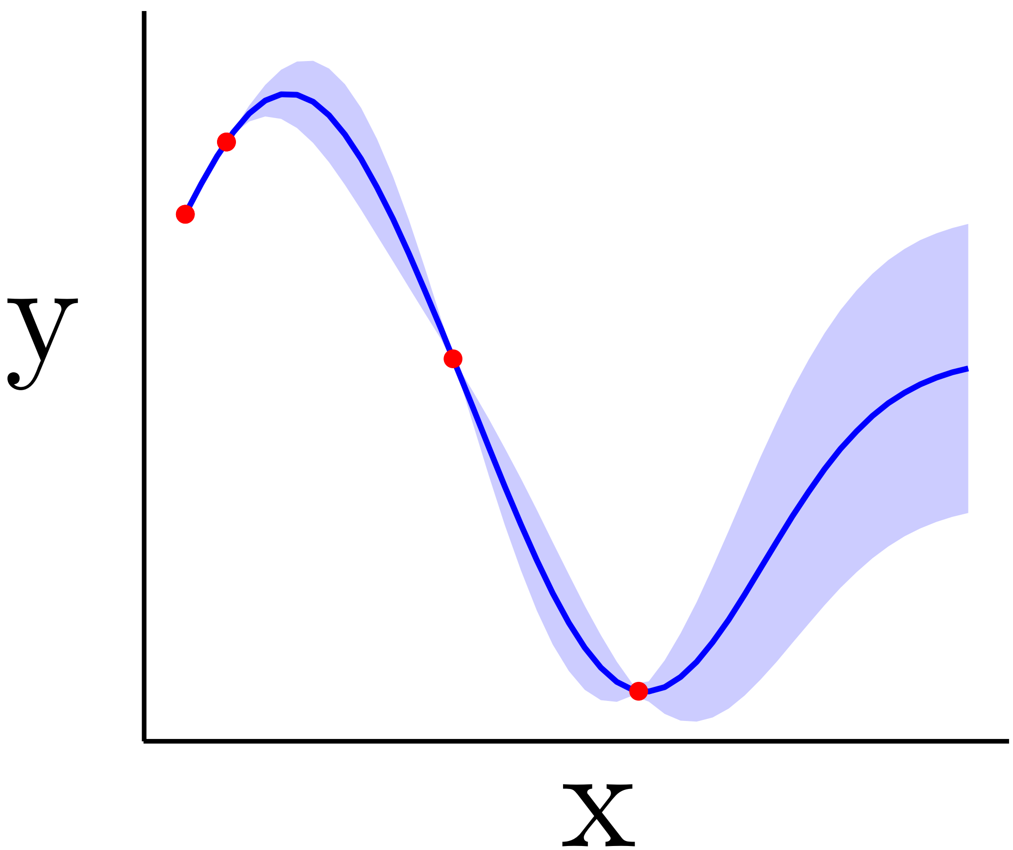
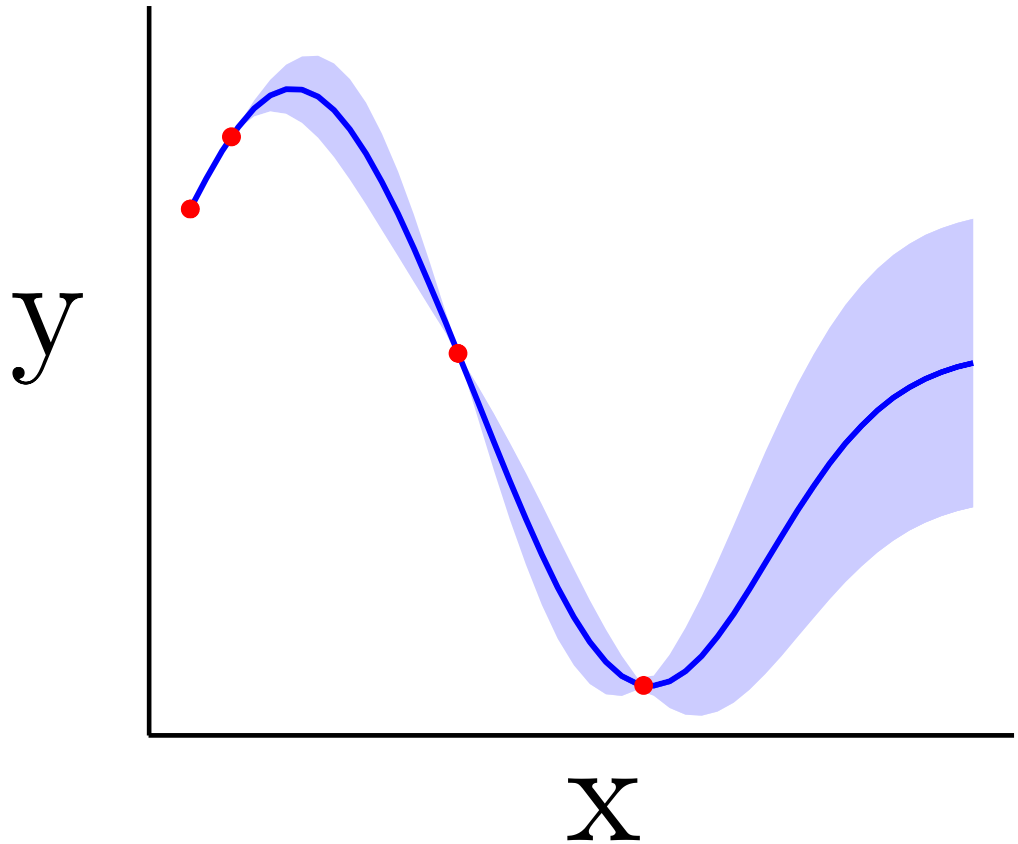


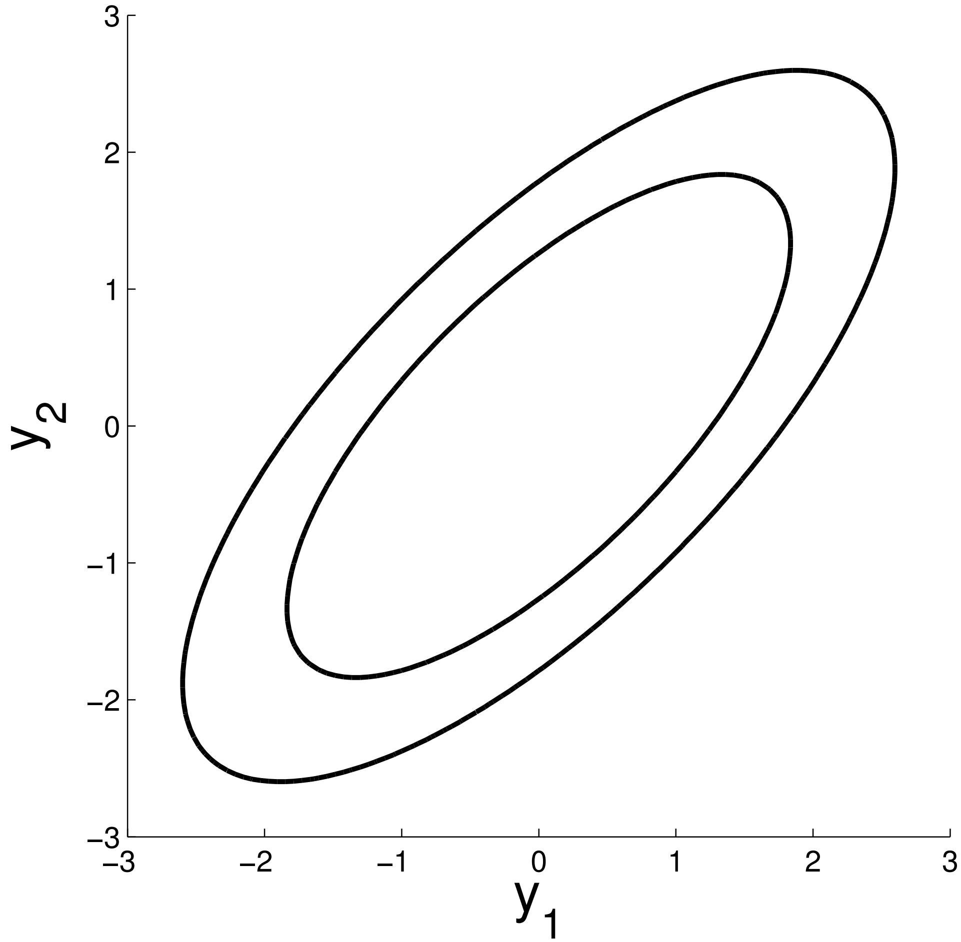
Motivation: non-linear regression

Can we do this with a plain old Gaussian?



Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\tfrac{1}{2}\mathbf{y}^\mathsf{T}\Sigma^{-1}\mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & .7 \\ .7 & 1 \end{bmatrix}$$



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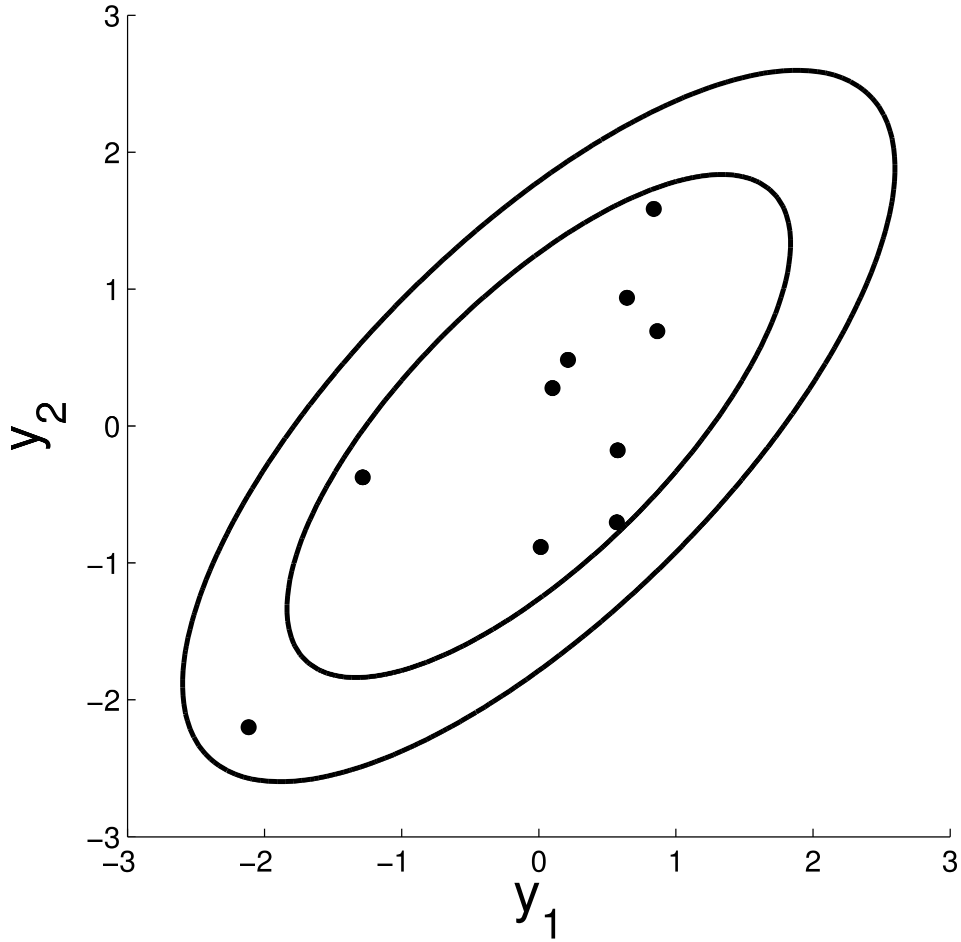
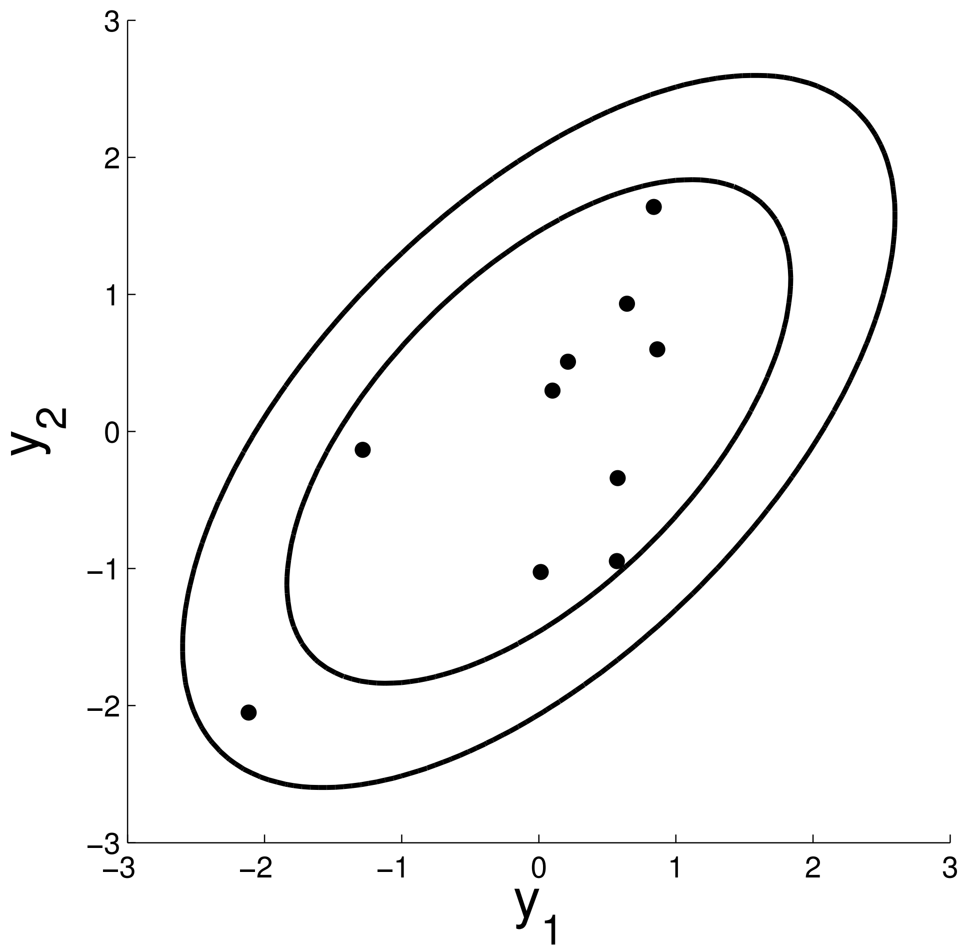


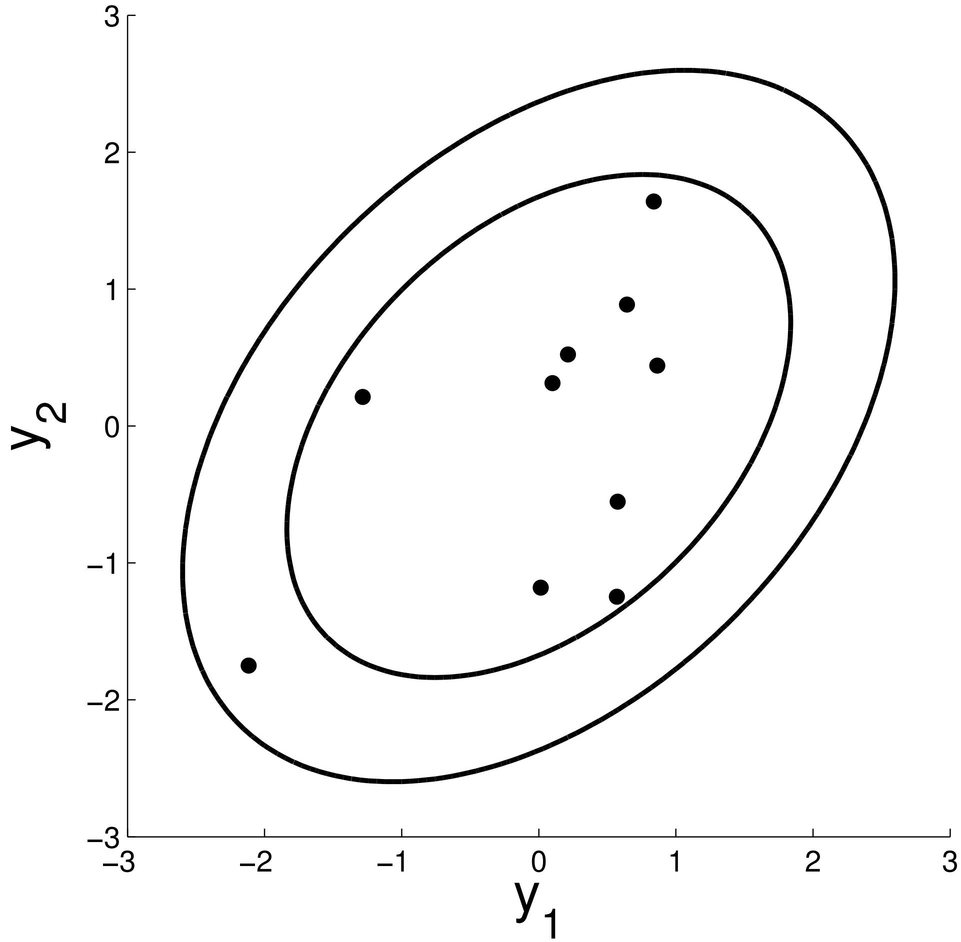
Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\tfrac{1}{2}\mathbf{y}^\intercal \Sigma^{-1} \mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & .6 \\ .6 & 1 \end{bmatrix}$$



Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\tfrac{1}{2}\mathbf{y}^{\mathsf{T}}\Sigma^{-1}\mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & .4 \\ .4 & 1 \end{bmatrix}$$



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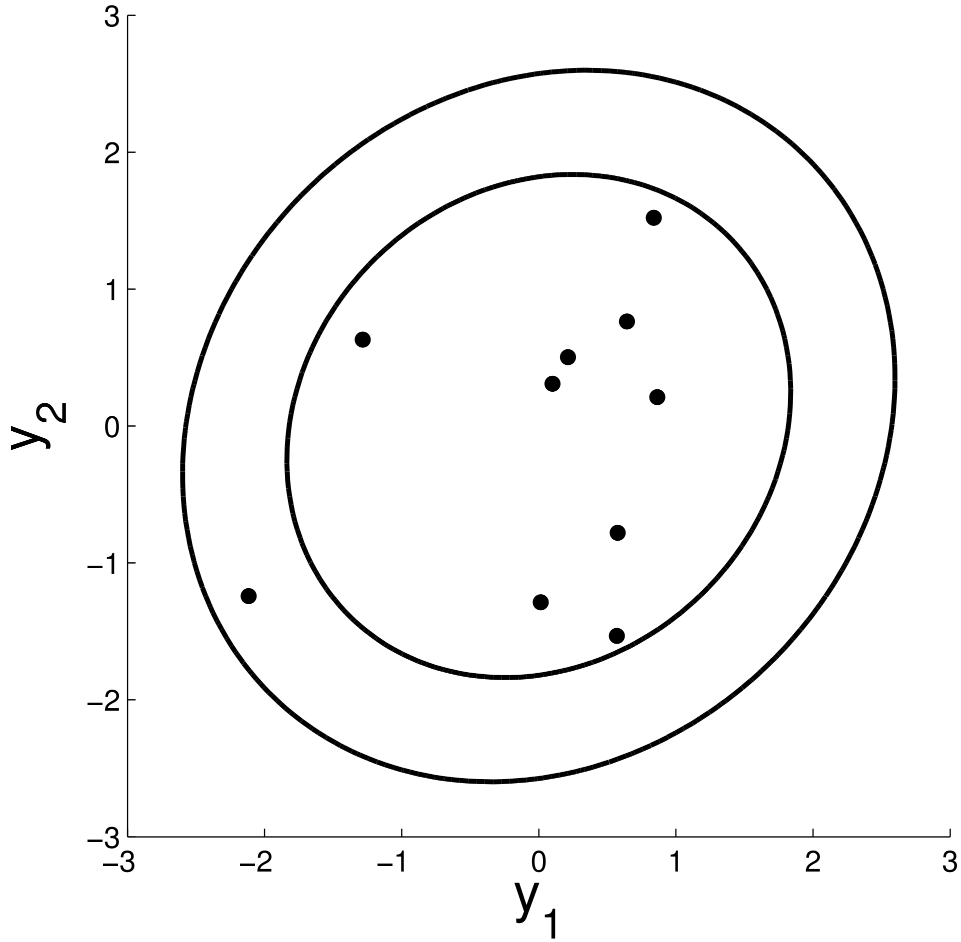

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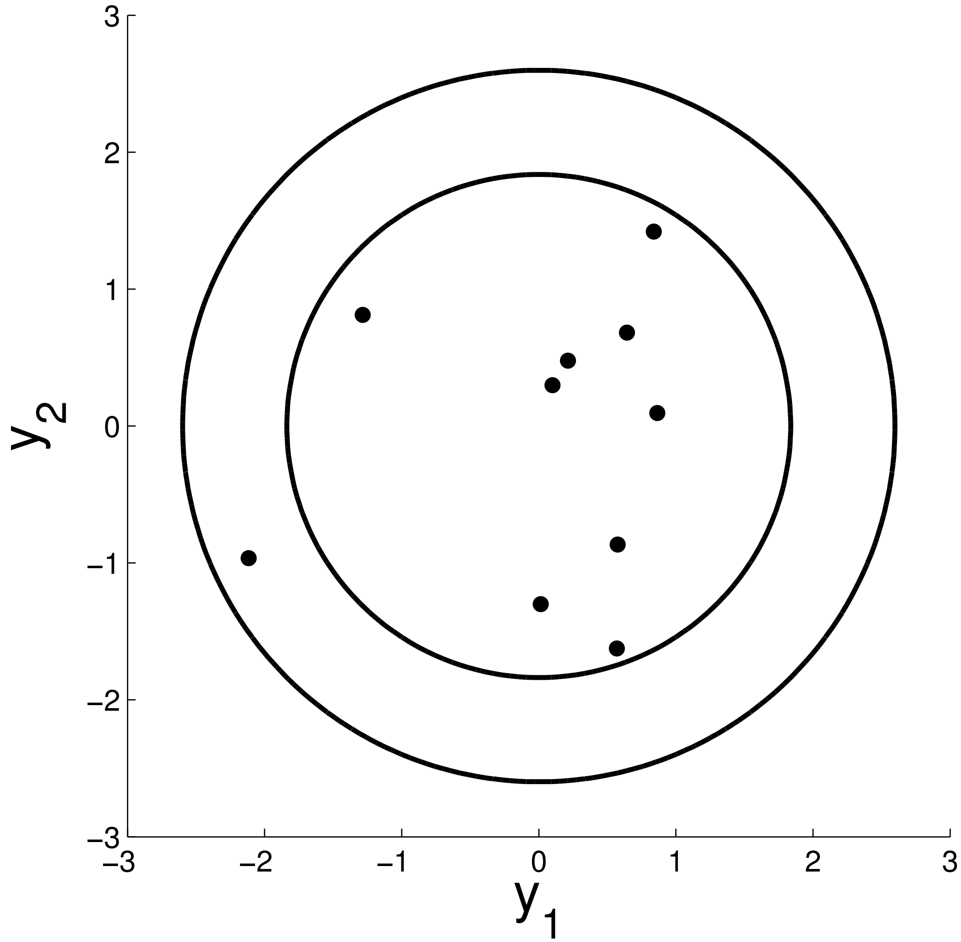

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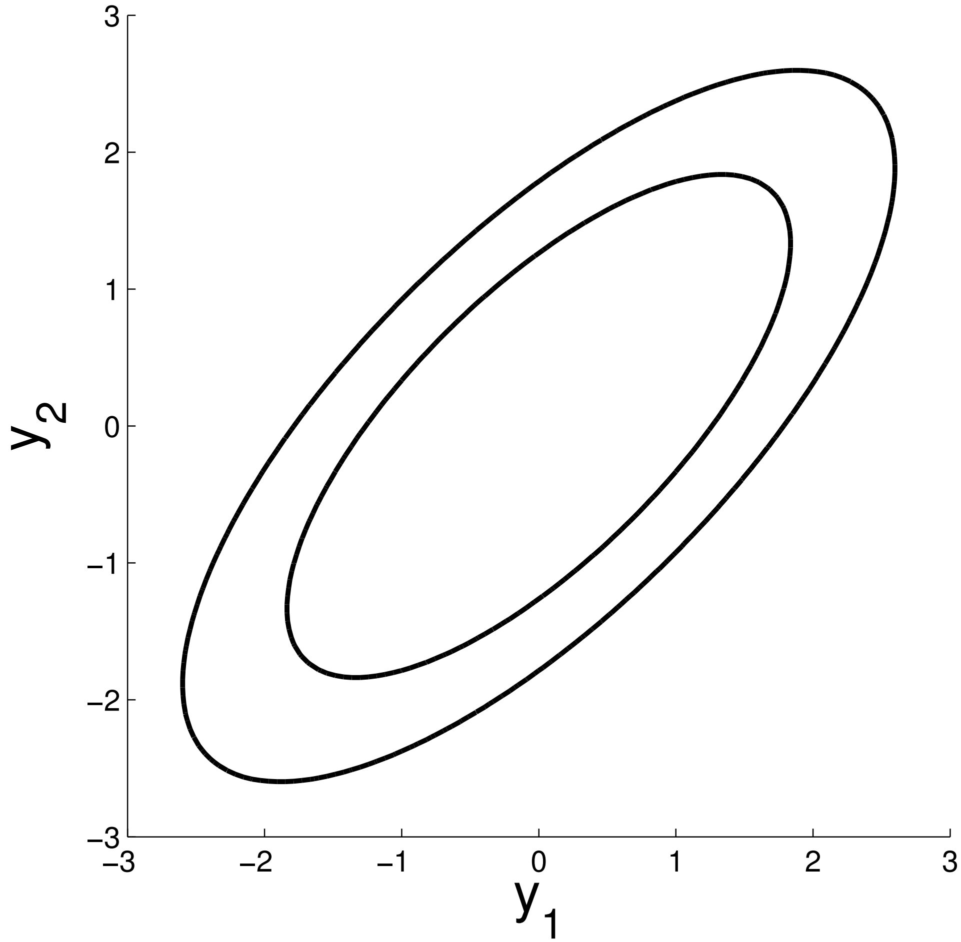

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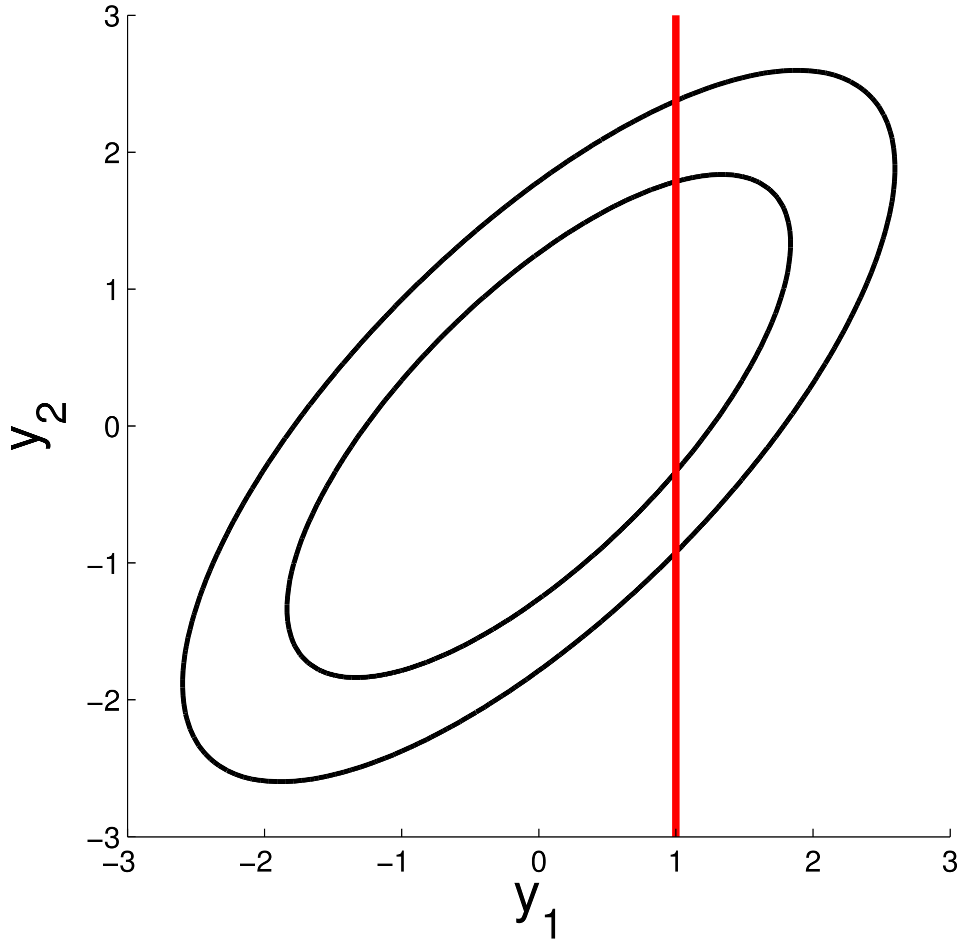

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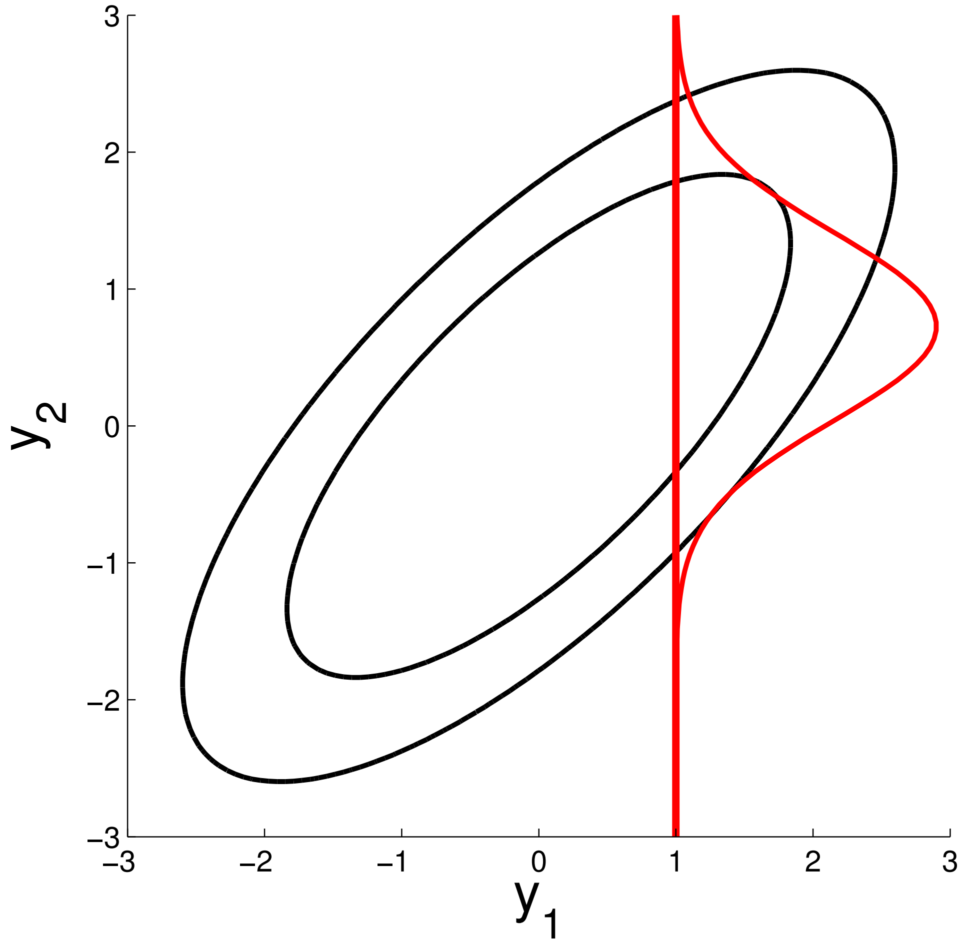
$$p(\mathsf{y}_2|\mathsf{y}_1,\Sigma) \propto \exp\left(-\frac{1}{2}(\mathsf{y}_2 - \mu_*)\Sigma_*{}^{-1}(\mathsf{y}_2 - \mu_*)\right)$$



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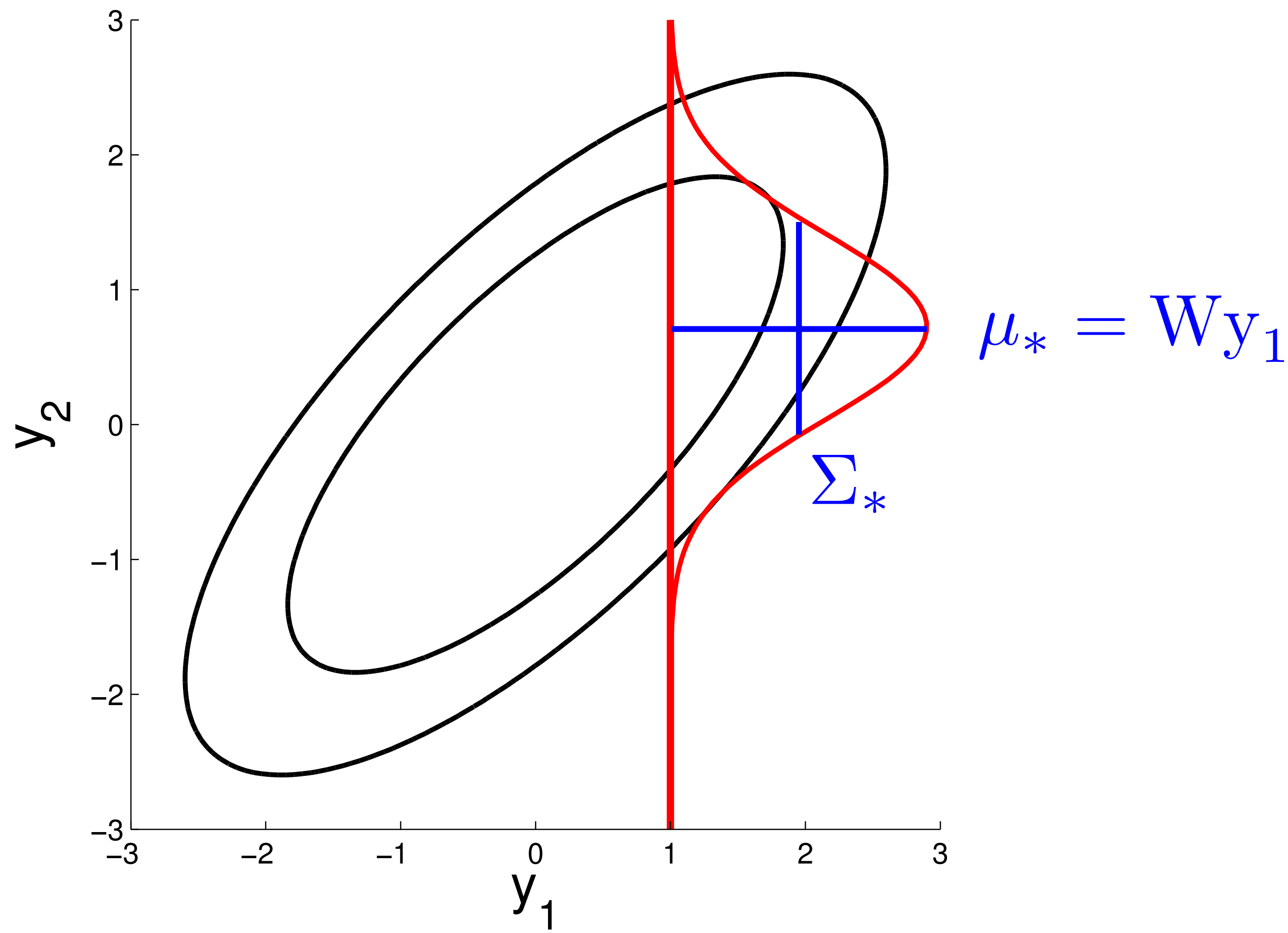

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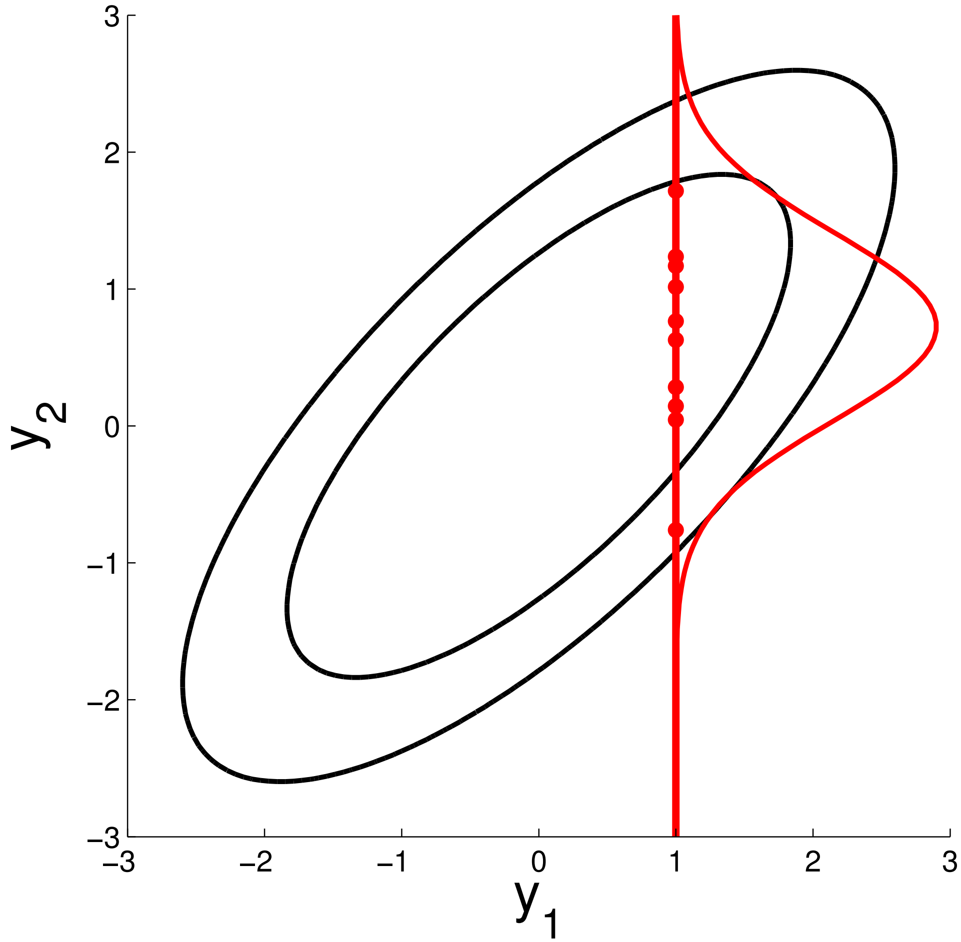

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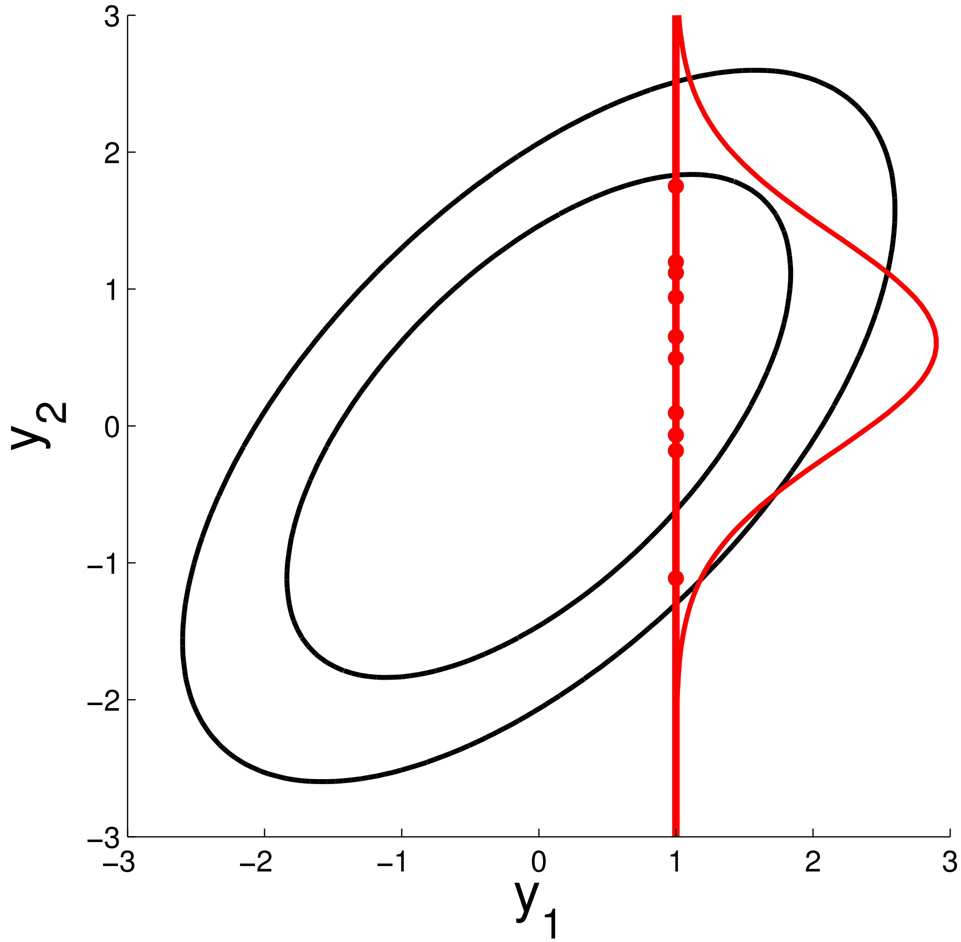

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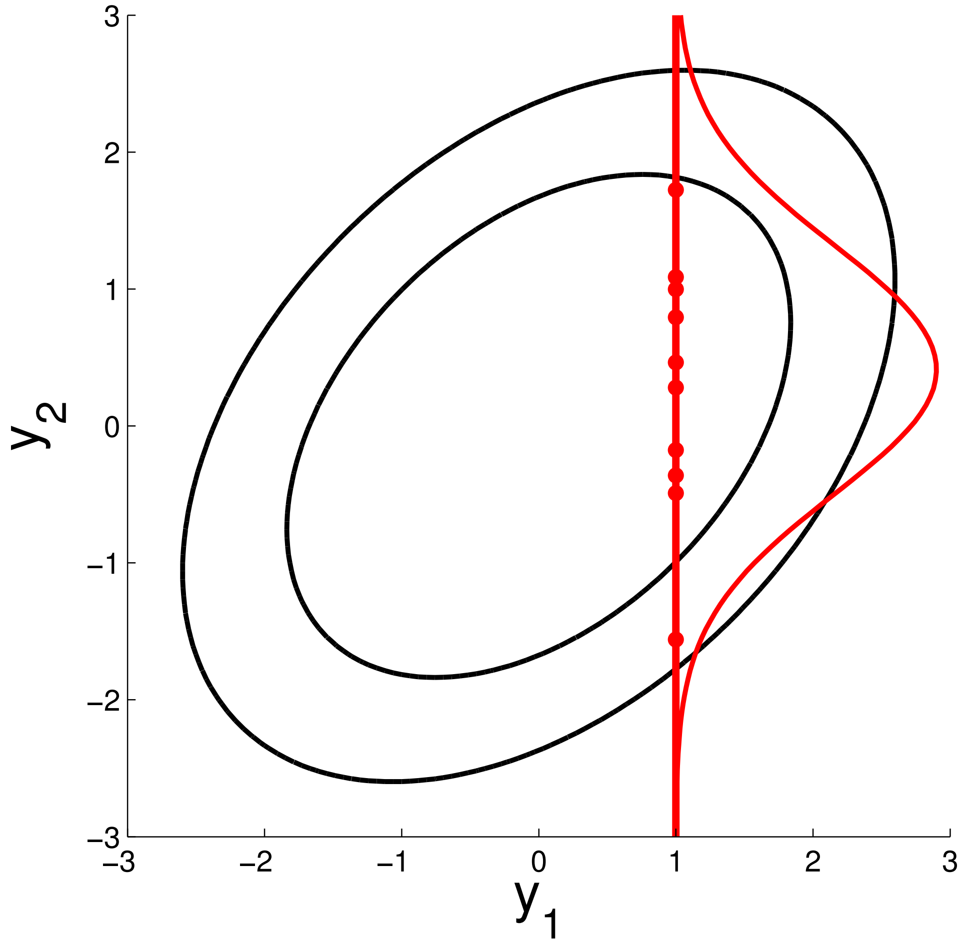

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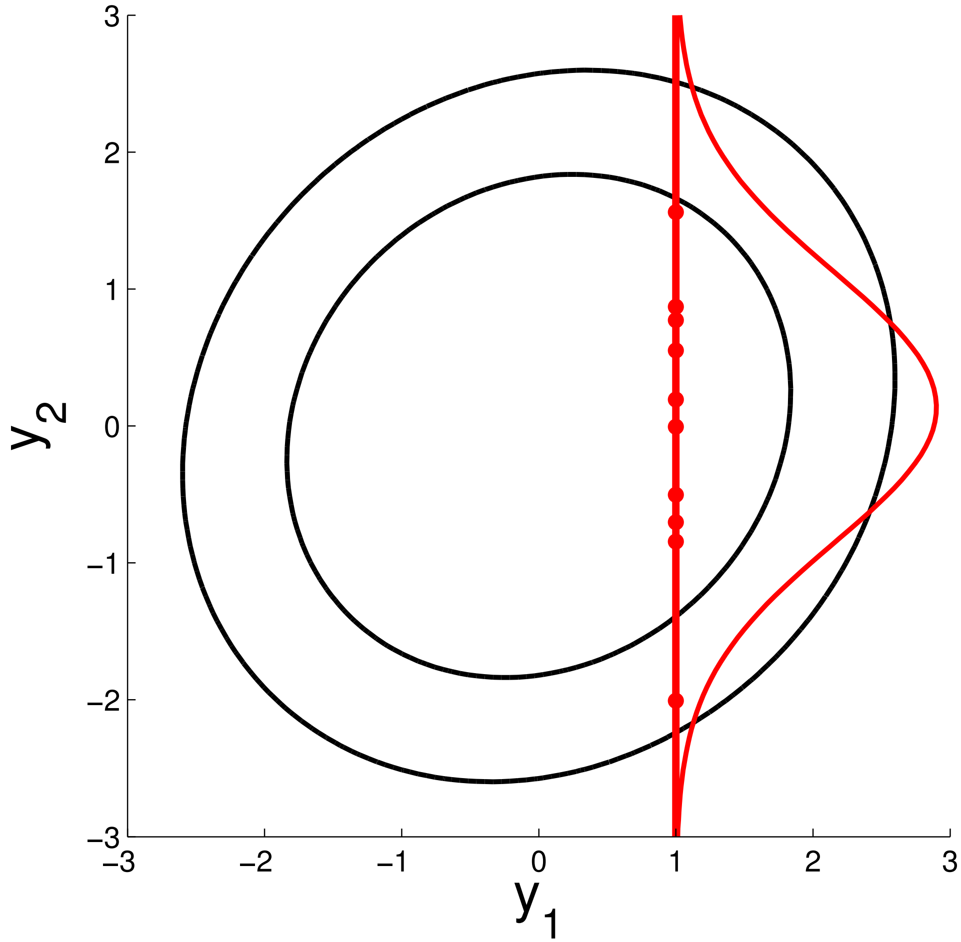

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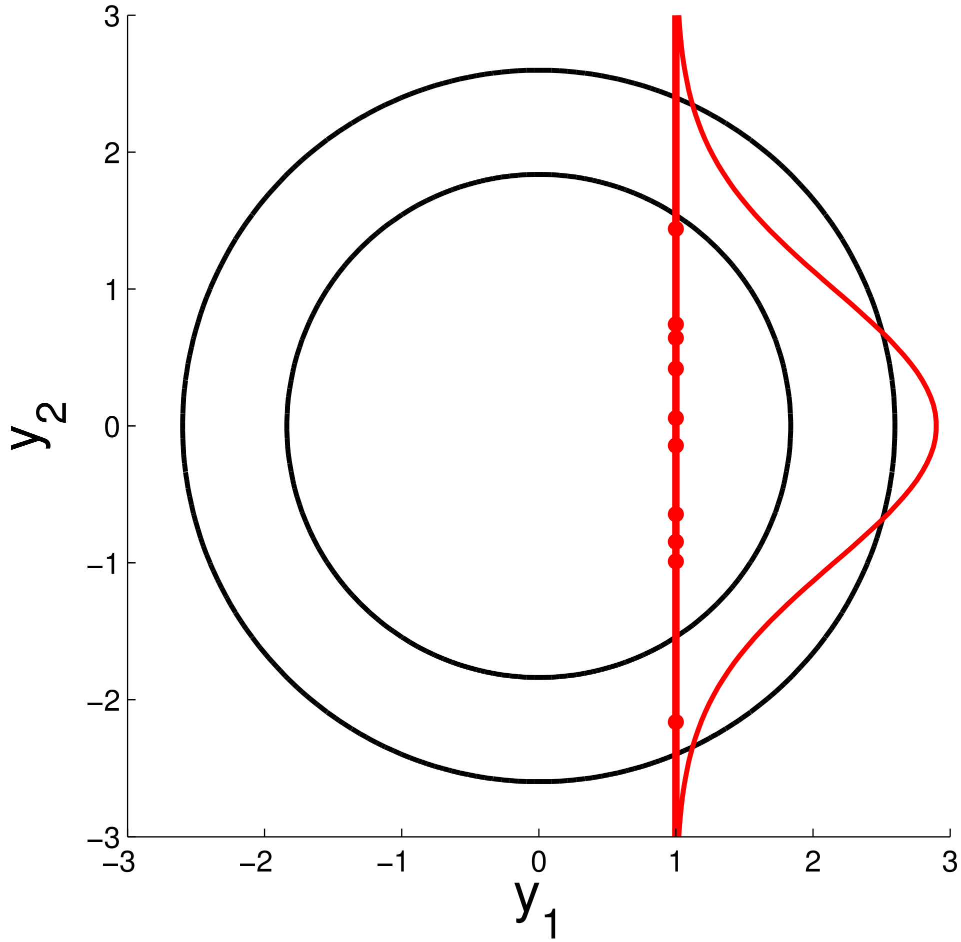
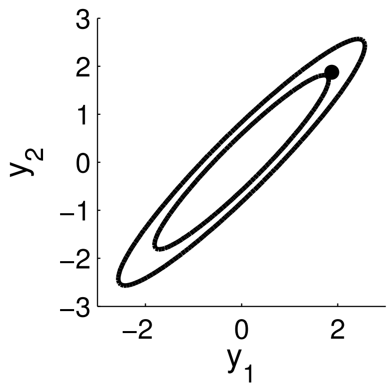


New visualisation



$$\Sigma = \begin{bmatrix} 1 & .9 \\ .9 & 1 \end{bmatrix}$$

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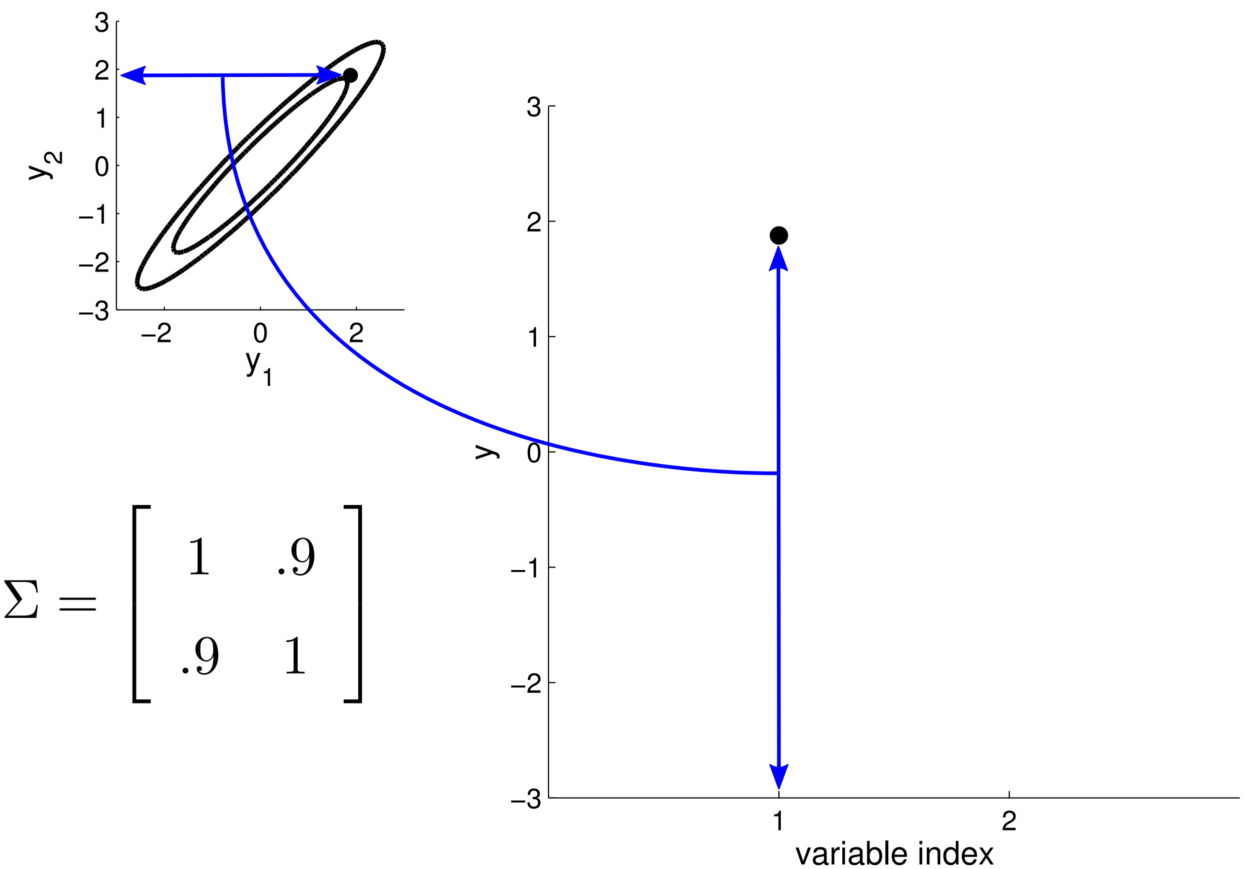

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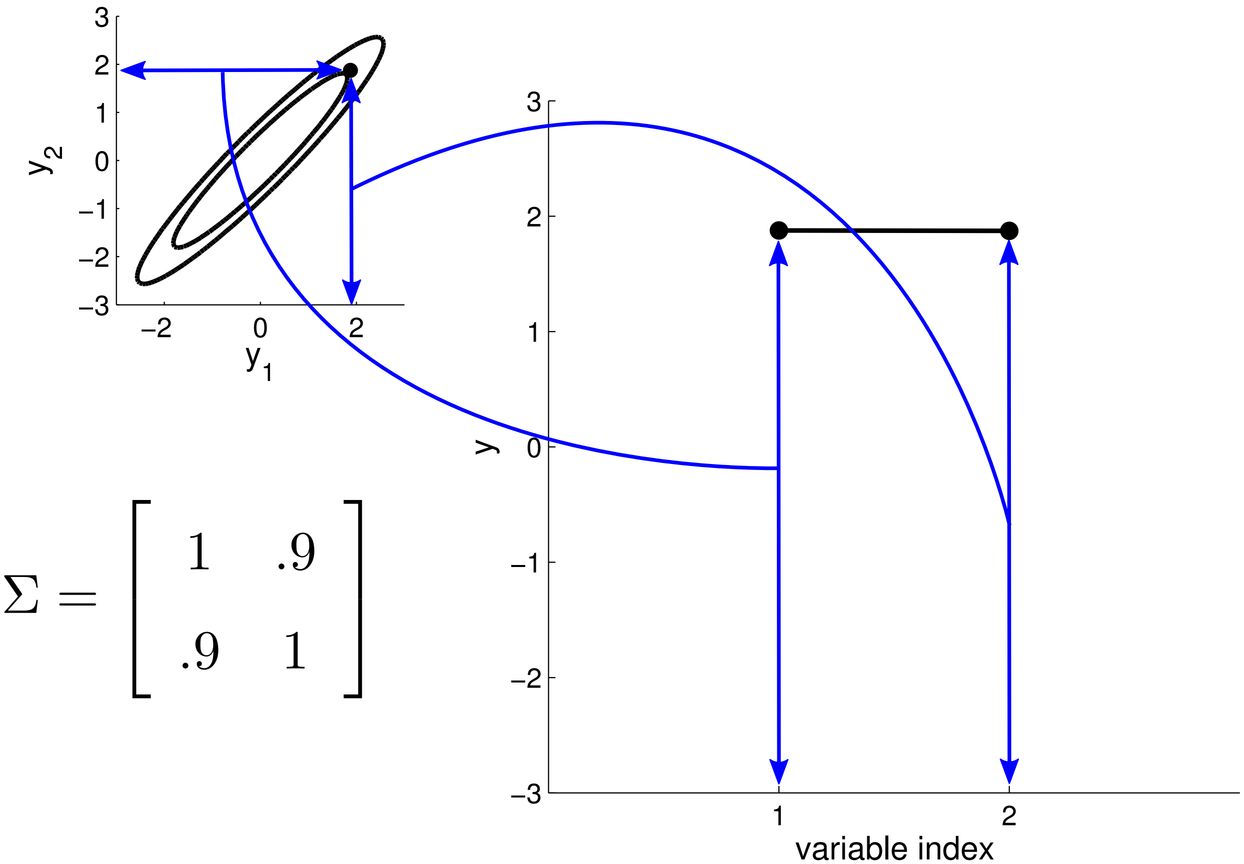

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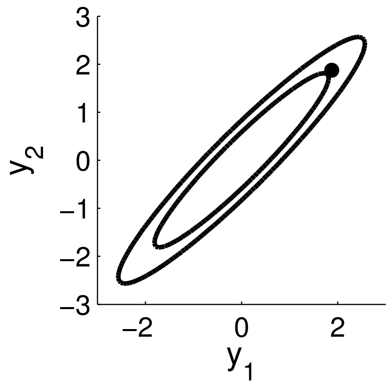

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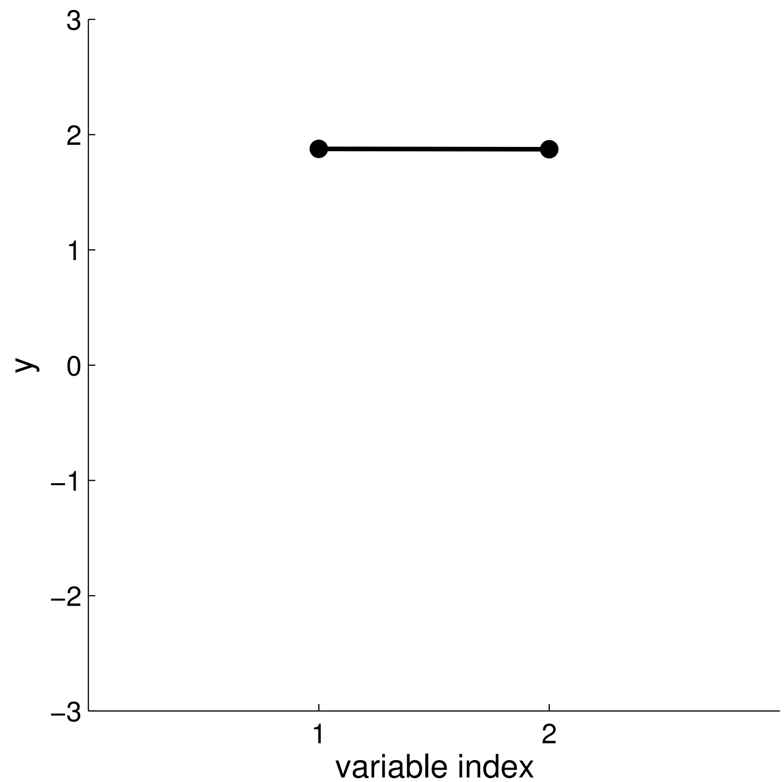

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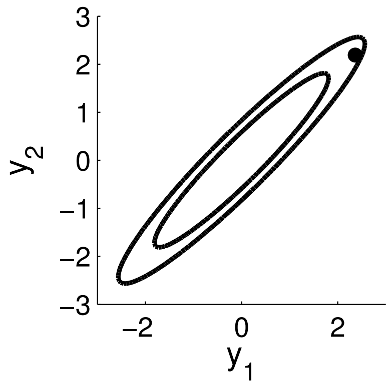

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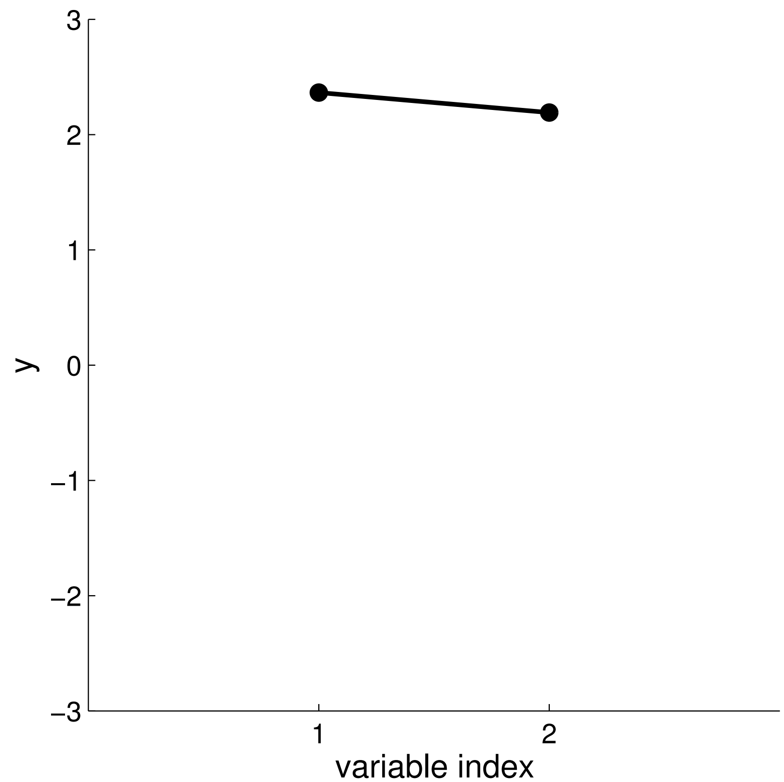

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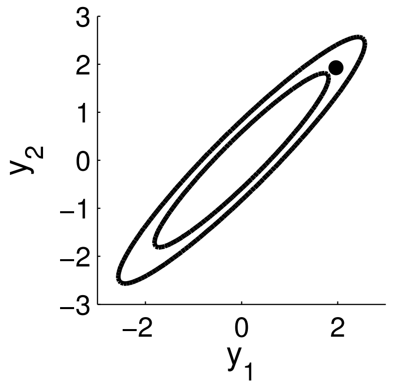

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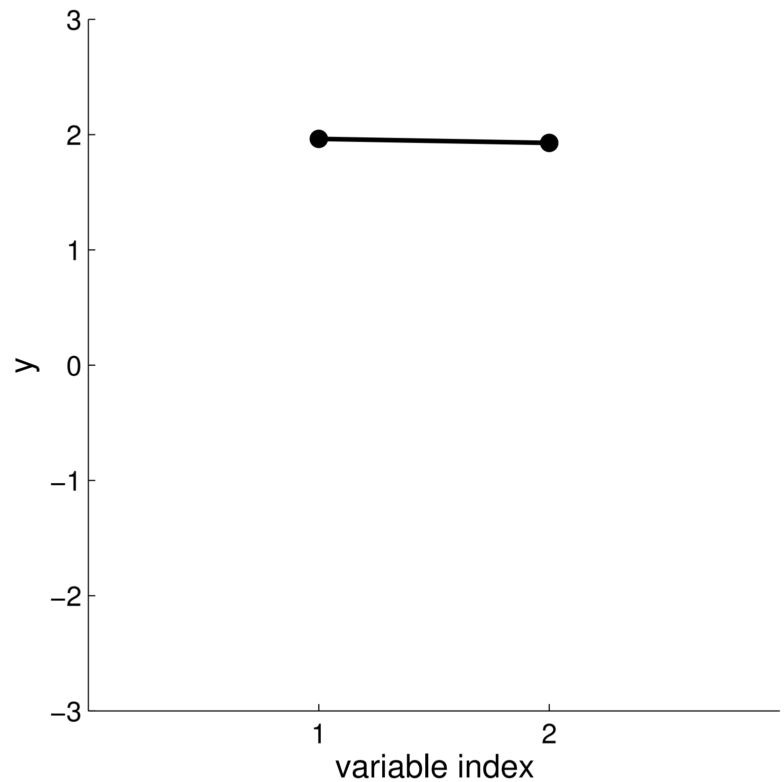

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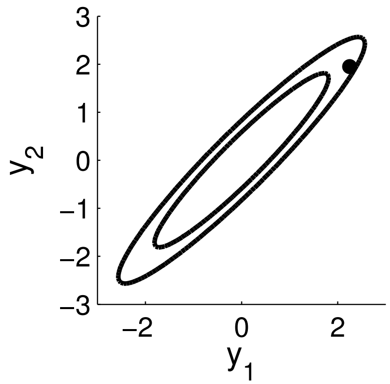

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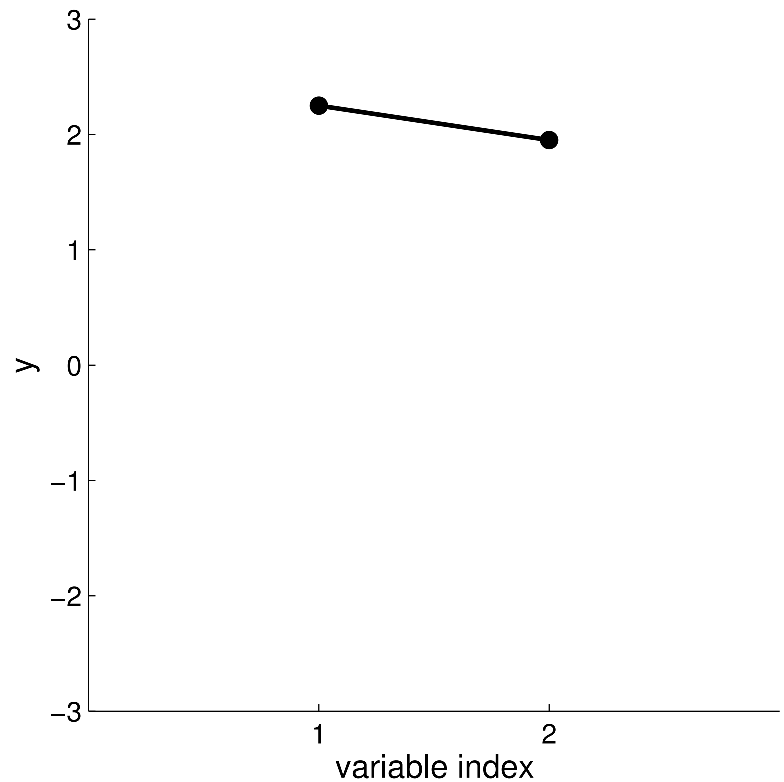

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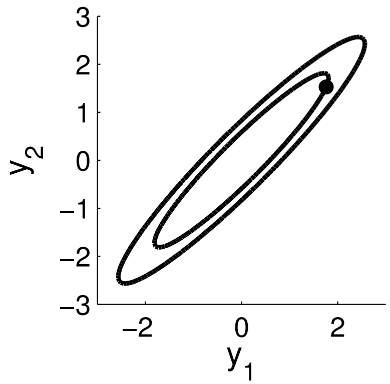

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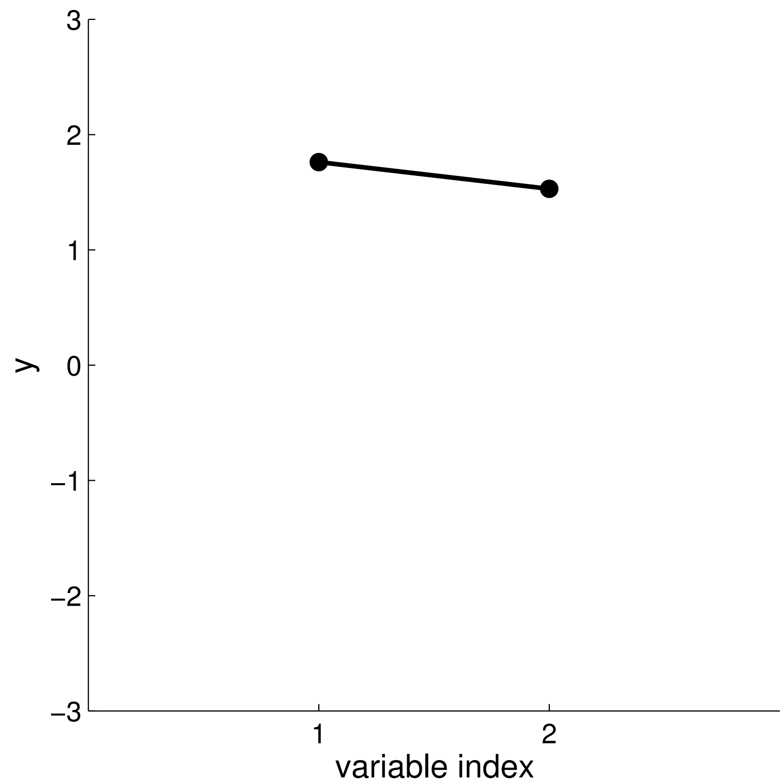

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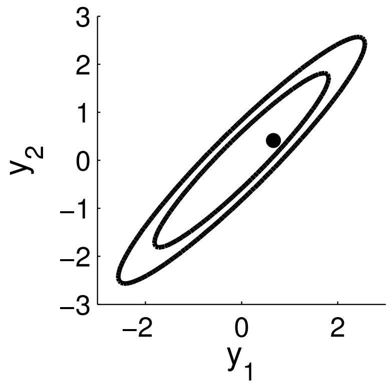

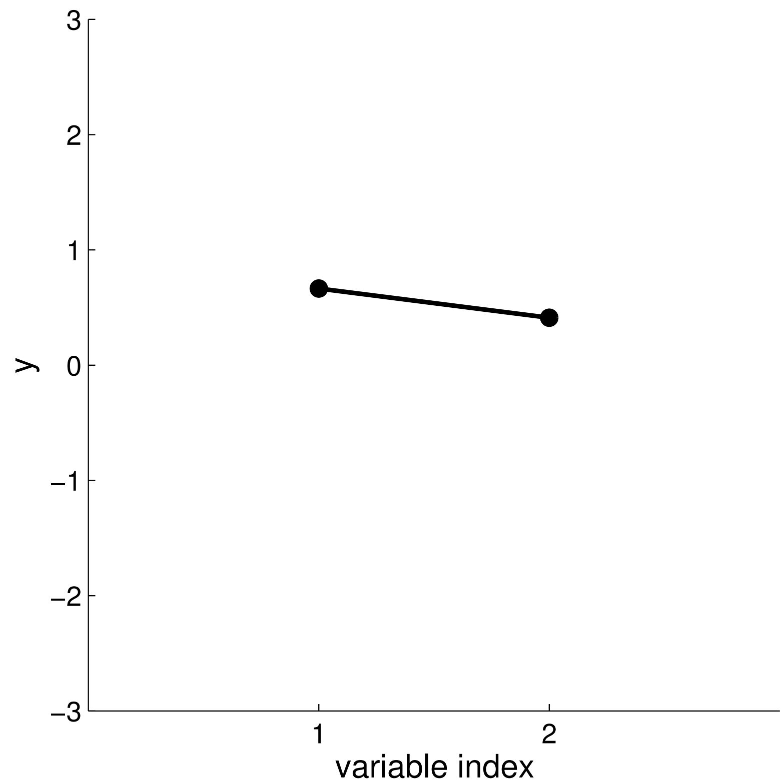

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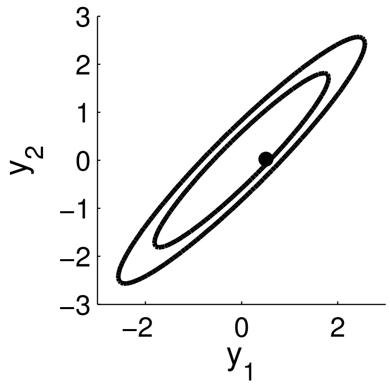

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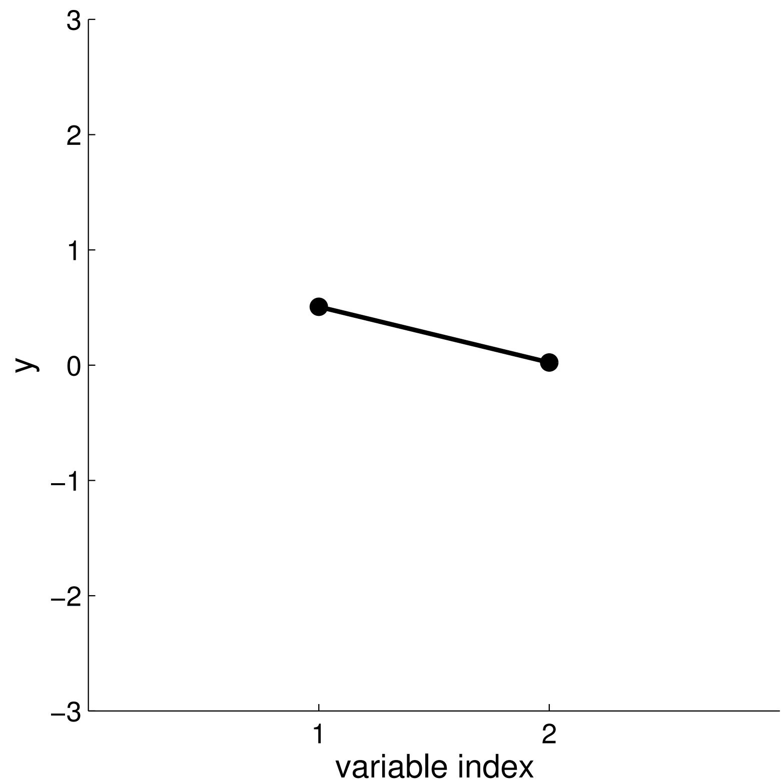

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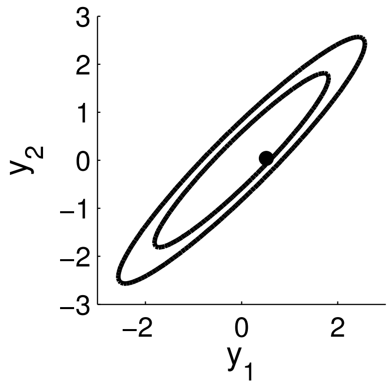

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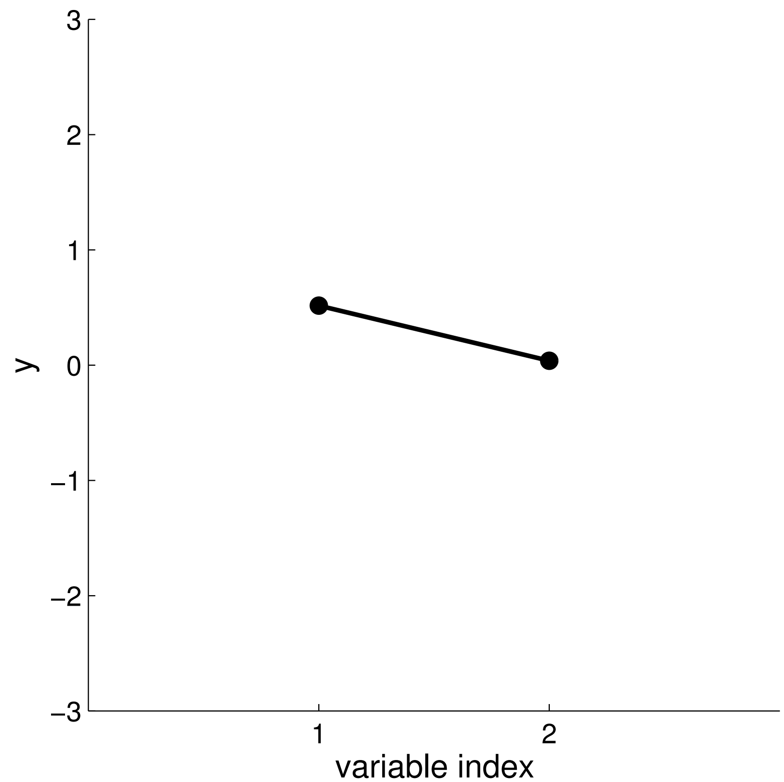

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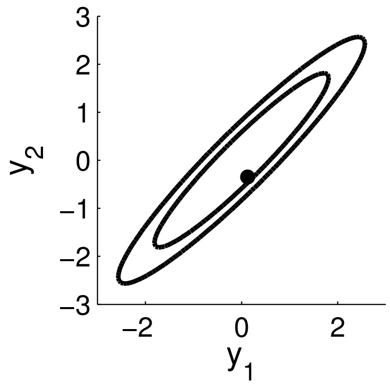

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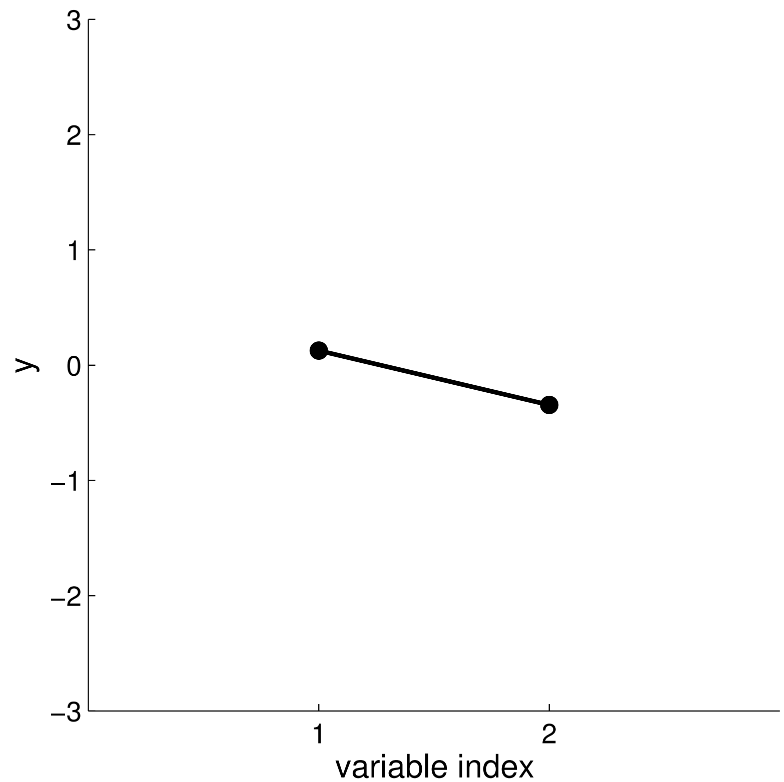

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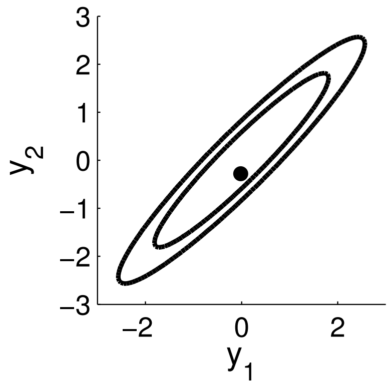

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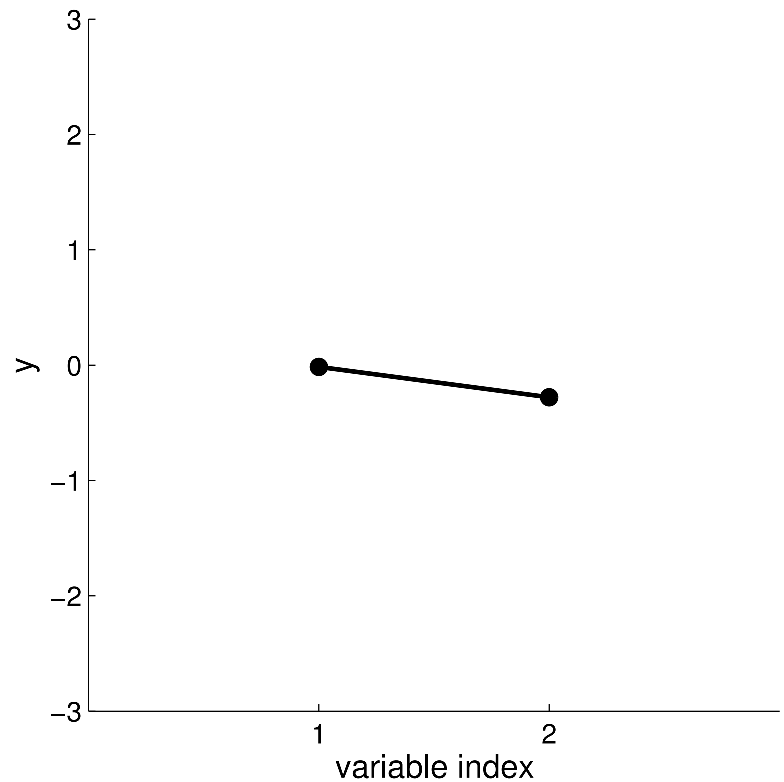

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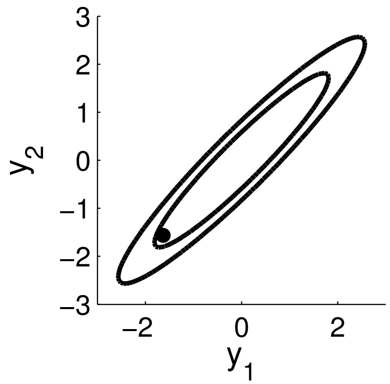

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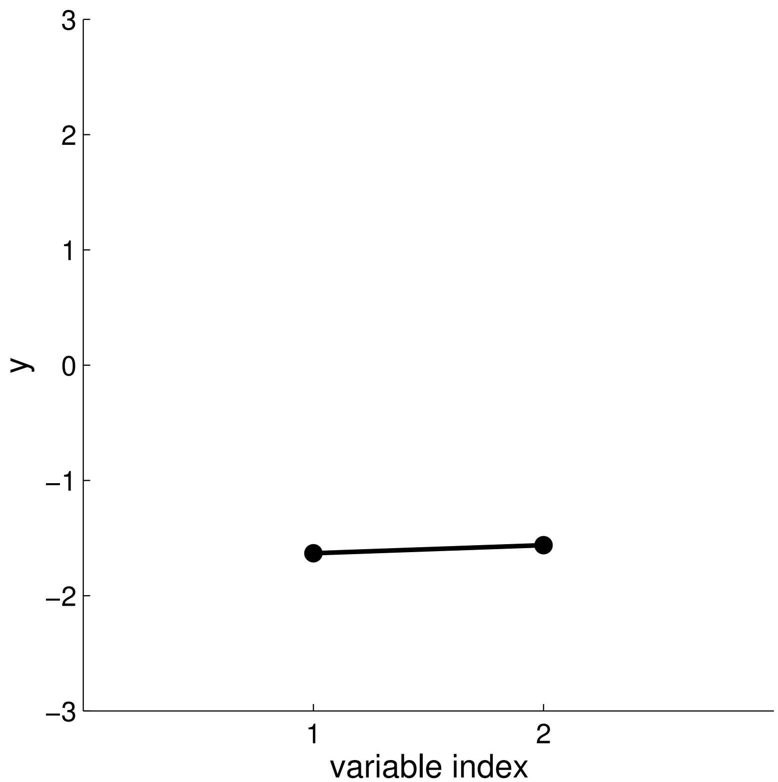

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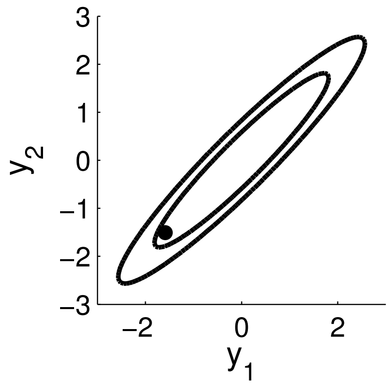

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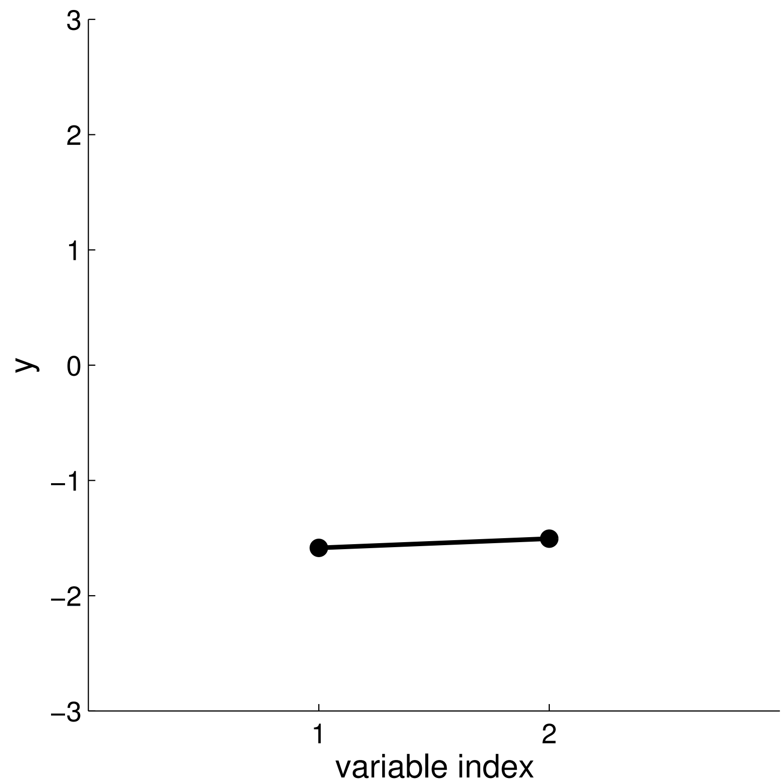

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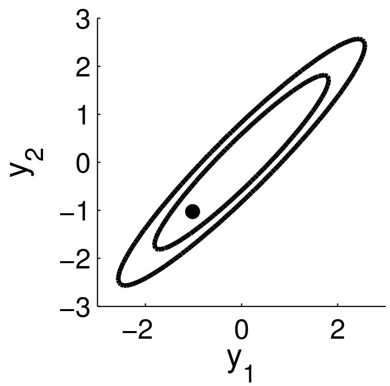

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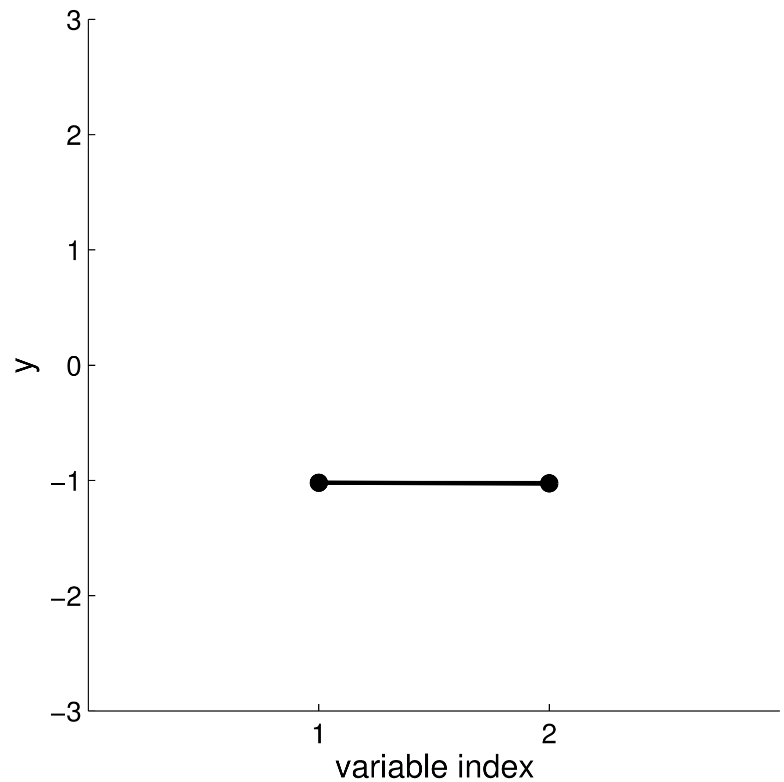

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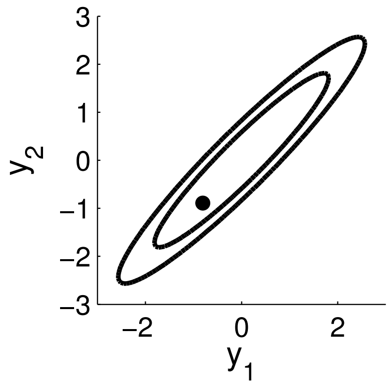

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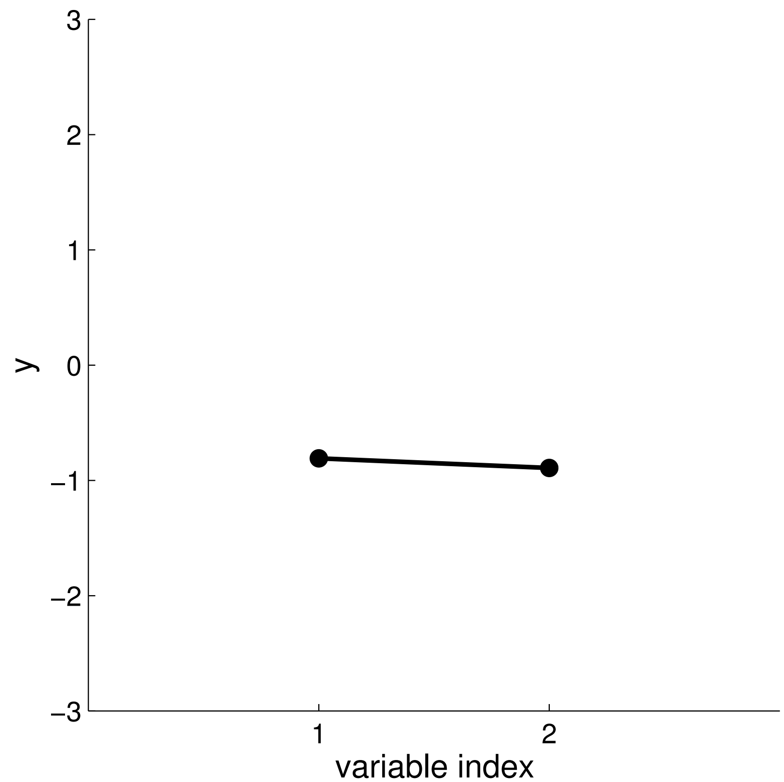
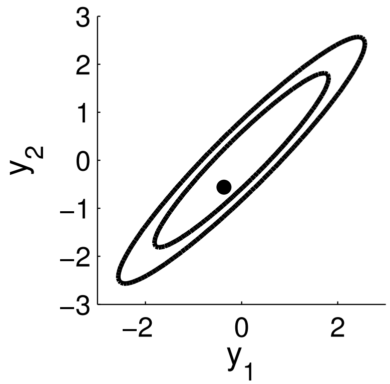


New visualisation



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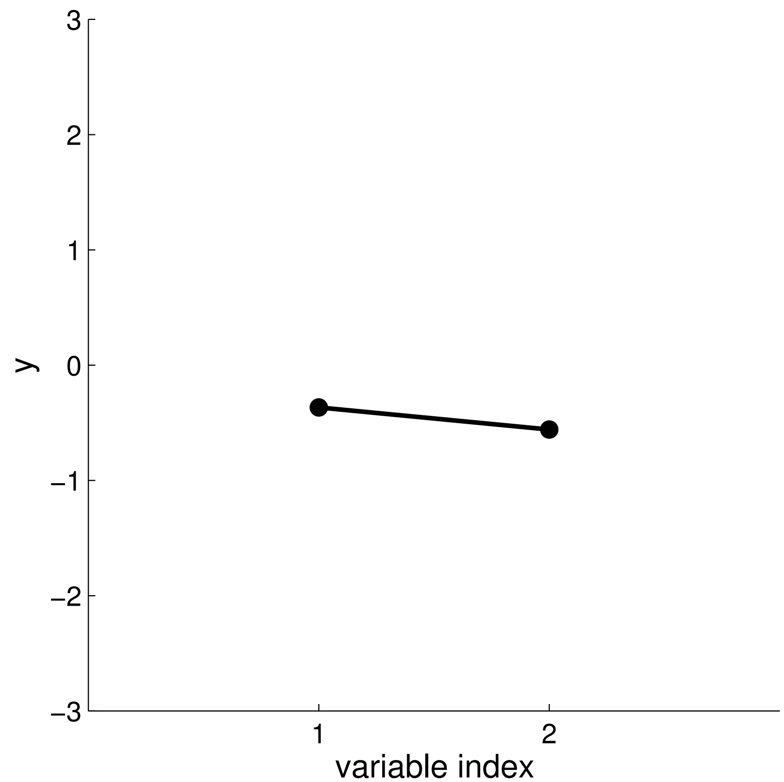

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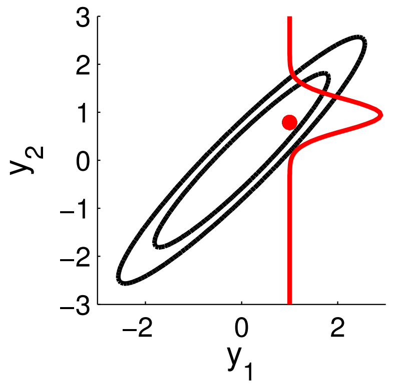

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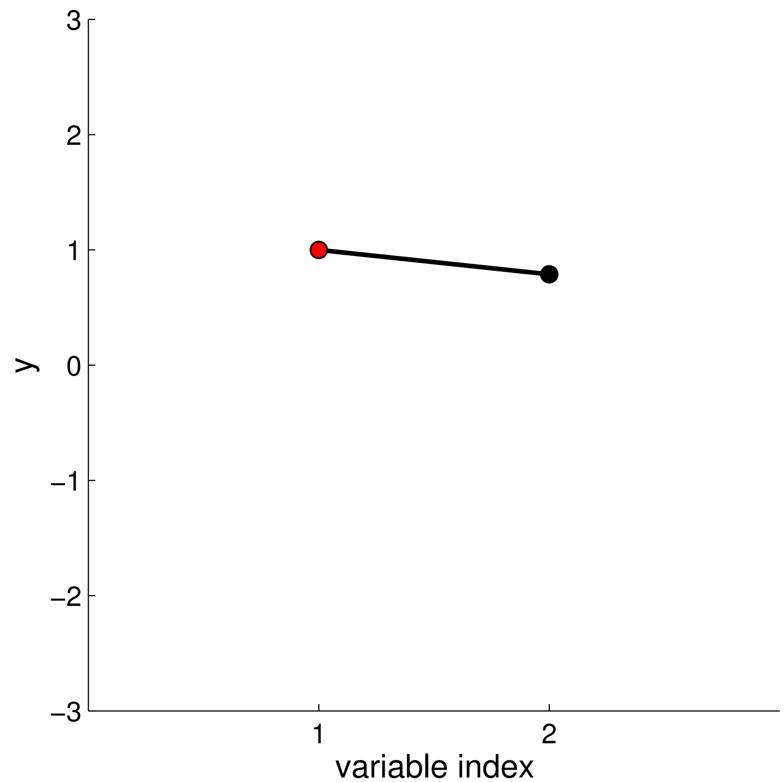

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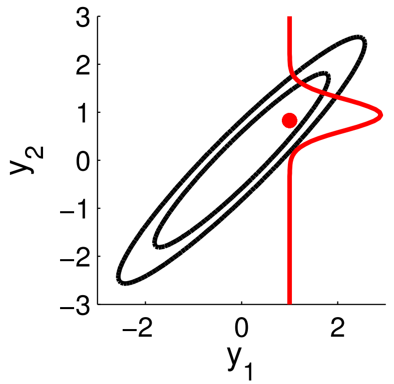

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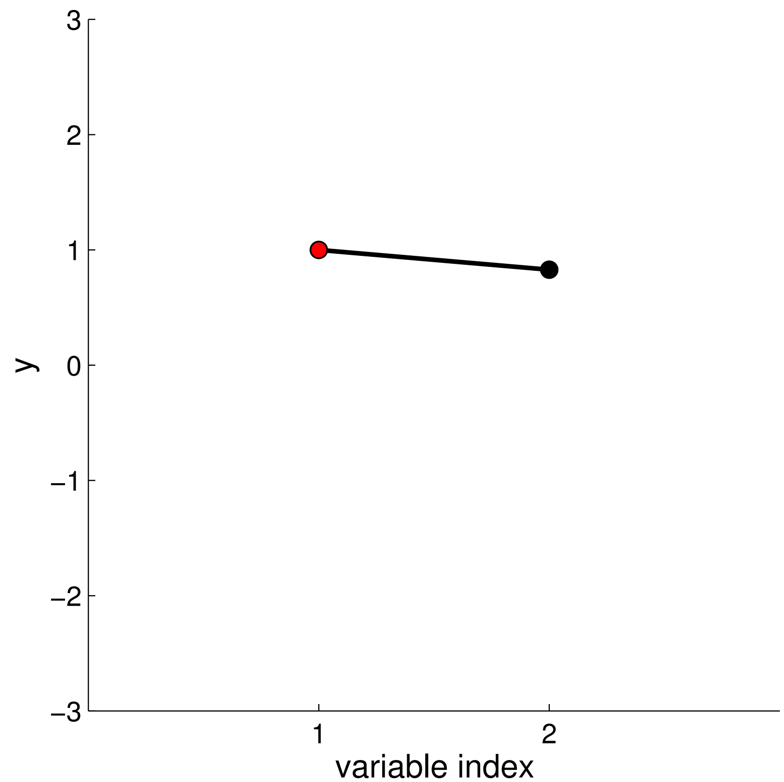

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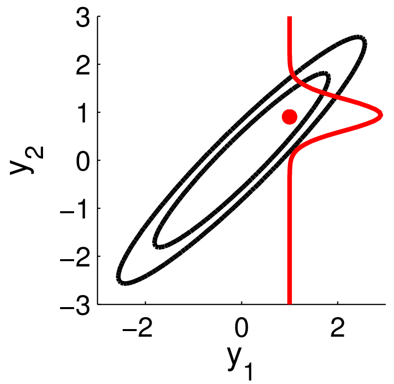

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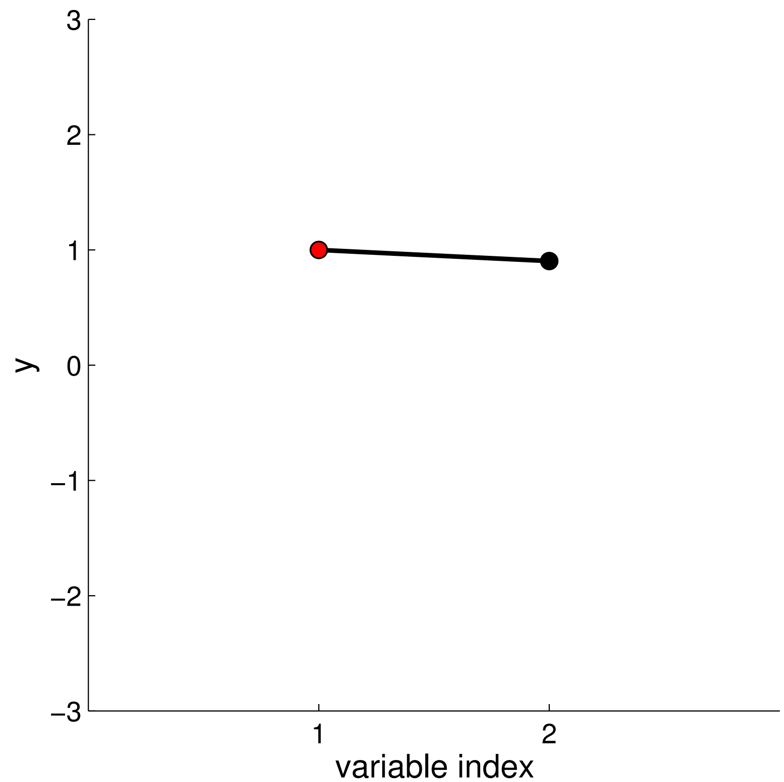

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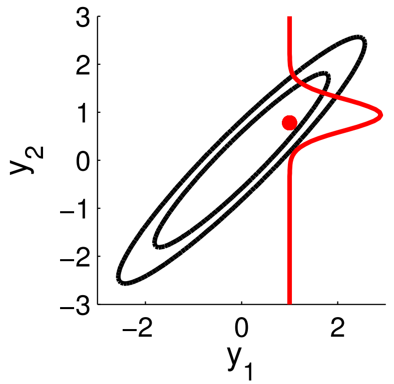

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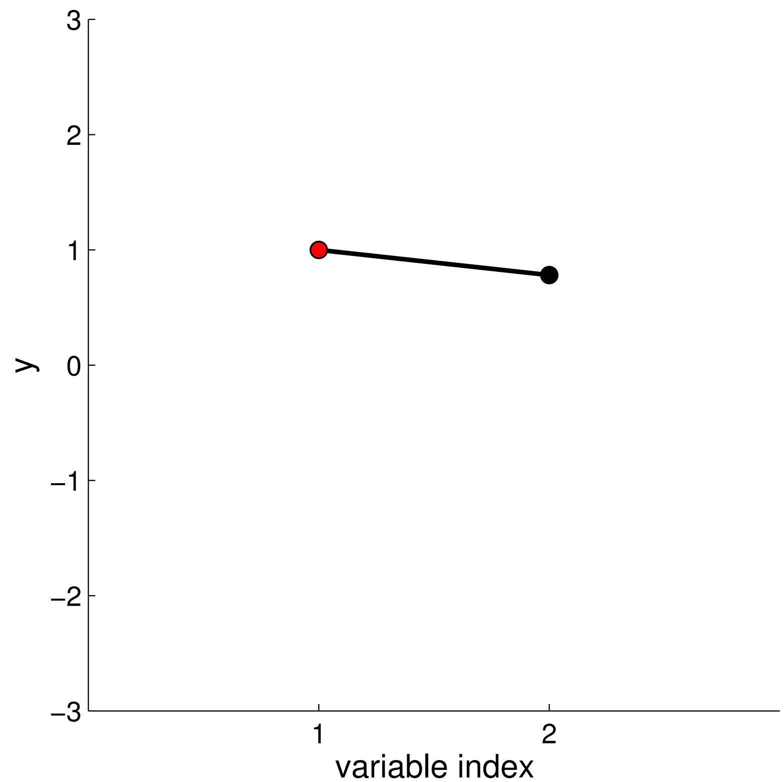

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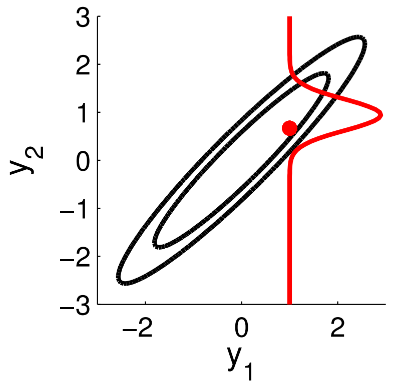

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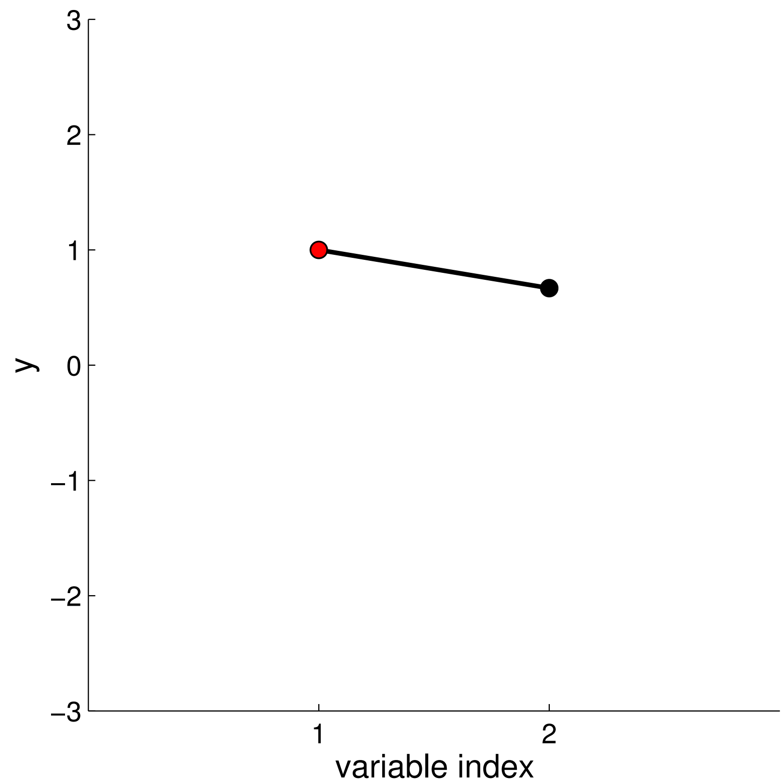

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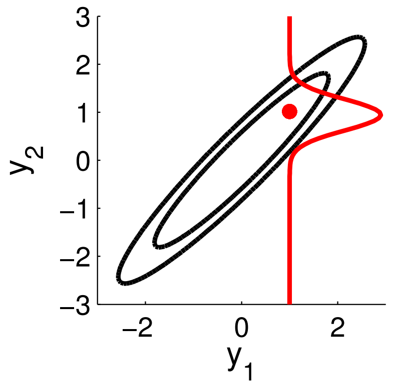

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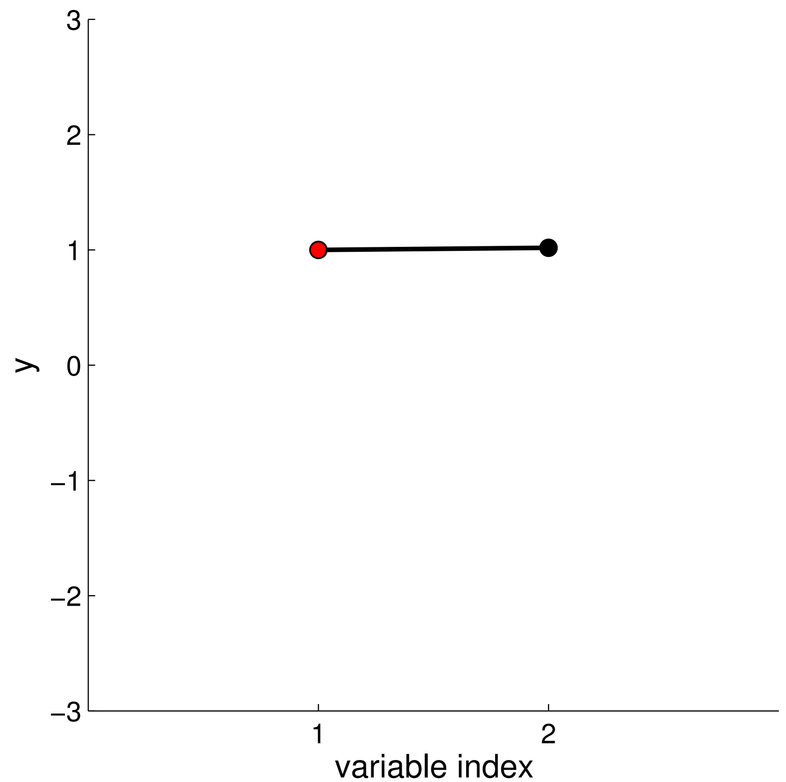

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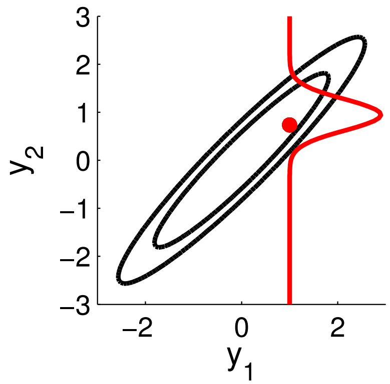

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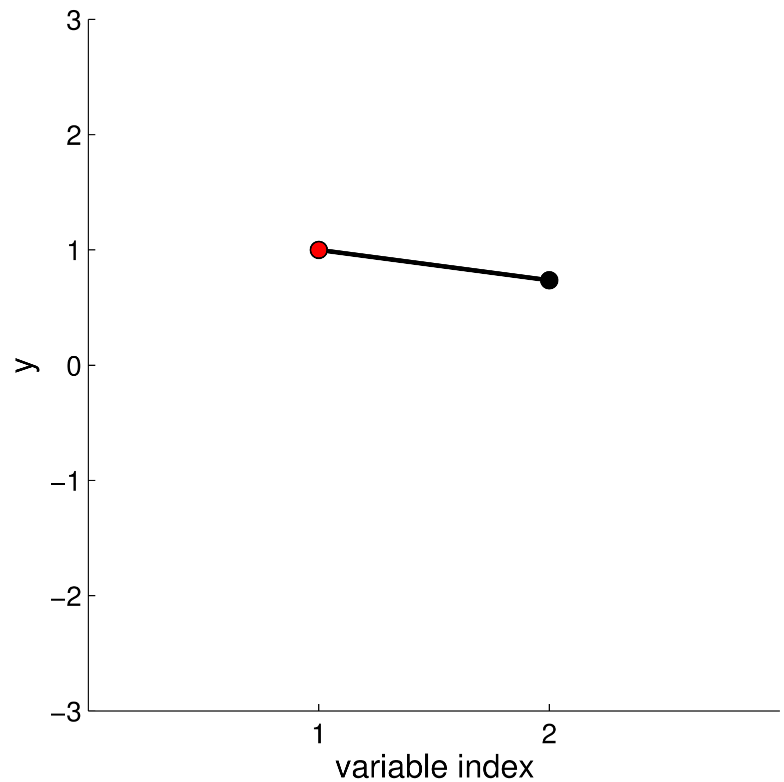

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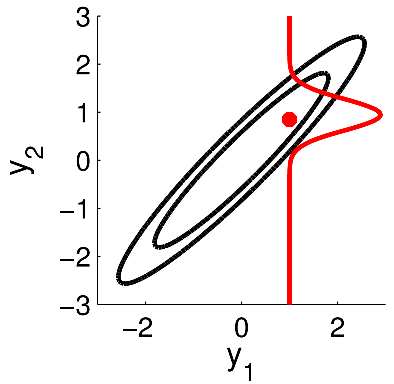

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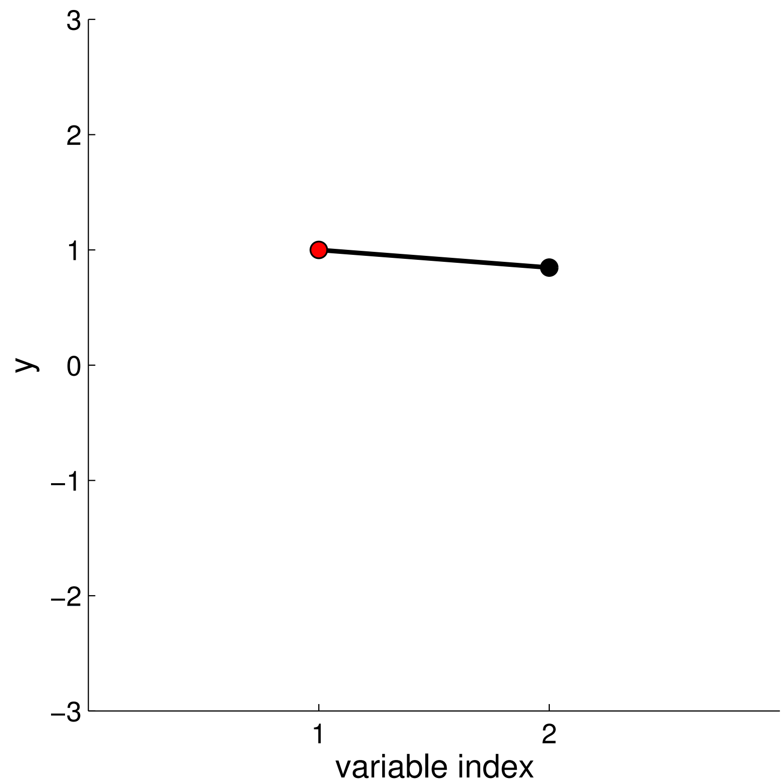

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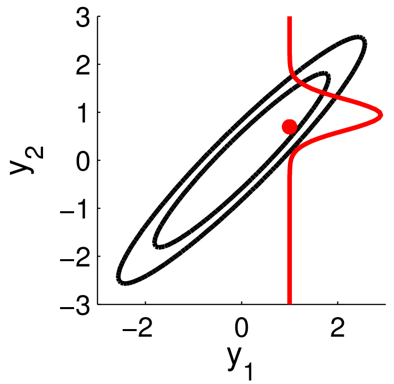

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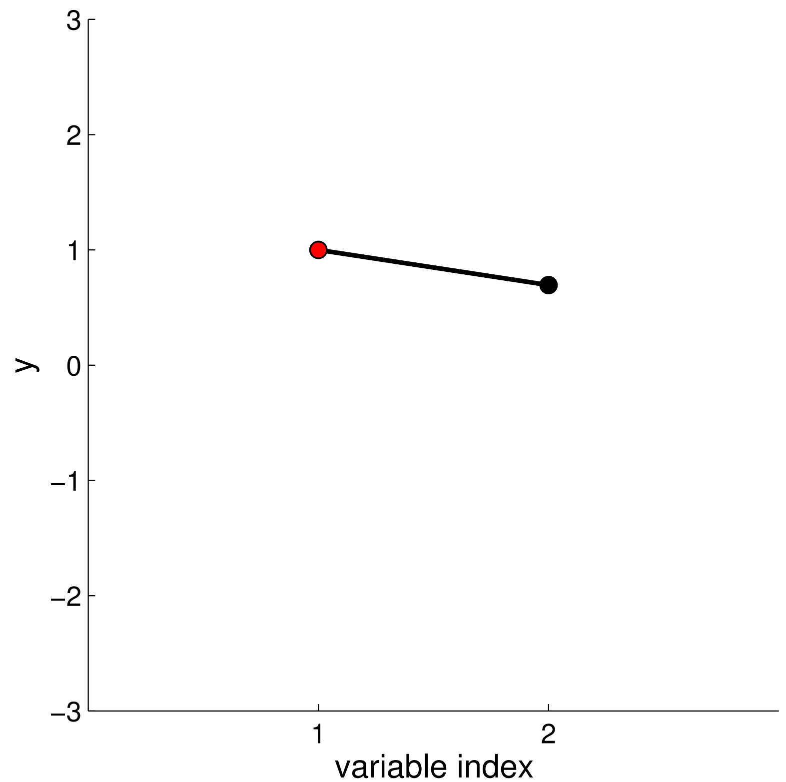

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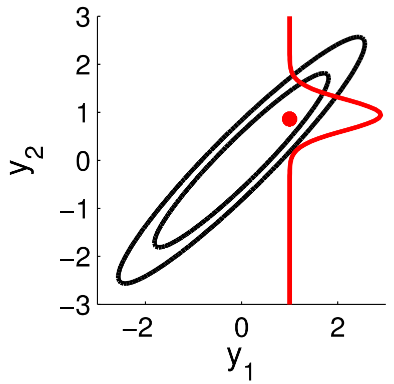

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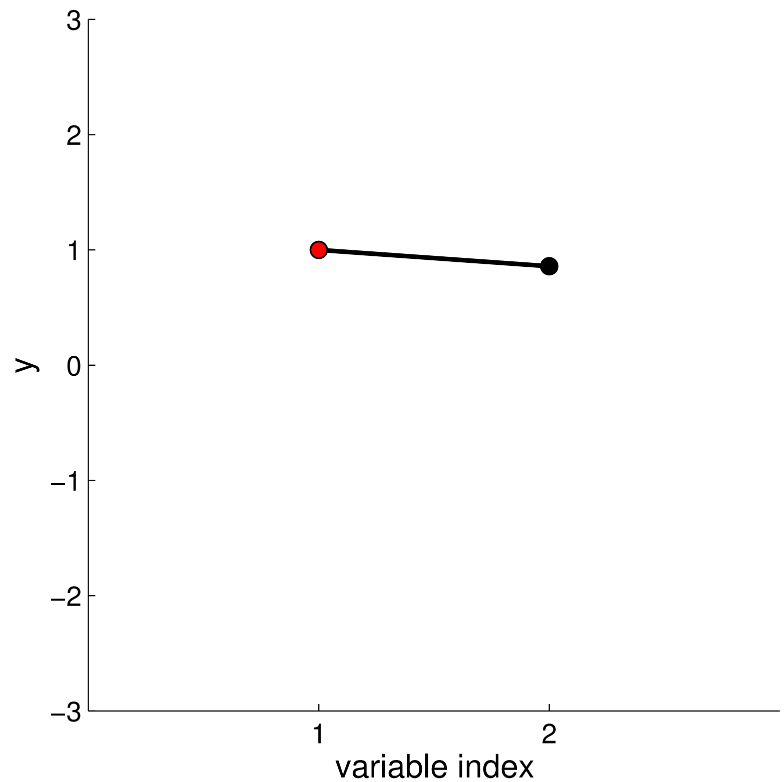

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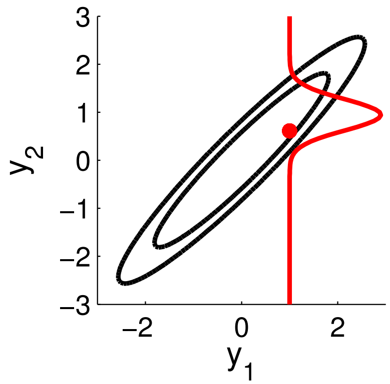

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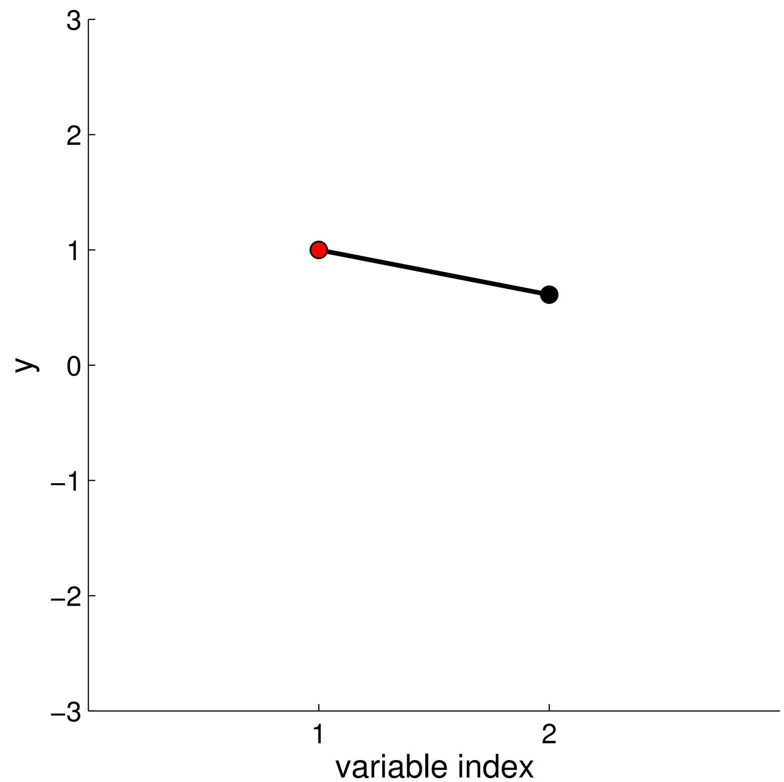

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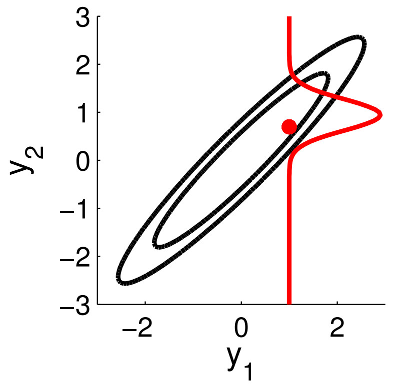

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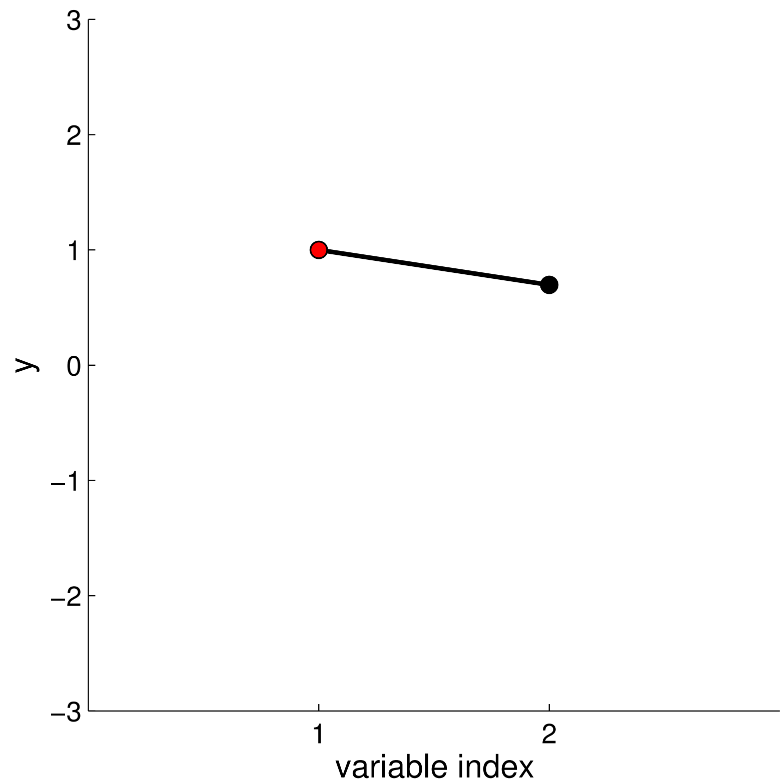

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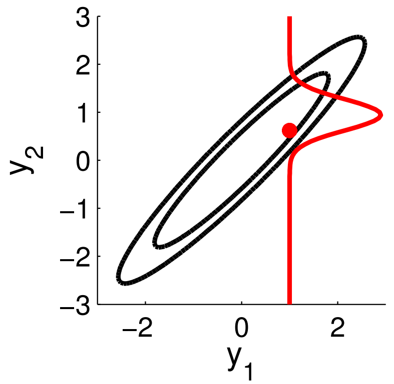

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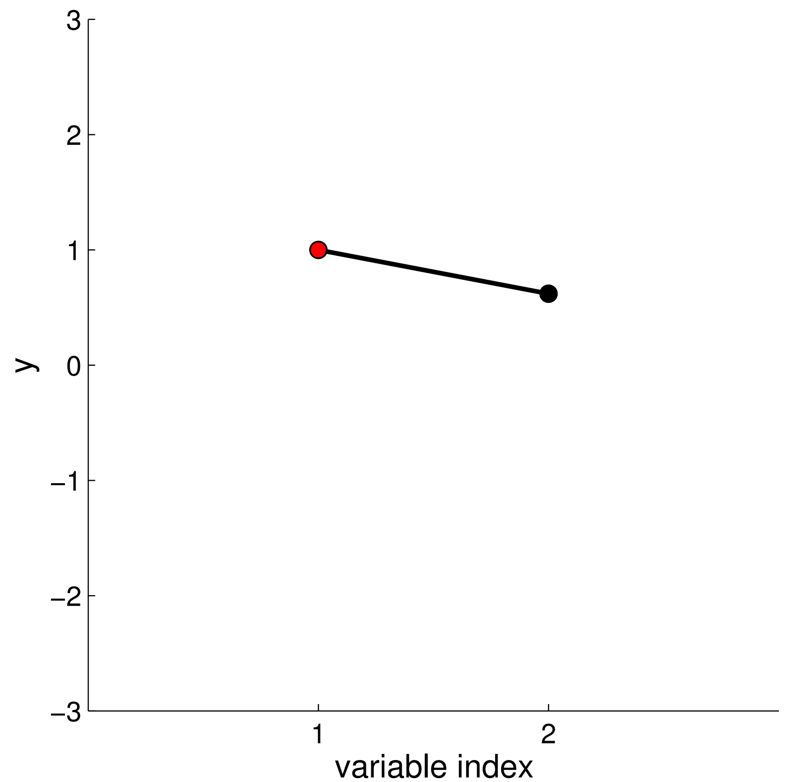

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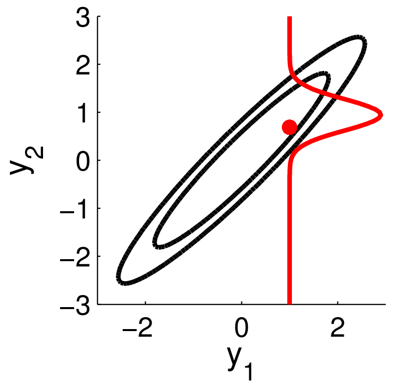

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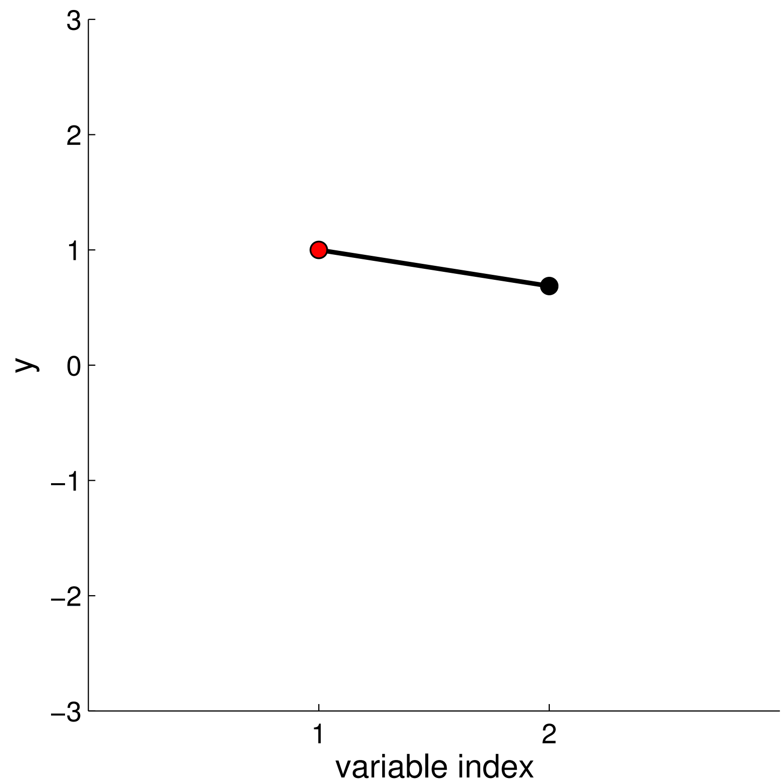

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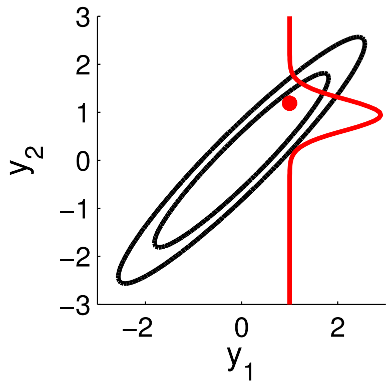

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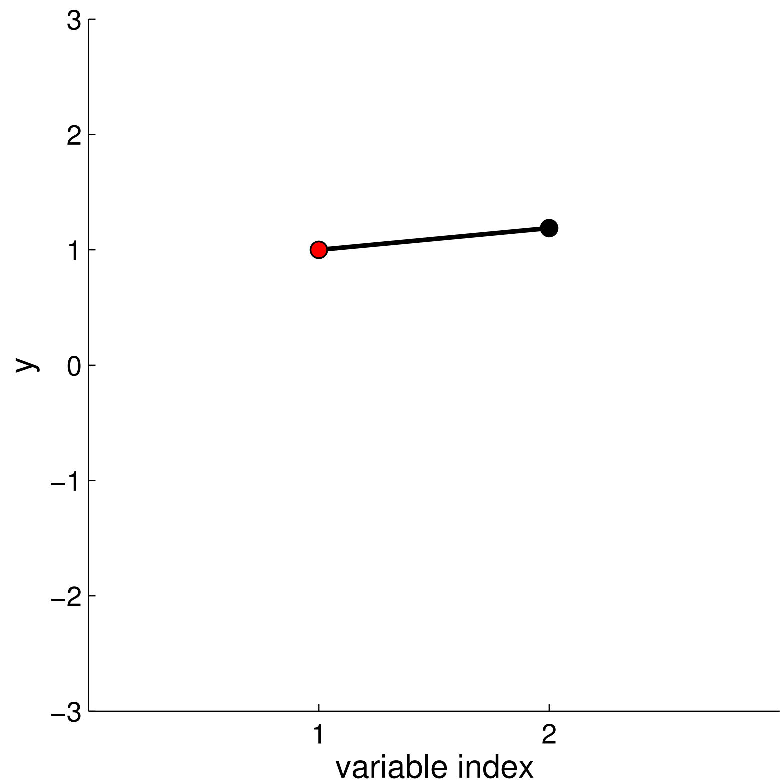

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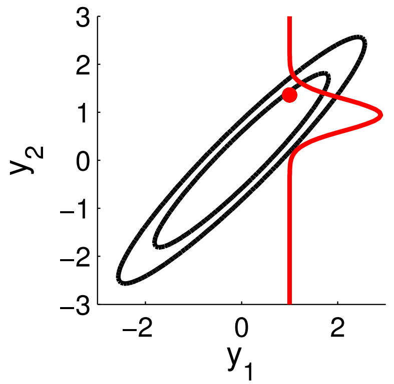

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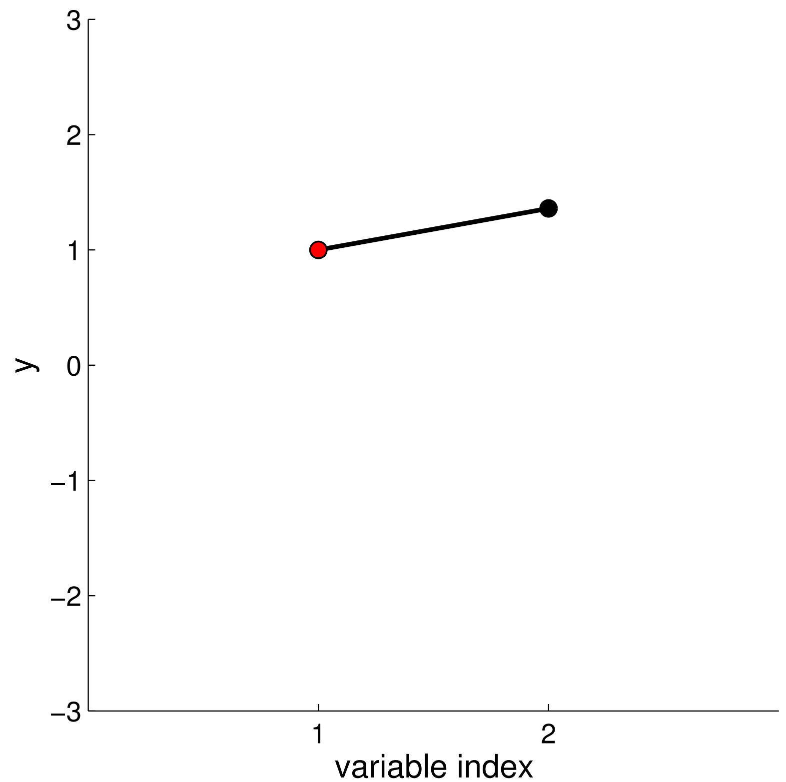

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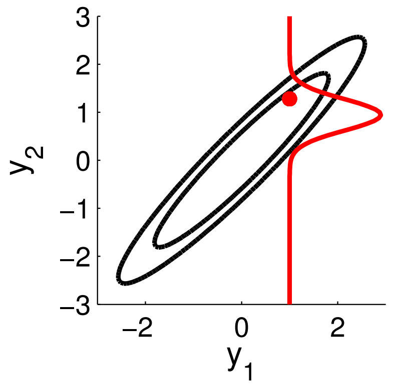

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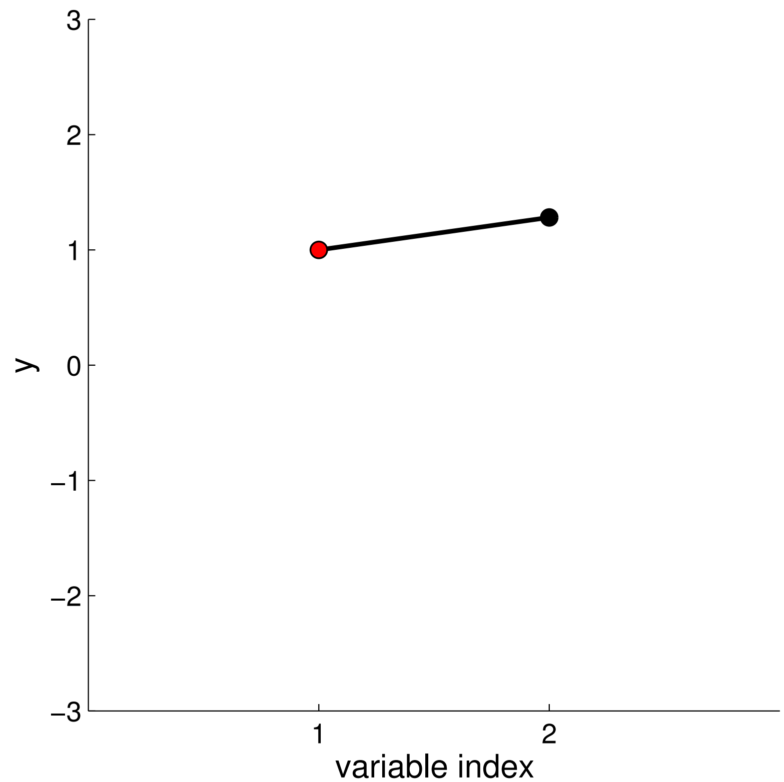

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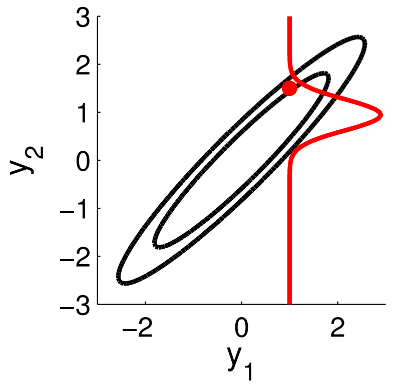

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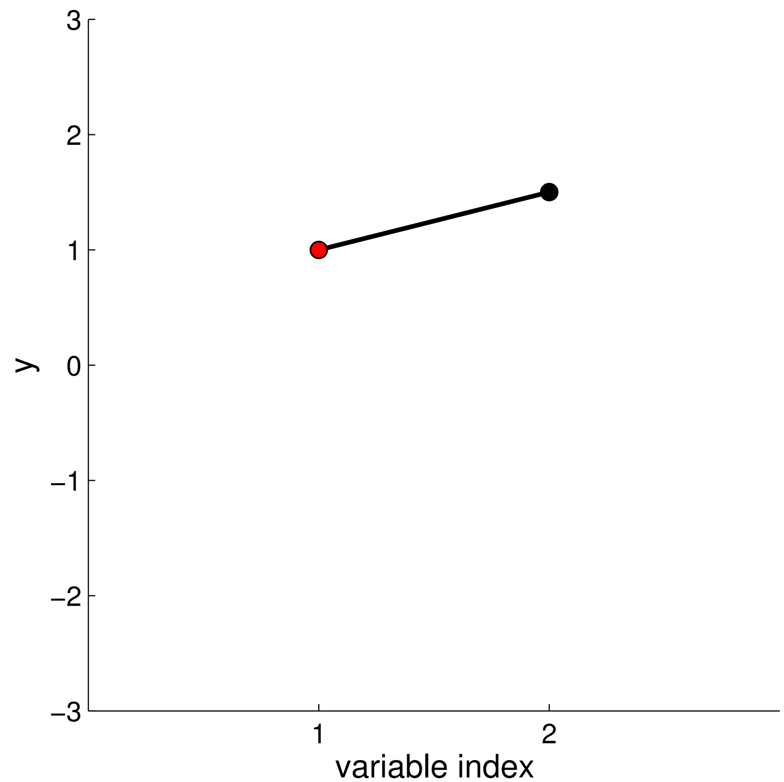

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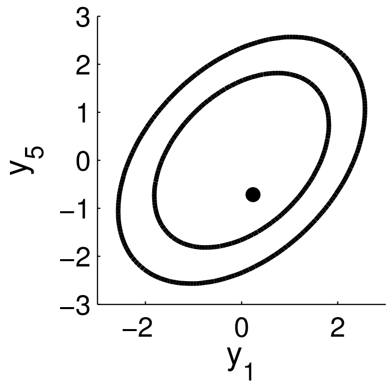

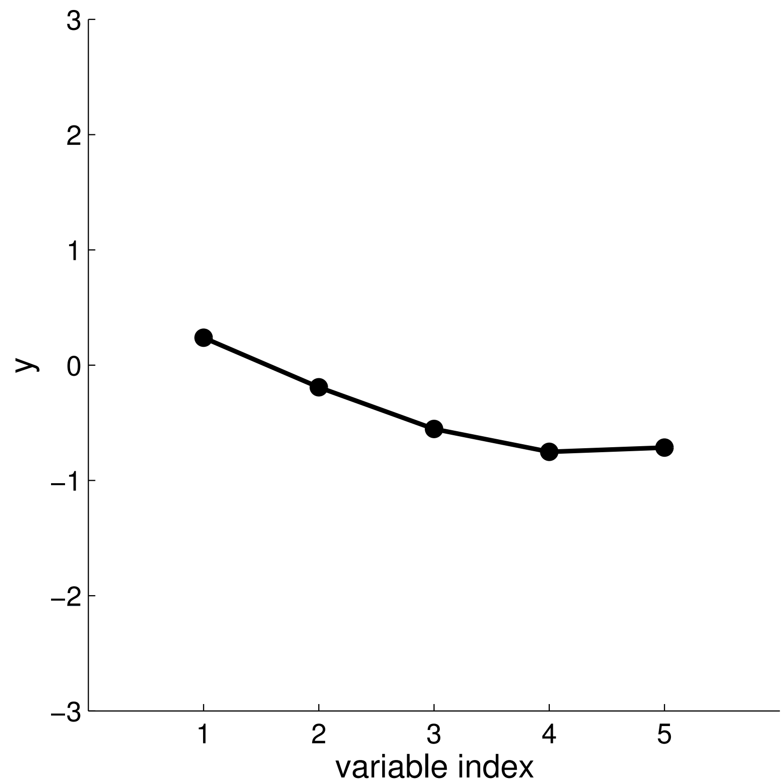

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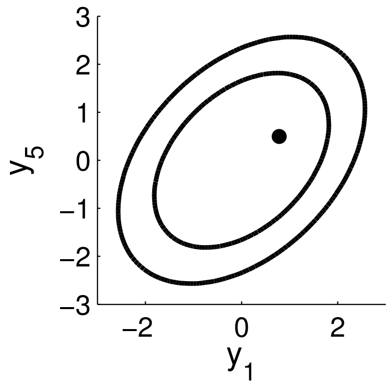
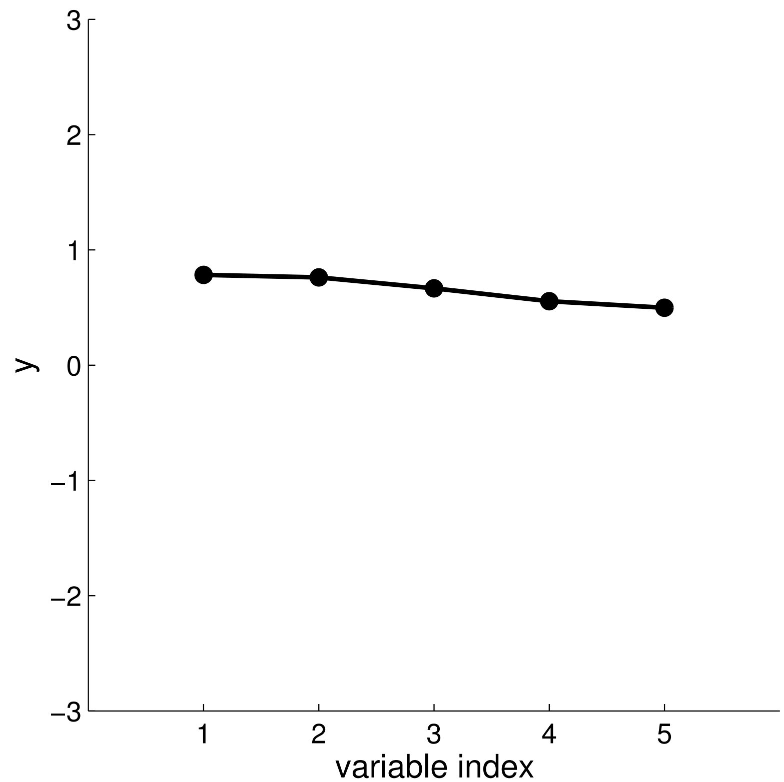
New visualisation





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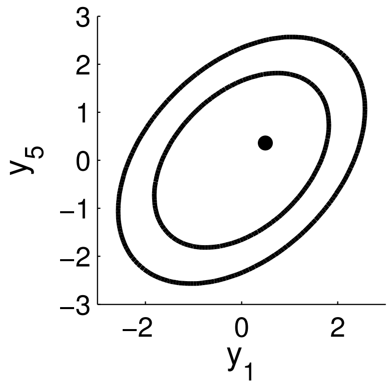

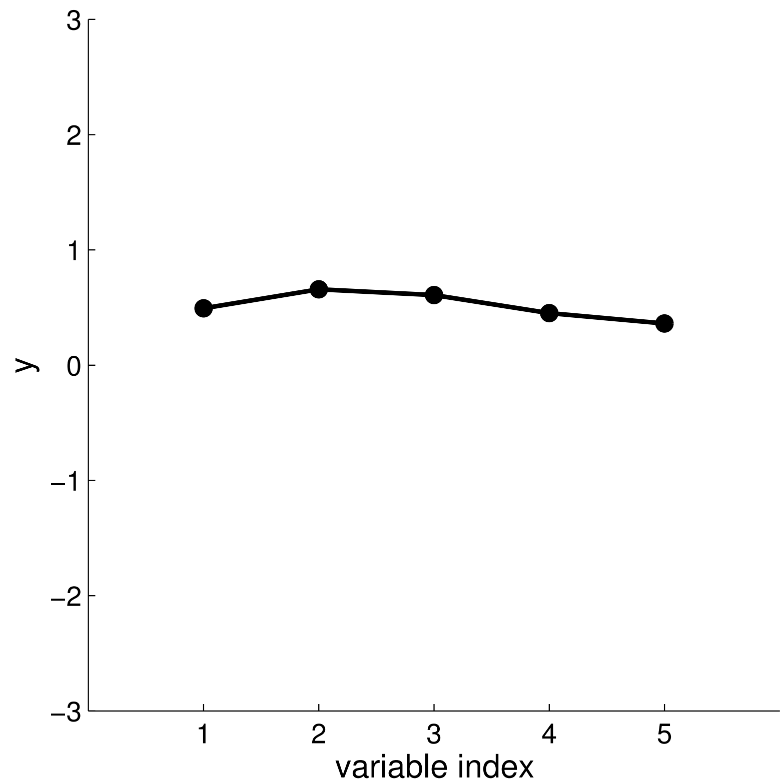

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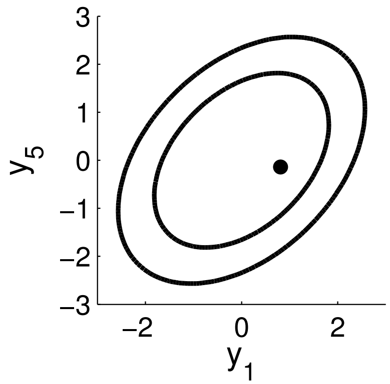
New visualisation



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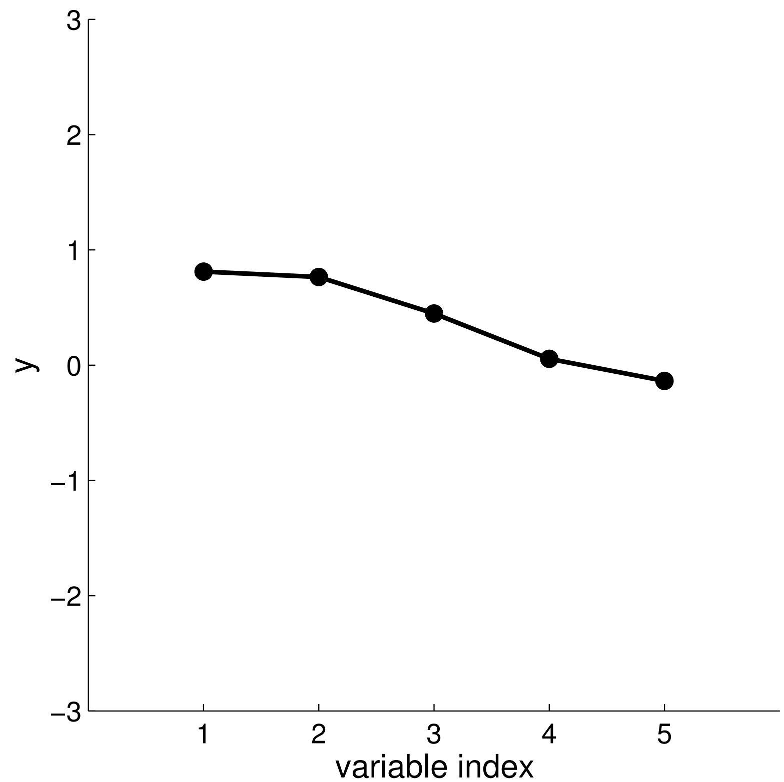

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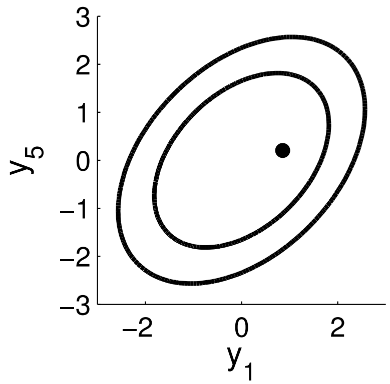

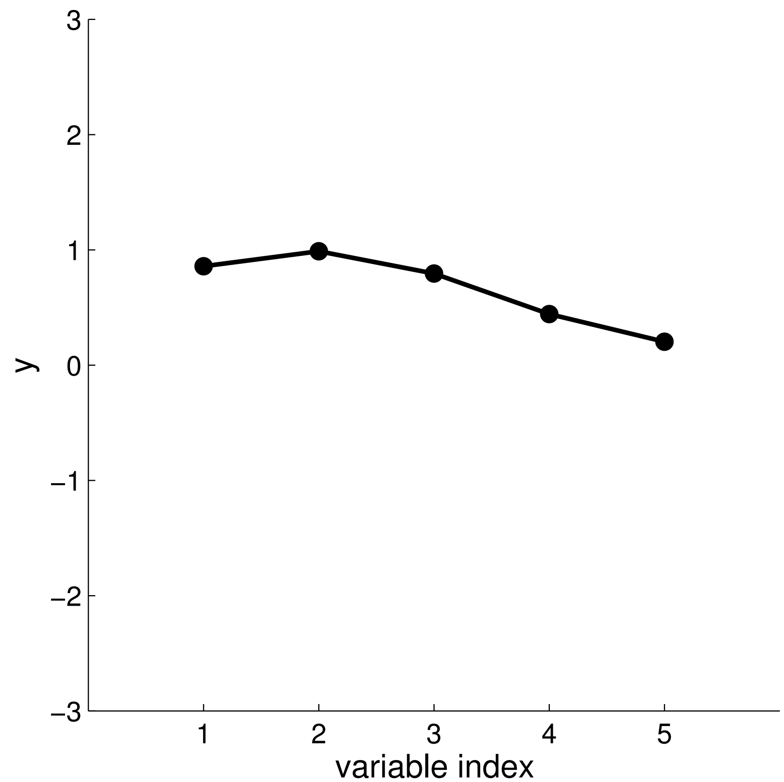

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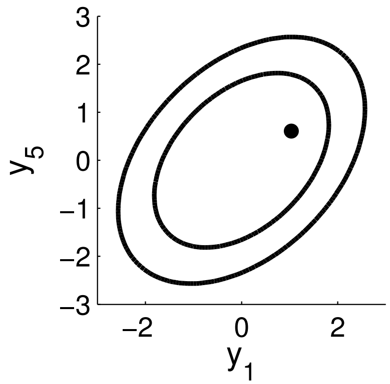
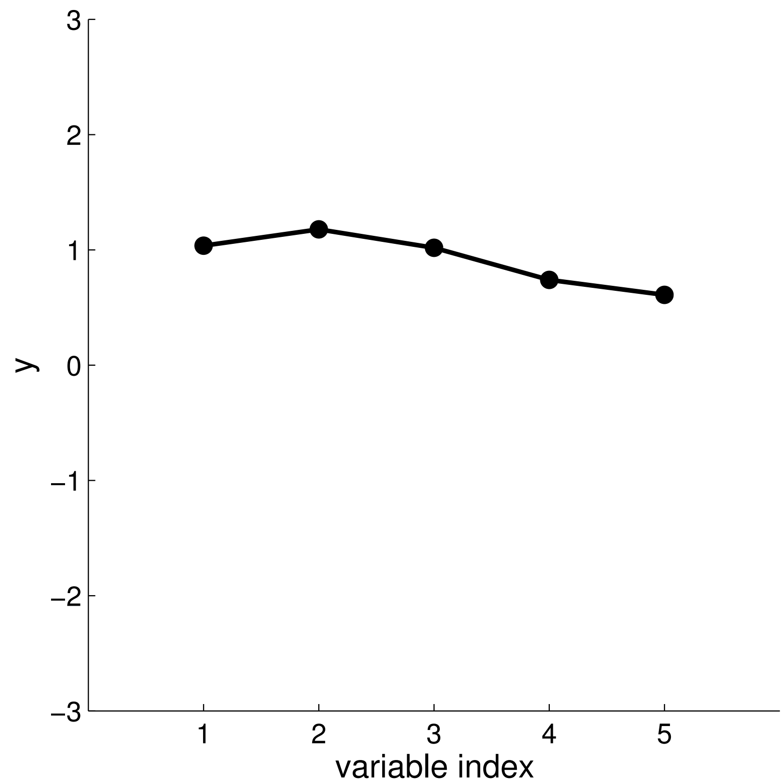
New visualisation





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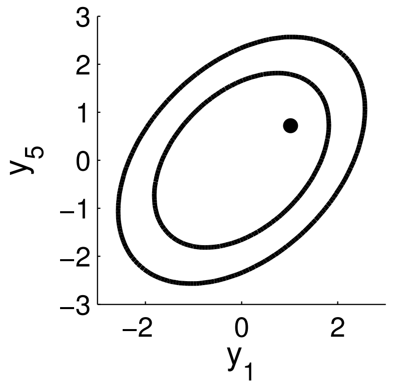
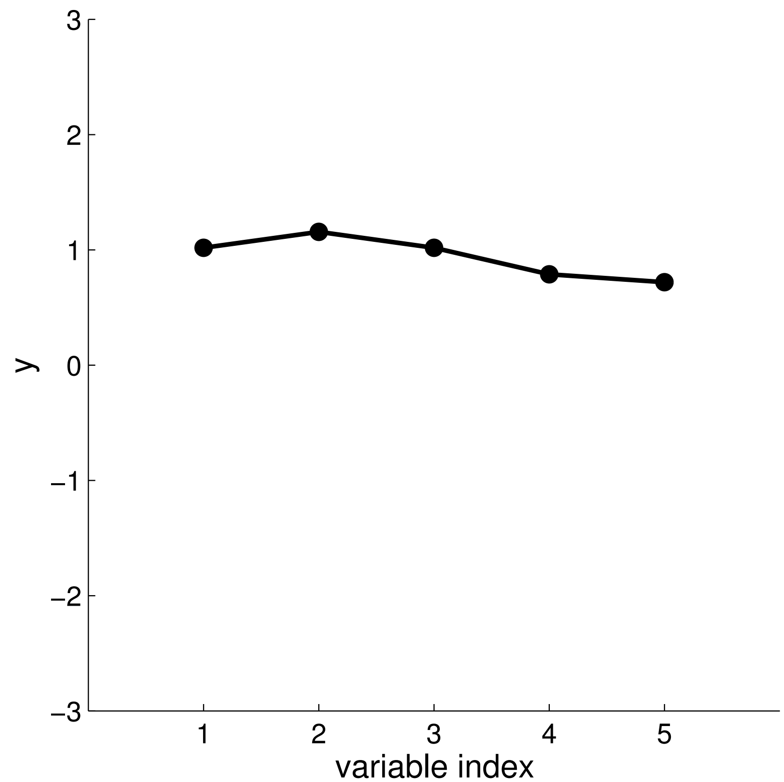
New visualisation





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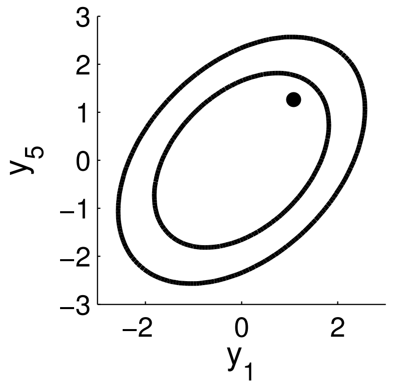
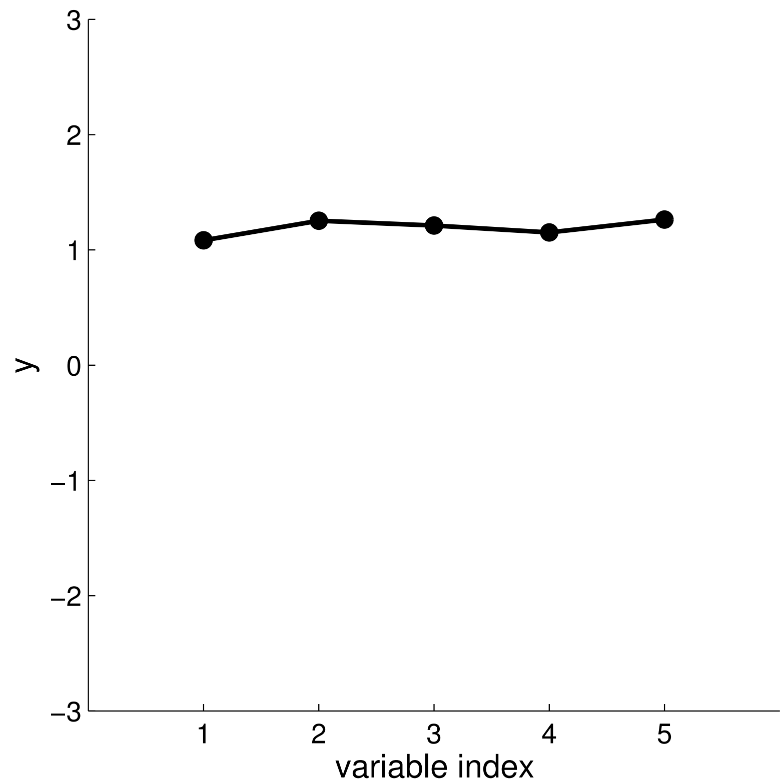
New visualisation





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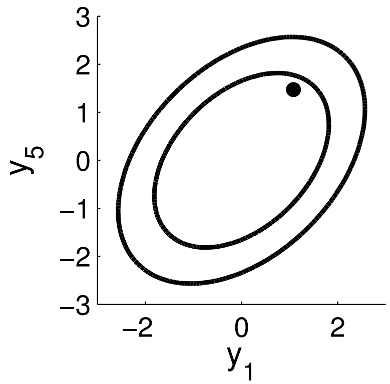
New visualisation



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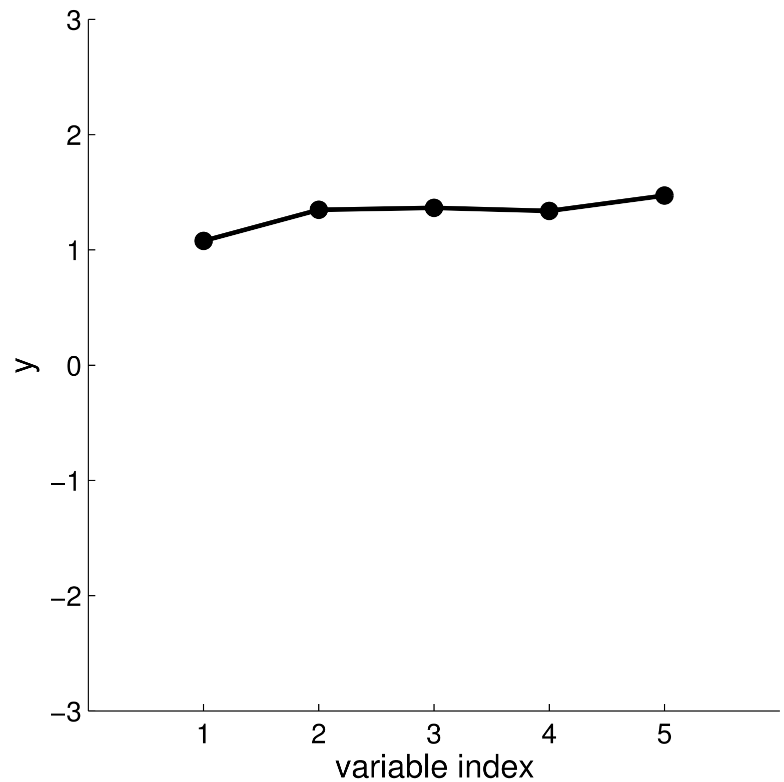

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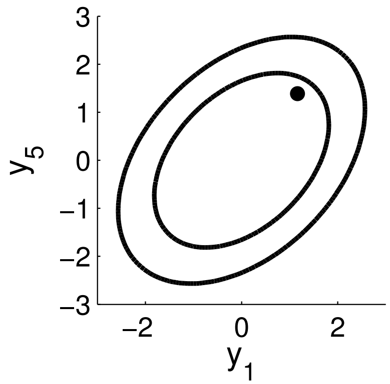

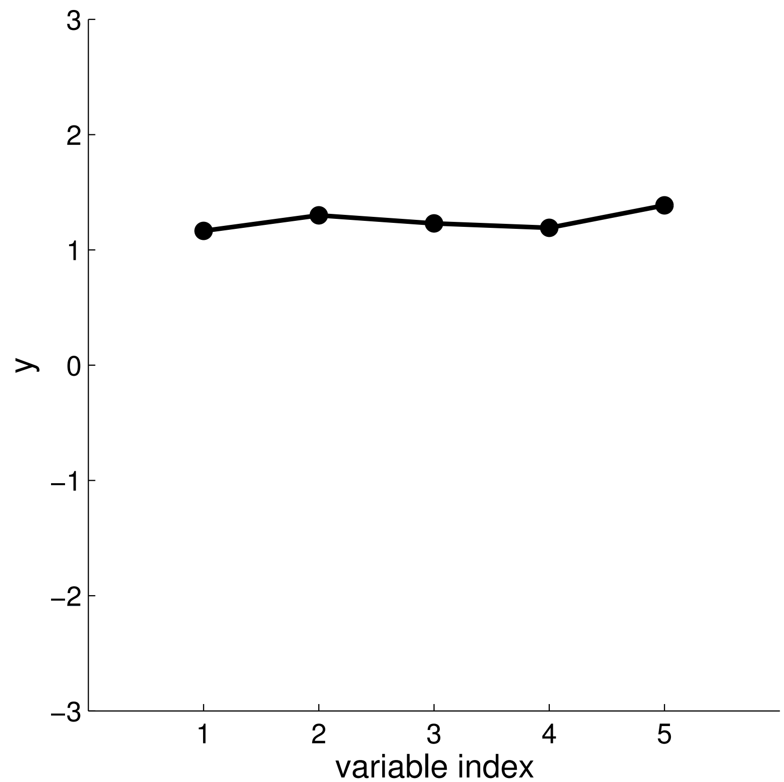

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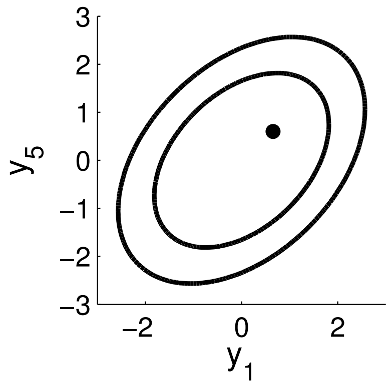
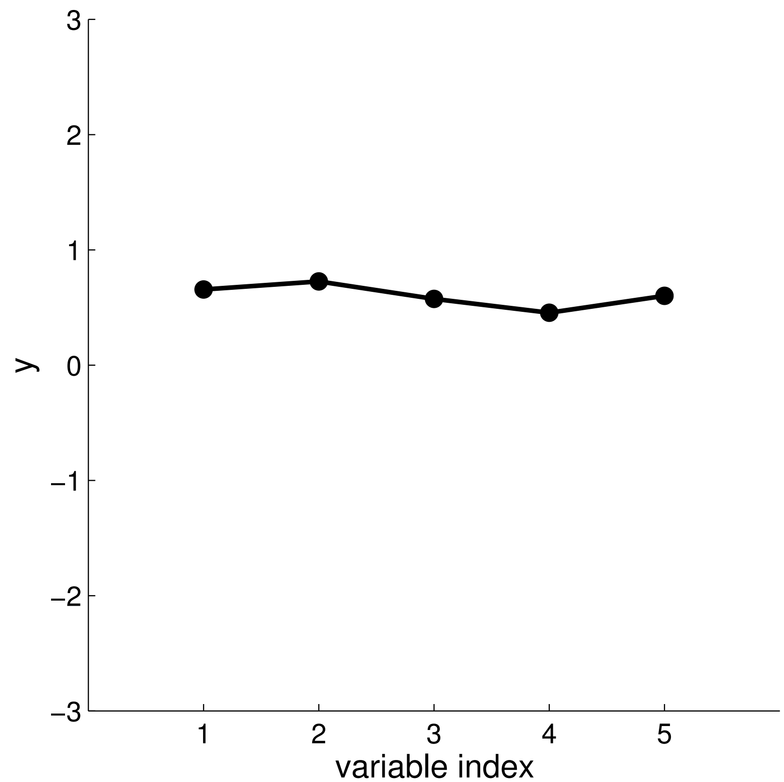
New visualisation





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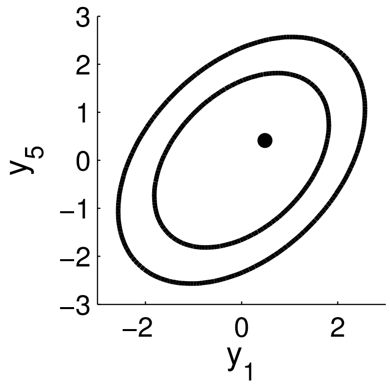
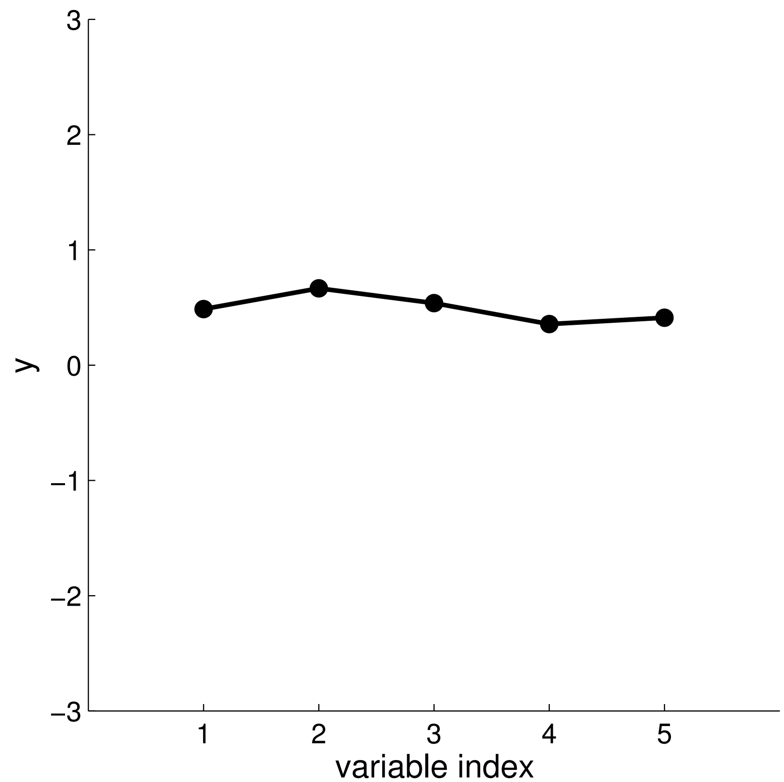
New visualisation





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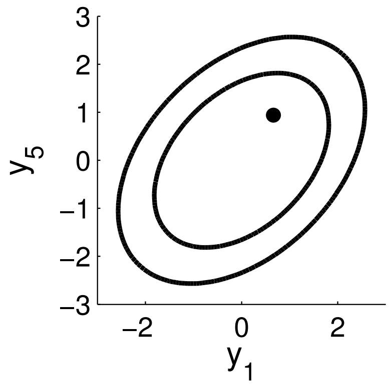
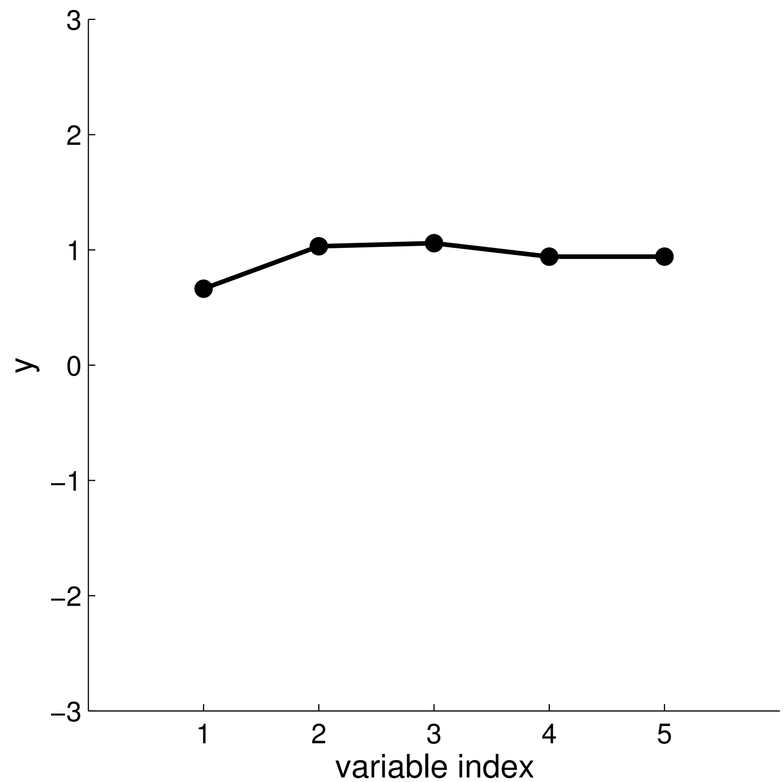
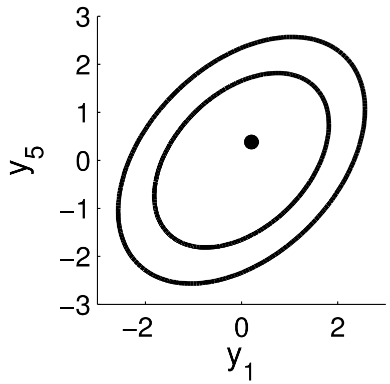
New visualisation

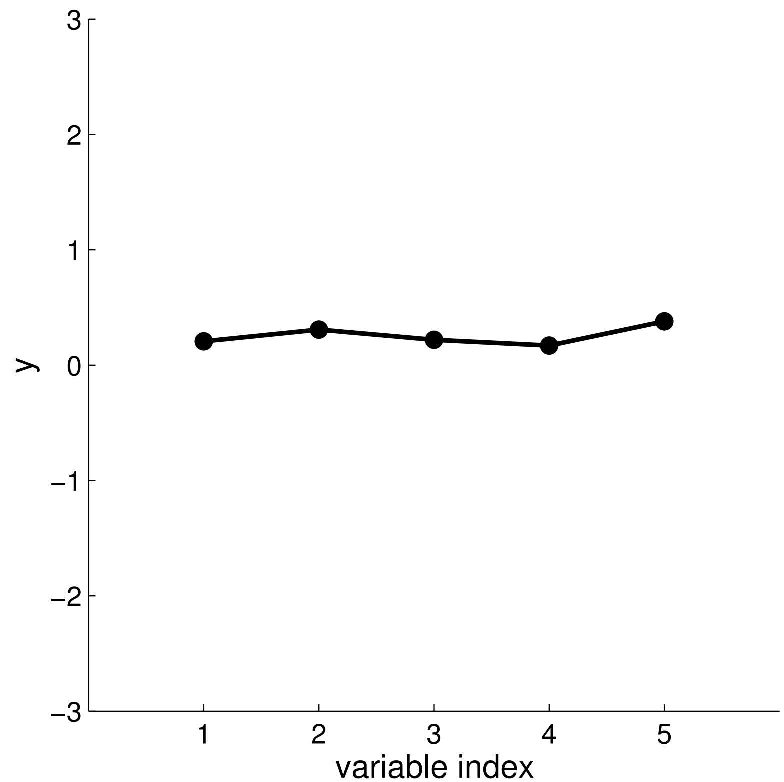




$$\Sigma = \begin{bmatrix} 1 & .9 & .8 & .6 & .4 \\ .9 & 1 & .9 & .8 & .6 \\ .8 & .9 & 1 & .9 & .8 \\ .6 & .8 & .9 & 1 & .9 \\ .4 & .6 & .8 & .9 & 1 \end{bmatrix}$$

New visualisation





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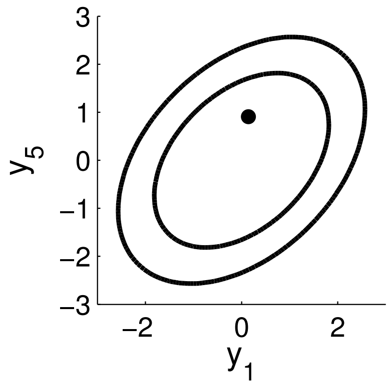
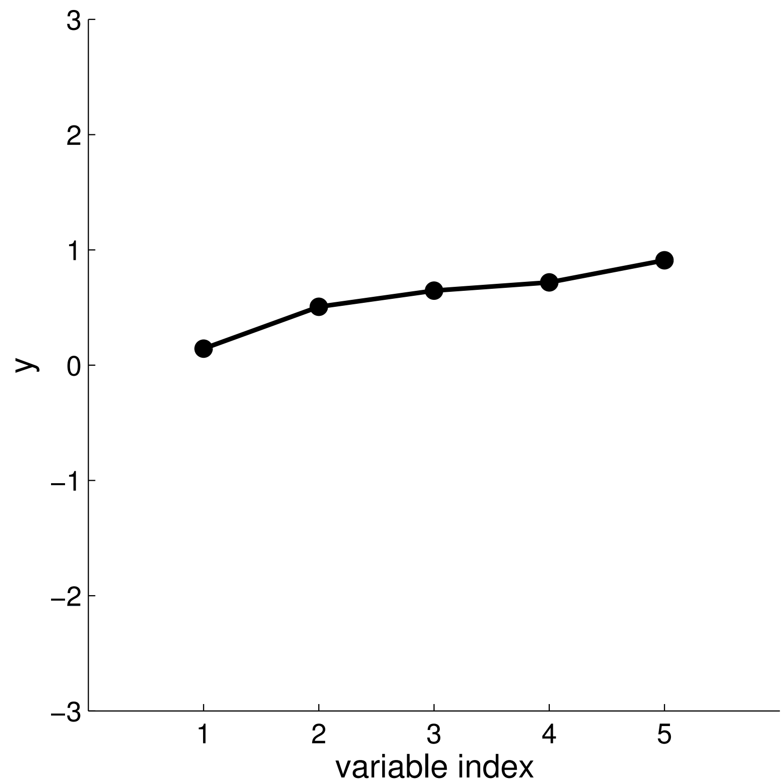
New visualisation





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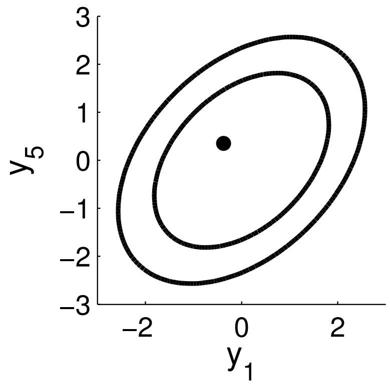
New visualisation



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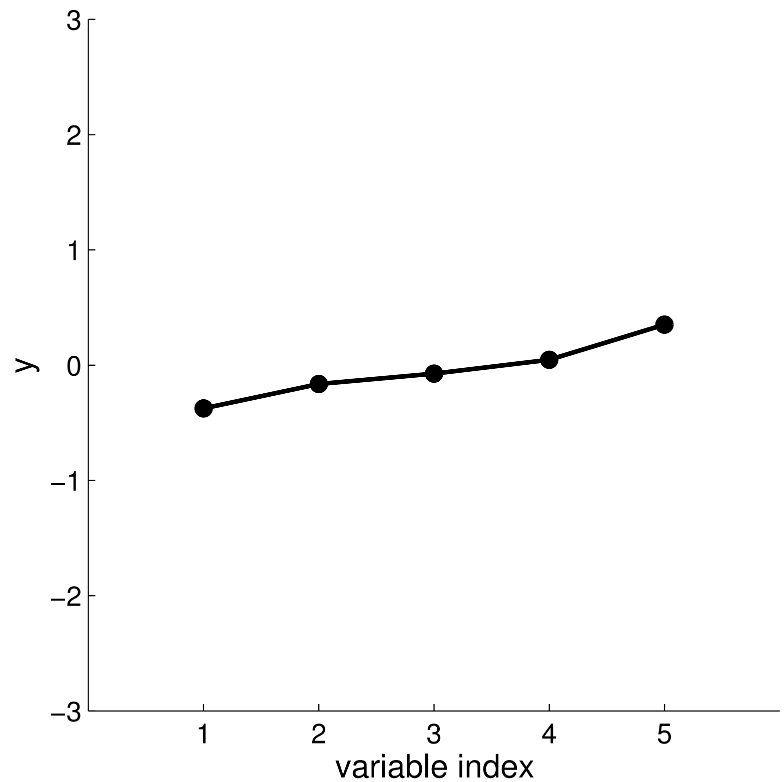
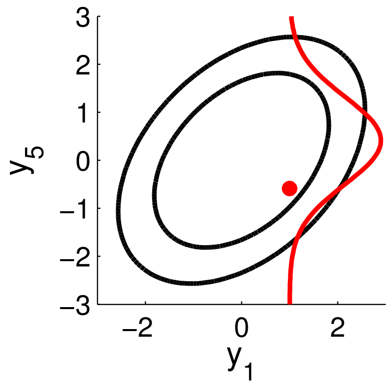
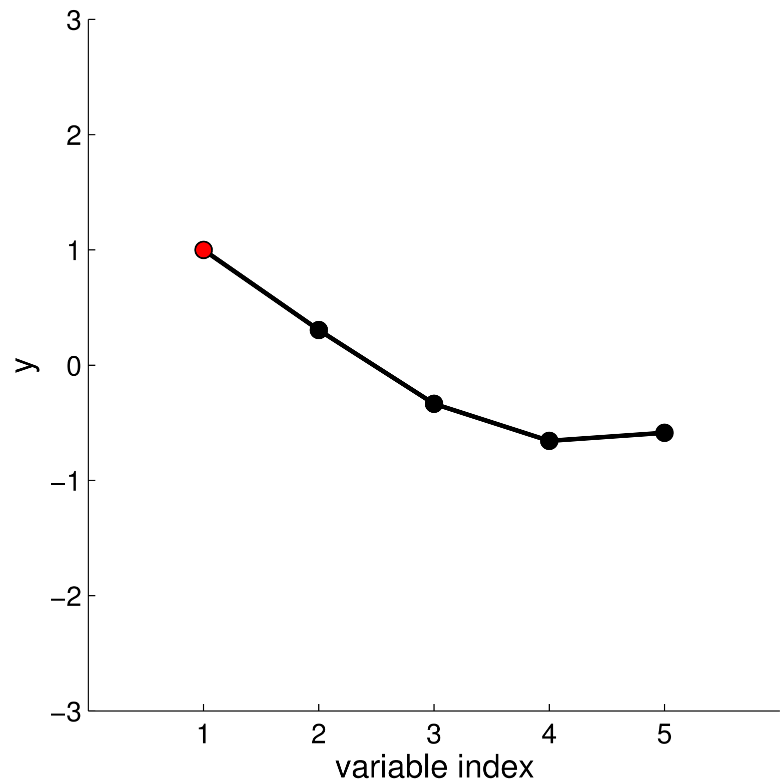


New visualisation





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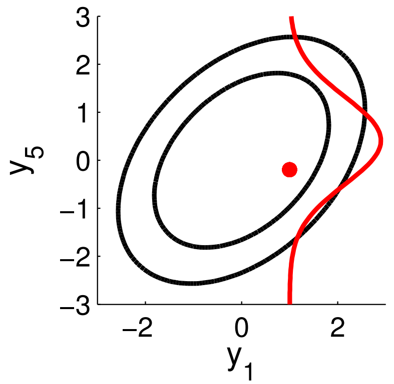
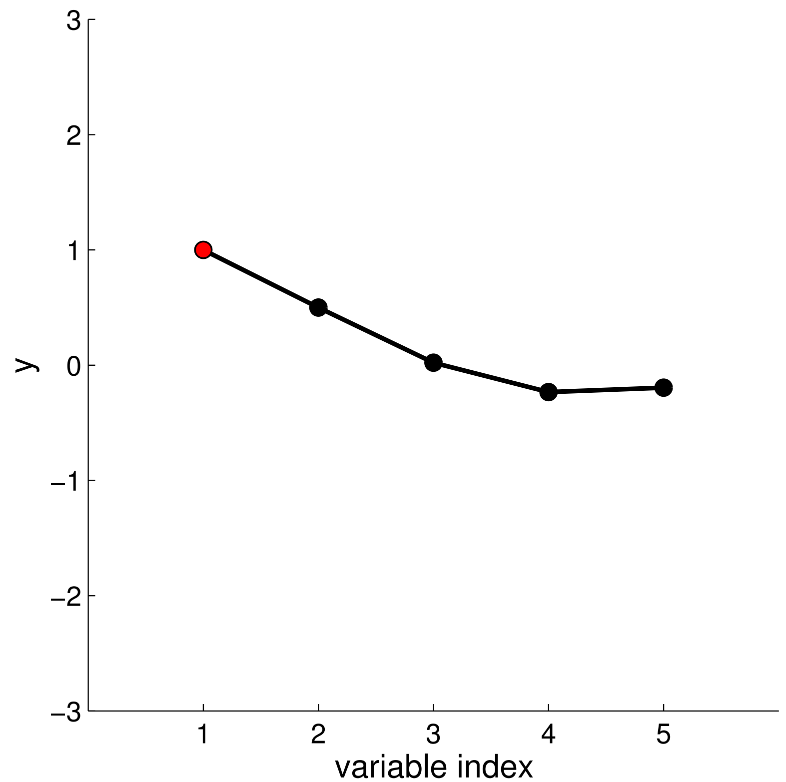
New visualisation





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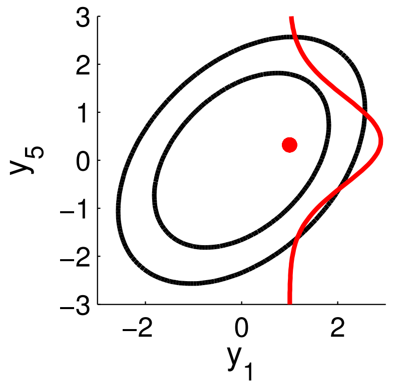
New visualisation



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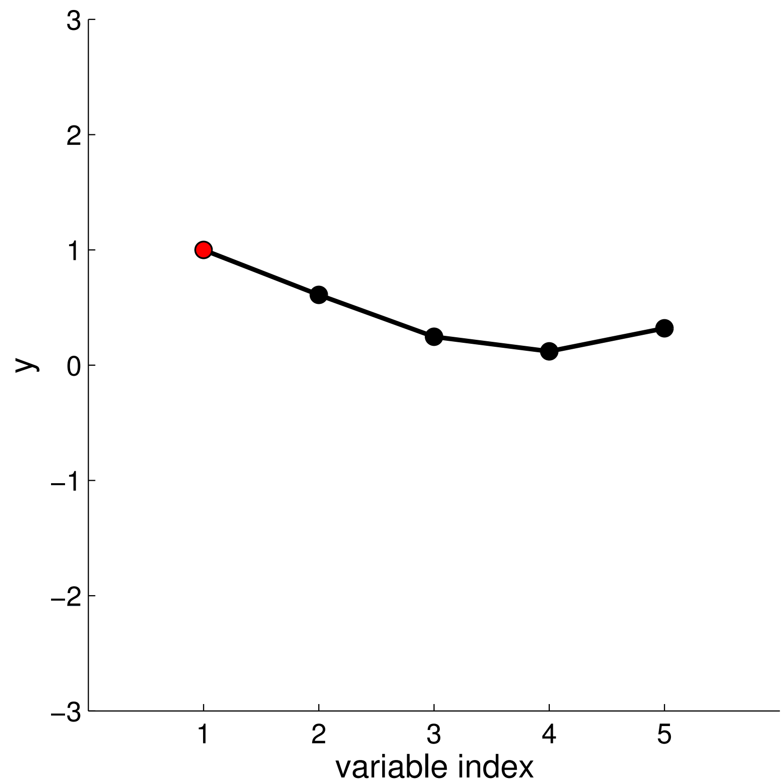
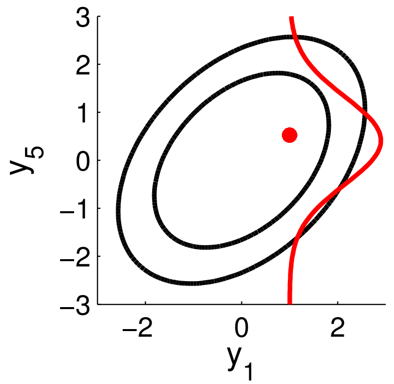
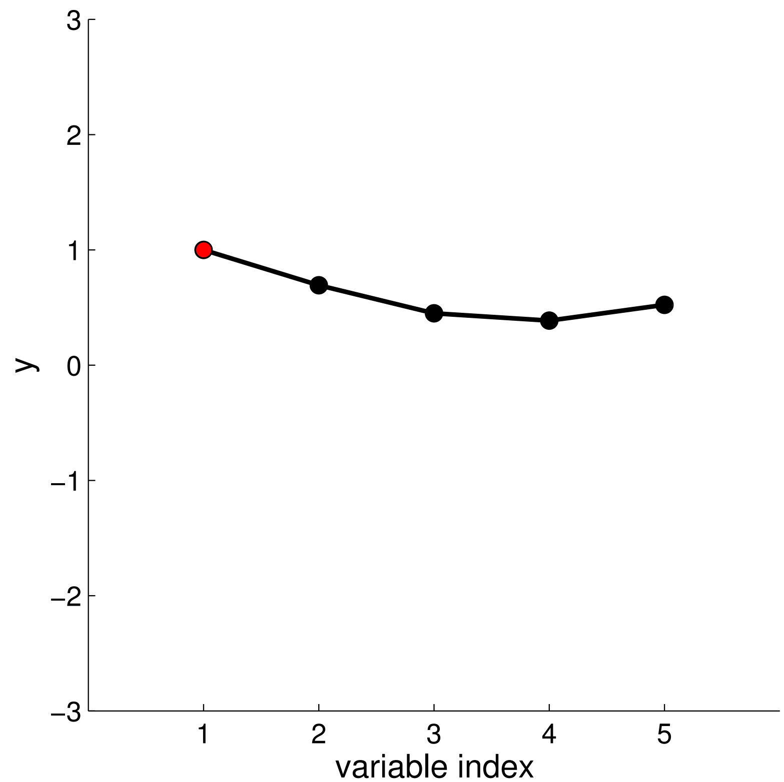


New visualisation





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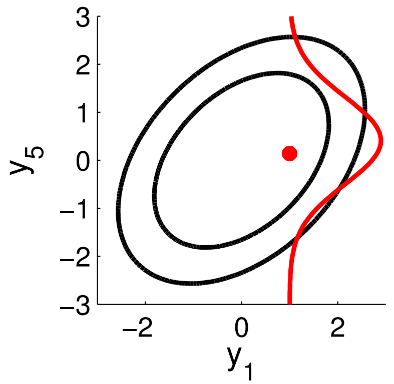
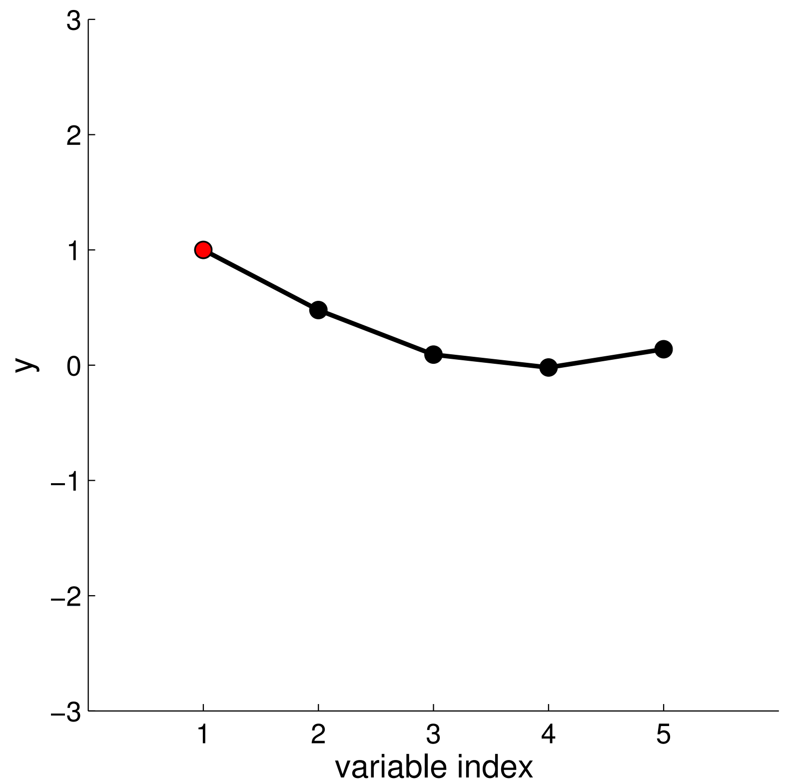
New visualisation





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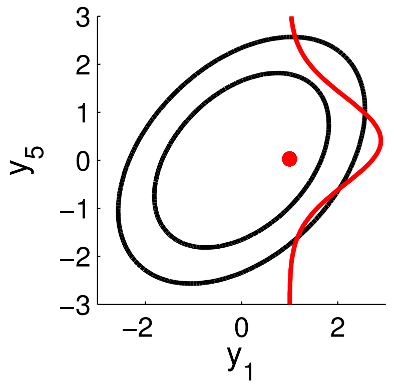
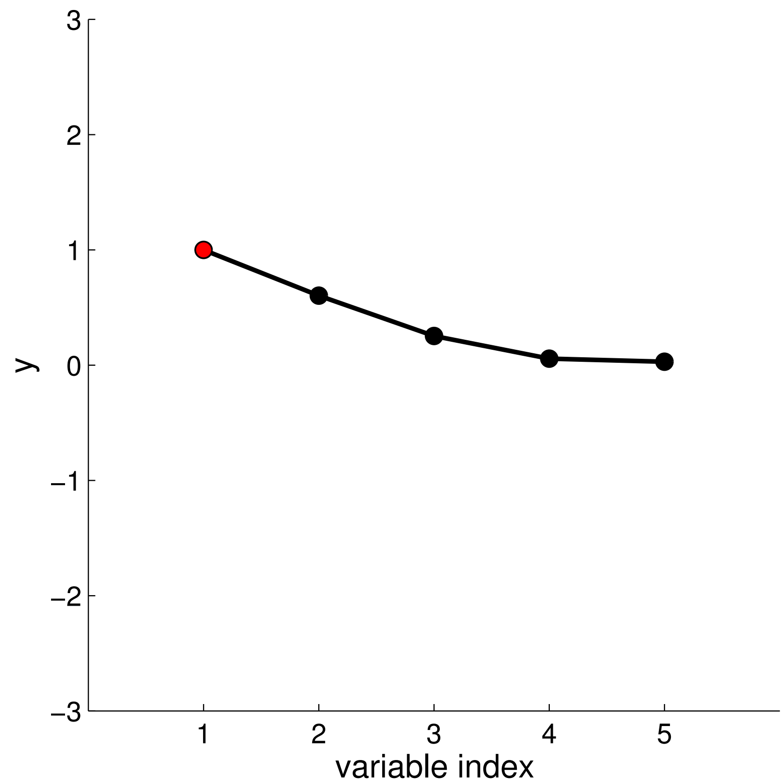
New visualisation





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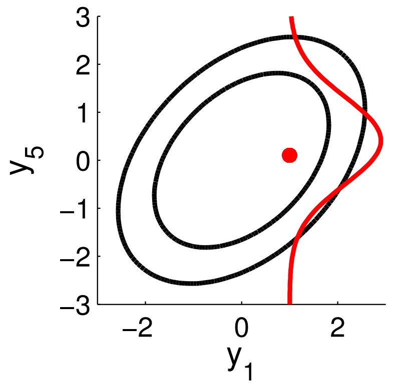
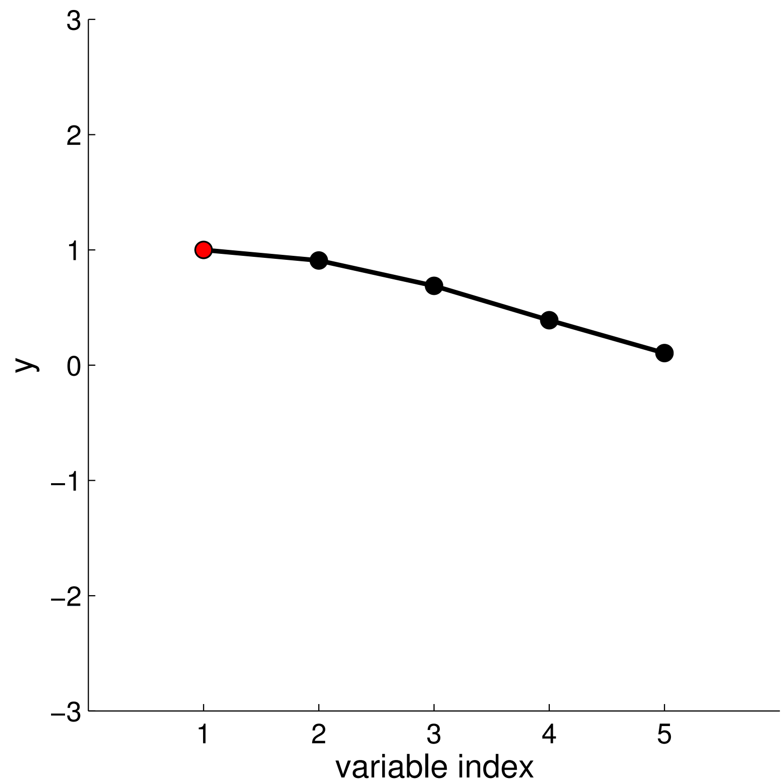
New visualisation





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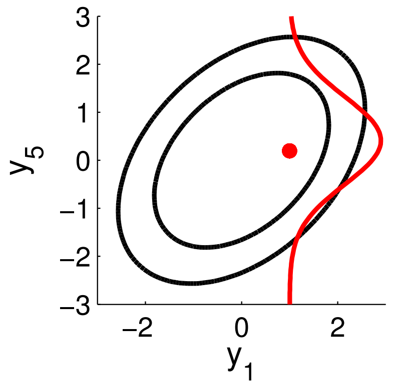
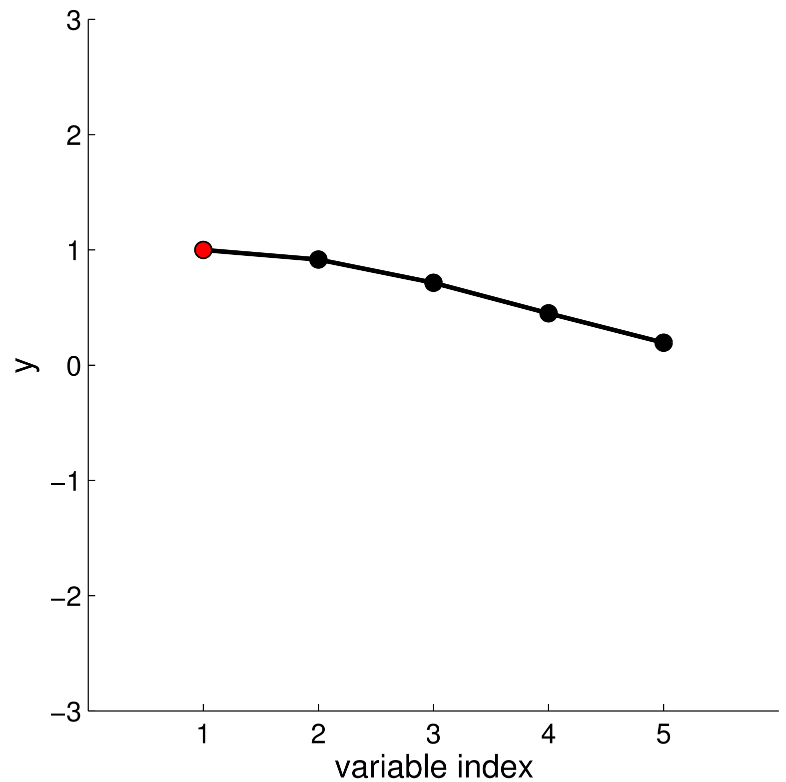
New visualisation





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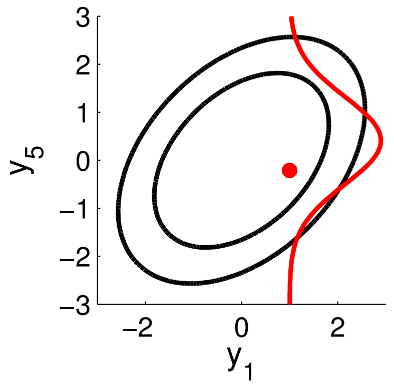
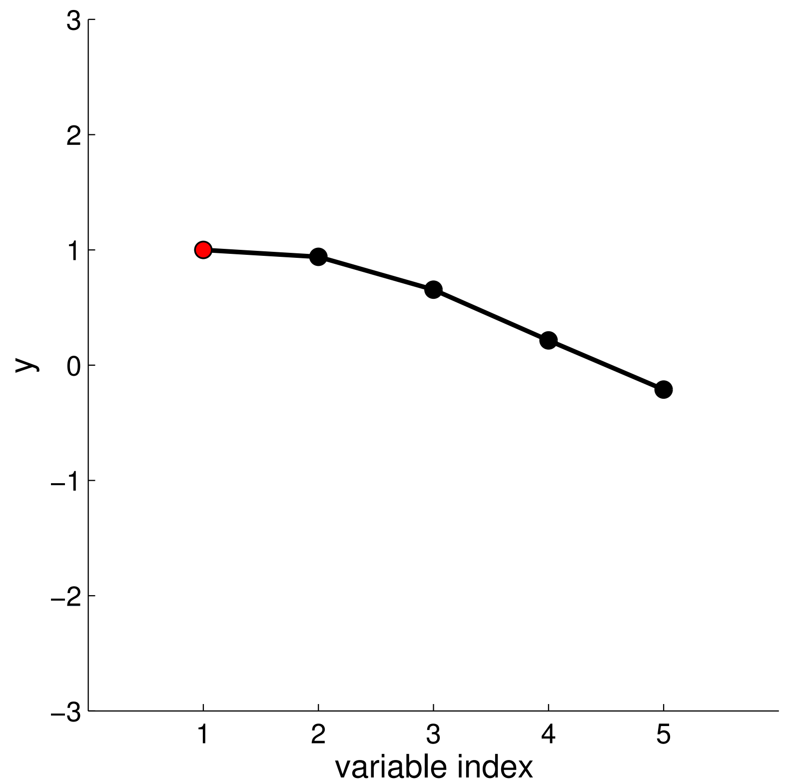
New visualisation





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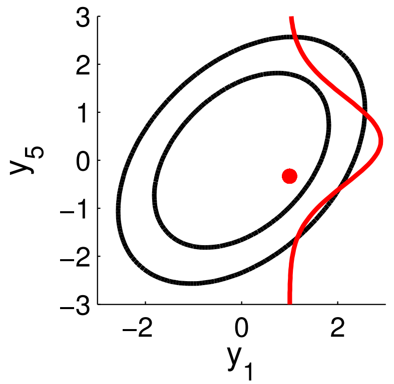
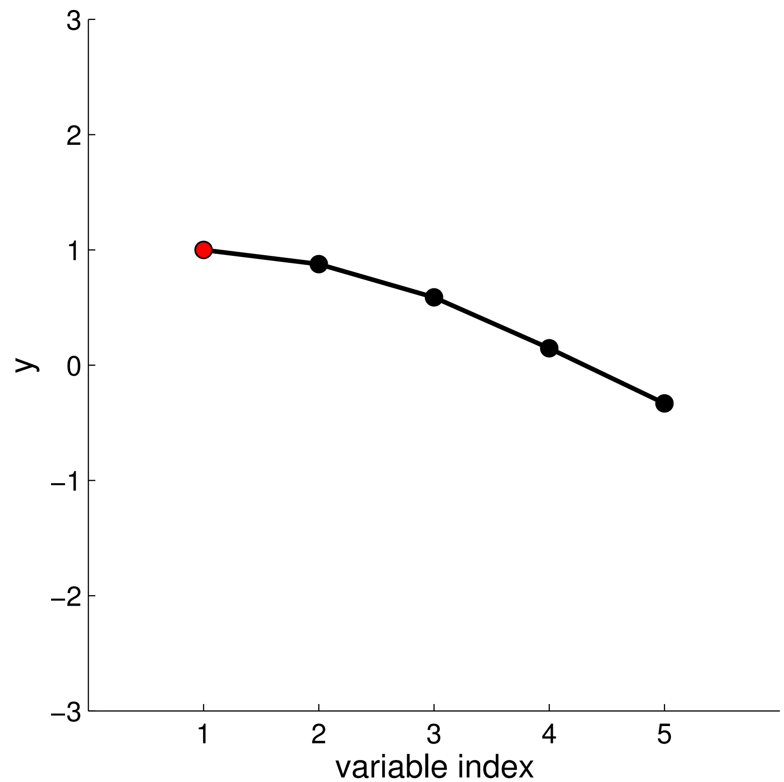
New visualisation





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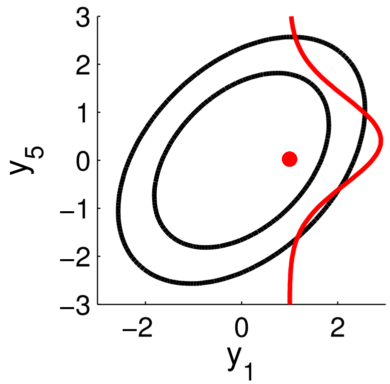
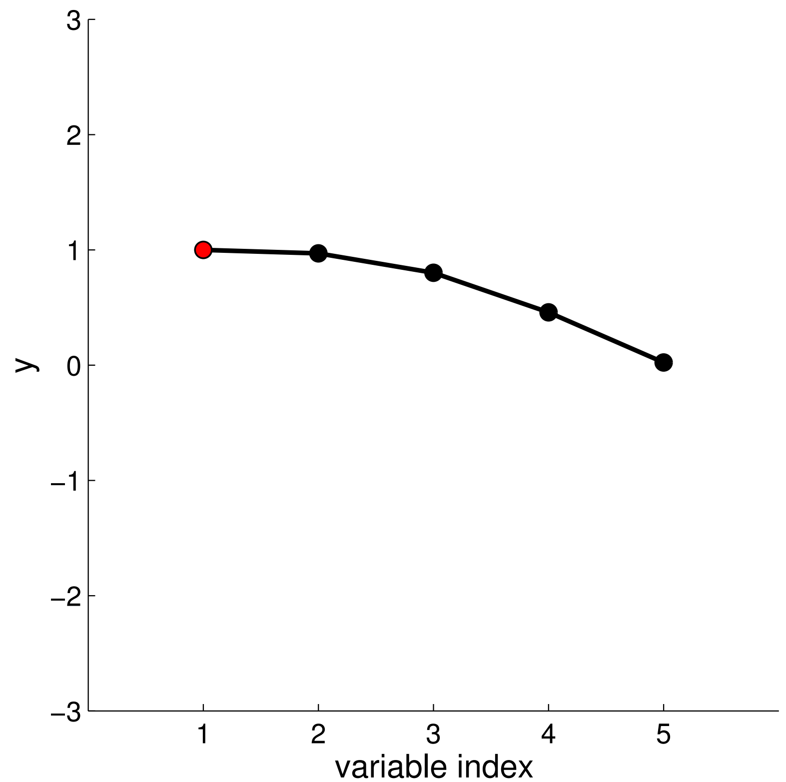
New visualisation





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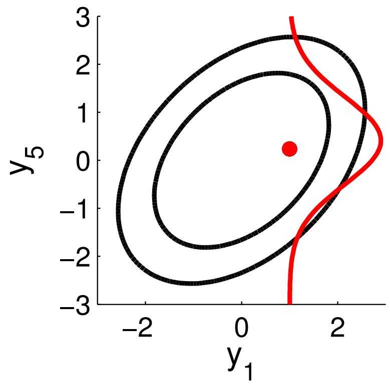
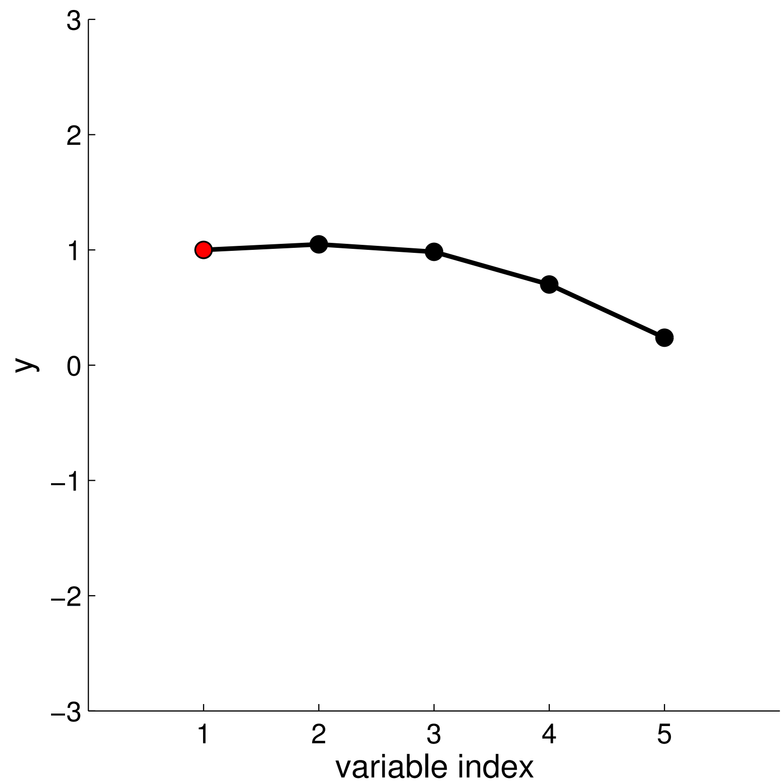
New visualisation





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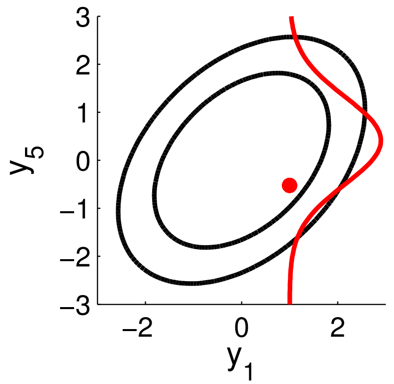
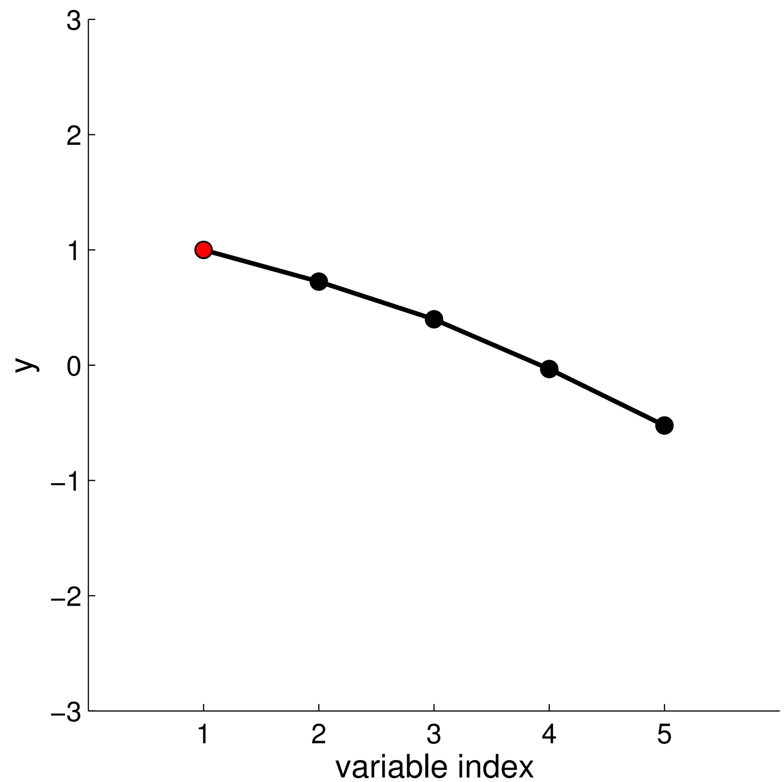
New visualisation





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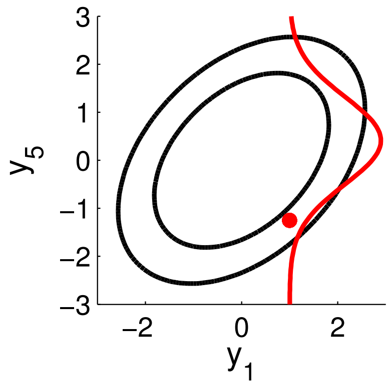
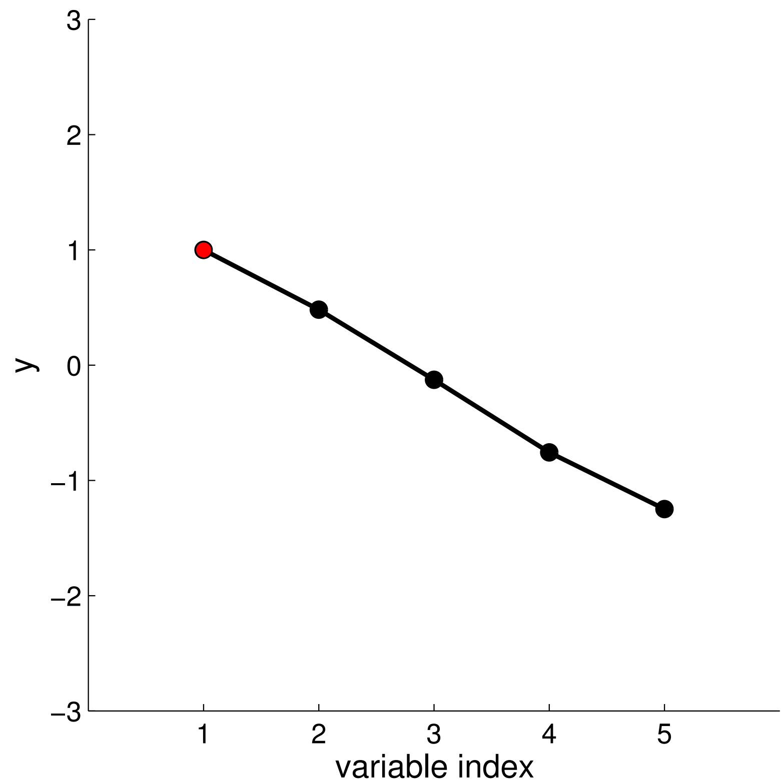
New visualisation





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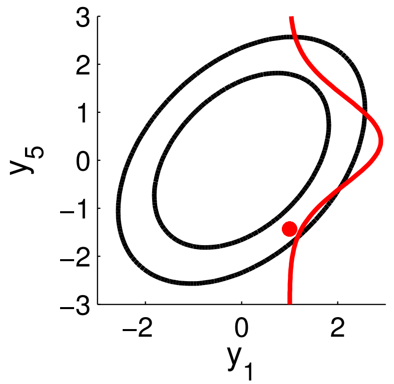
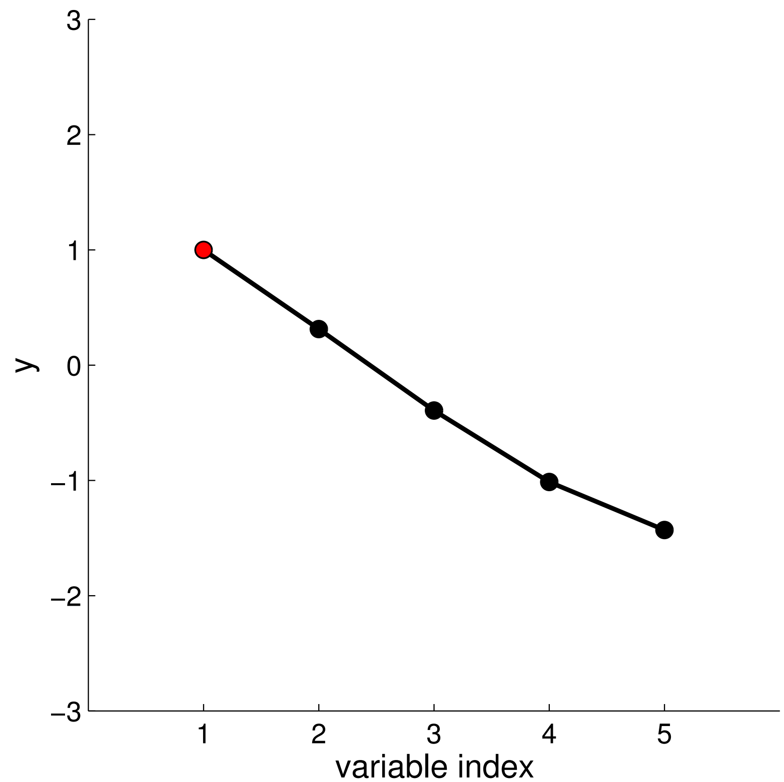
New visualisation





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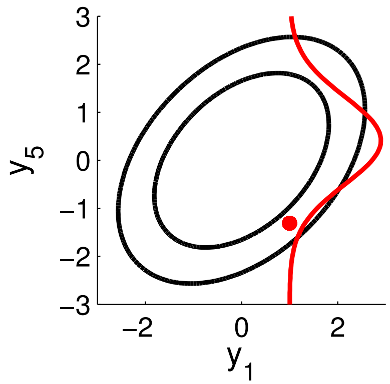
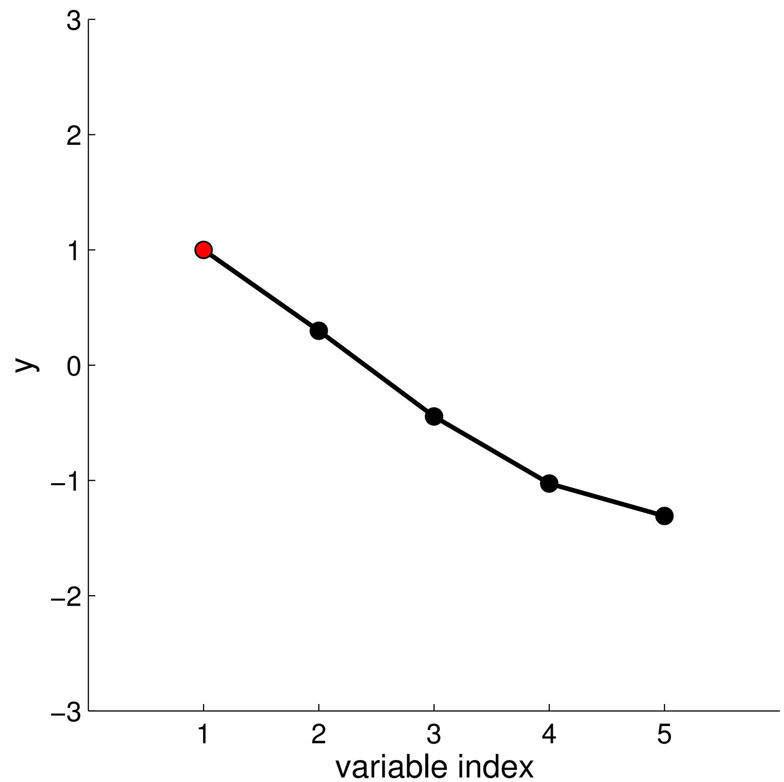
New visualisation





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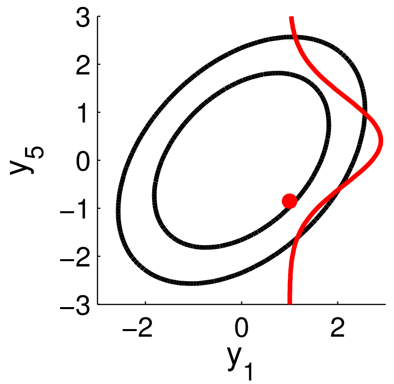
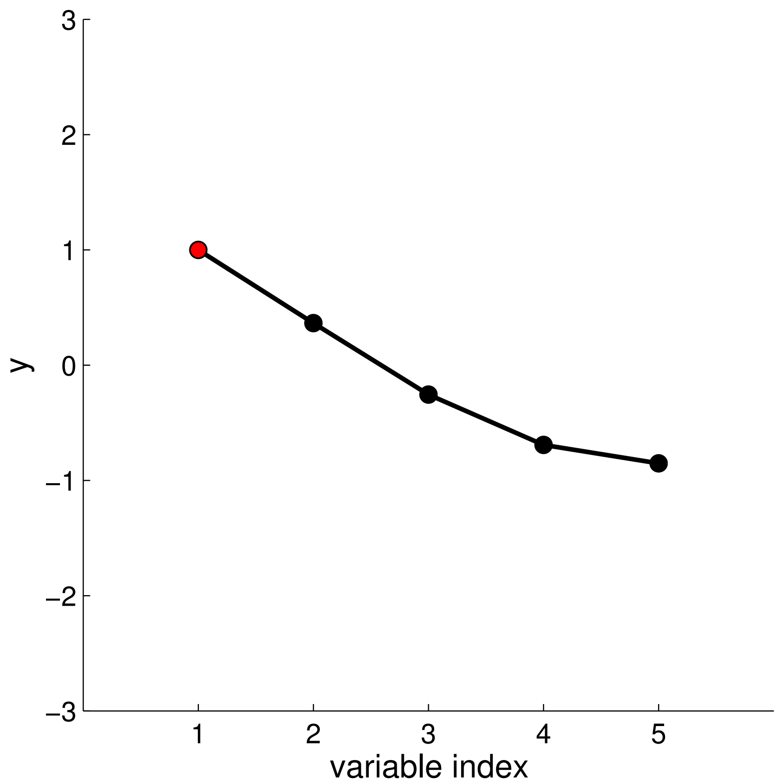
New visualisation





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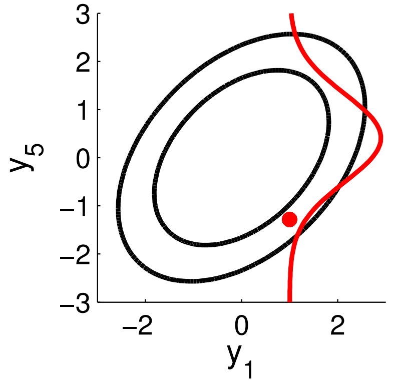
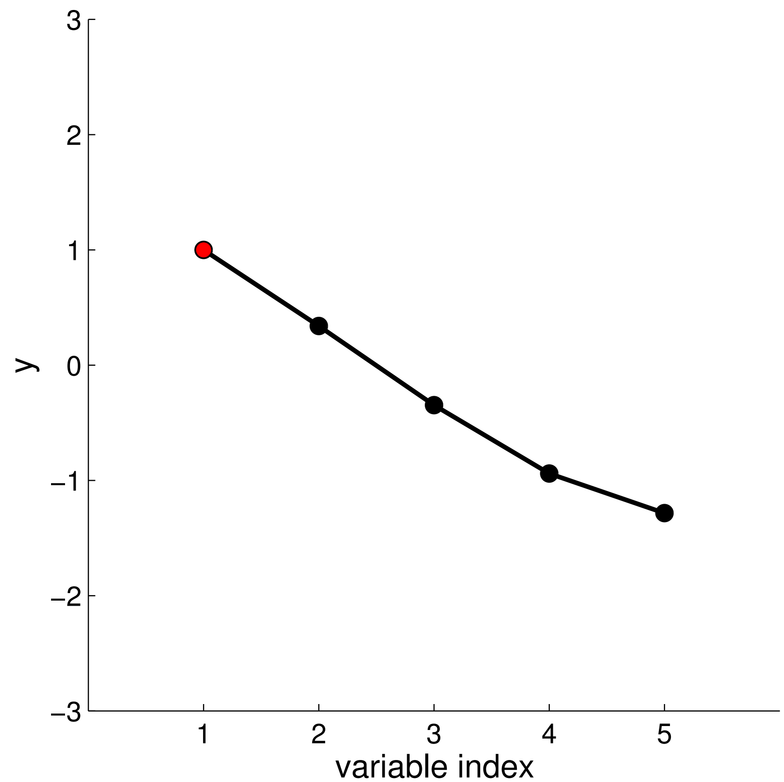
New visualisation





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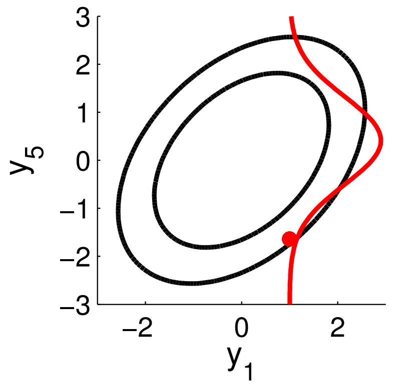
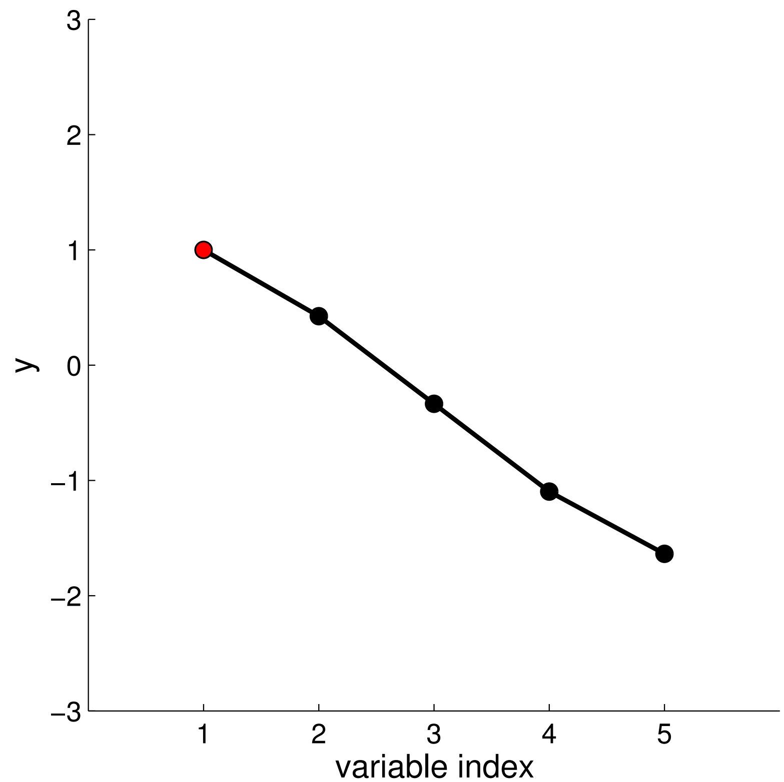
New visualisation





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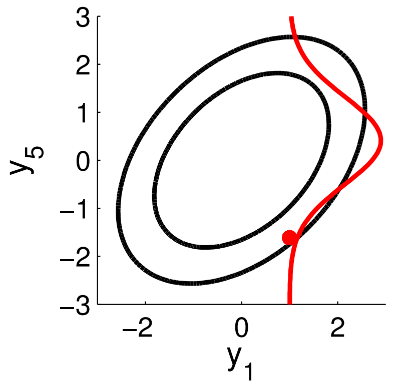
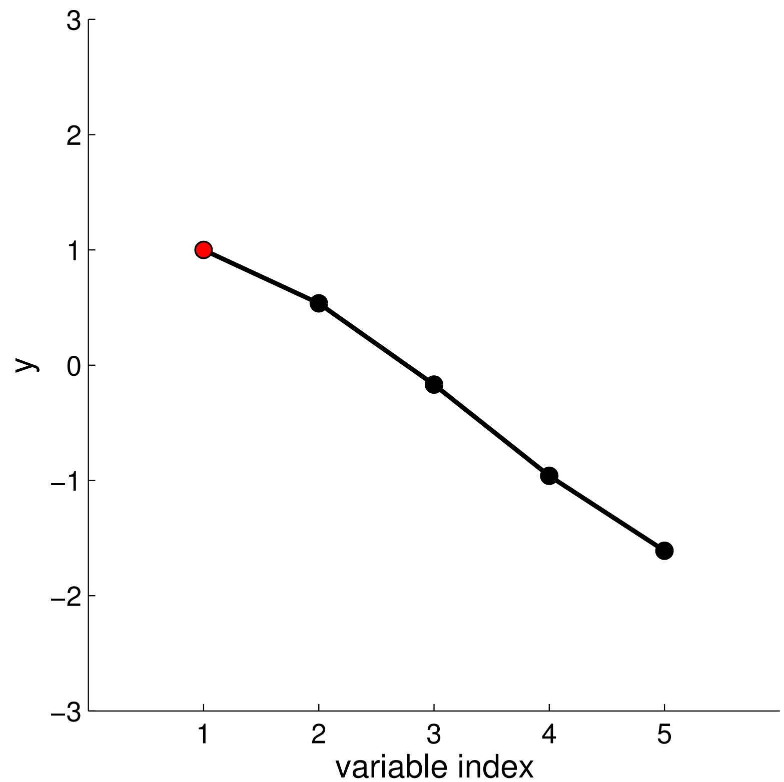
New visualisation





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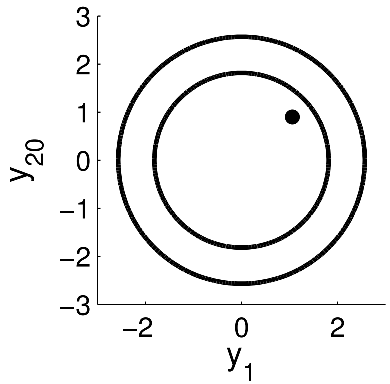
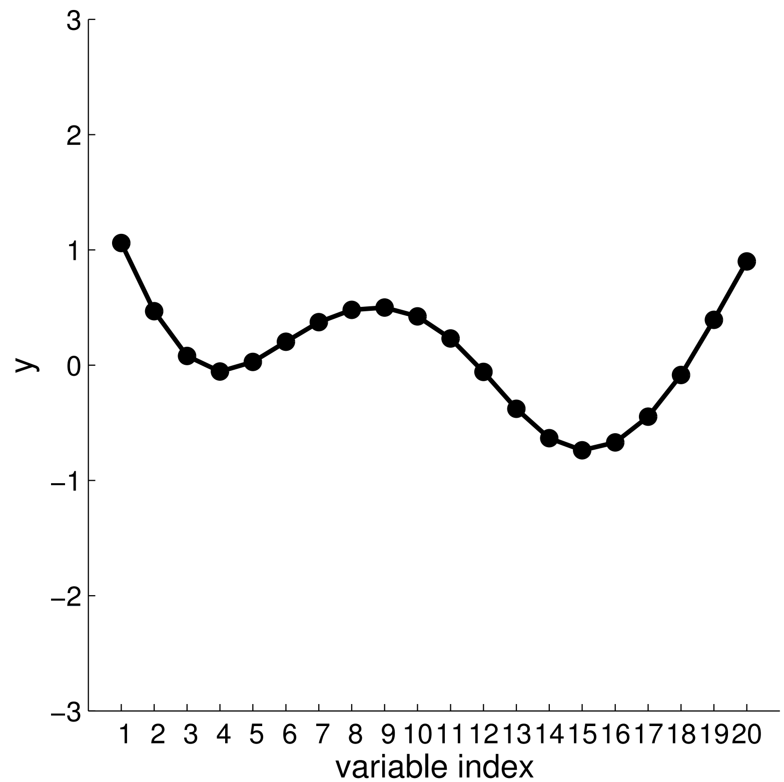
New visualisation





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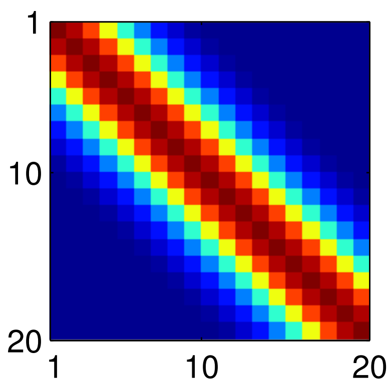
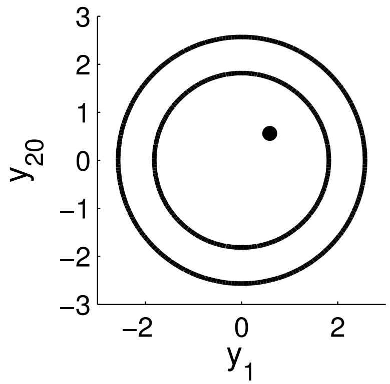
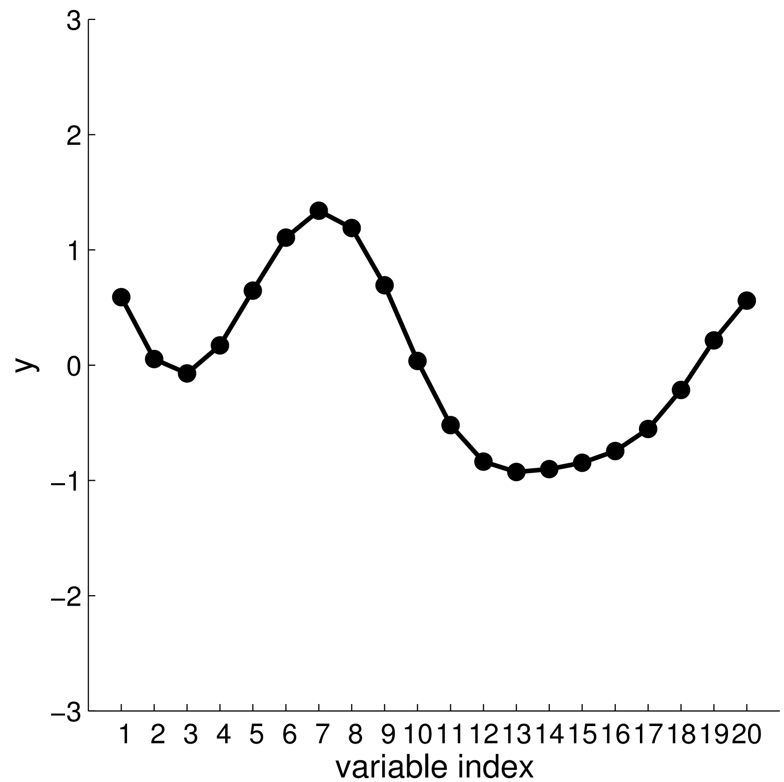


New visualisation





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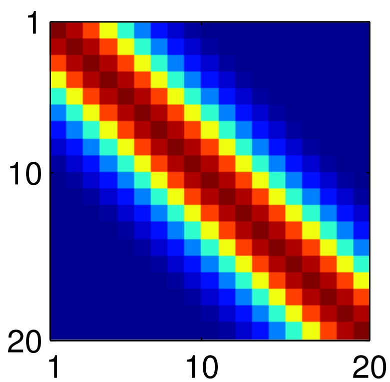
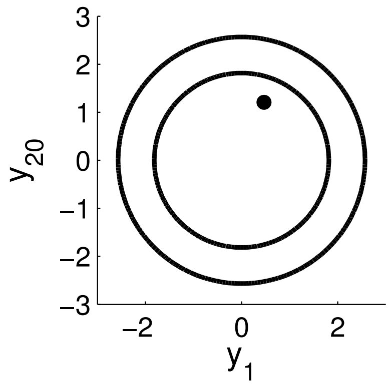


New visualisation



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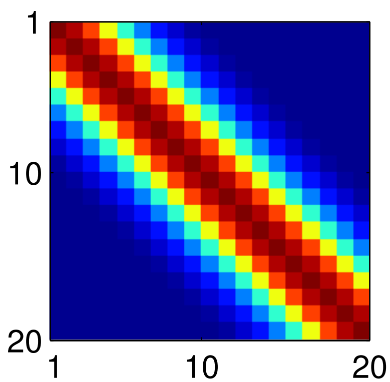

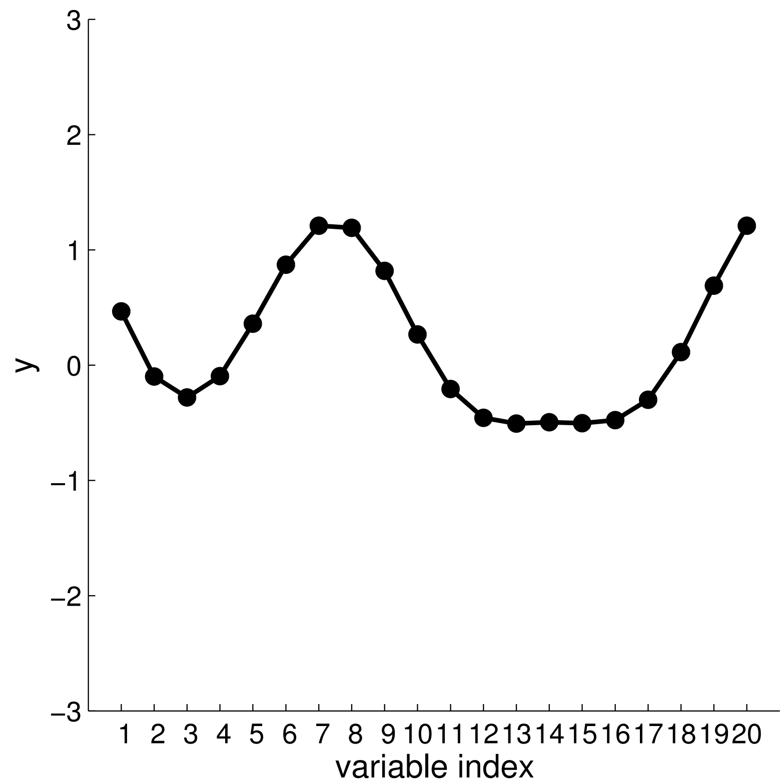

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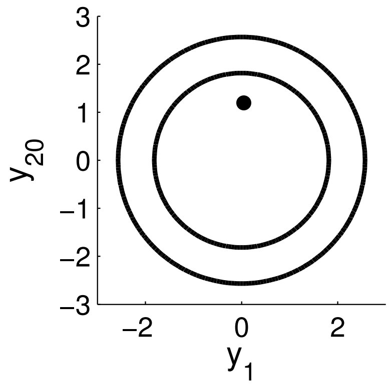

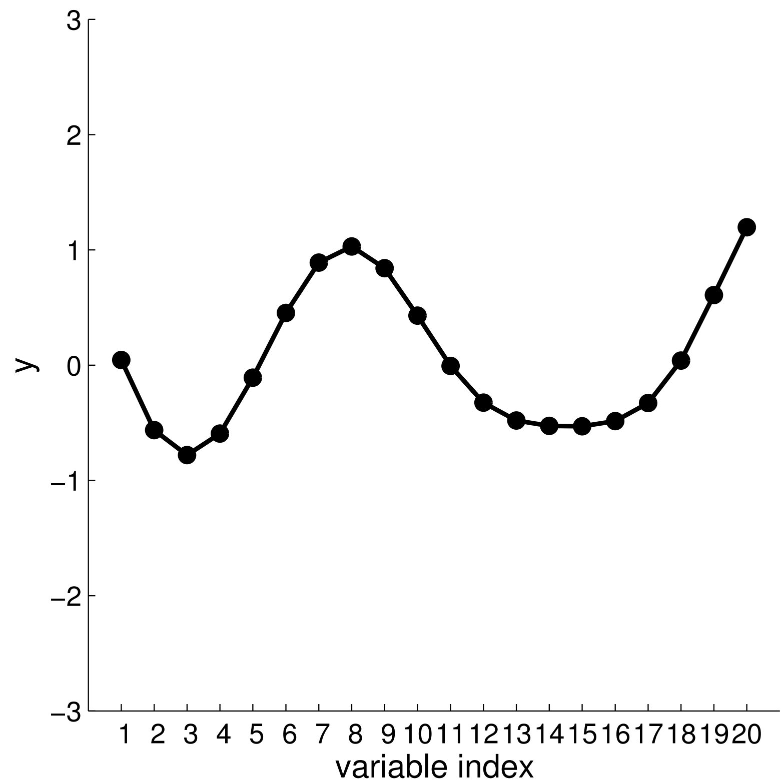

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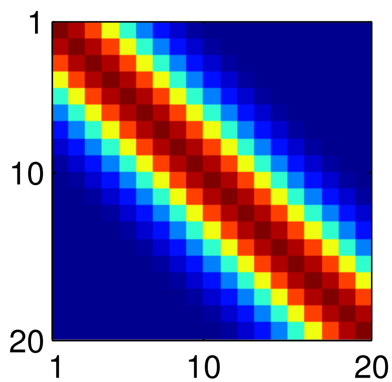

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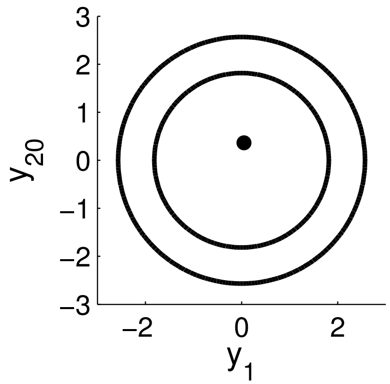

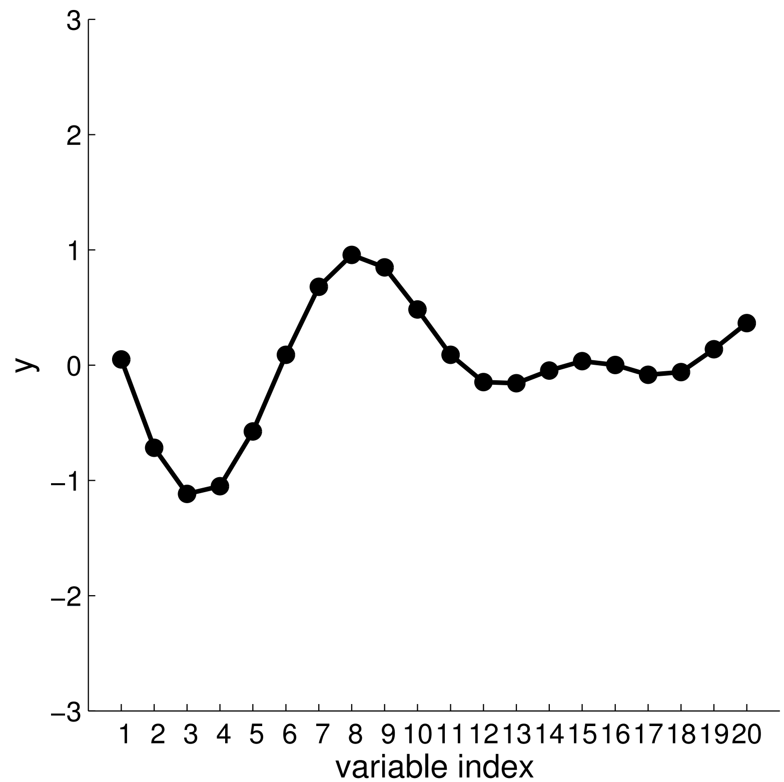

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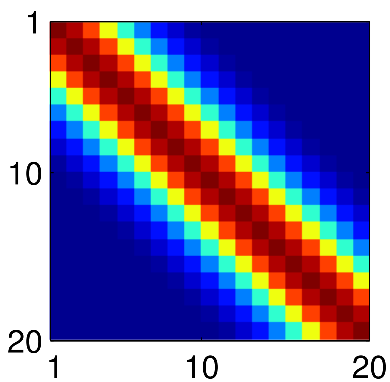

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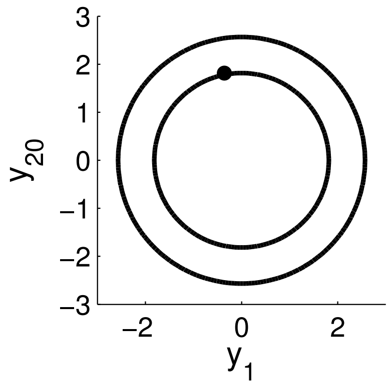

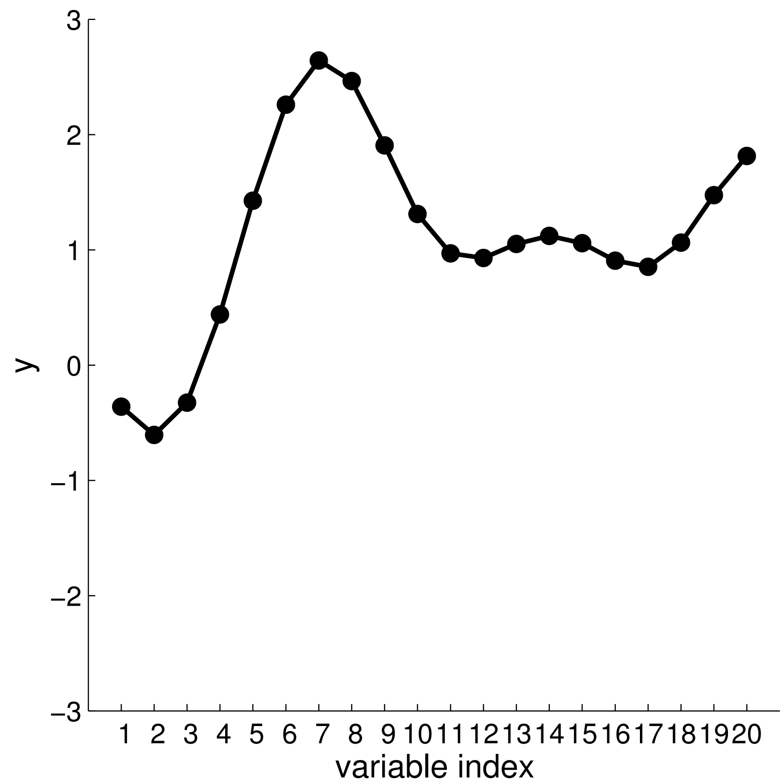

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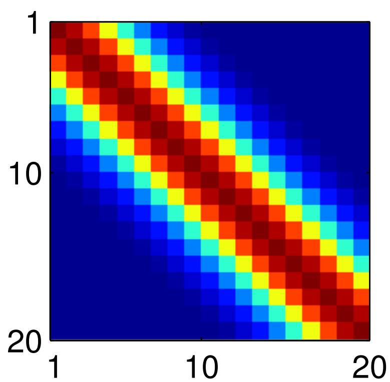

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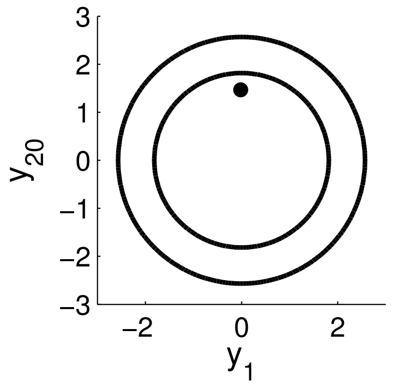

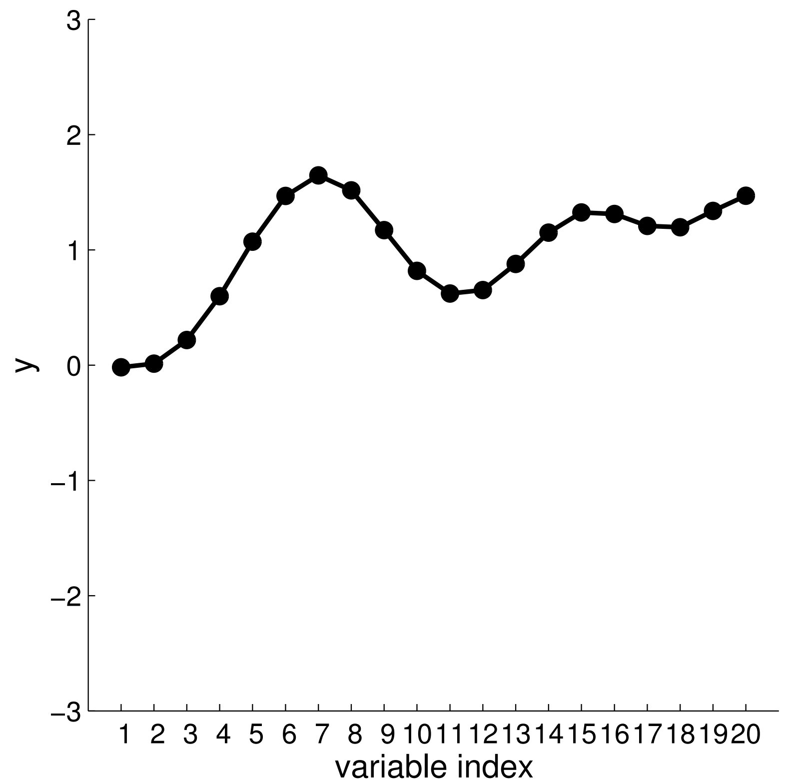

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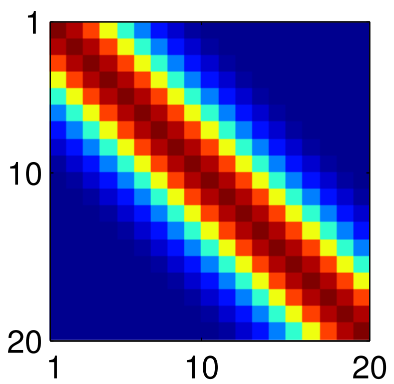

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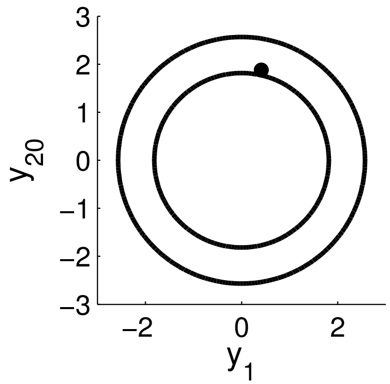

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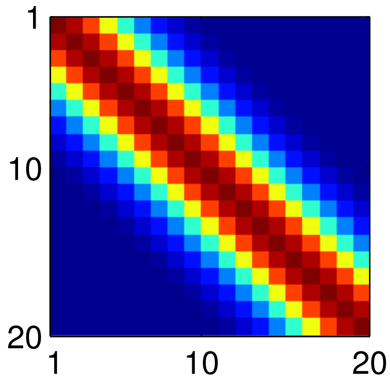

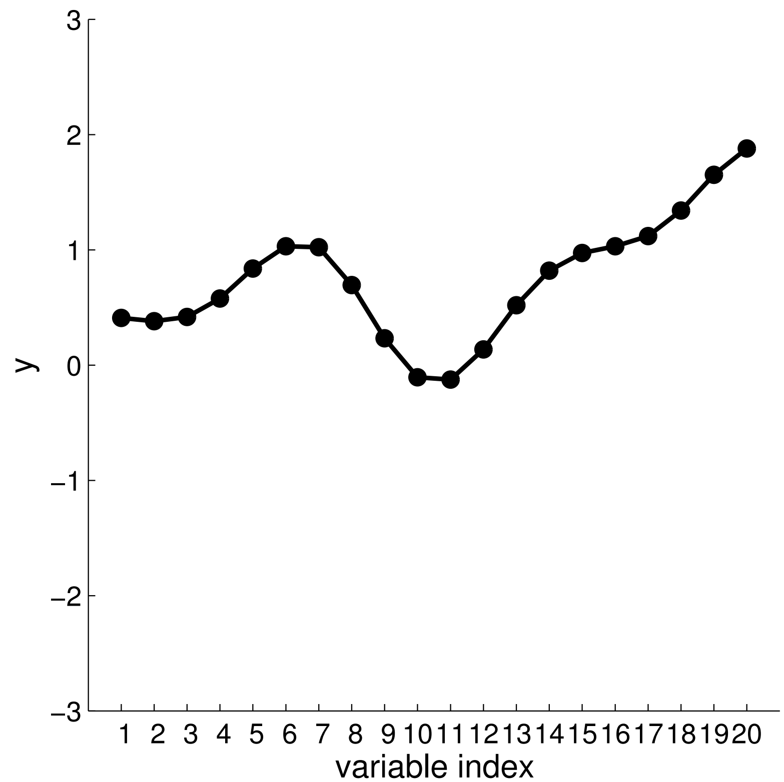

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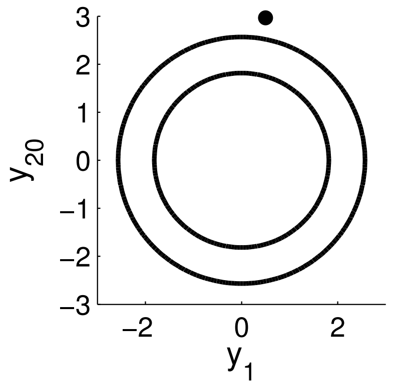

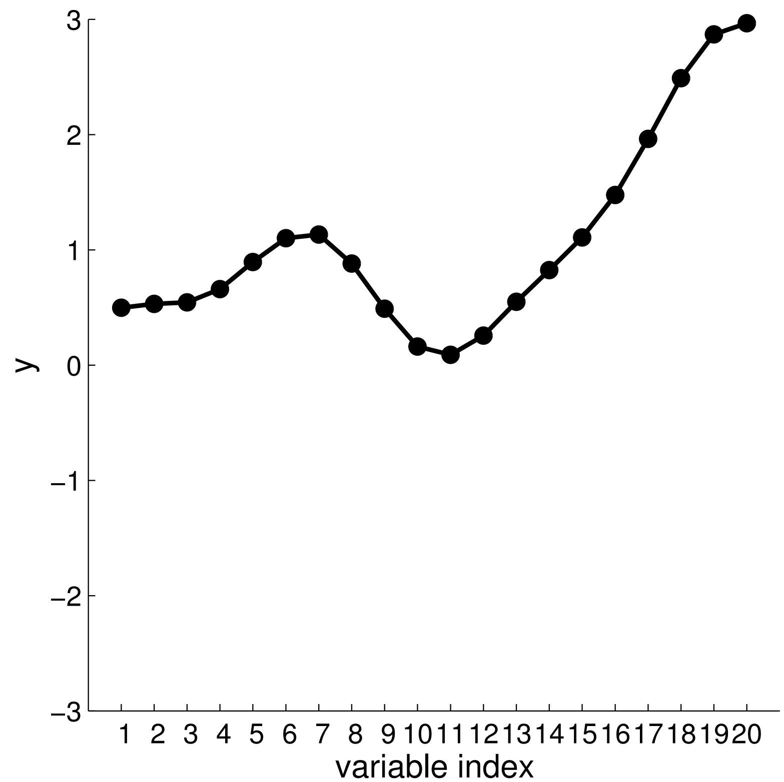

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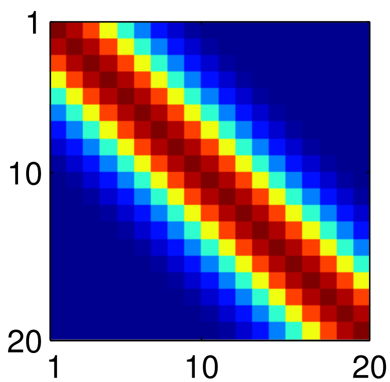

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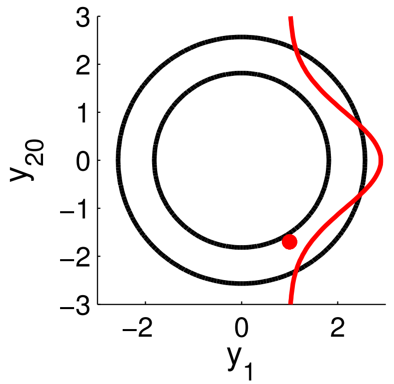

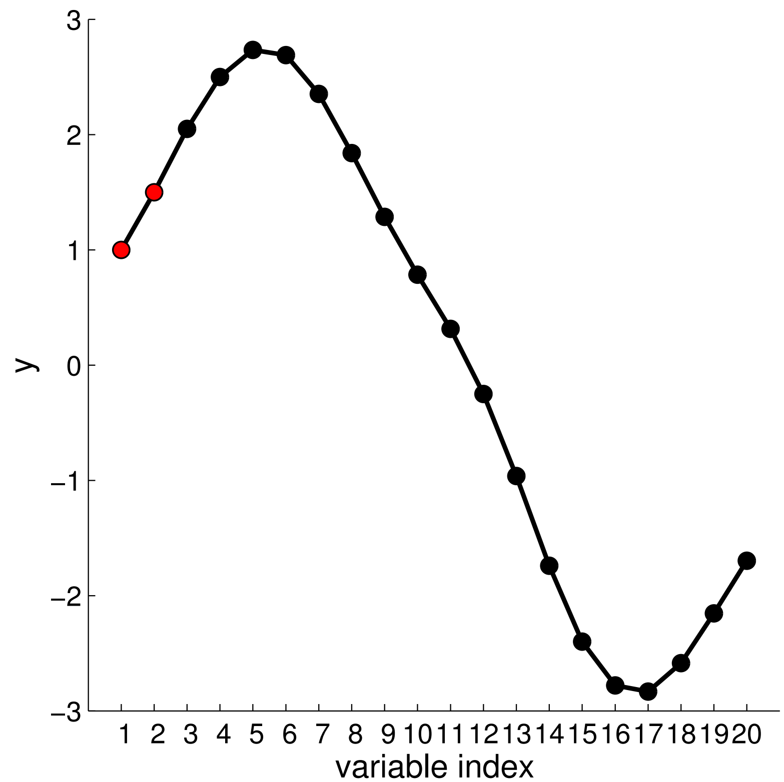

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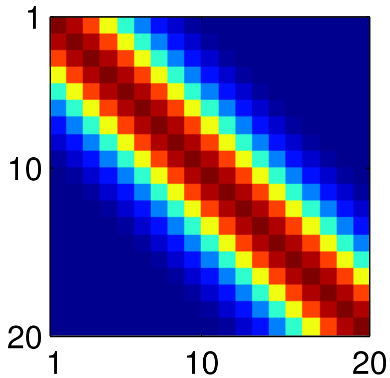

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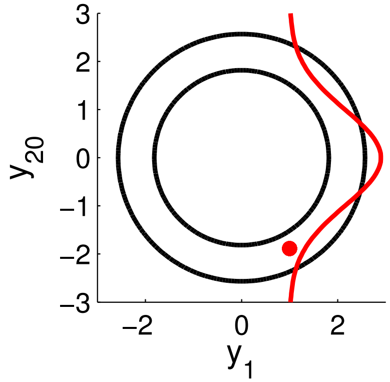
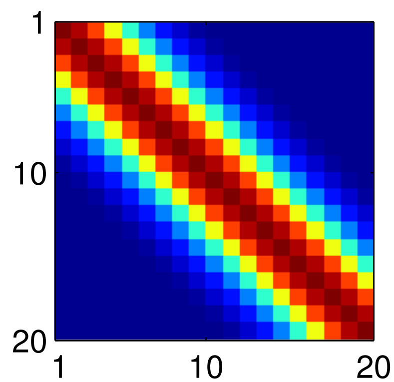
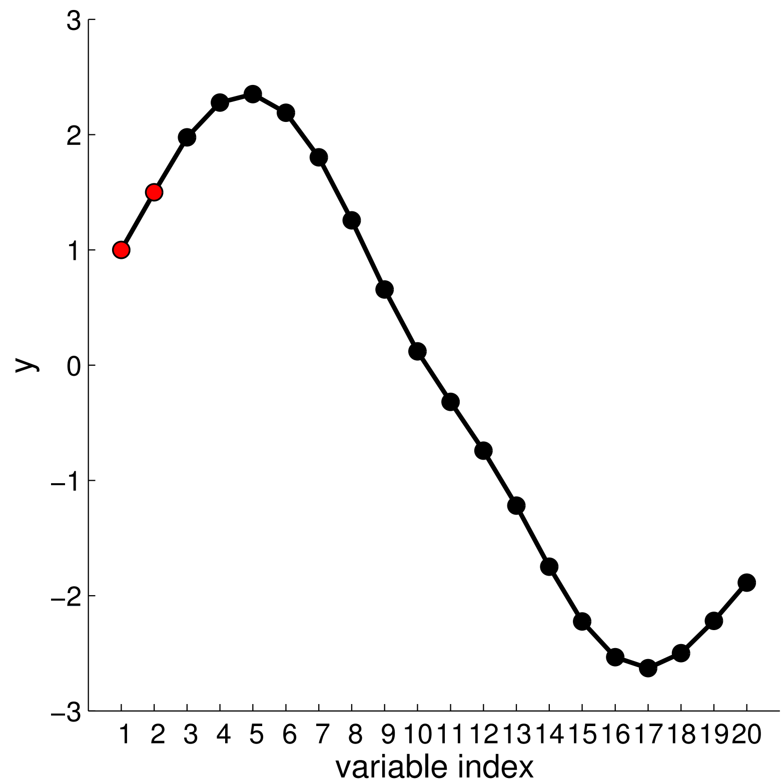


$\Sigma =$ 



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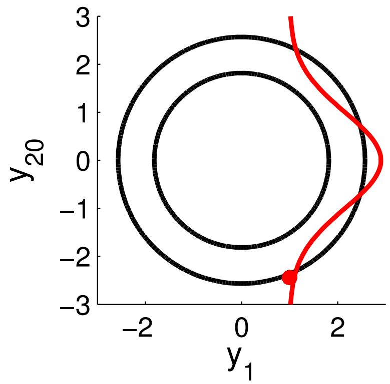

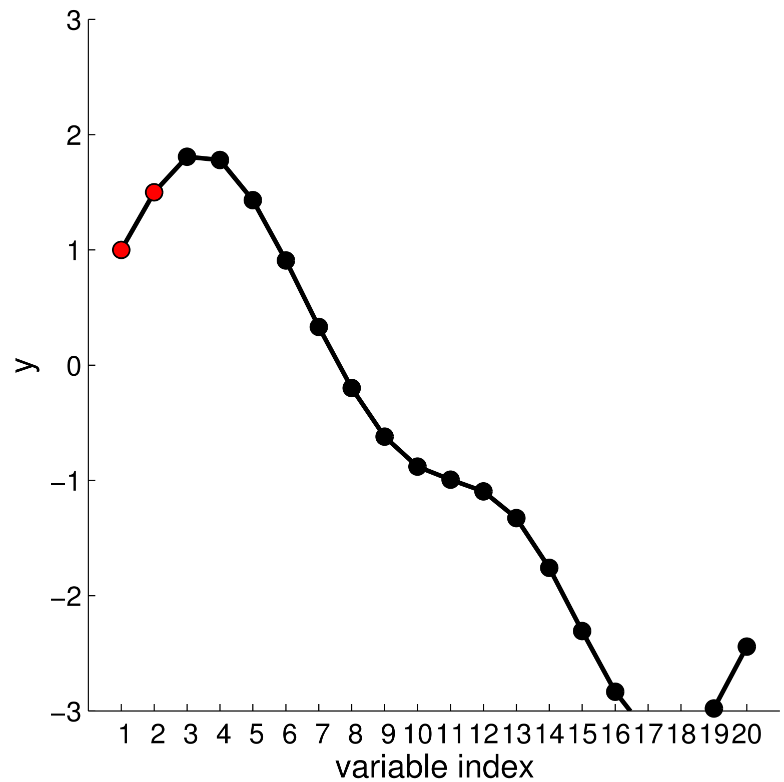

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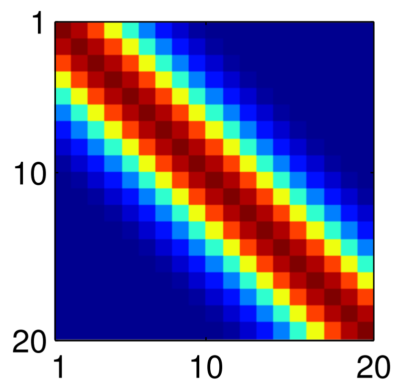

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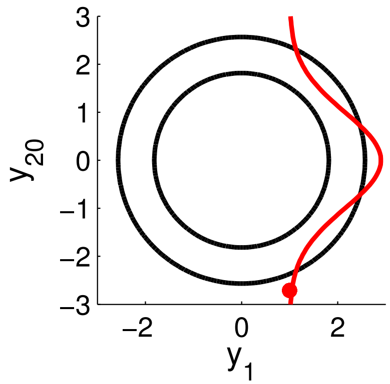

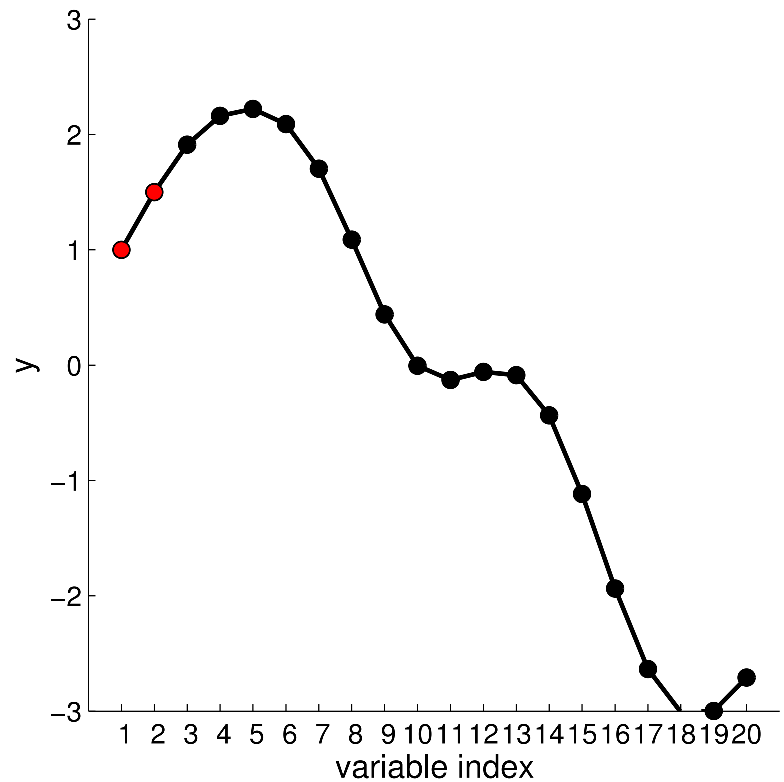

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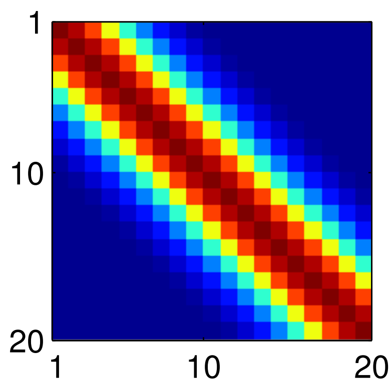

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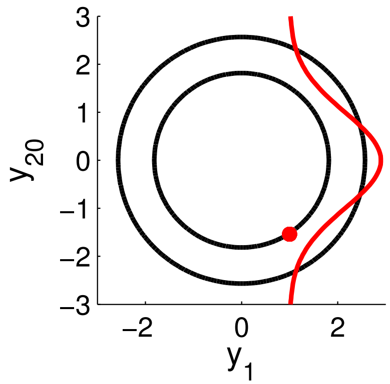

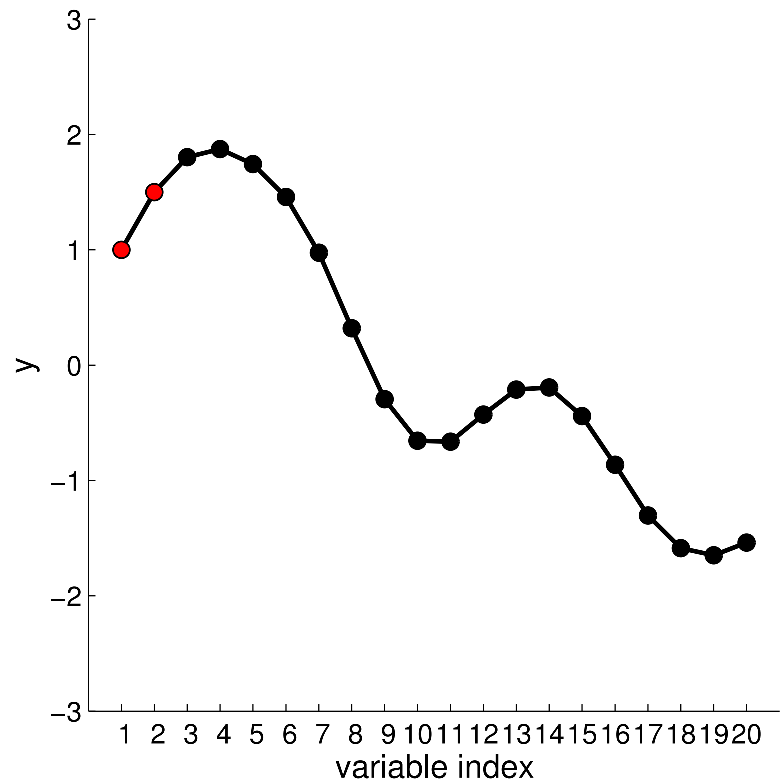

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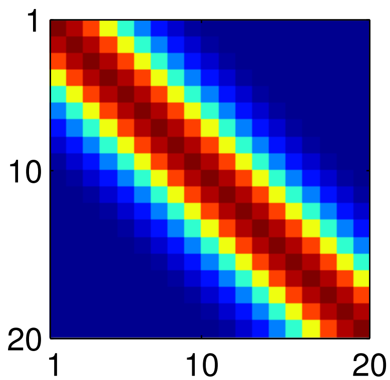

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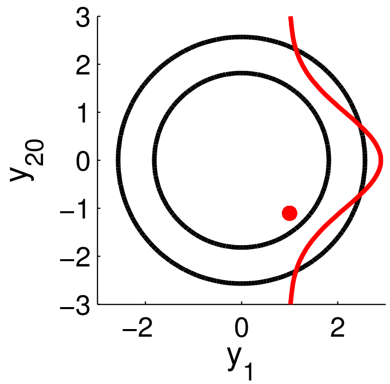

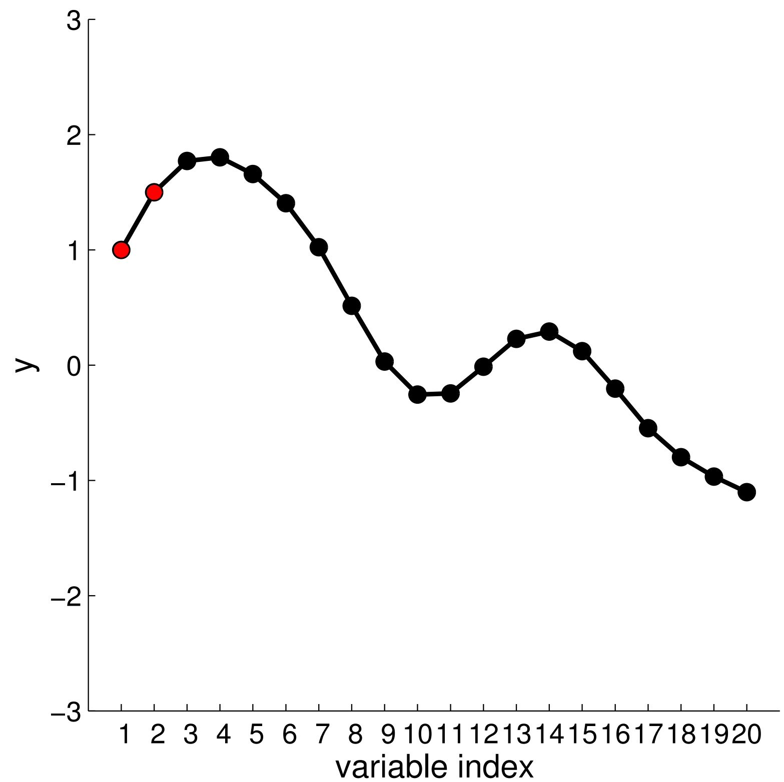

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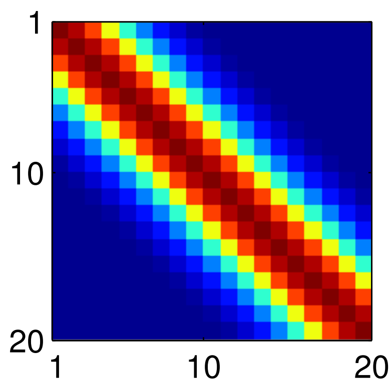

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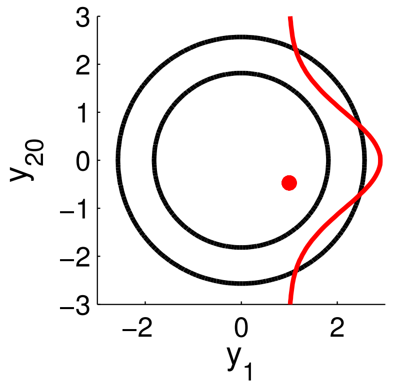

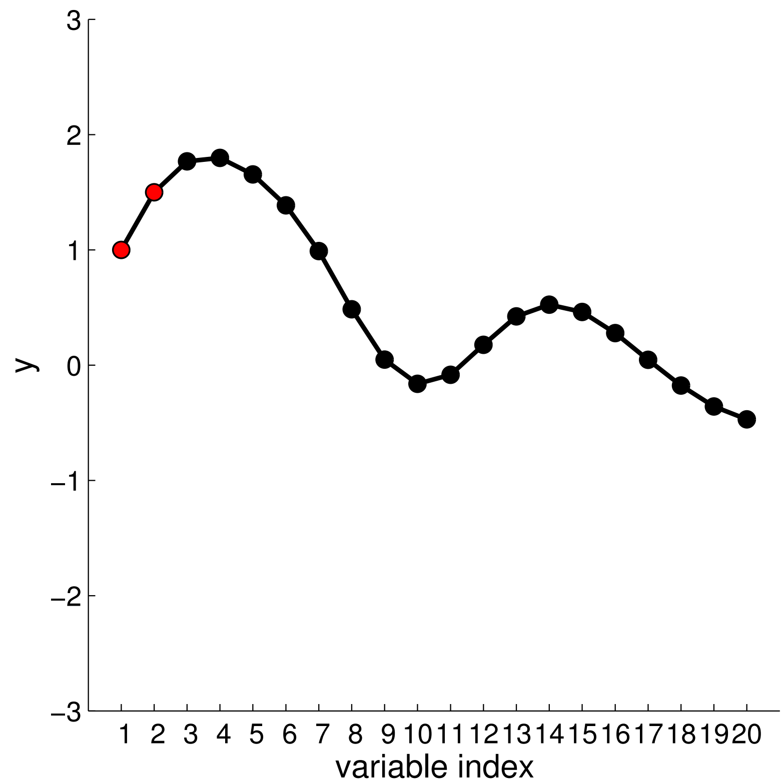

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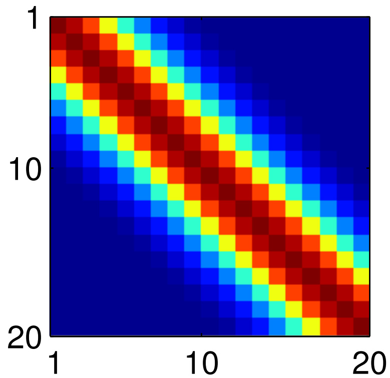

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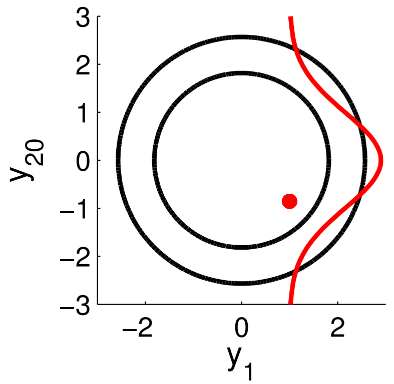

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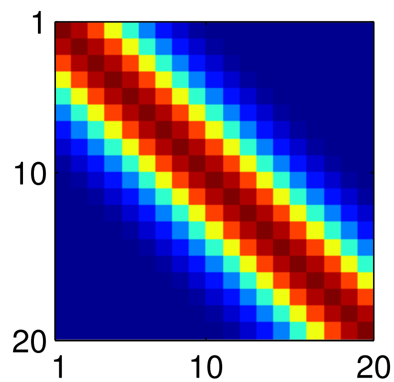

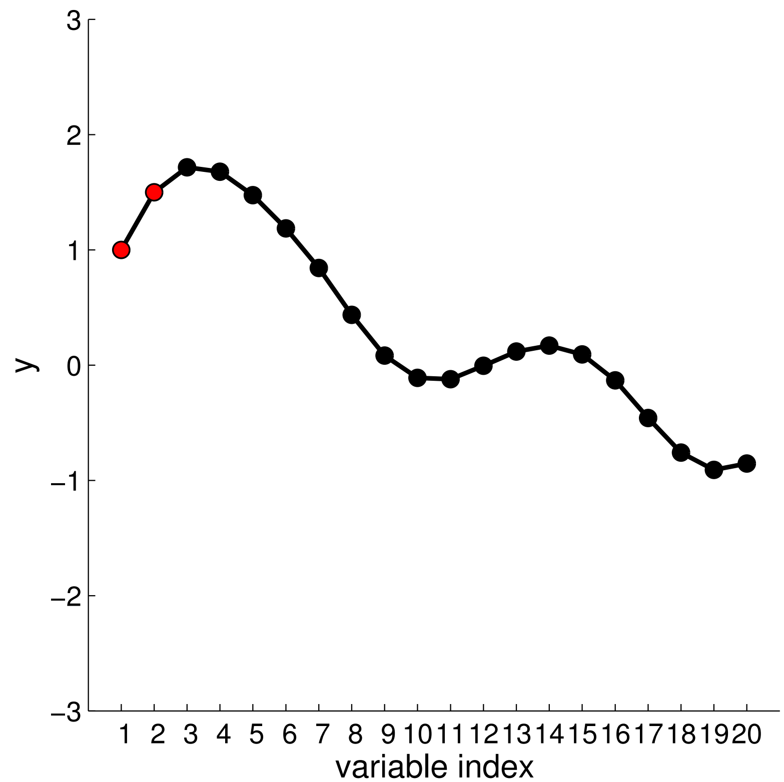

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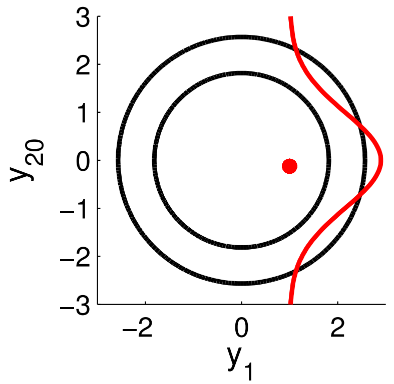

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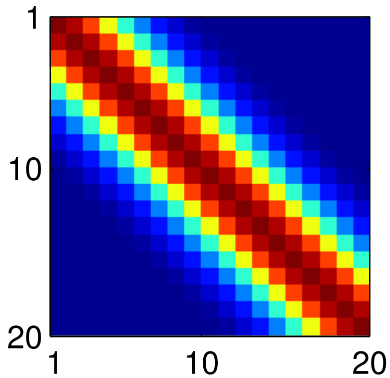

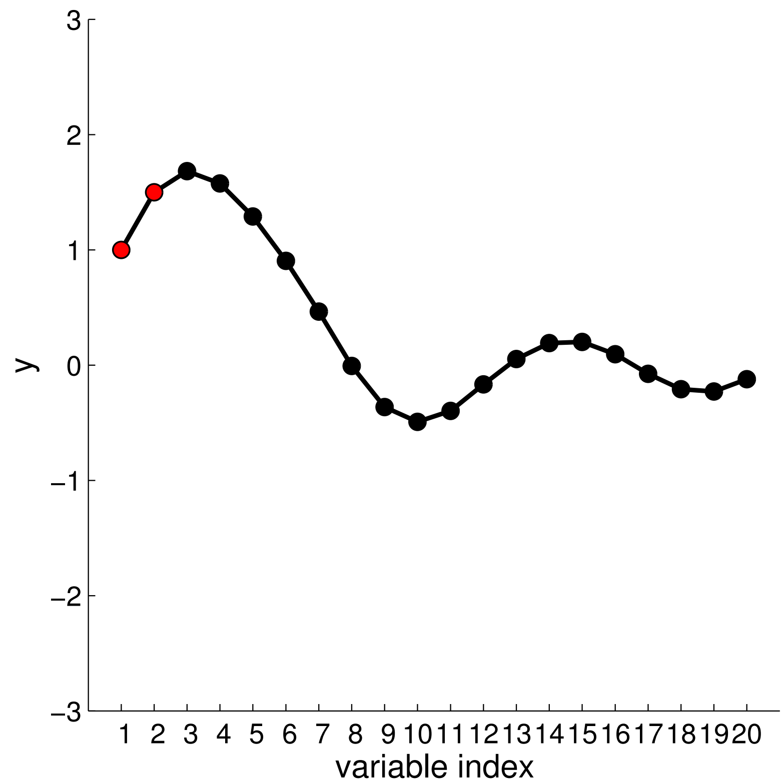

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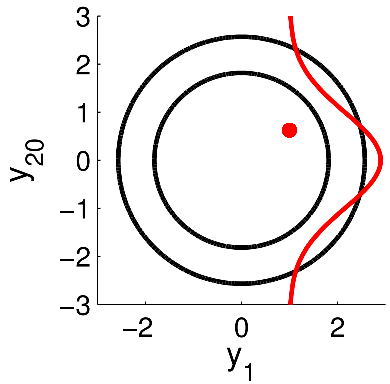

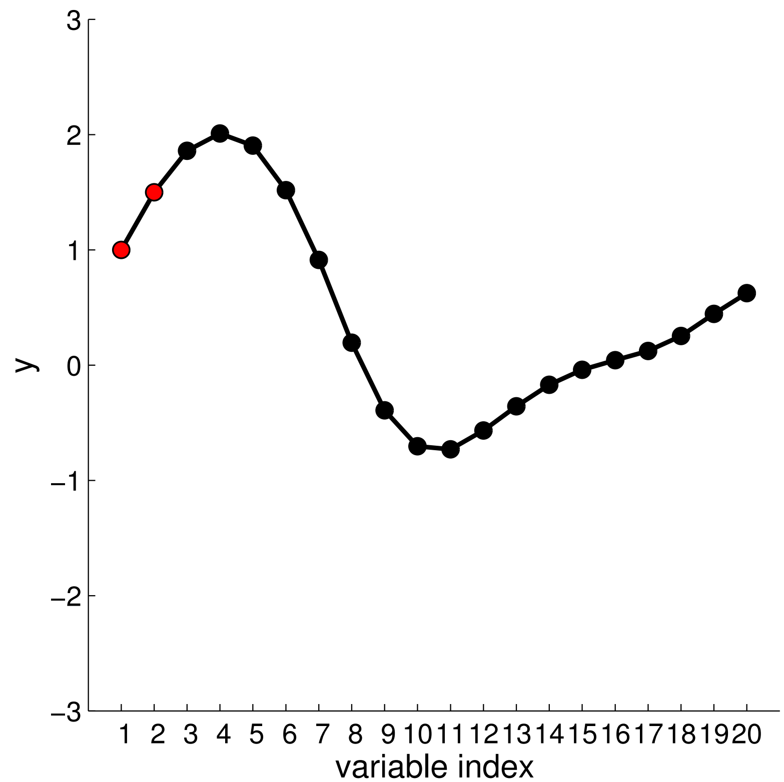

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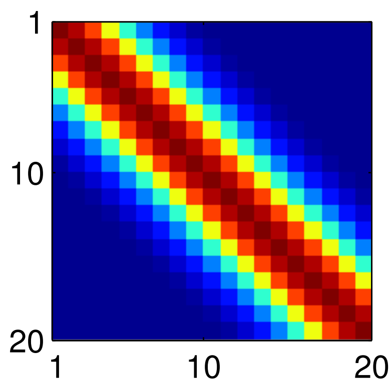

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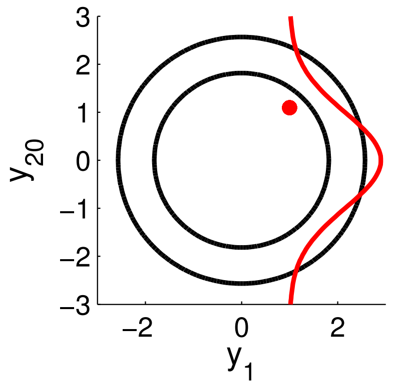

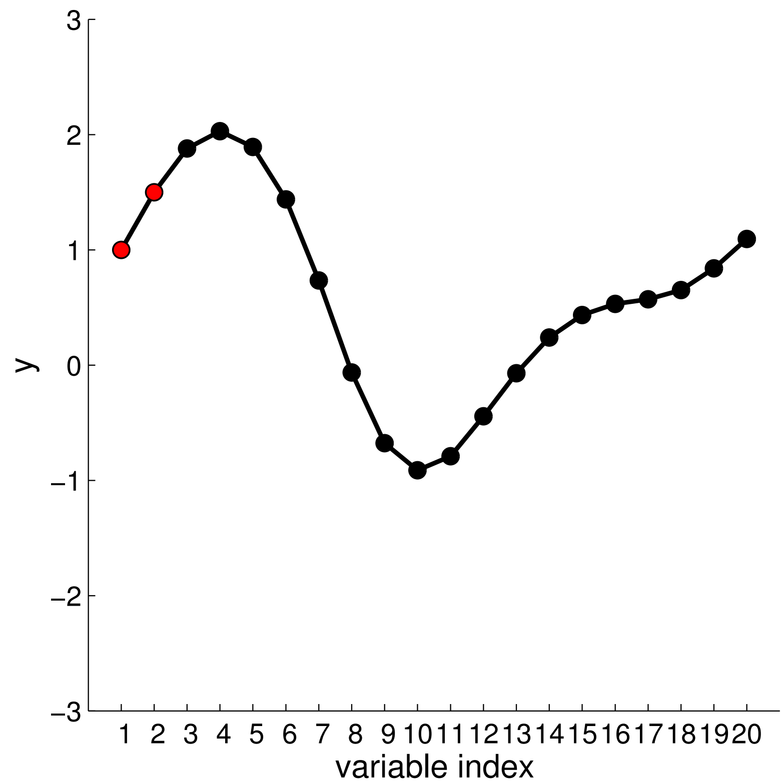

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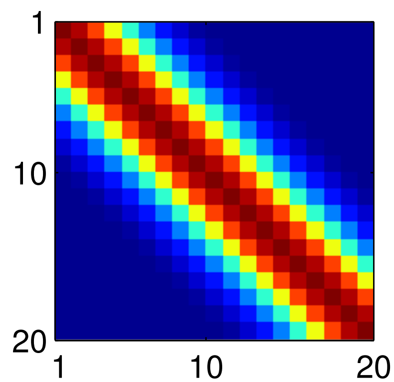

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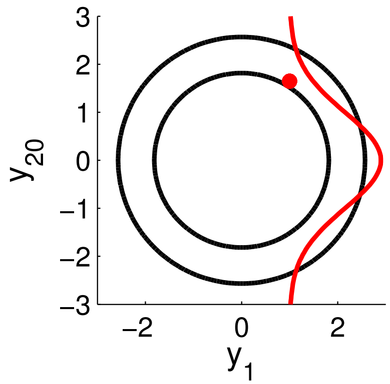

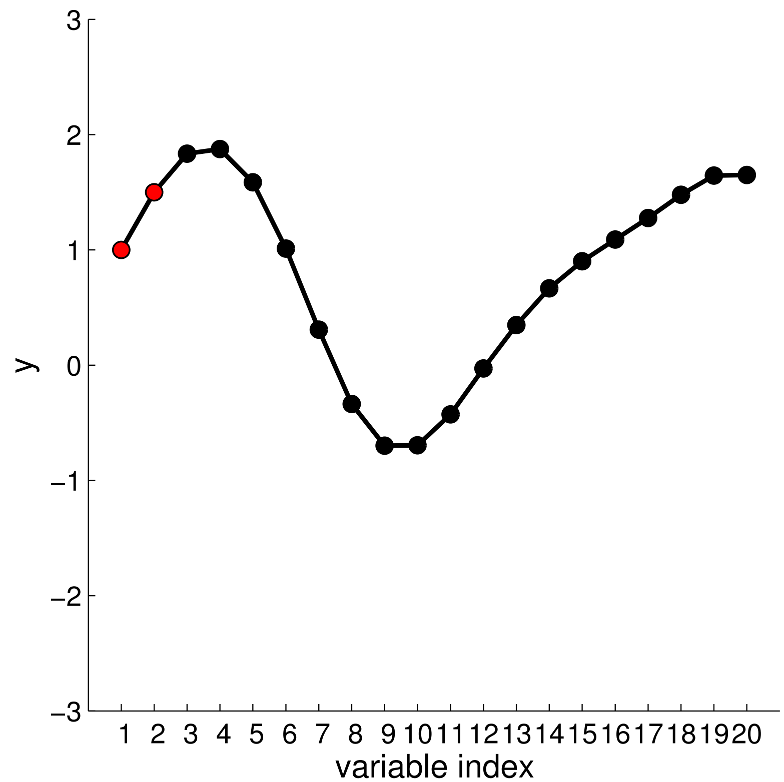

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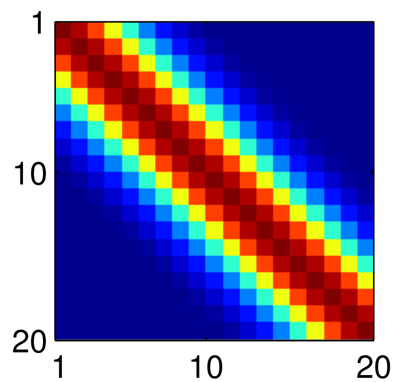

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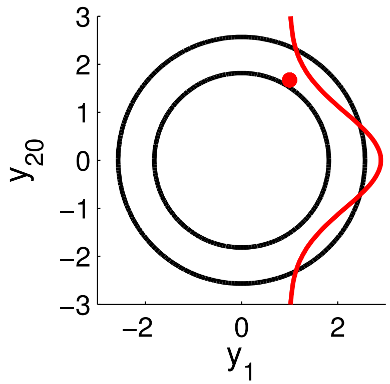

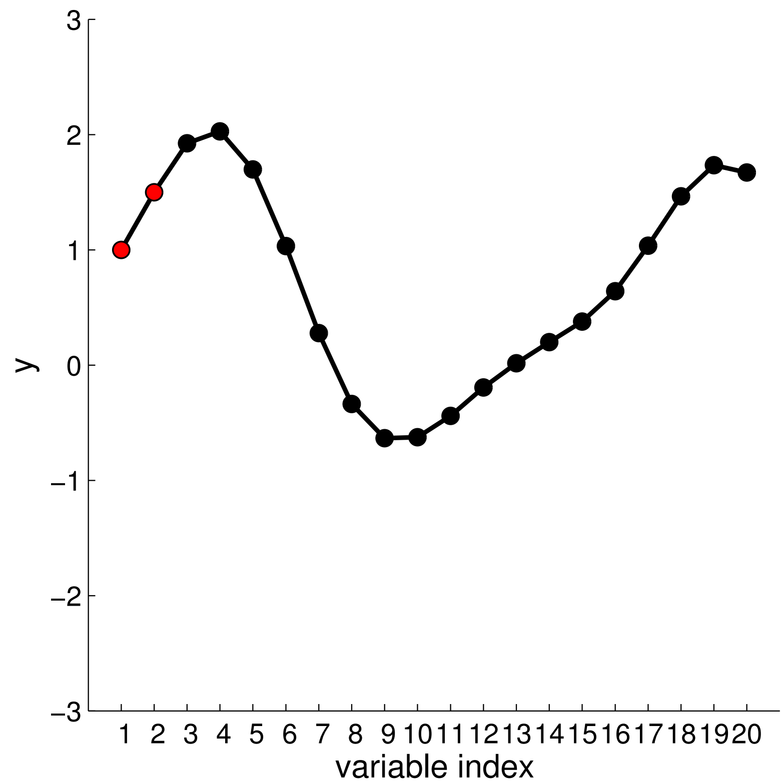

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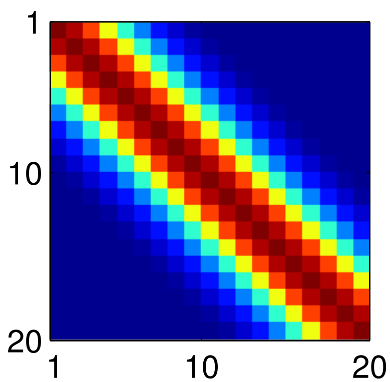

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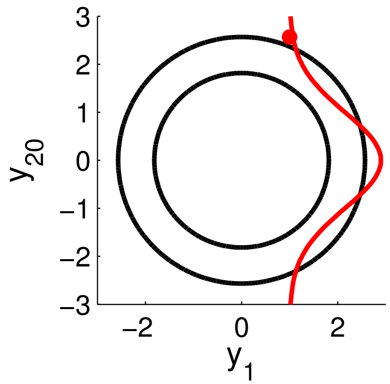

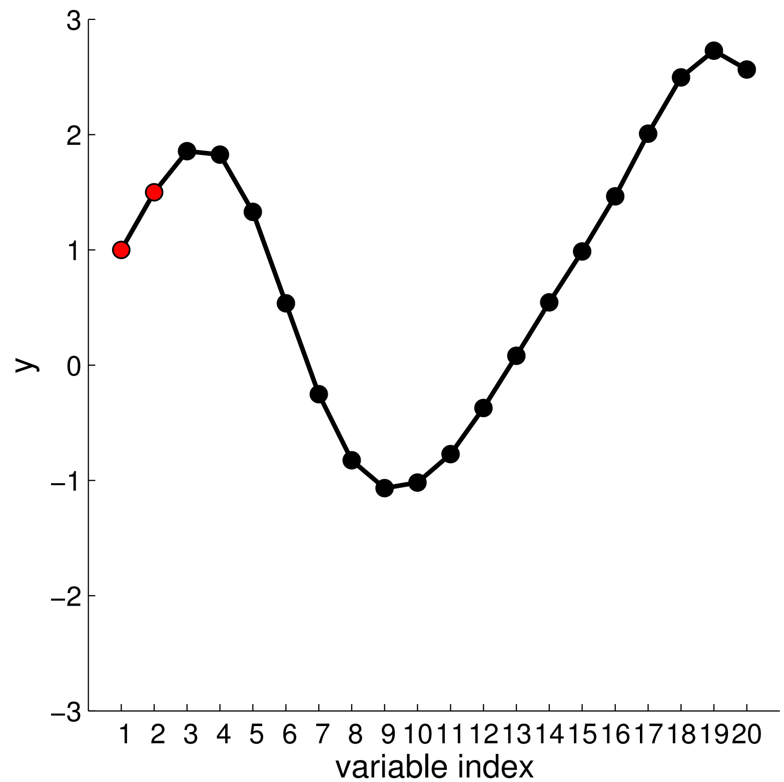

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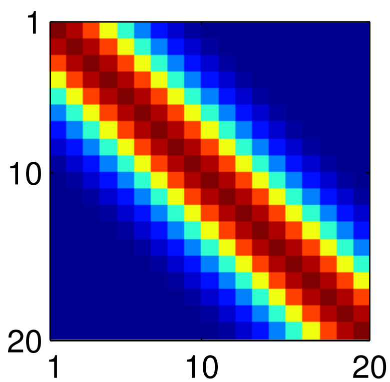

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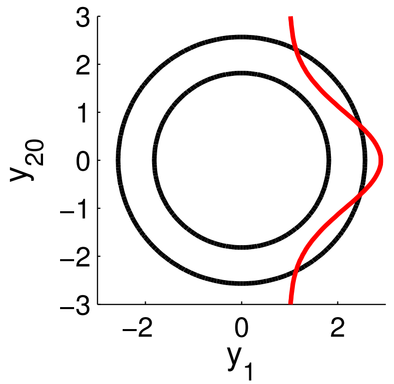

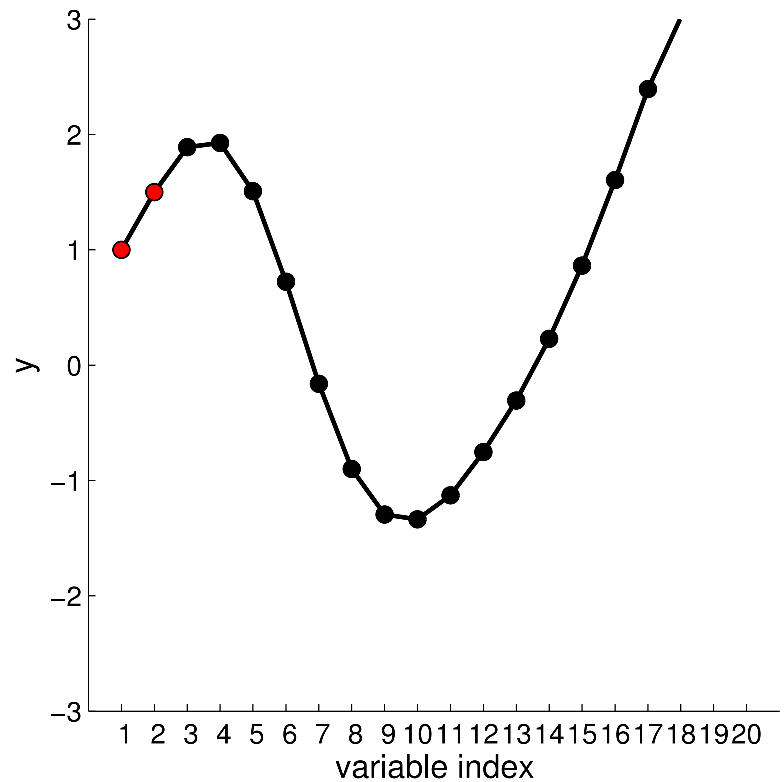

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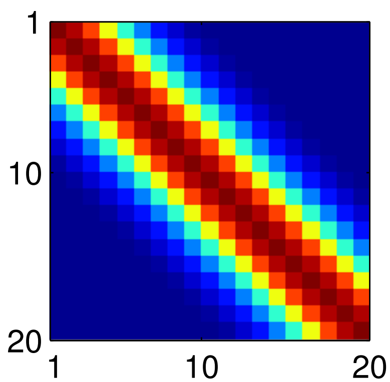
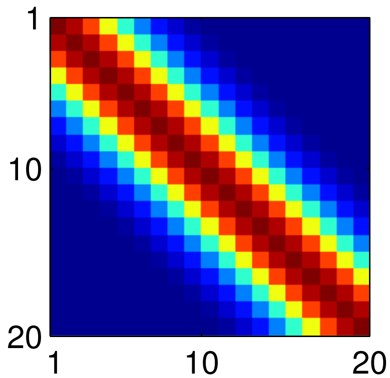
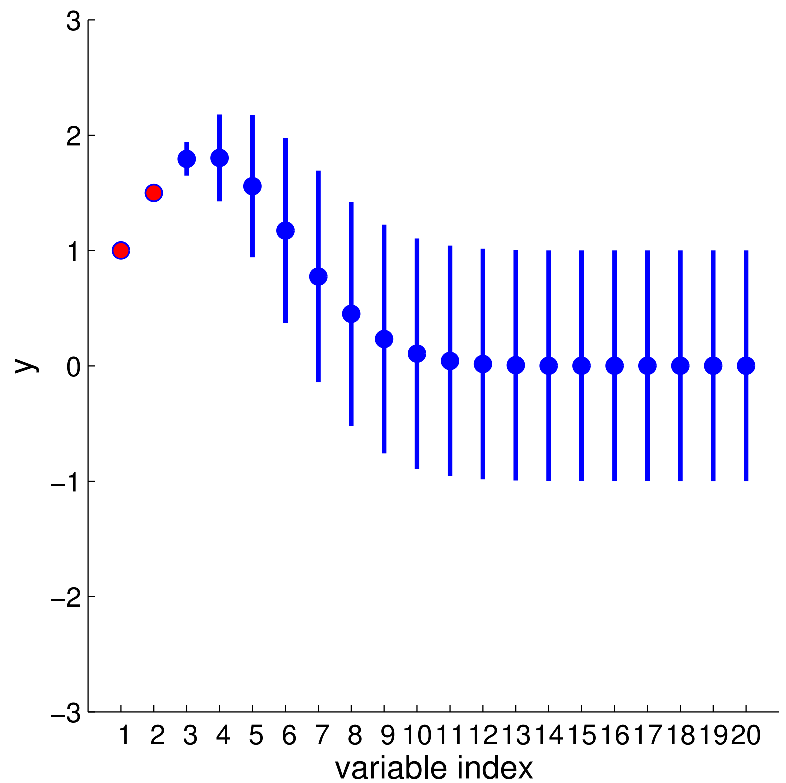


Regression using Gaussians

$$\Sigma =$$





Regression using Gaussians

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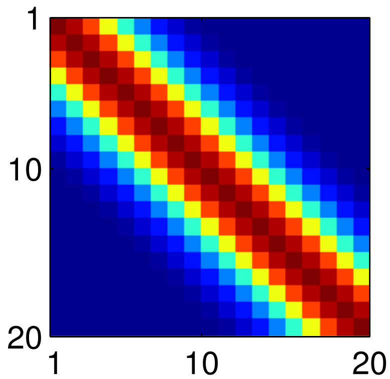

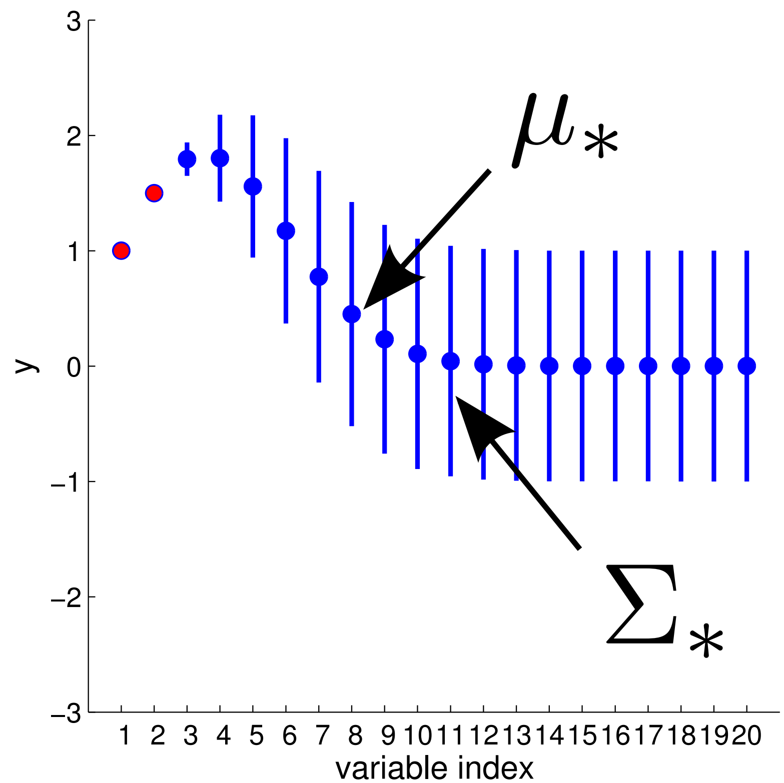

Regression using Gaussians

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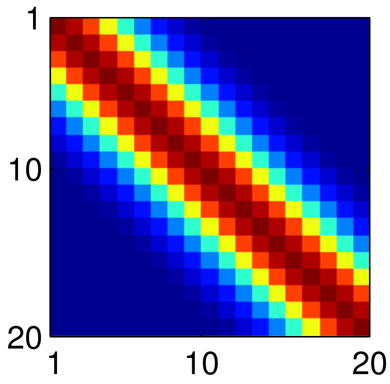
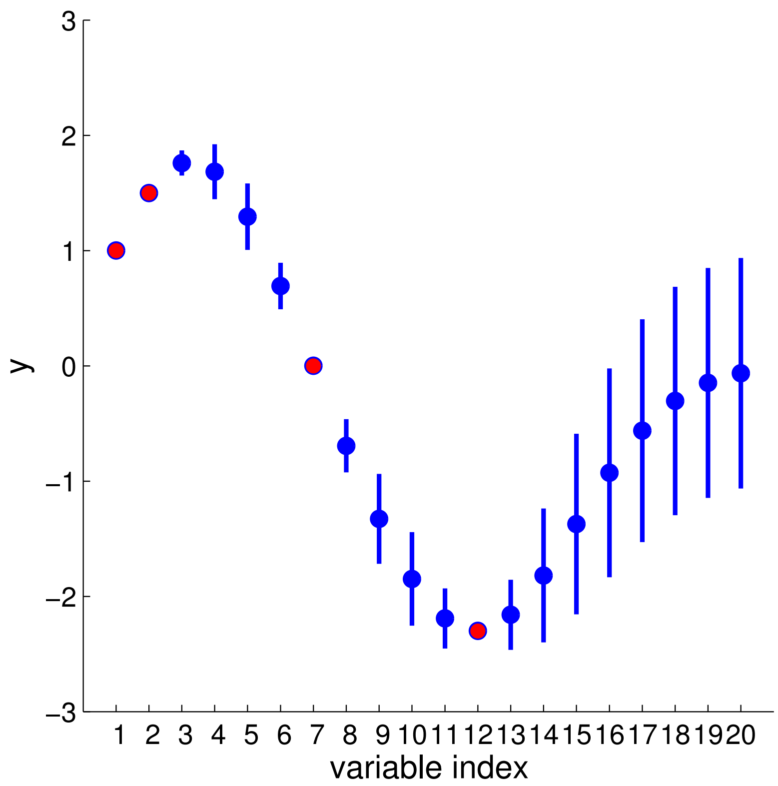


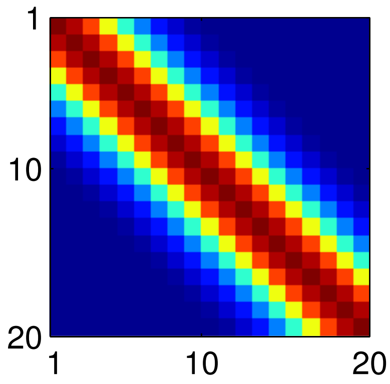


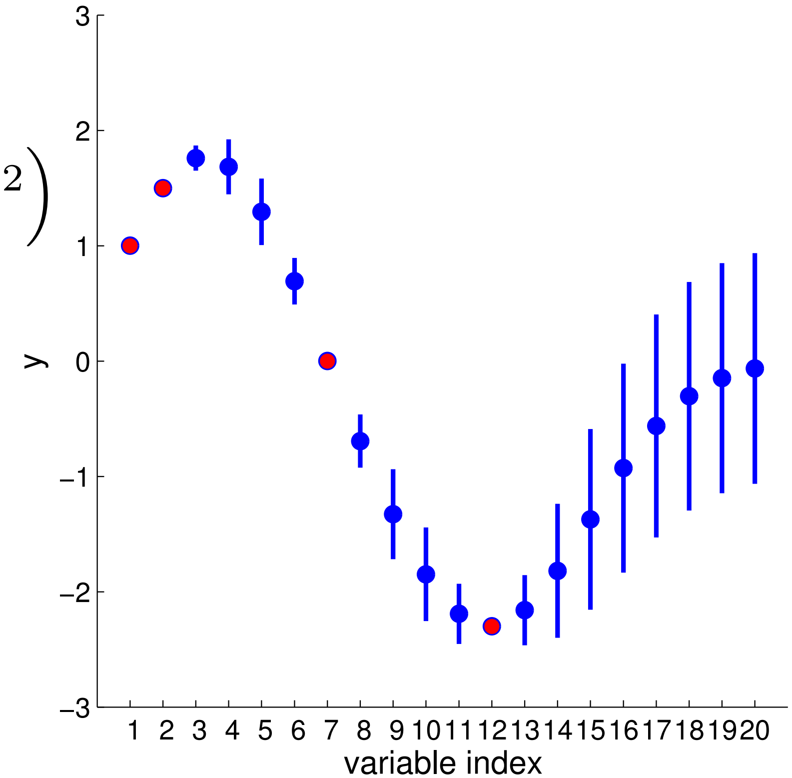
Regression using Gaussians

$$\Sigma(\mathsf{x}_1, \mathsf{x}_2) = \mathsf{K}(\mathsf{x}_1, \mathsf{x}_2) + \mathsf{I}\sigma_{\mathsf{y}}^2$$

$$\mathsf{K}(\mathsf{x}_1, \mathsf{x}_2) = \sigma^2 \exp\left(-\frac{1}{2\mathsf{l}^2}(\mathsf{x}_1 - \mathsf{x}_2)^2\right)$$

$\Sigma =$

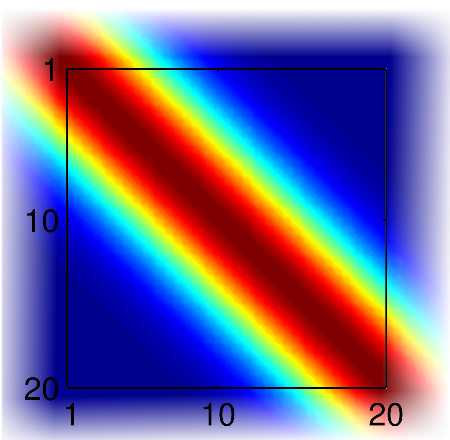


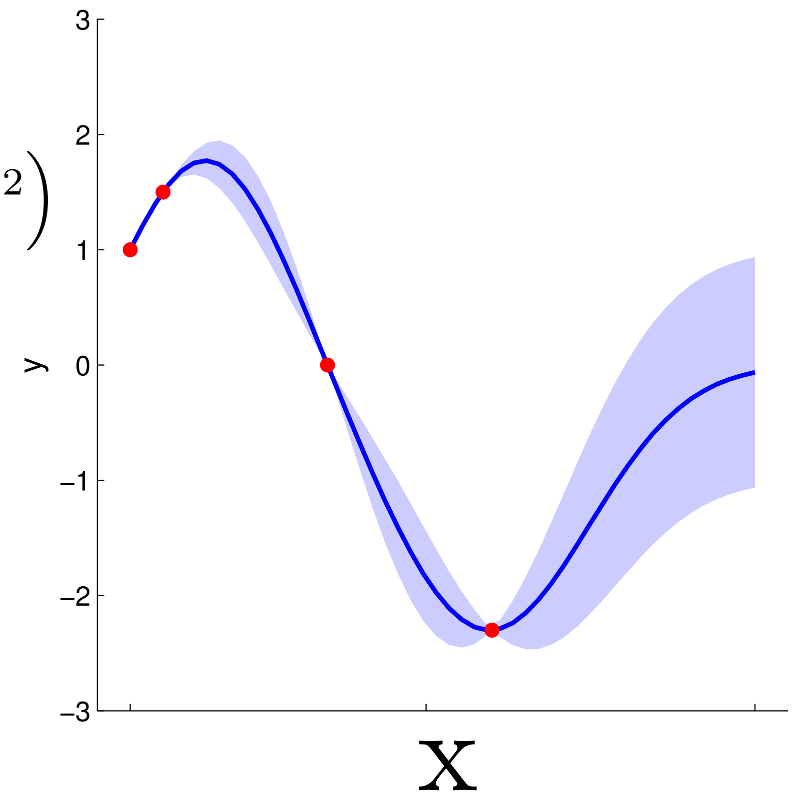


Regression: probabilistic inference in function space

$$\Sigma(\mathsf{x}_1, \mathsf{x}_2) = \mathsf{K}(\mathsf{x}_1, \mathsf{x}_2) + \mathsf{I}\sigma_{\mathsf{y}}^2$$

$$\mathsf{K}(\mathsf{x}_1, \mathsf{x}_2) = \sigma^2 \exp\left(-\frac{1}{2\mathsf{l}^2}(\mathsf{x}_1 - \mathsf{x}_2)^2\right)$$

$\Sigma =$ 



Regression: probabilistic inference in function space

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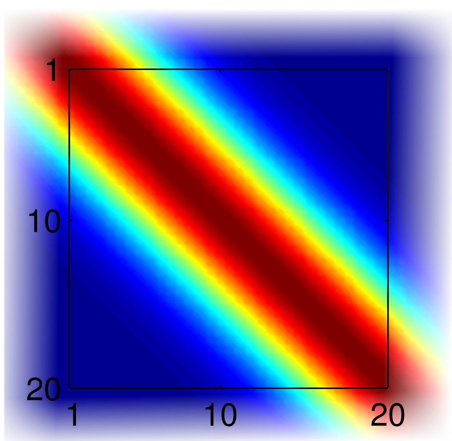

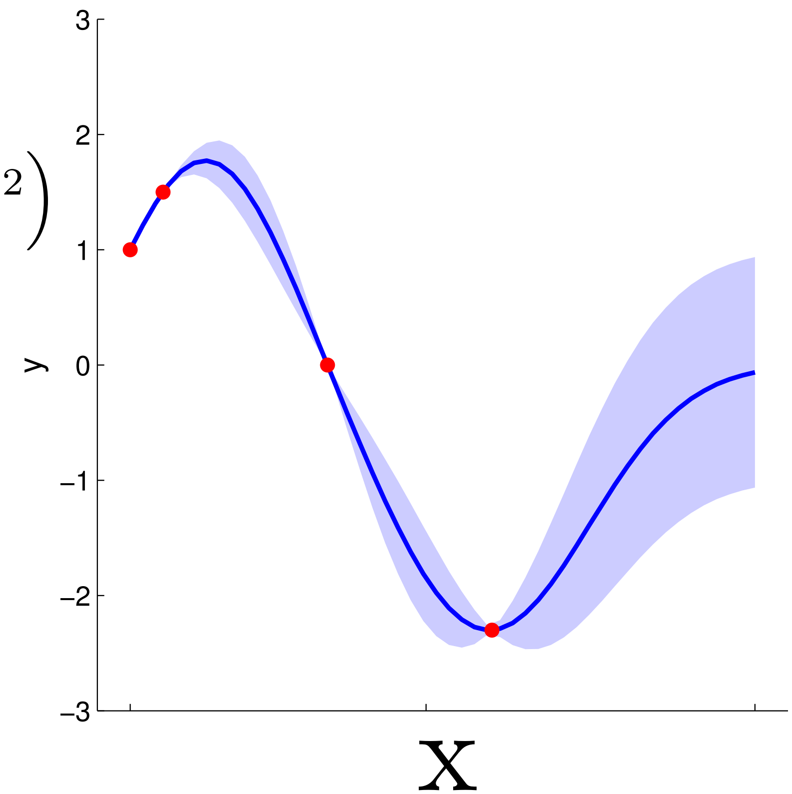

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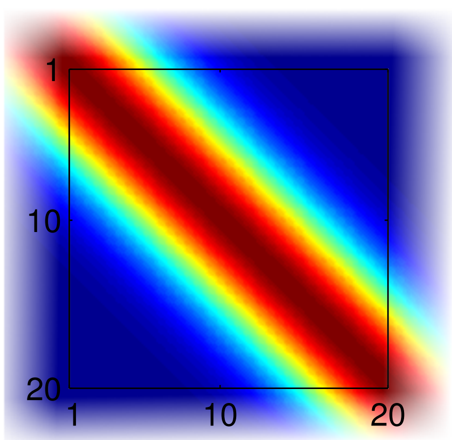
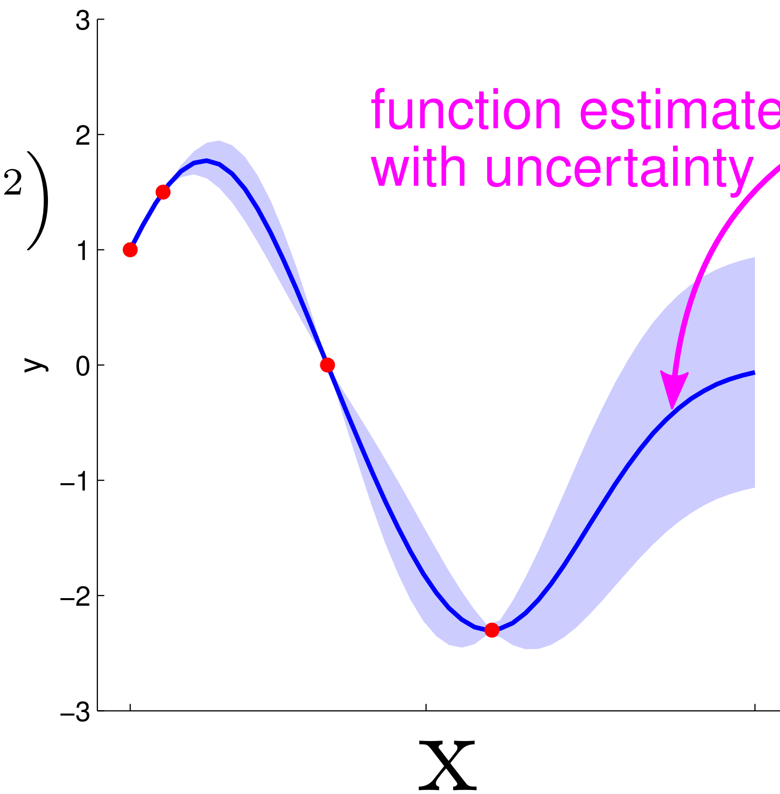
$\Sigma =$ 



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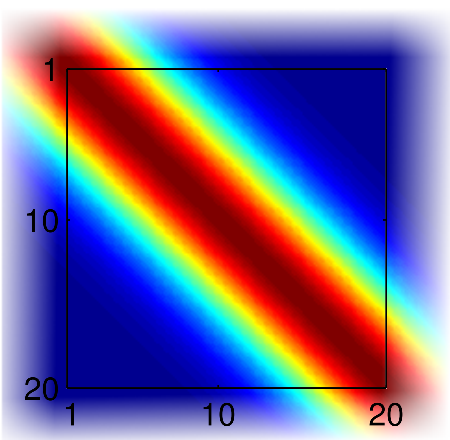
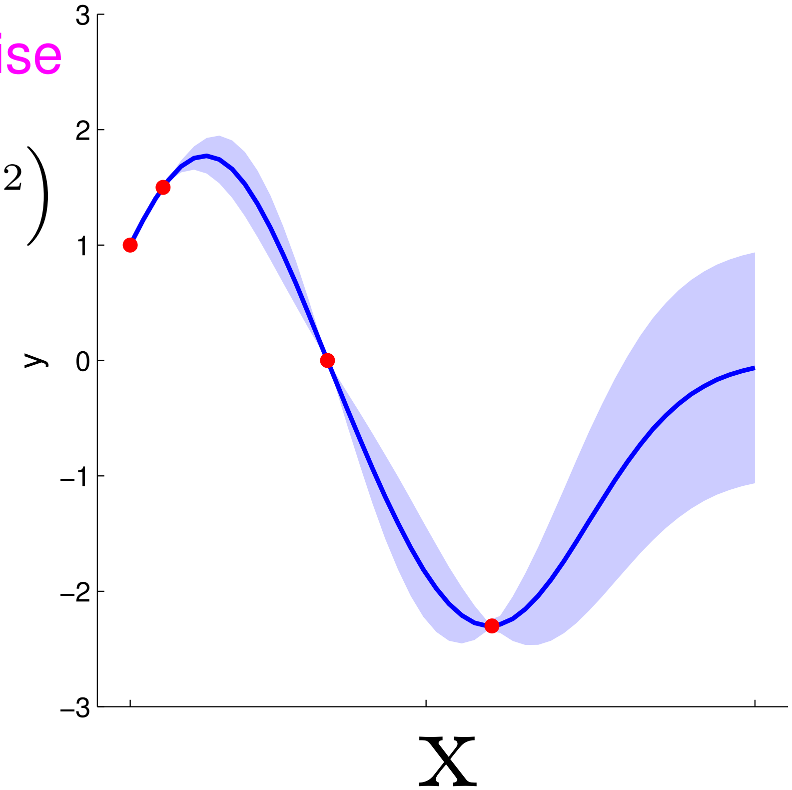
$\Sigma =$ 



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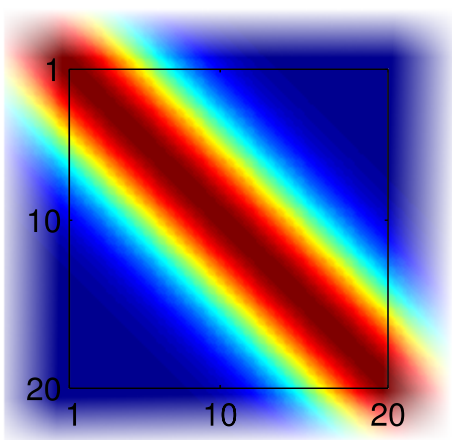
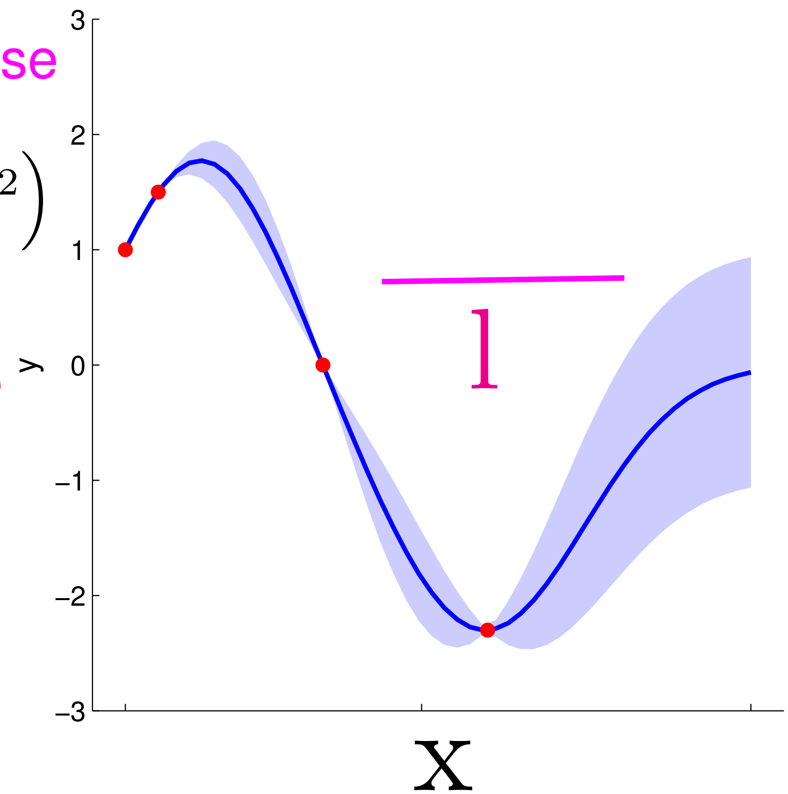
↑ horizontal-scale

$\Sigma =$ 



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vertical-scale   horizontal-scale

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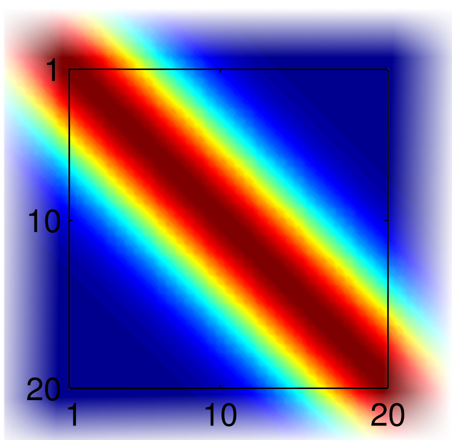

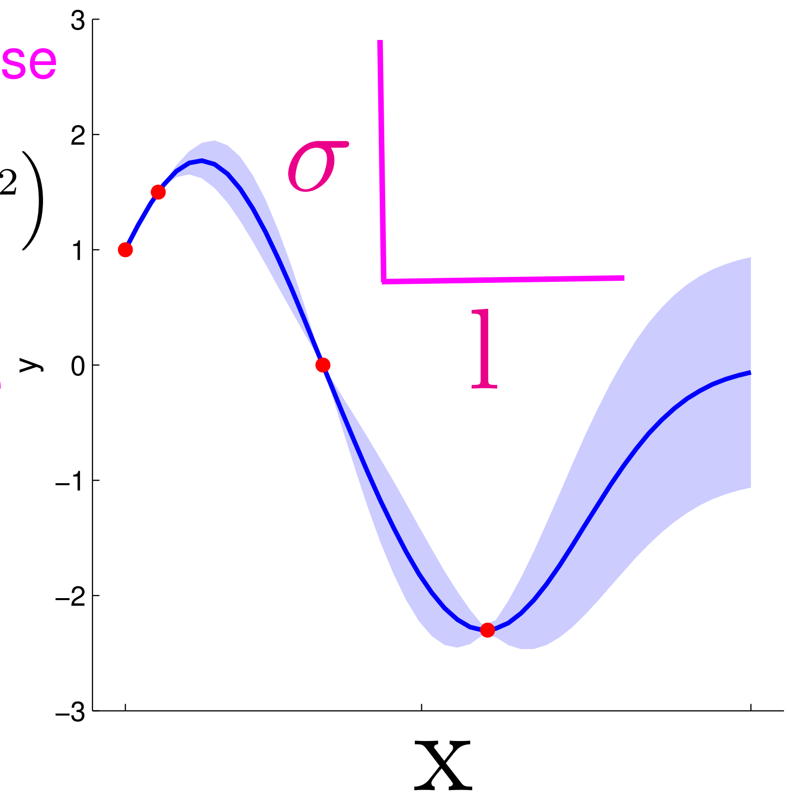

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Mathematical Foundations: Definition

Gaussian process = generalization of multivariate Gaussian distribution to infinitely many variables.

Definition: a Gaussian process is a collection of random variables, any finite number of which have (consistent) Gaussian distributions.

A Gaussian distribution is fully specified by a mean vector, $\boldsymbol{\mu}$, and covariance matrix Σ:

$$\mathbf{f} = (\mathsf{f}_1, \ldots, \mathsf{f}_n) \sim \mathcal{N}(\mu, \Sigma), \quad \text{indices} \quad i = 1, \ldots, n$$

A Gaussian process is fully specified by a mean function $m(\mathbf{x})$ and covariance function $\mathsf{K}(\mathbf{x}, \mathbf{x}')$:

$$f(\mathbf{x}) \sim \mathcal{GP}\left(m(\mathbf{x}), \mathsf{K}(\mathbf{x}, \mathbf{x}')\right), \quad \text{indices} \quad \mathbf{x}$$

Mathematical Foundations: Marginalisation

Q1. A GP is "like" a Gaussian distribution with an infinitely long mean vector and an "infinite by infinite" covariance matrix, so how do we represent it on a computer?

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We are saved by the marginalisation property:

$$p(\mathbf{y}_1) = \int p(\mathbf{y}_1, \mathbf{y}_2) \mathrm{d}\mathbf{y}_2$$

$$p(\mathbf{y}_1, \mathbf{y}_2) = \mathcal{N}\left(\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}, \begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{B}^\mathsf{T} & \mathrm{C} \end{bmatrix} \right) \quad \Longrightarrow \quad p(\mathbf{y}_1) = \mathcal{N}(\mathbf{a}, \mathrm{A})$$

$\Longrightarrow$ Only need to represent finite dimensional projections of GPs on computer.

nonparametric model: complexity increases with size of the data

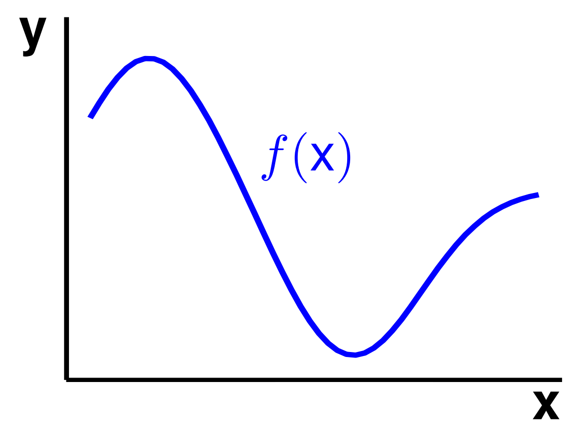
blackboard: revisiting ARIMA

Mathematical Foundations: Regression

Q3. What's the formal justification for how we were using GPs for regression?

Generative model (like non-linear regression)

$$\mathrm{y}(\mathrm{x}) = f(\mathrm{x}) + \epsilon\sigma_{\mathrm{y}}$$



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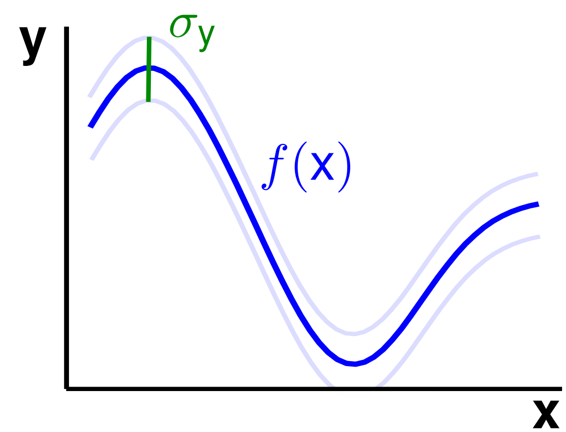

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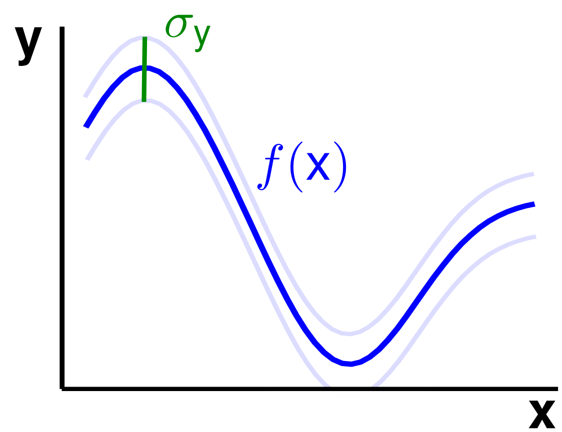

place GP prior over the non-linear function

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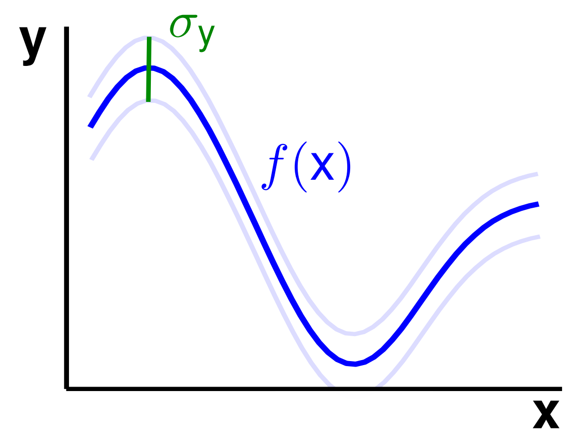

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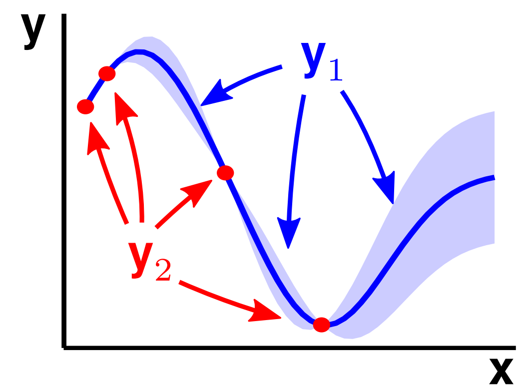
since the sum of two Gaussians is a Gaussian, the model induces a GP over $\mathsf{y}(\mathsf{x})$

$$p(\mathsf{y}(\mathsf{x})|\theta) = \mathcal{GP}(0, \mathsf{K}(\mathsf{x}, \mathsf{x}') + \mathrm{I}\sigma_{\mathsf{y}}^2)$$

Mathematical foundations: Prediction

Q4. How do we make predictions?



$$p(\mathbf{y}_1|\mathbf{y}_2) = \frac{p(\mathbf{y}_1, \mathbf{y}_2)}{p(\mathbf{y}_2)}$$

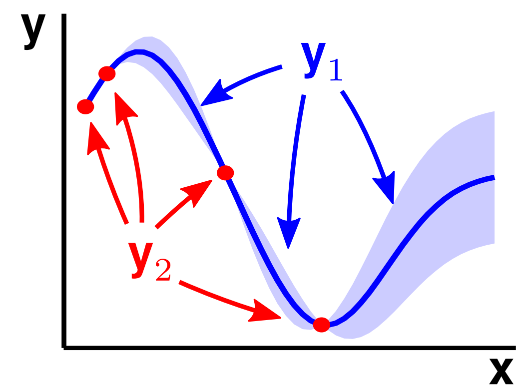
Mathematical foundations: Prediction

Q4. How do we make predictions?

$$p(\mathbf{y}_1, \mathbf{y}_2) = \mathcal{N}\left(\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}, \begin{bmatrix} \mathsf{A} & \mathsf{B} \\ \mathsf{B}^{\mathsf{T}} & \mathsf{C} \end{bmatrix} \right)$$

$$p(\mathbf{y}_1 | \mathbf{y}_2) = \frac{p(\mathbf{y}_1, \mathbf{y}_2)}{p(\mathbf{y}_2)}$$

$$p(\mathbf{y}_2) = \mathcal{N}\left(\mathbf{b}, \mathsf{C}\right)$$



$$\Longrightarrow \; p(\mathbf{y}_1 | \mathbf{y}_2) = \mathcal{N}(\mathbf{a} + \mathsf{B}\mathsf{C}^{-1}(\mathbf{y}_2 - \mathbf{b}), \mathsf{A} - \mathsf{B}\mathsf{C}^{-1}\mathsf{B}^{\mathsf{T}})$$

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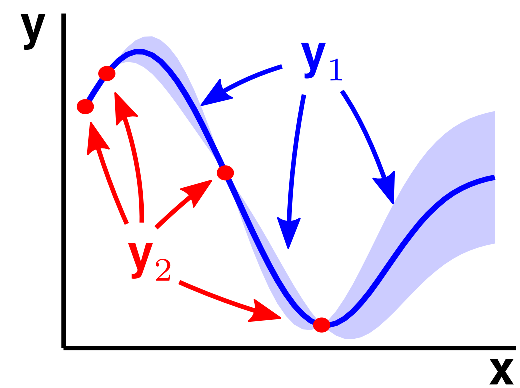

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predictive mean

$$\mu_{\mathbf{y}_1|\mathbf{y}_2} = \mathbf{a} + \mathsf{B}\mathsf{C}^{-1}(\mathbf{y}_2 - \mathbf{b})$$

$$= \mathsf{B}\mathsf{C}^{-1}\mathbf{y}_2$$

$$= \mathsf{W}\mathbf{y}_2$$

linear in the data

Mathematical foundations: Prediction

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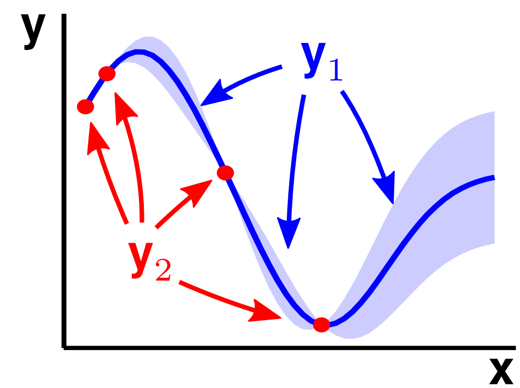

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predictive mean

$$\mu_{\mathbf{y}_1|\mathbf{y}_2} = \mathbf{a} + \mathsf{B}\mathsf{C}^{-1}(\mathbf{y}_2 - \mathbf{b})$$
$$= \mathsf{B}\mathsf{C}^{-1}\mathbf{y}_2$$
$$= \mathsf{W}\mathbf{y}_2$$

linear in the data

predictive covariance

$$\Sigma_{\mathbf{y}_1|\mathbf{y}_2} = \mathsf{A} - \mathsf{B}\mathsf{C}^{-1}\mathsf{B}^\mathsf{T}$$

predictive uncertainty = prior uncertainty − reduction in uncertainty

predictions more confident than prior

What effect do the hyper-parameters have?

Non-parametric (∞-parametric)

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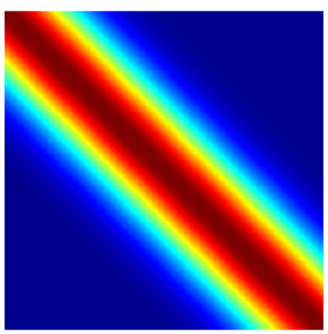
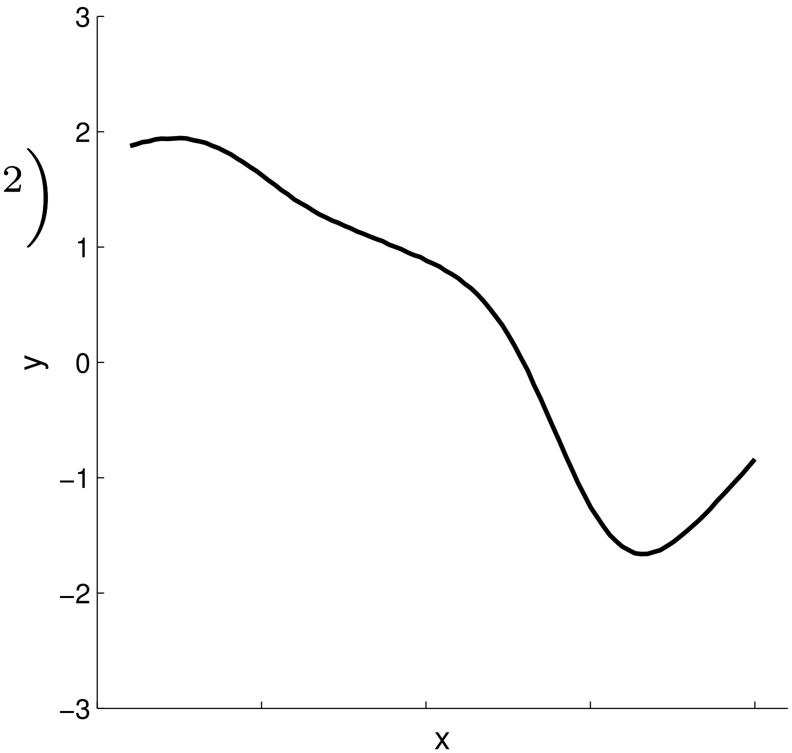
$\Sigma =$ 



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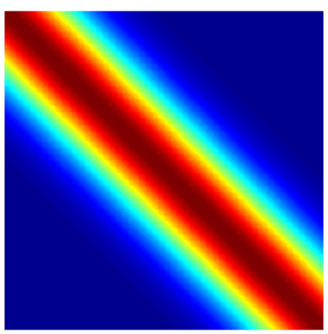
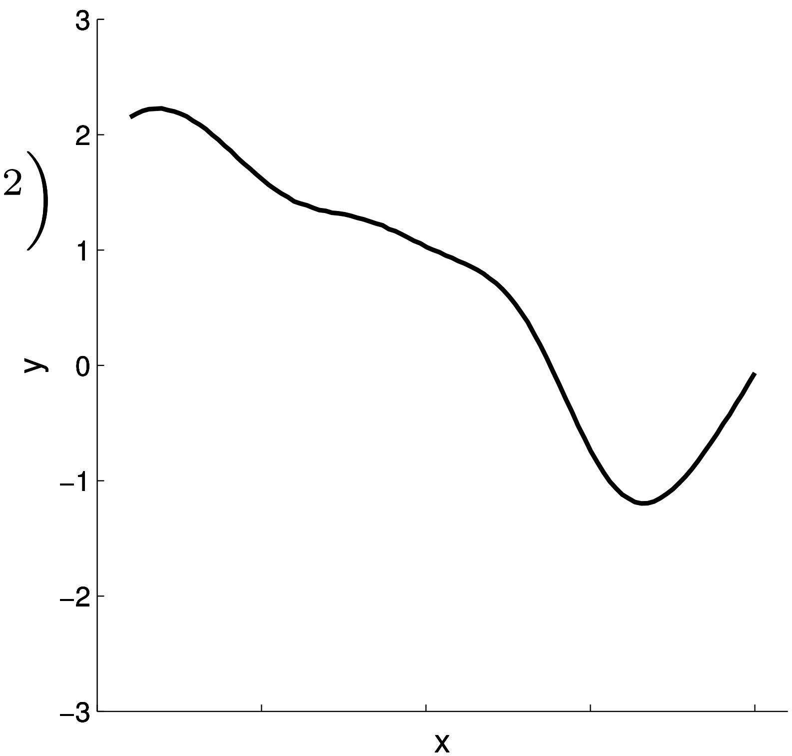
$\Sigma =$ 



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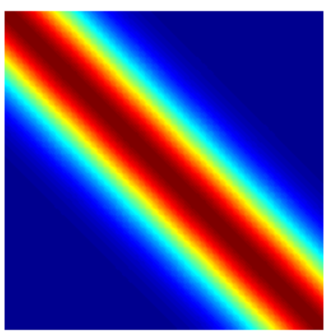
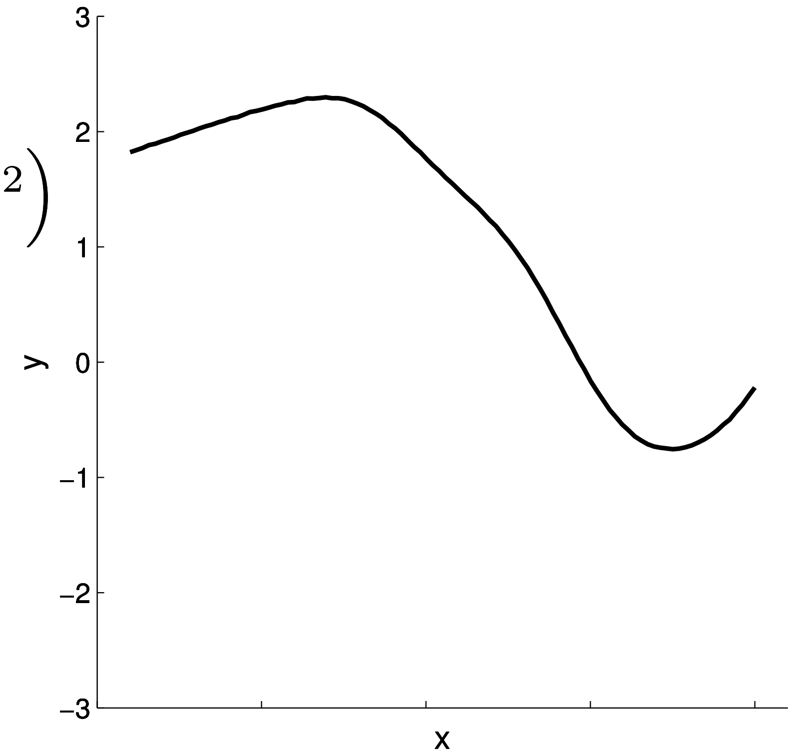
$\Sigma =$ 



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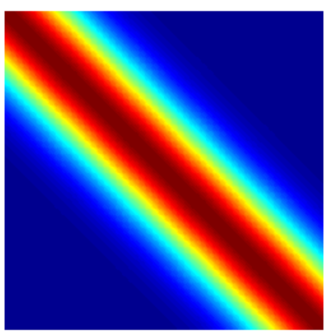

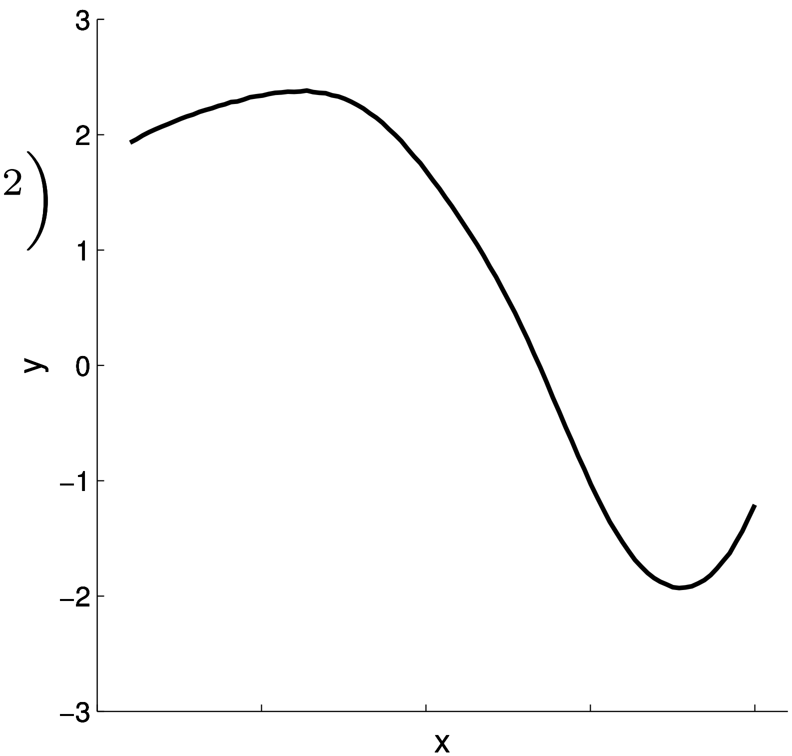

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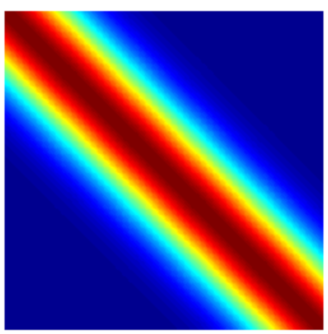
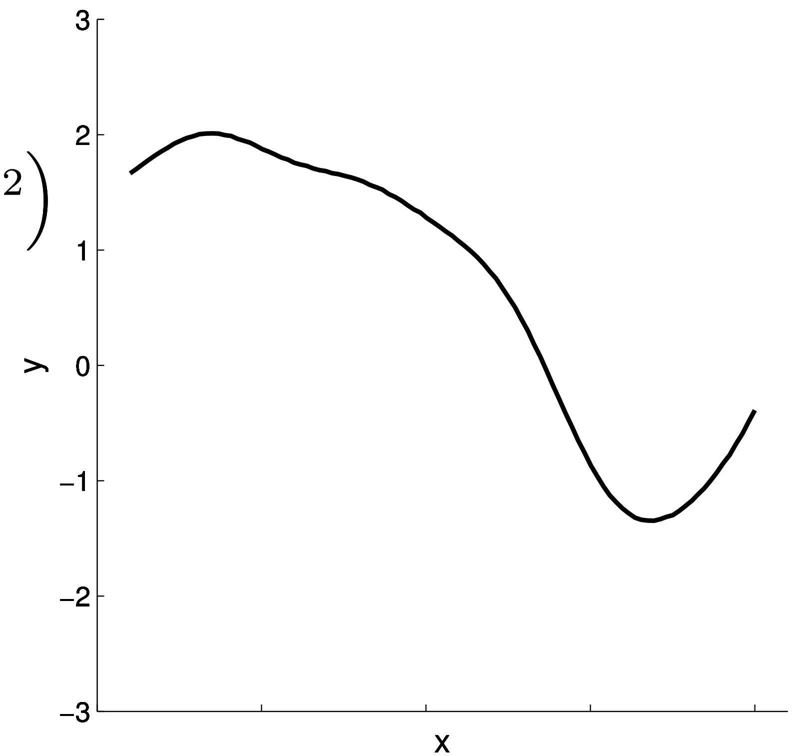
$\Sigma =$ 



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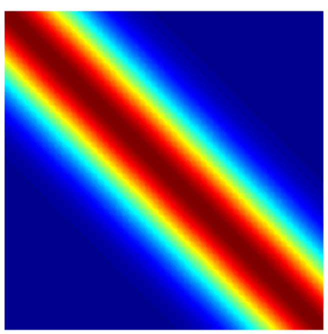

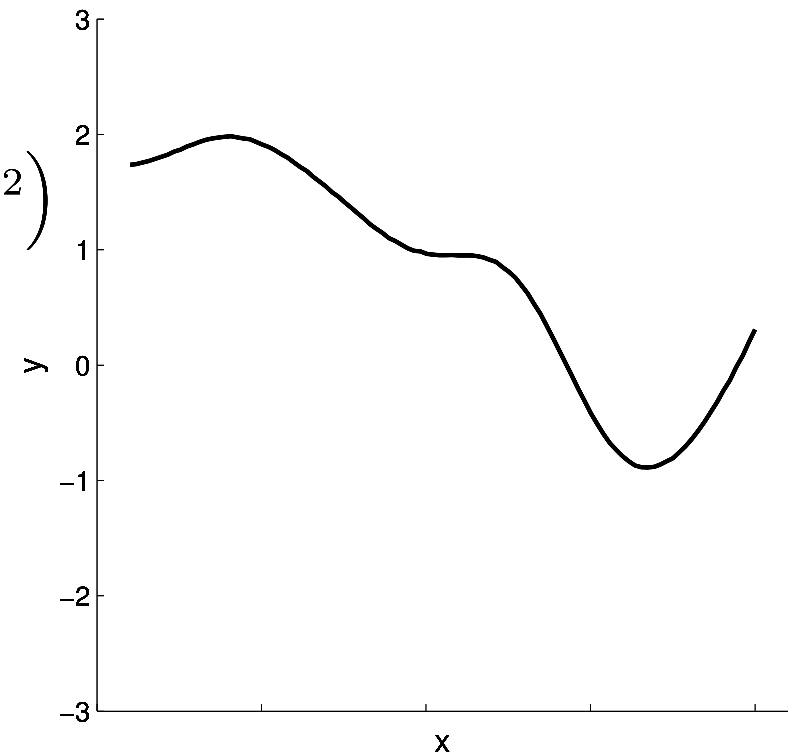

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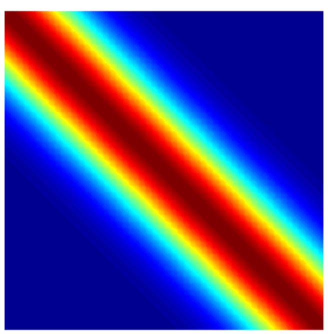
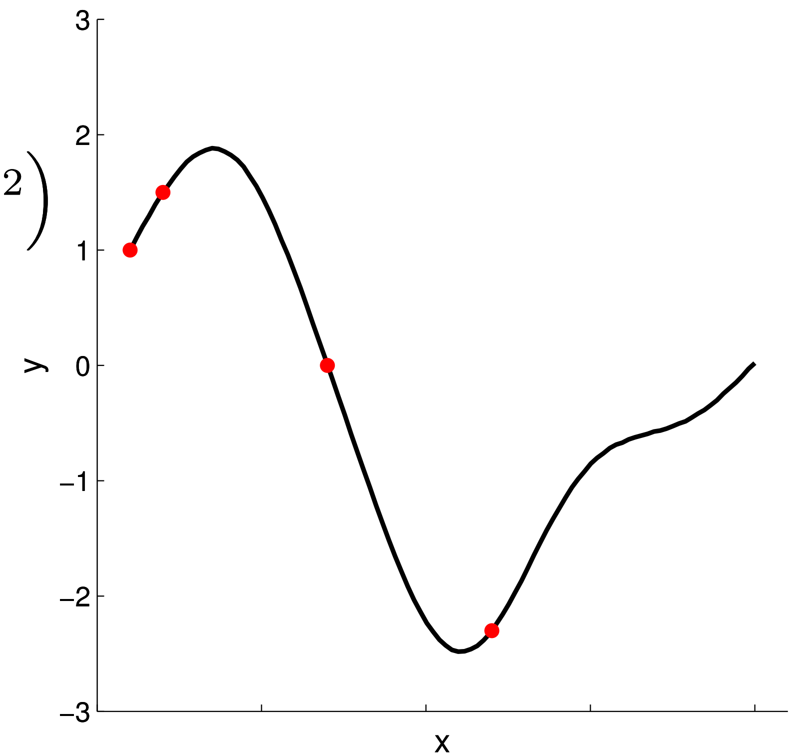
$\Sigma =$ 



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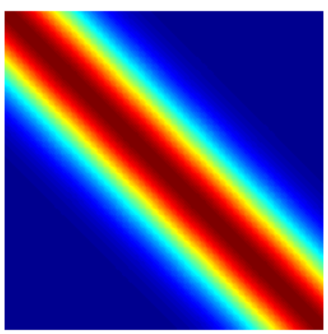
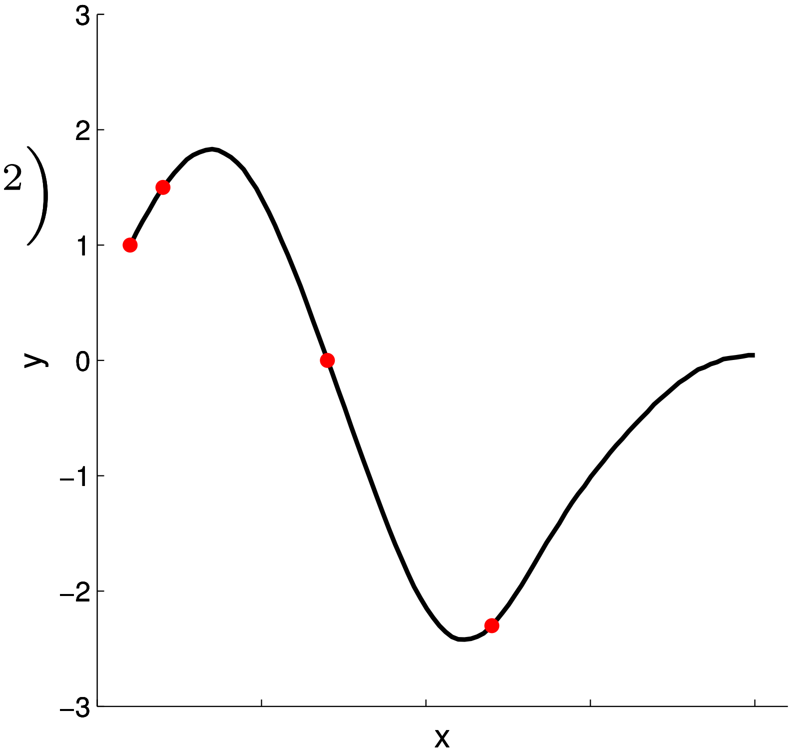
$\Sigma =$ 



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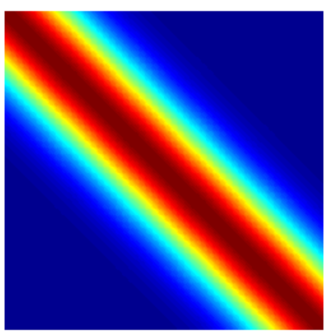
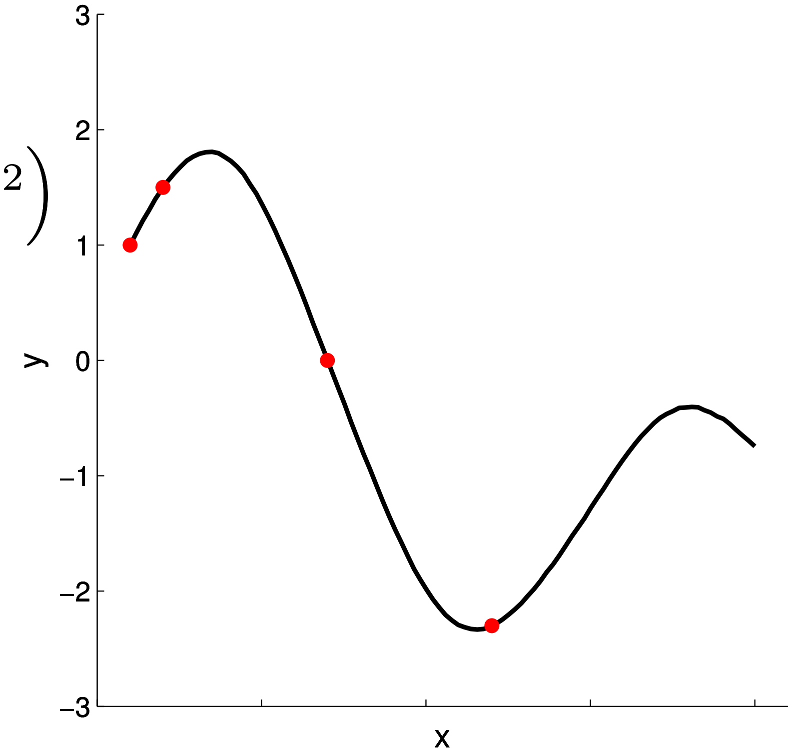
$\Sigma =$ 



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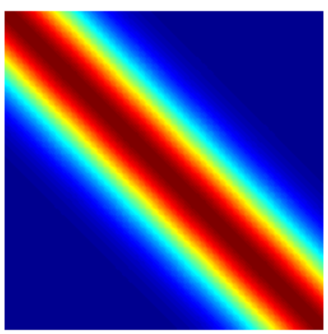
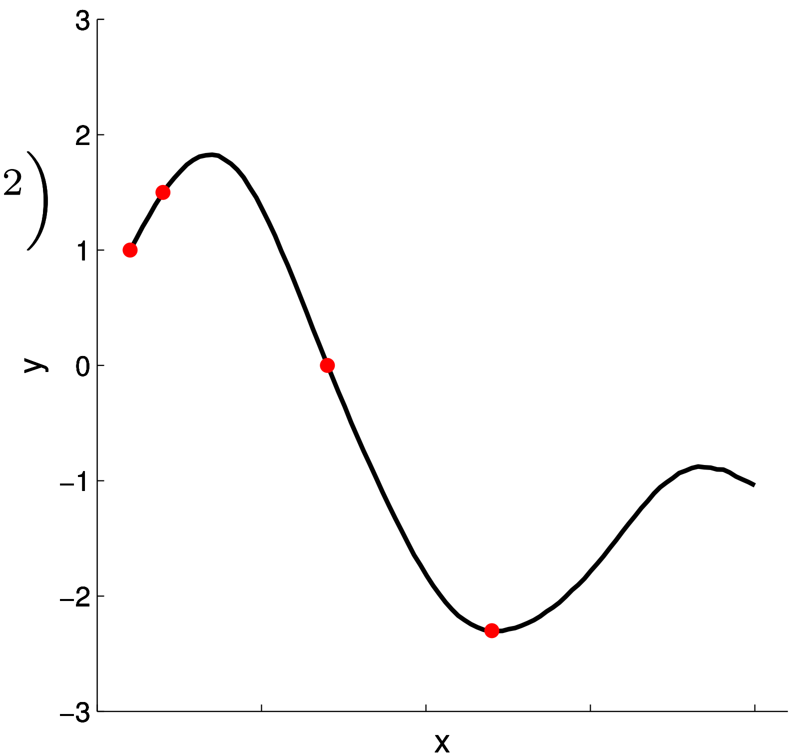
$\Sigma =$ 



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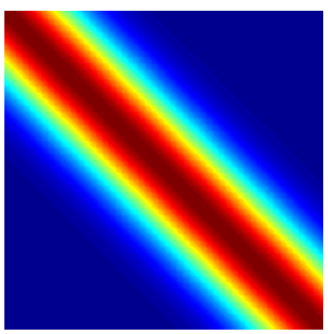
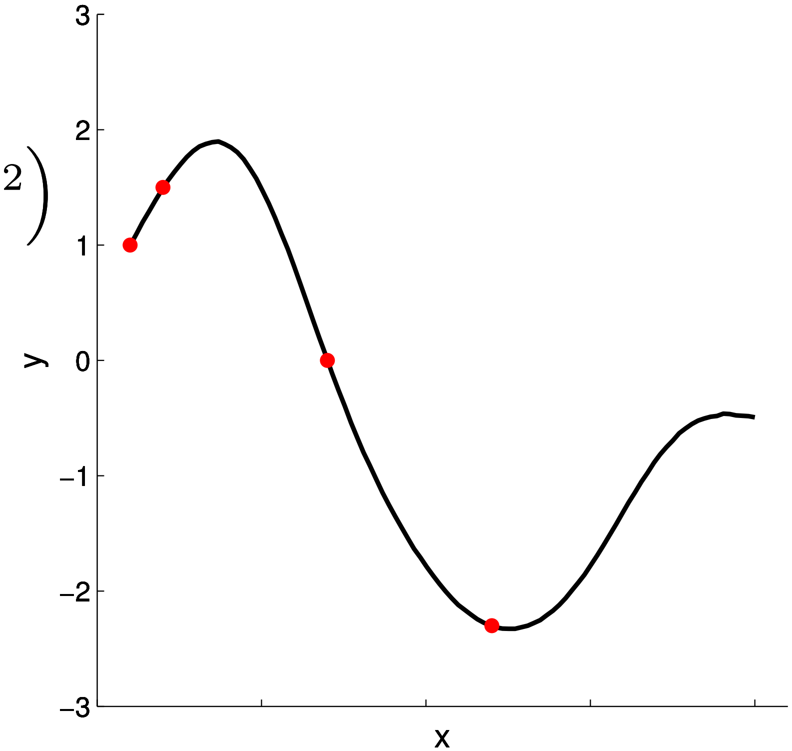
$\Sigma =$ 



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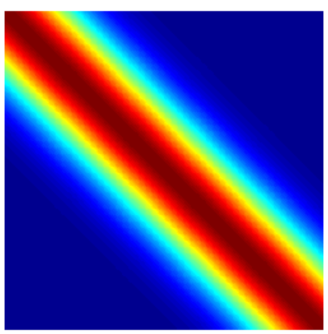
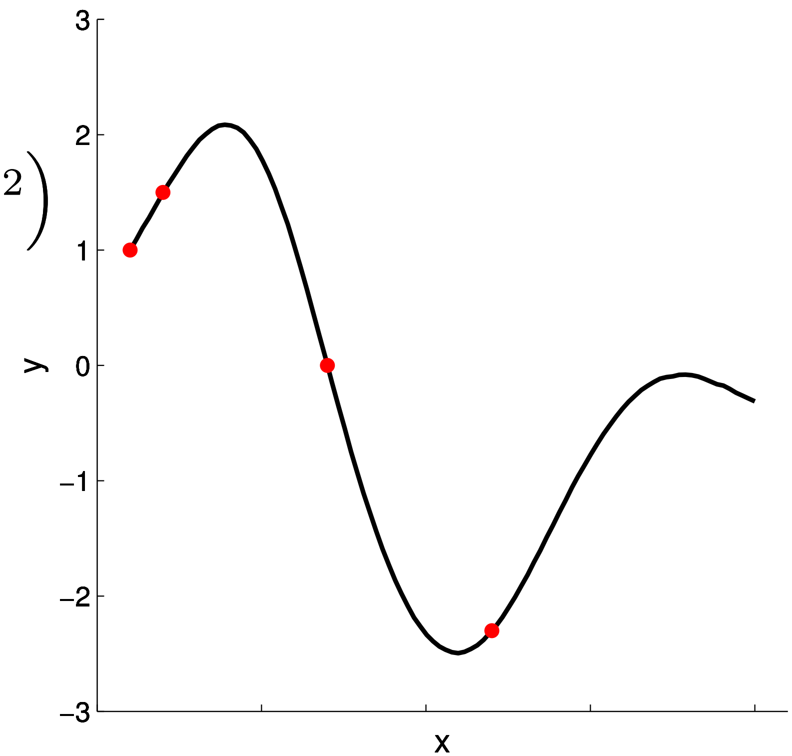
$\Sigma =$ 



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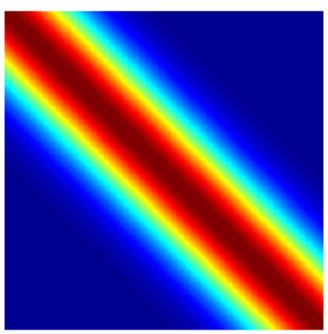
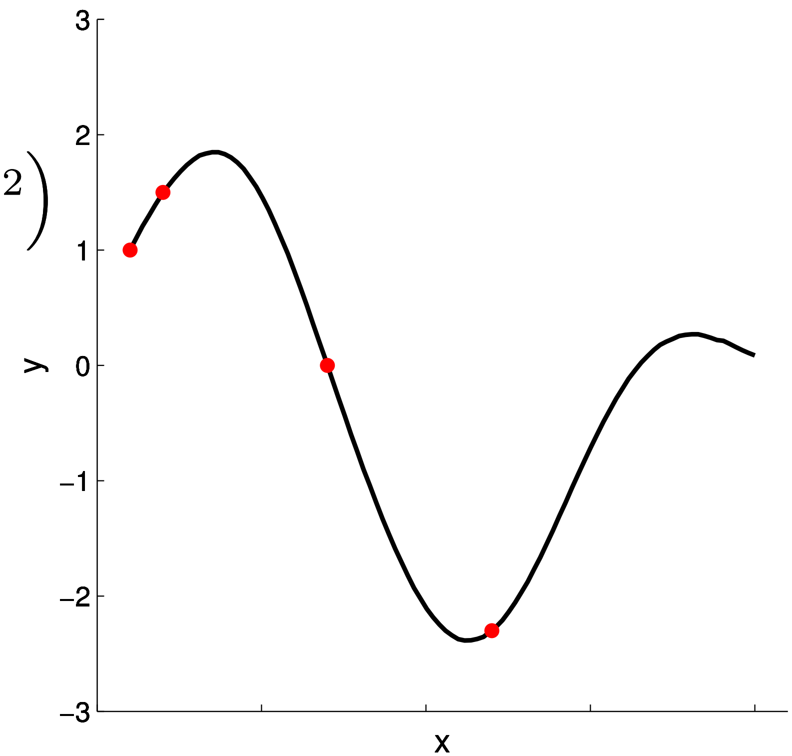
$\Sigma =$ 



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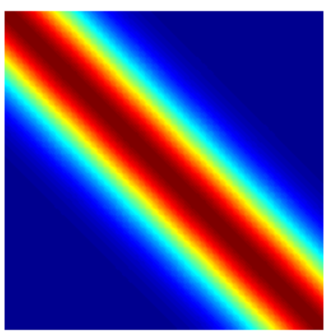
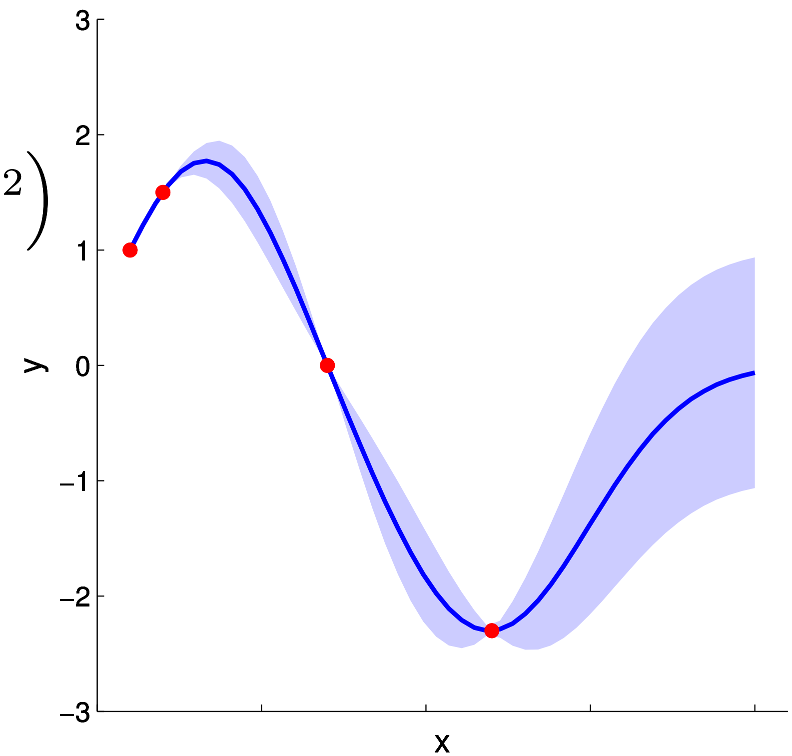
$\Sigma =$ 



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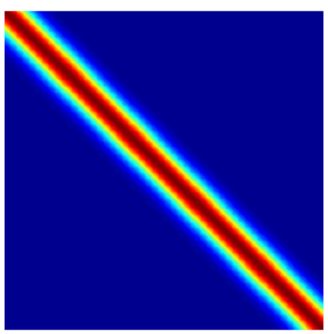
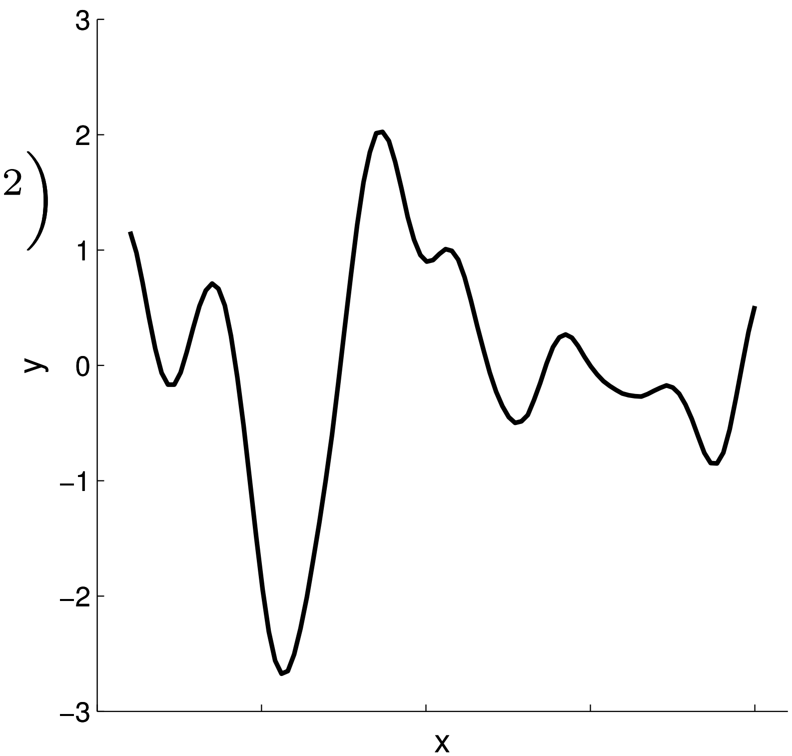
$\Sigma =$ 



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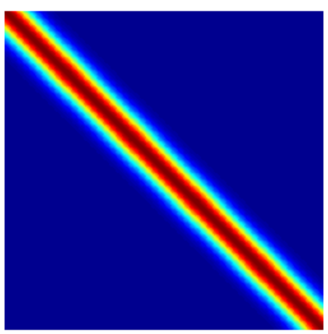
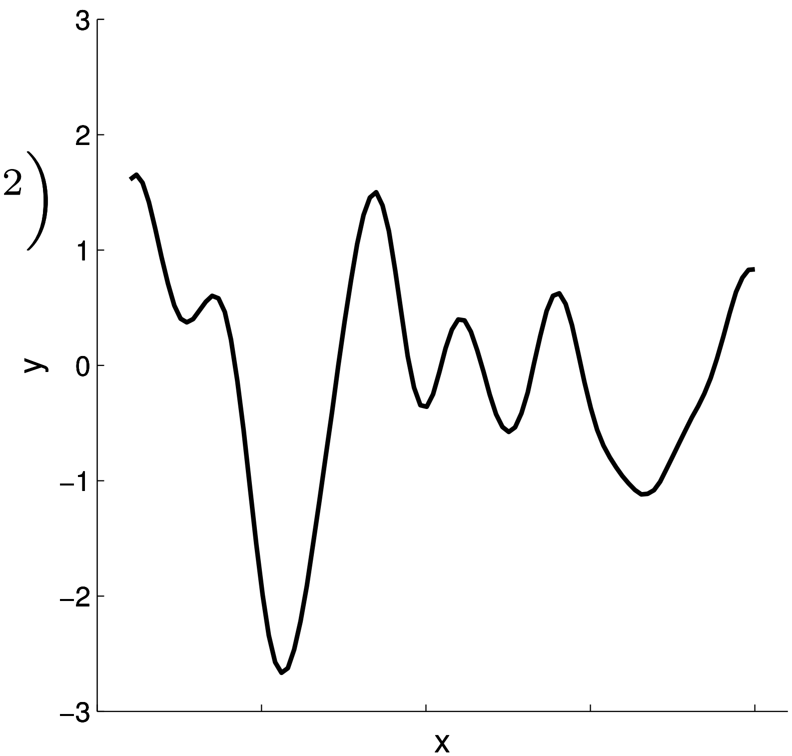
$\Sigma =$ 



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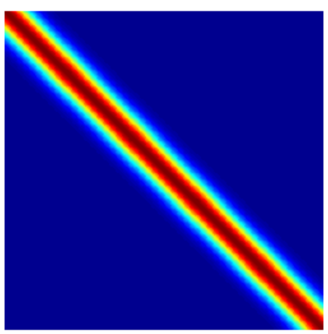
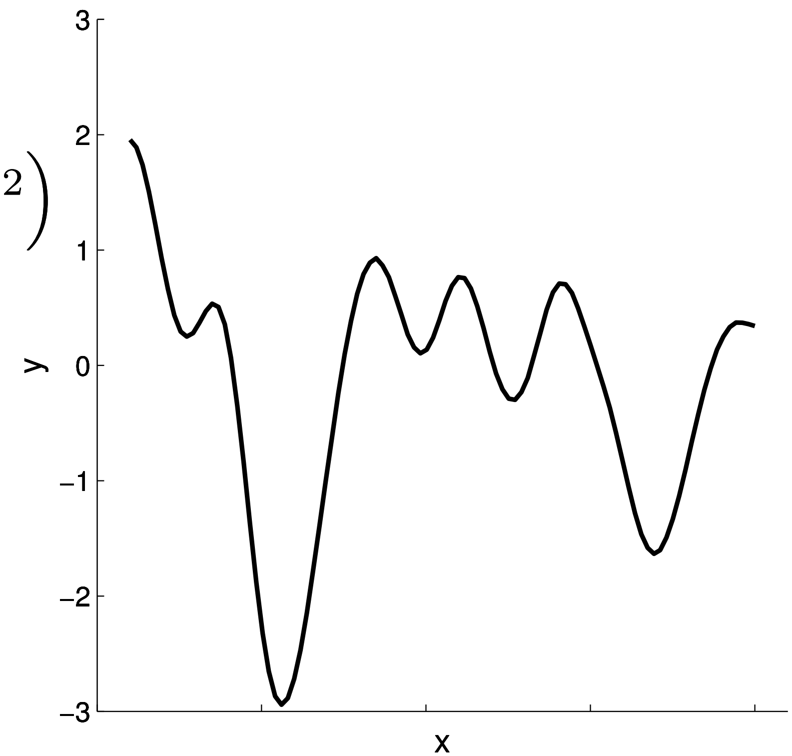
$\Sigma =$ 



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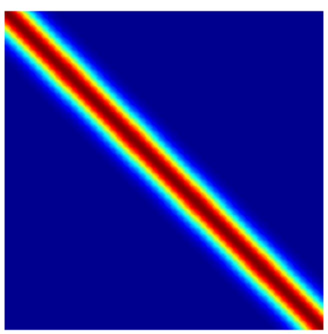
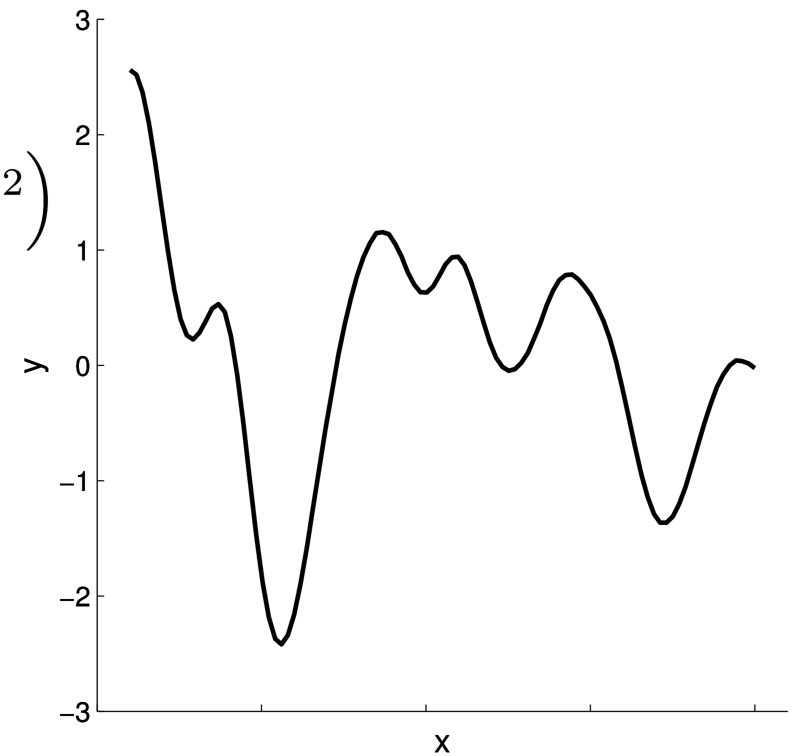
$\Sigma =$ 



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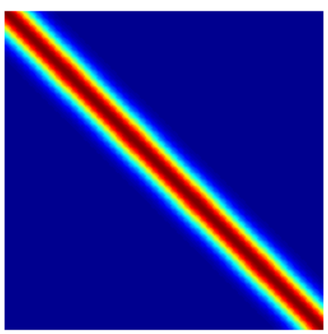

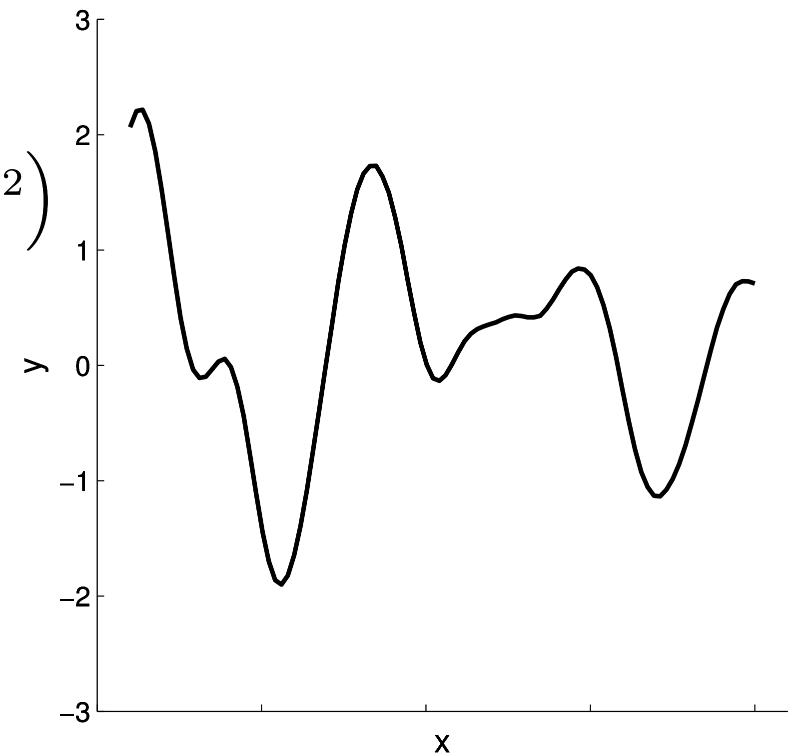

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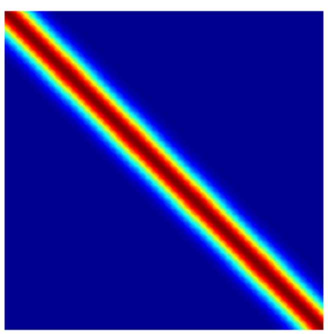
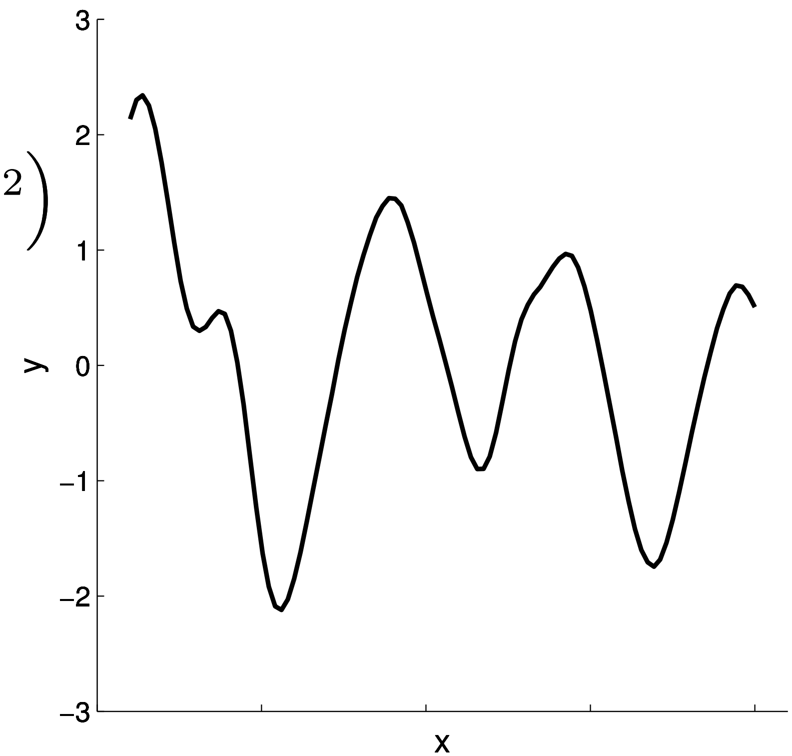
$\Sigma =$ 



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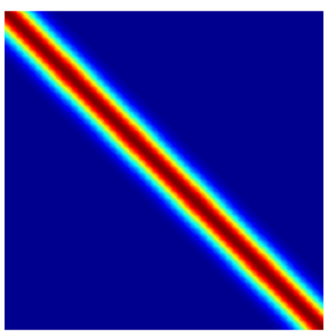
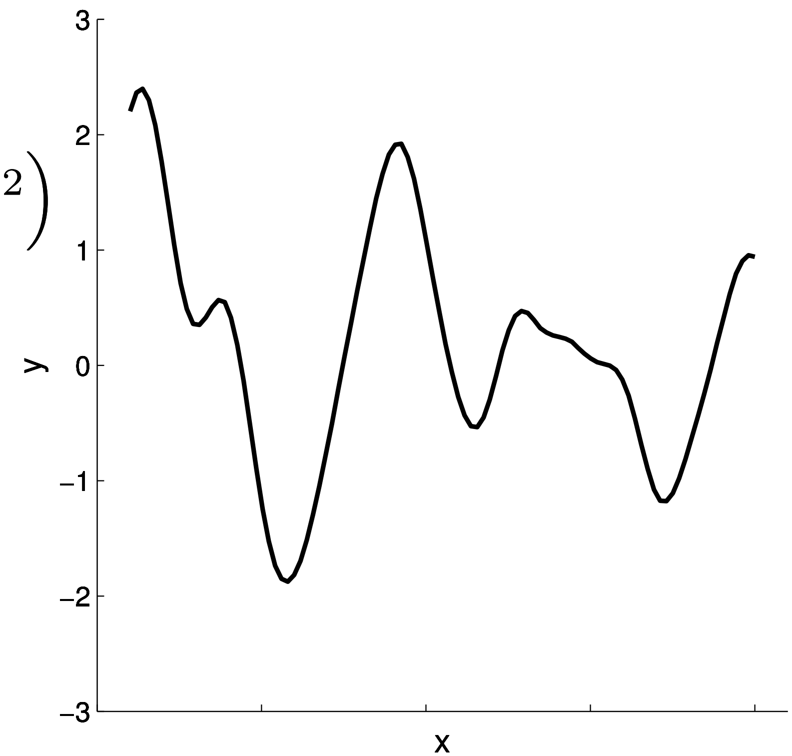
$\Sigma =$ 



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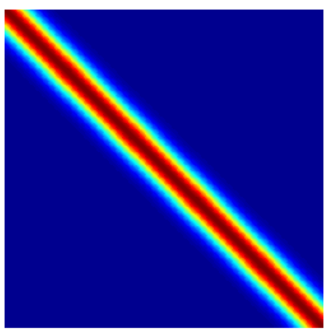
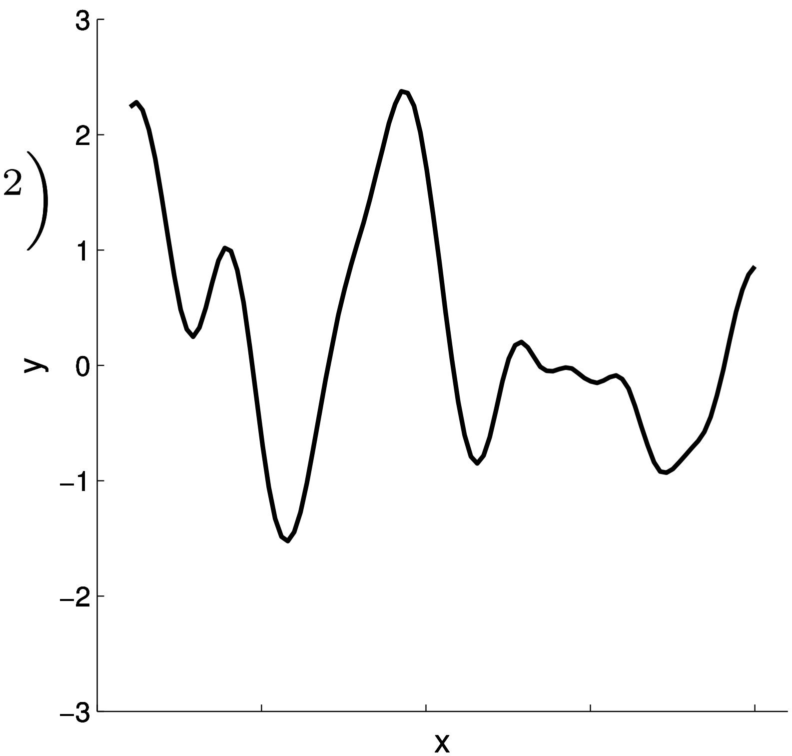
$\Sigma =$ 



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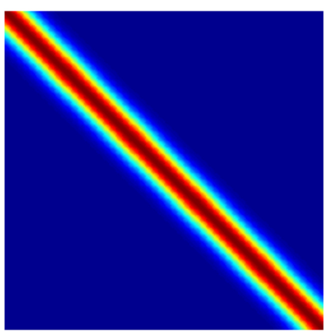
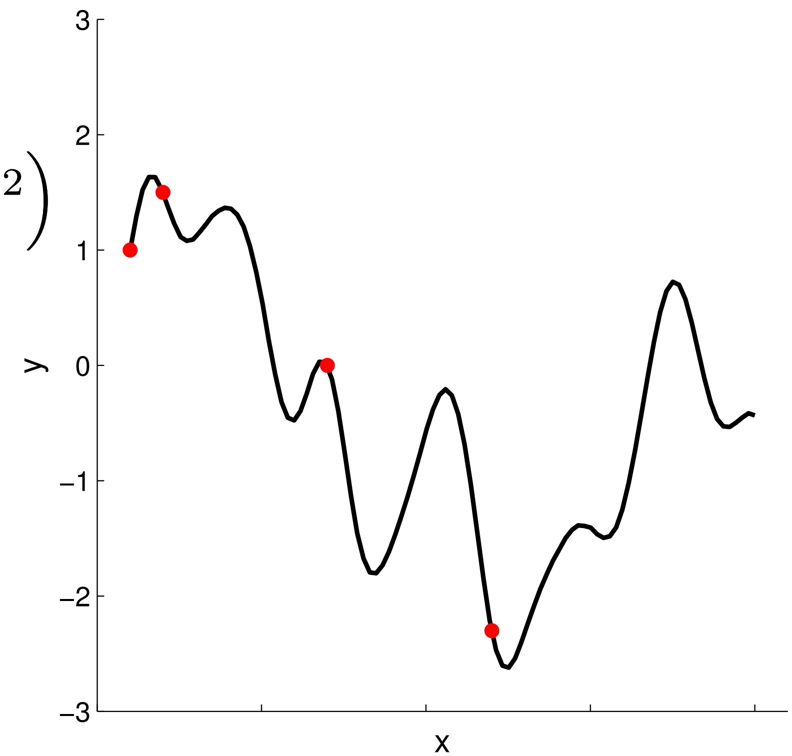
$\Sigma =$ 



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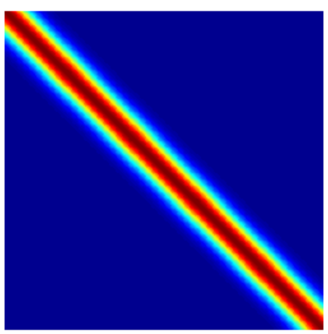
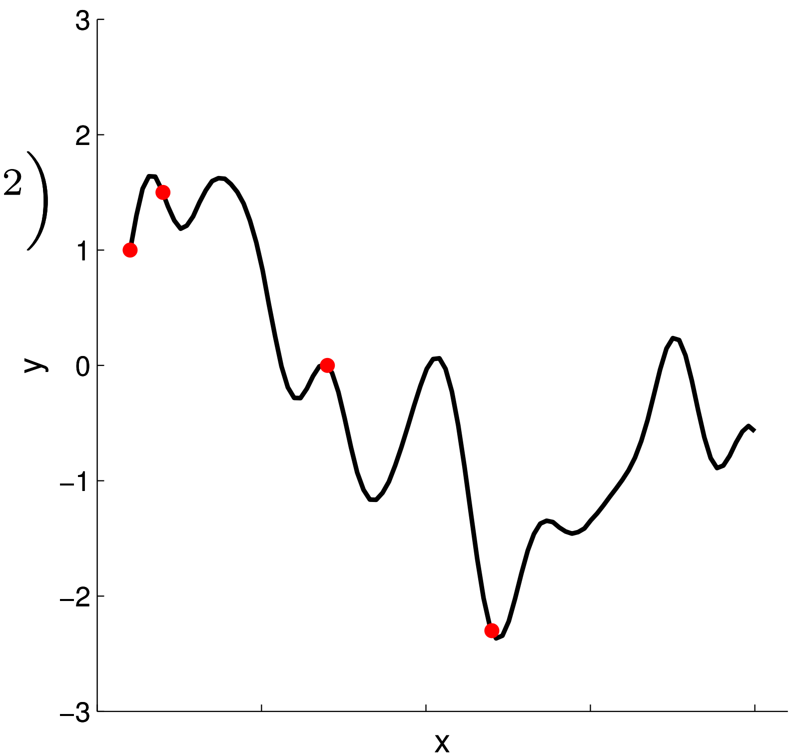
$\Sigma =$ 



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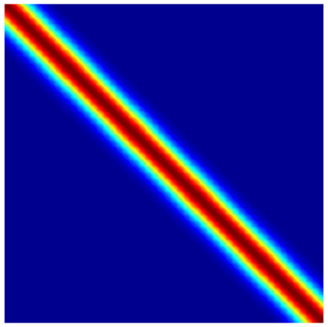
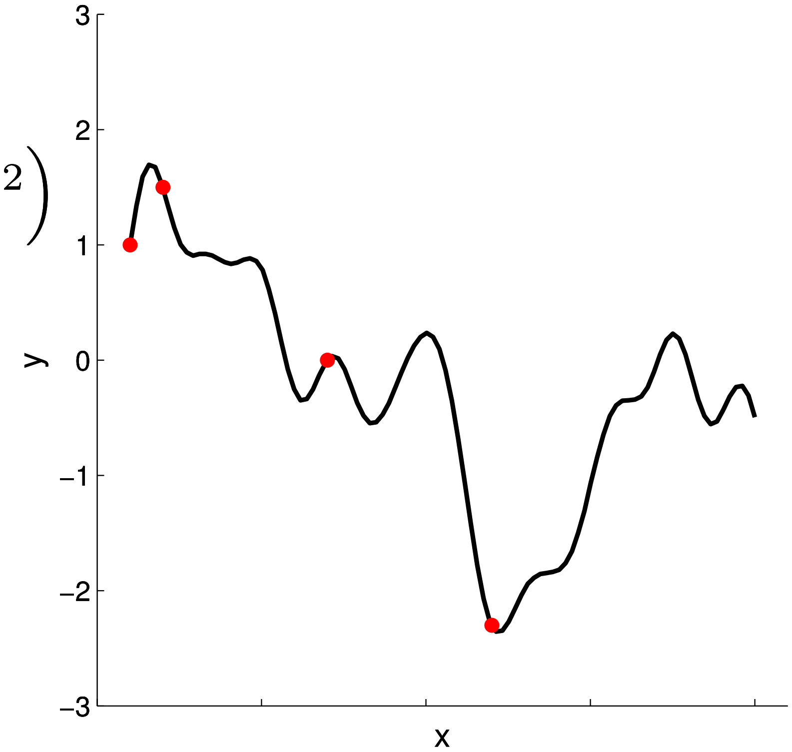
$\Sigma =$ 



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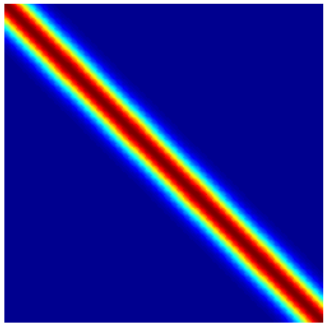

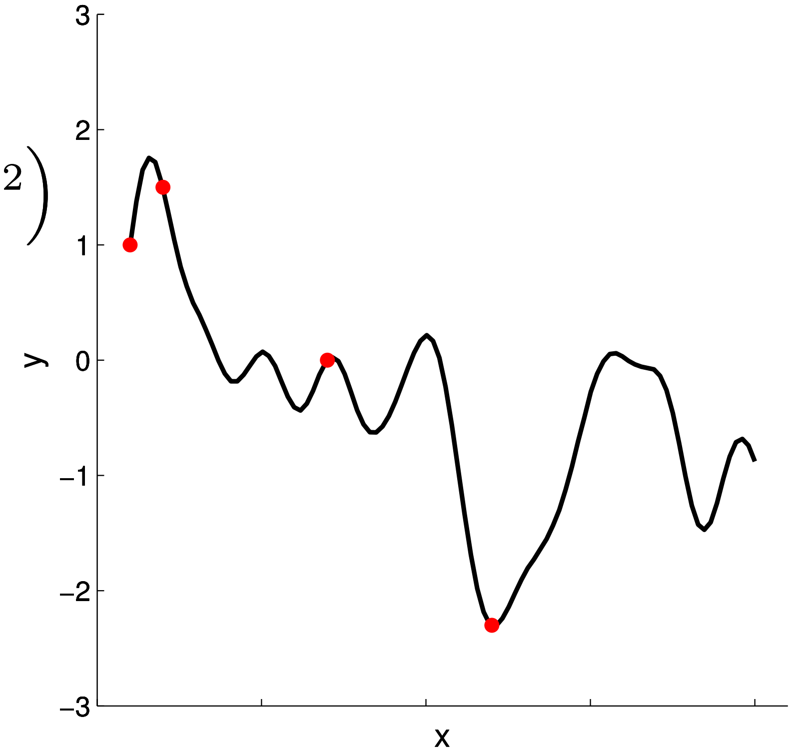

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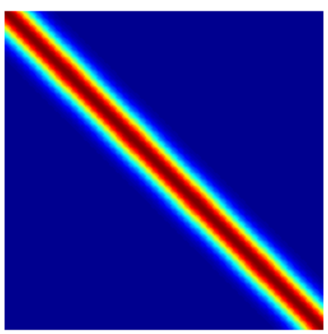
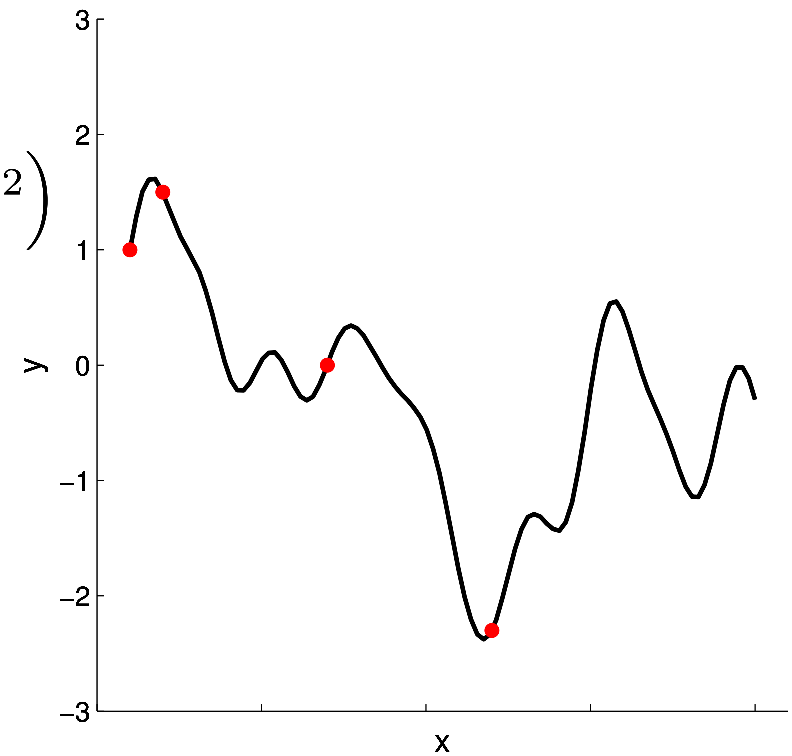
$\Sigma =$ 



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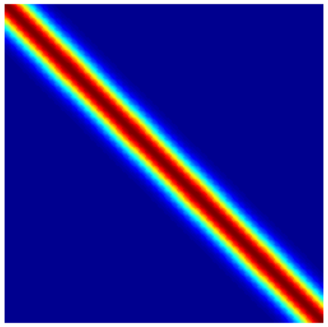

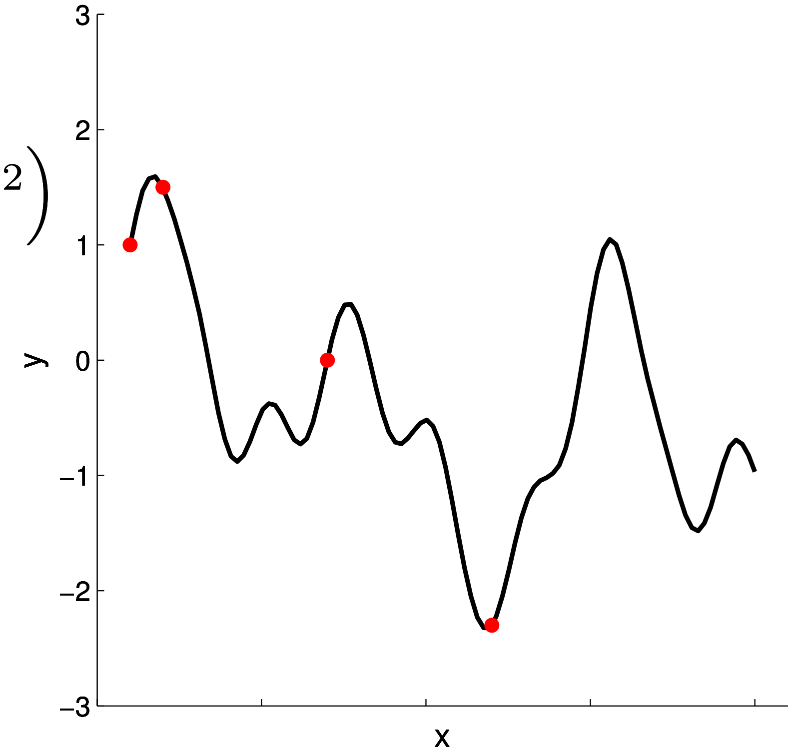

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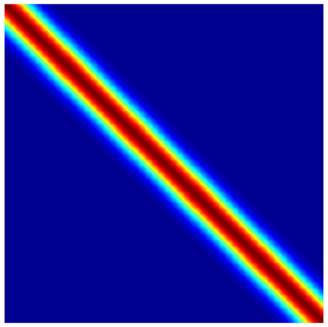

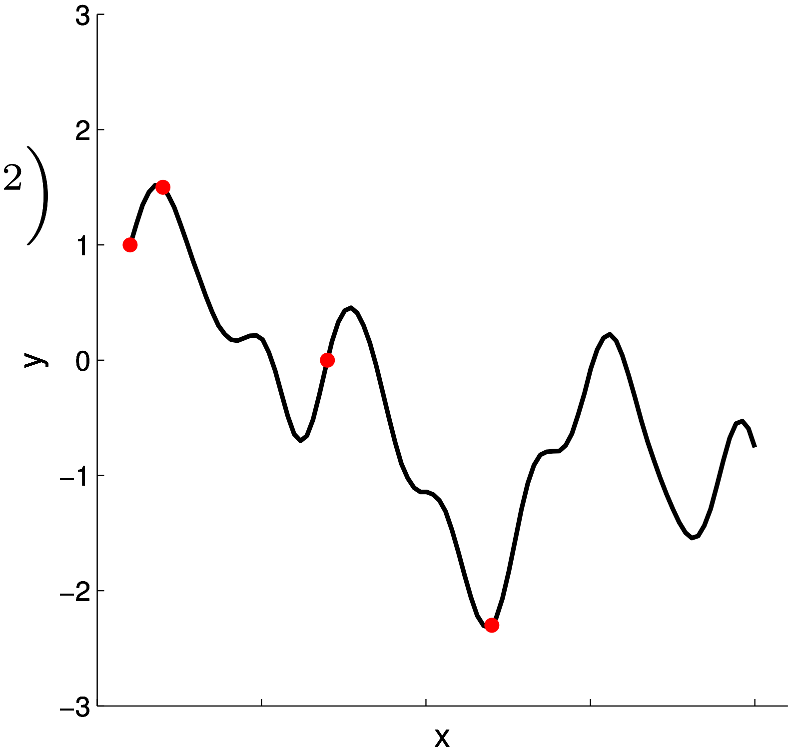

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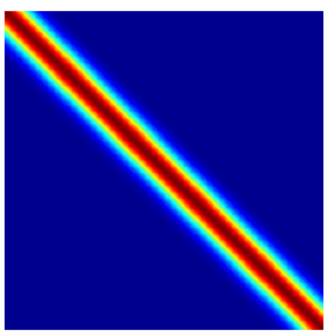

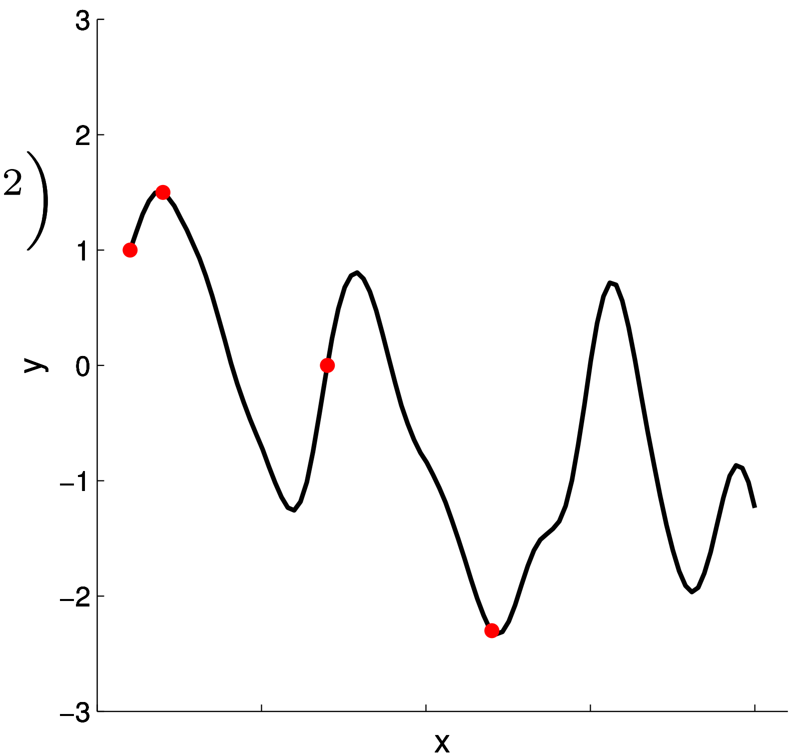

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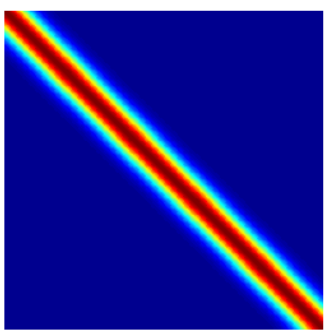
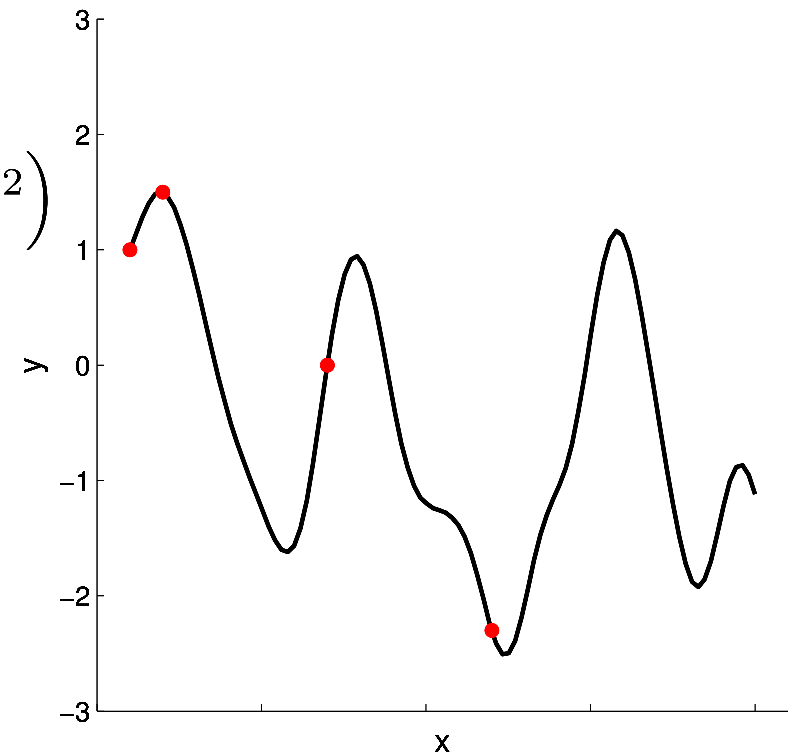
$\Sigma =$ 



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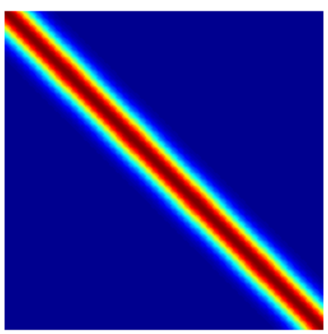
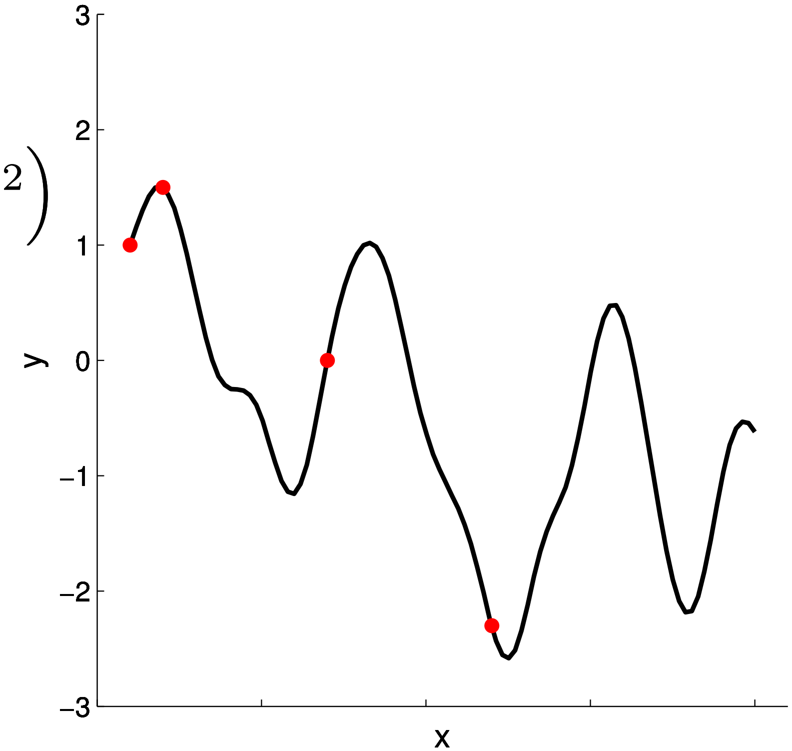
$\Sigma =$ 



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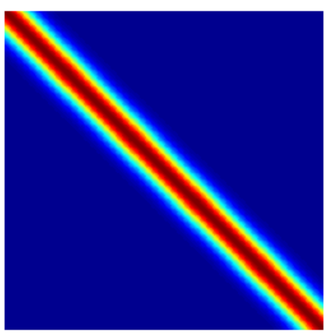
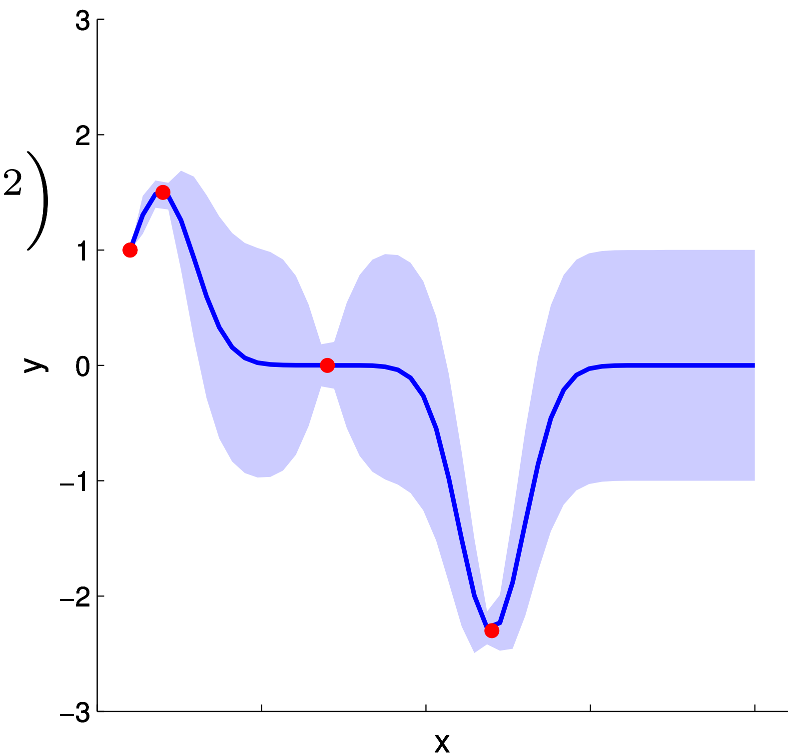
$\Sigma =$ 



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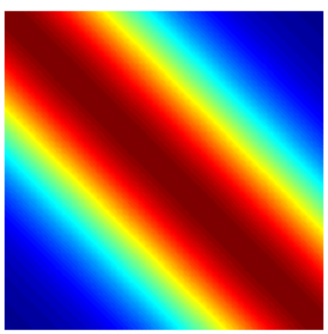
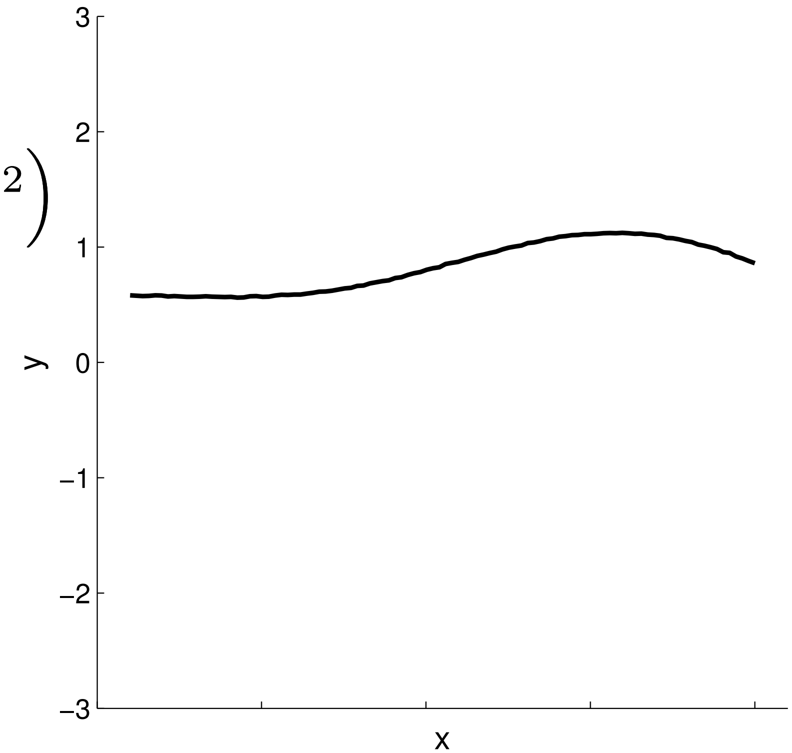
$\Sigma =$ 



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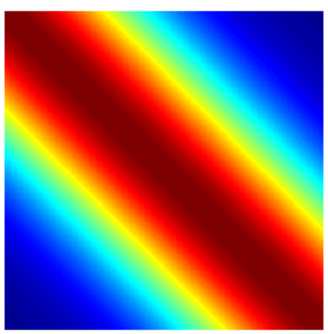
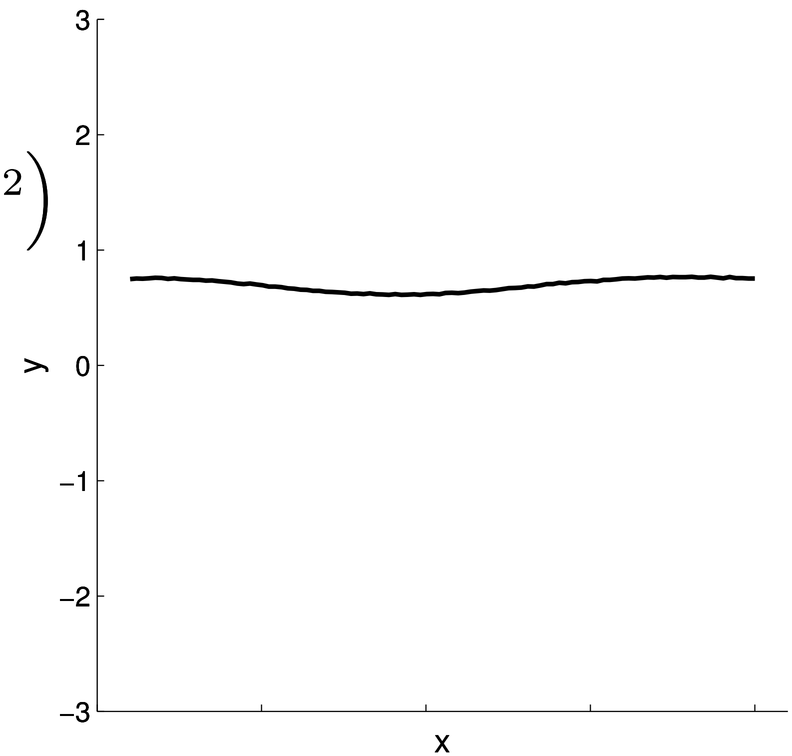
$\Sigma =$ 



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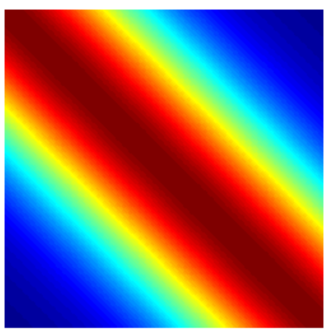
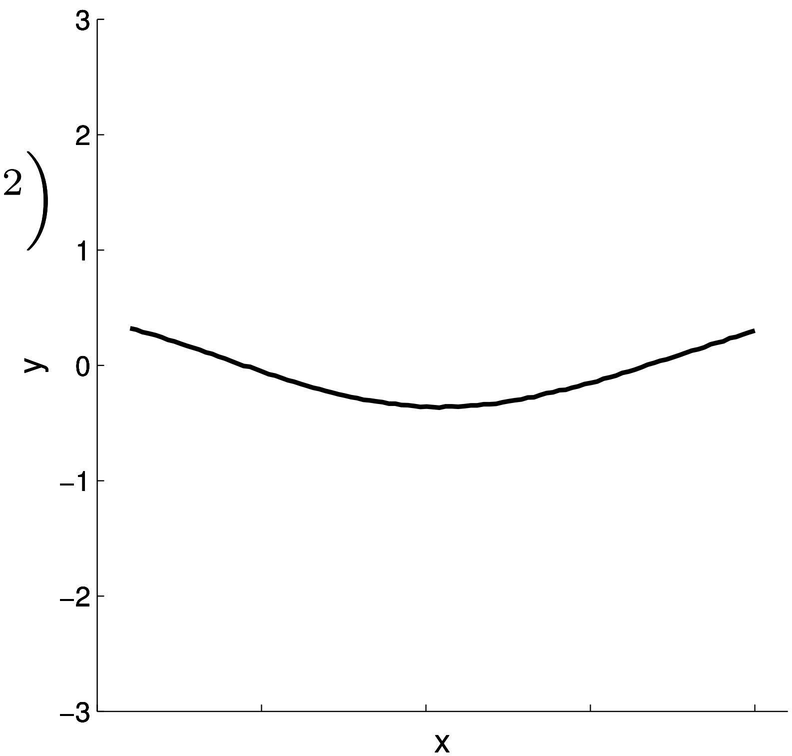
$\Sigma =$ 



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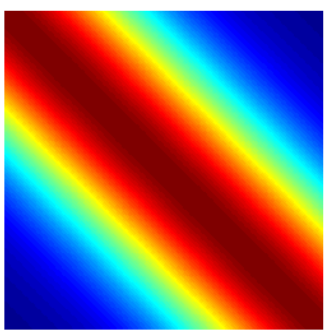
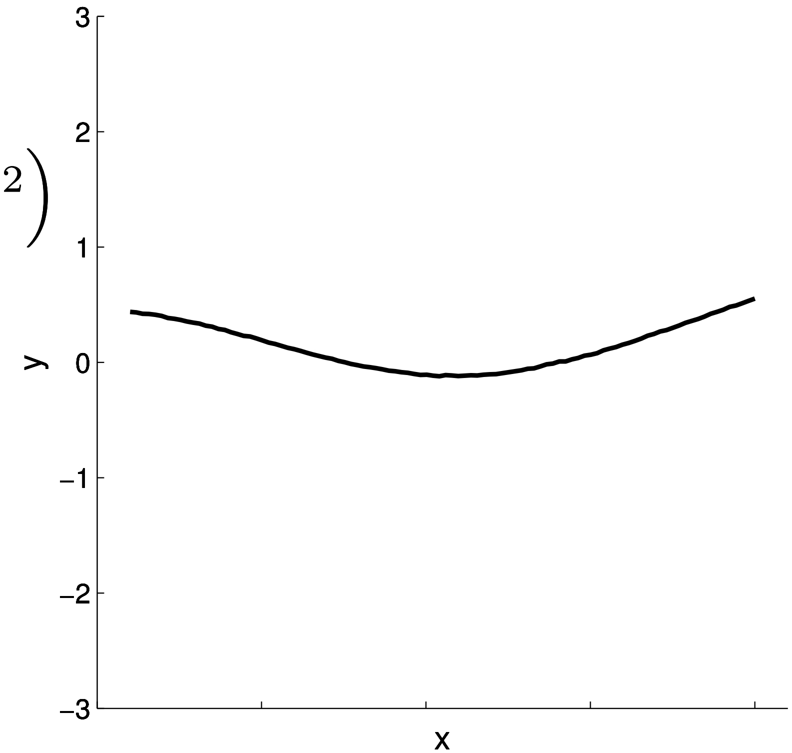
$\Sigma =$ 



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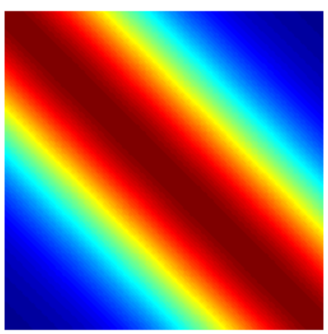
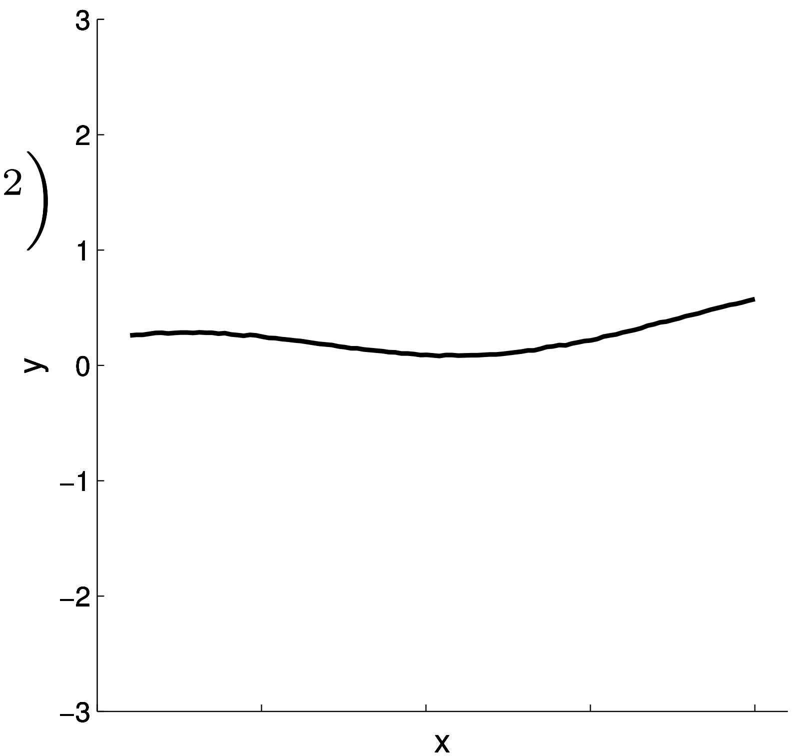
$\Sigma =$ 



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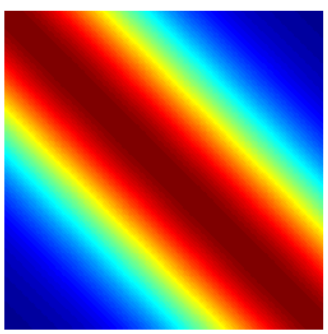
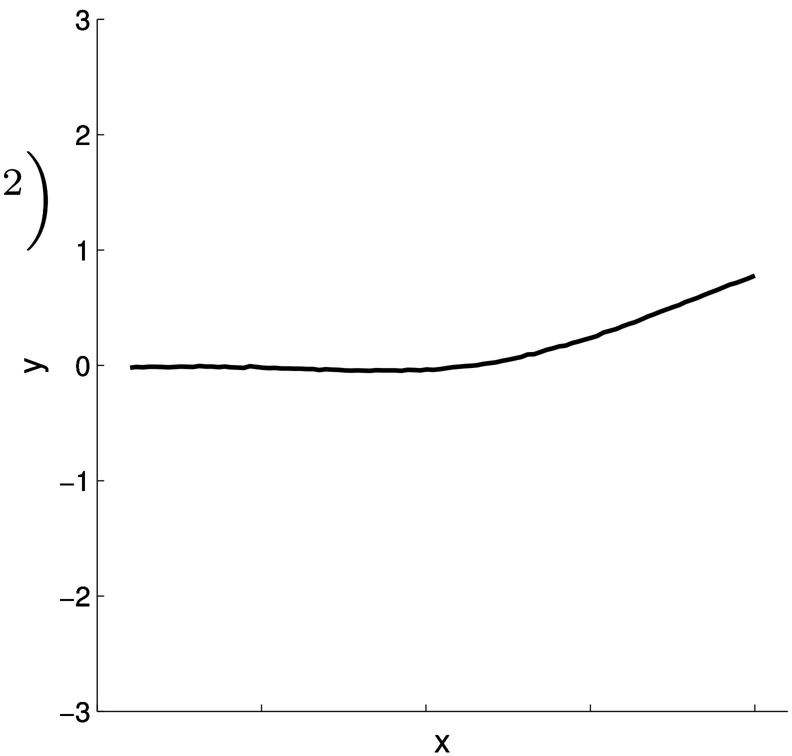
$\Sigma =$ 



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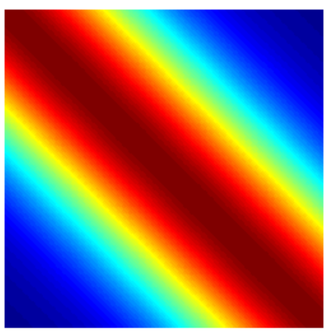
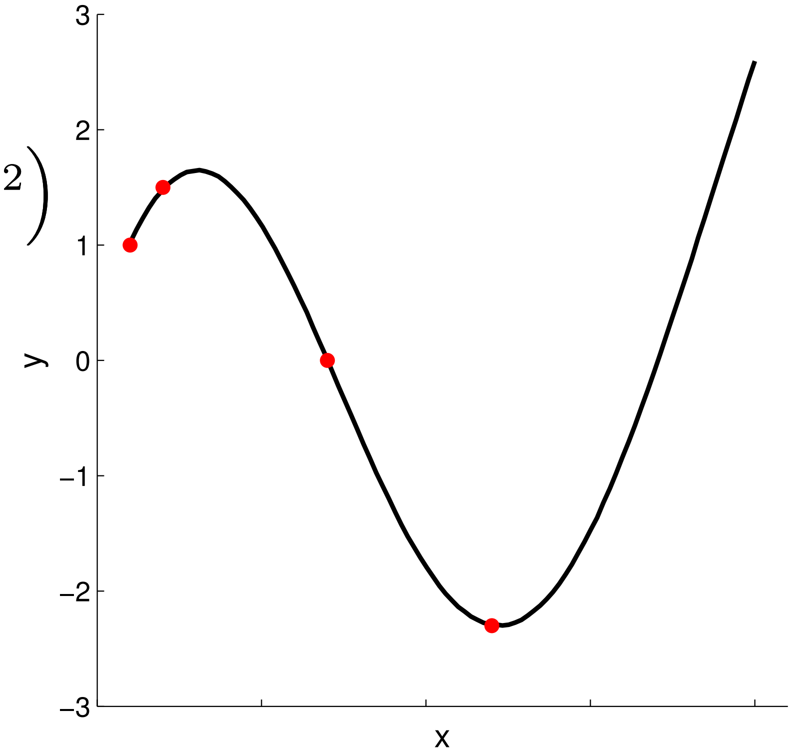
$\Sigma =$ 



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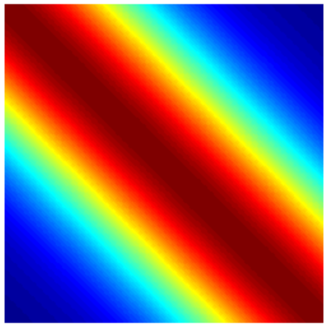
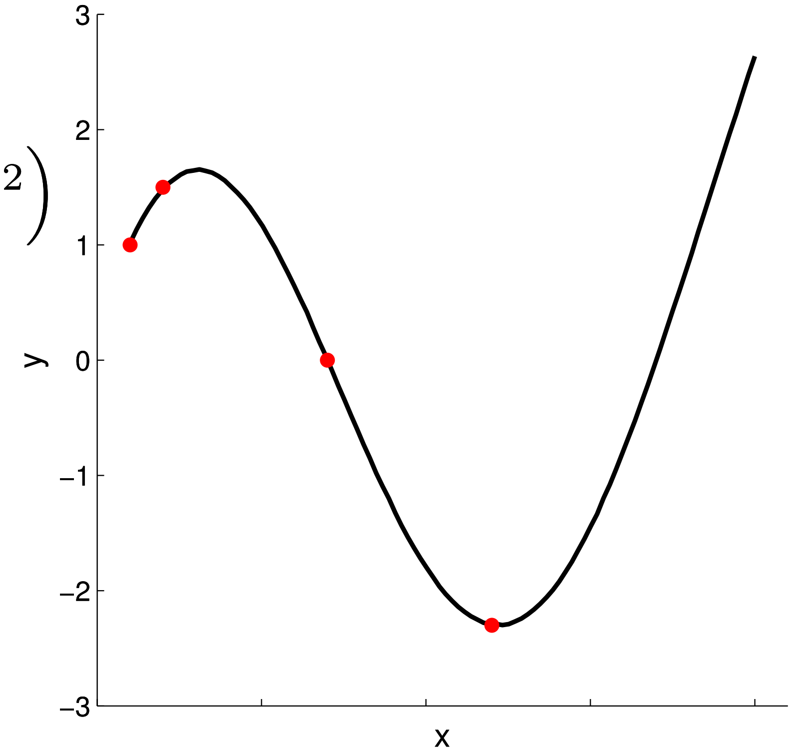
$\Sigma =$ 



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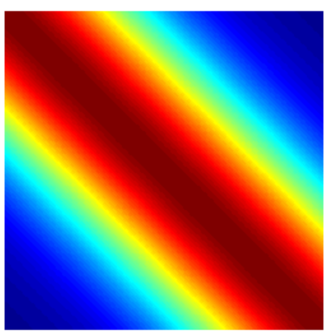

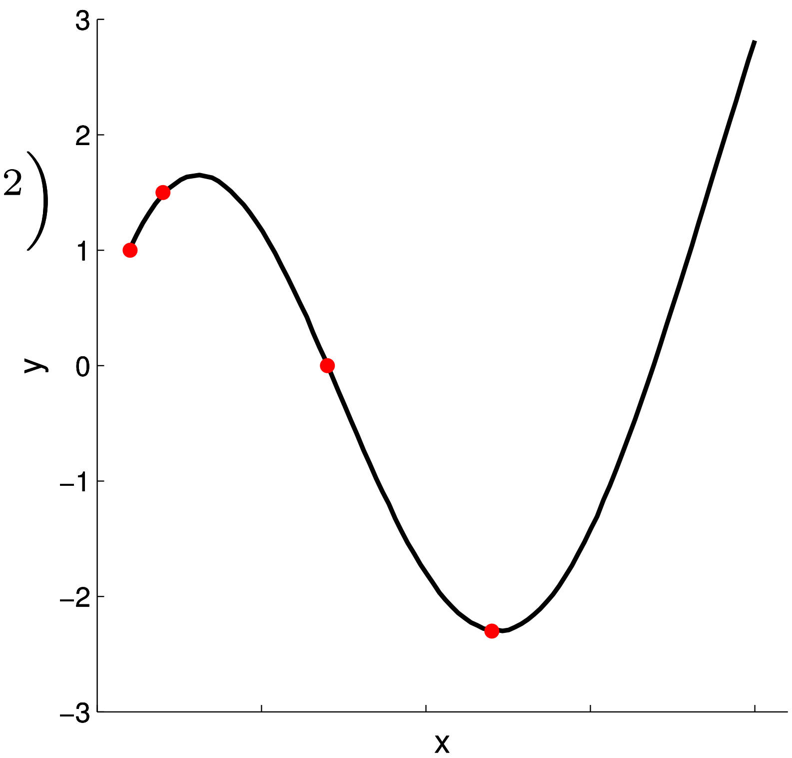

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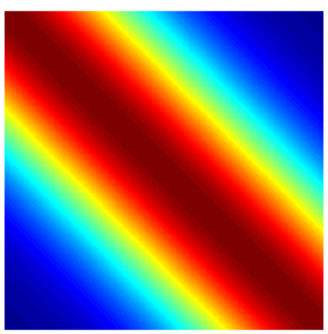
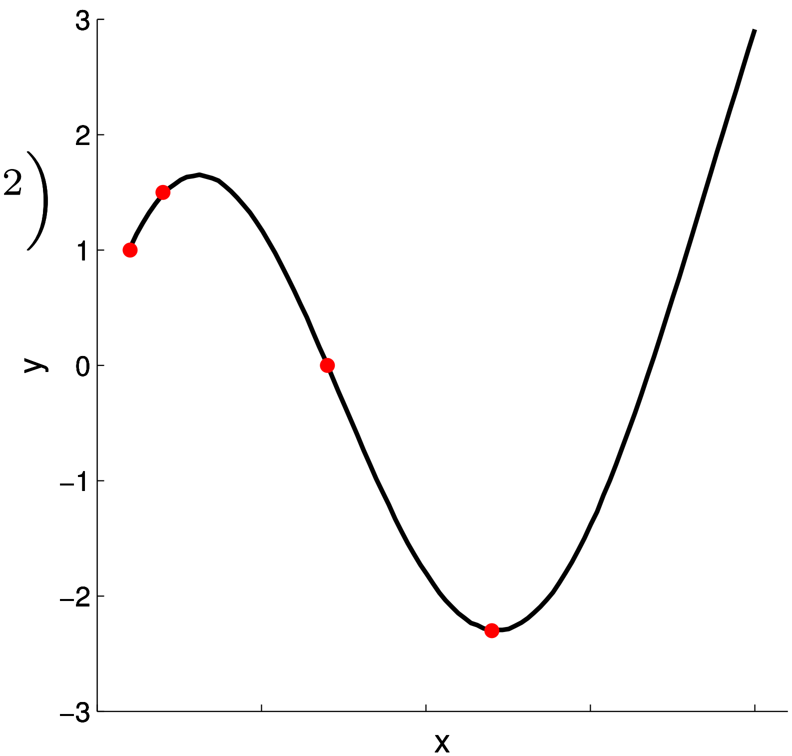
$\Sigma =$ 



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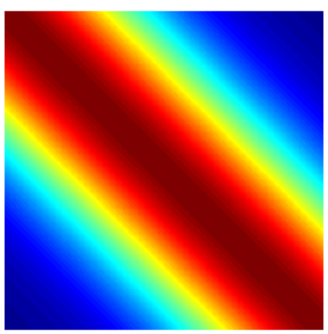
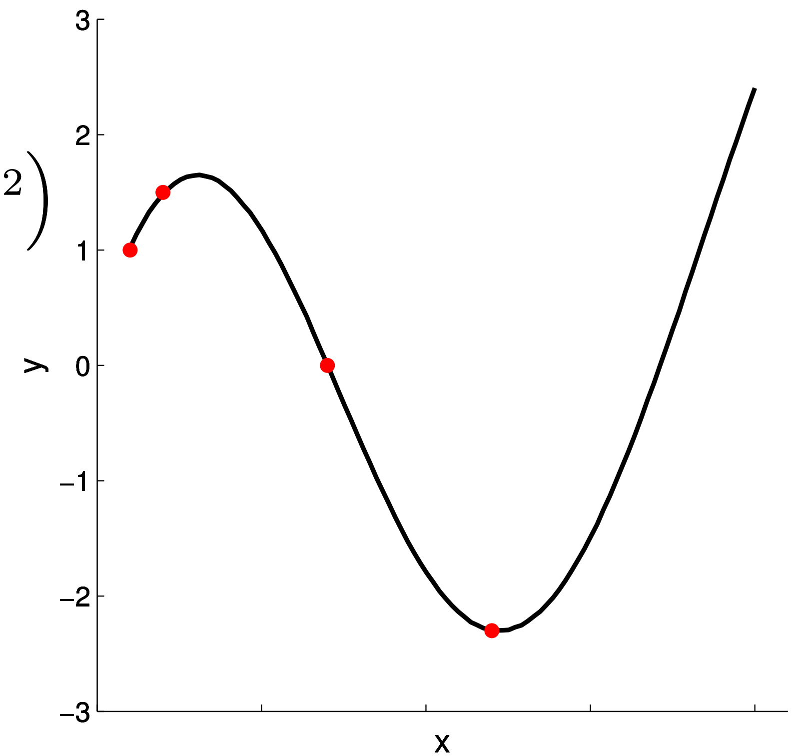
$\Sigma =$ 



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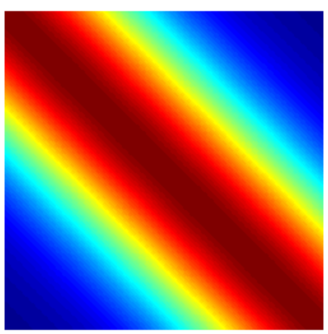
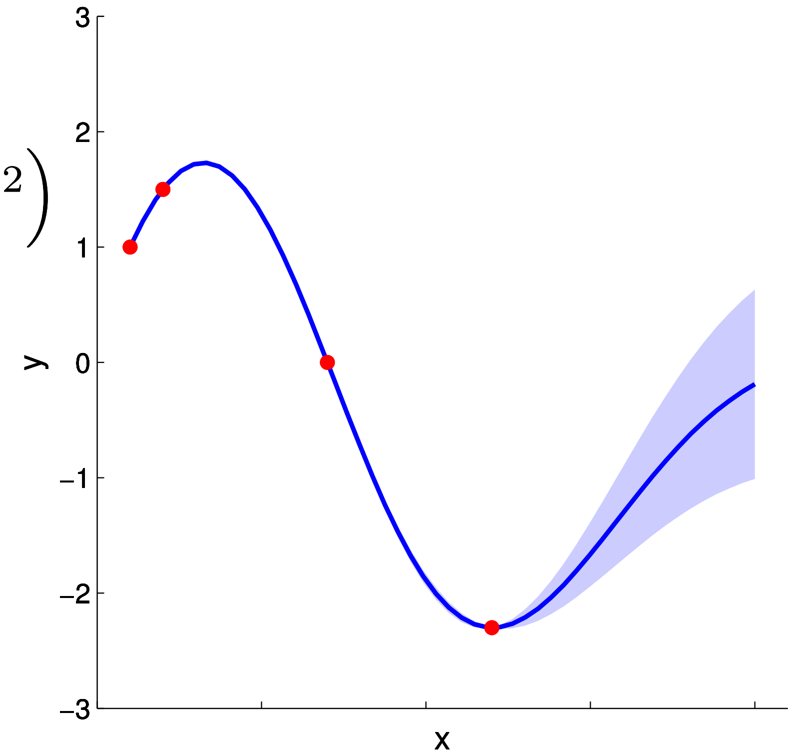
$\Sigma =$ 



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What effect do the hyper-parameters have?

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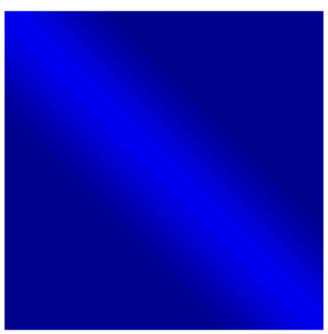
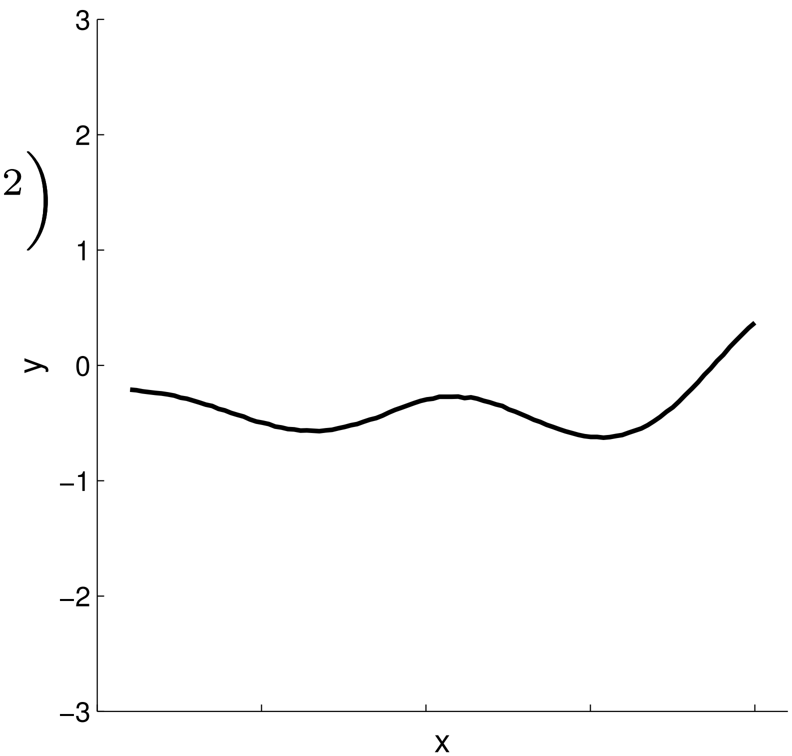
$\Sigma =$ 



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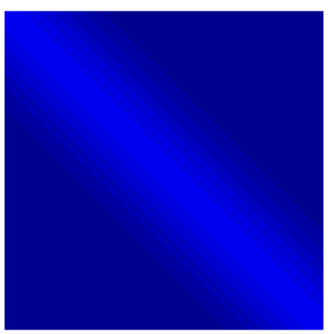

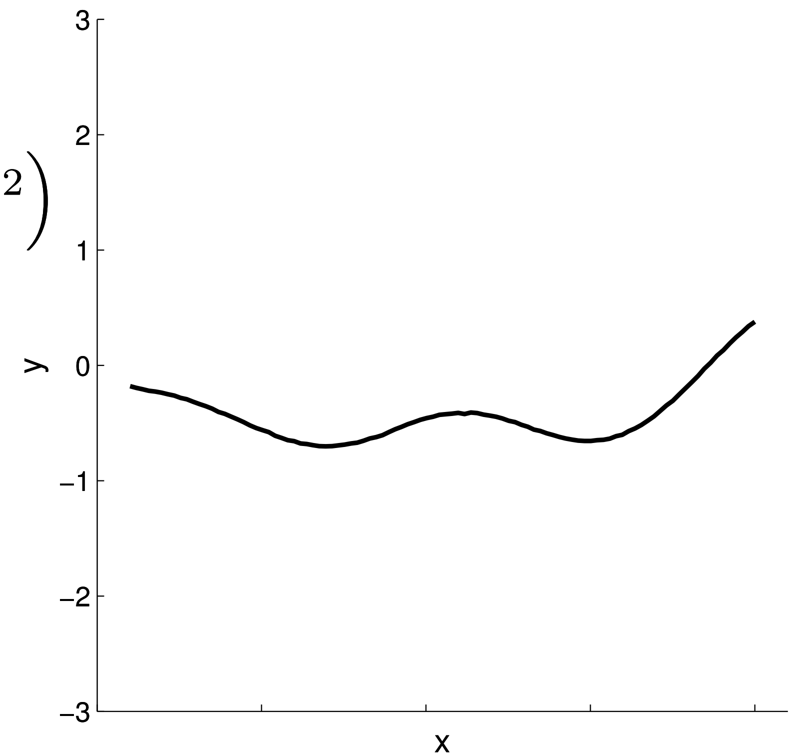

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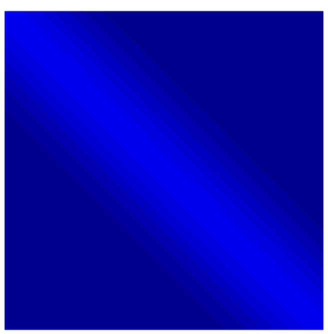

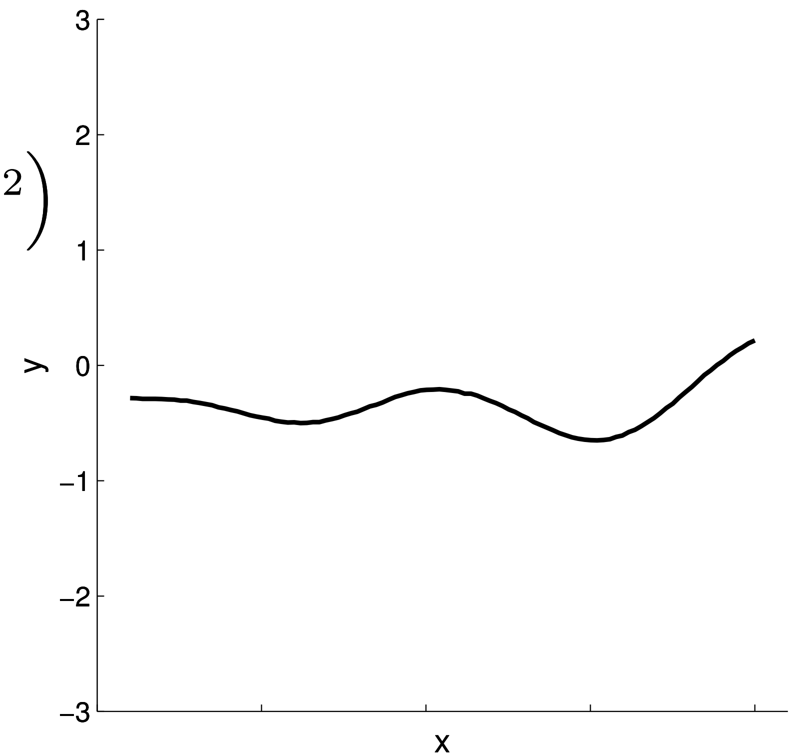

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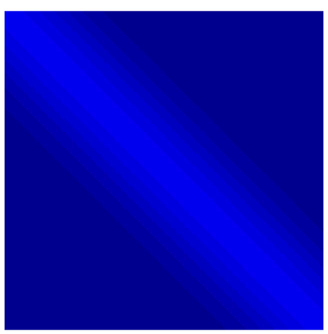
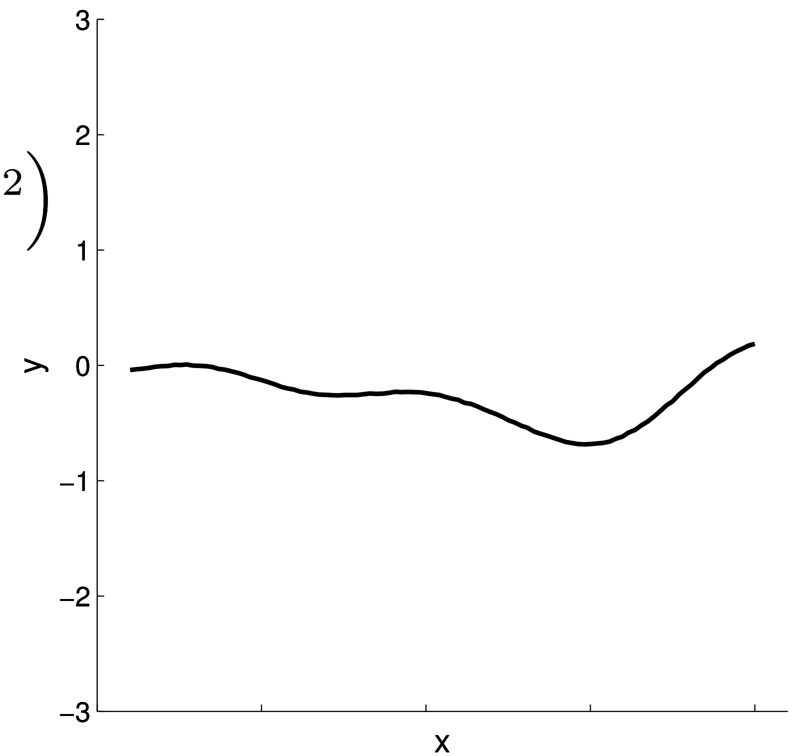
$\Sigma =$ 



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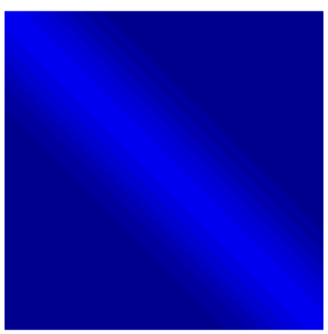
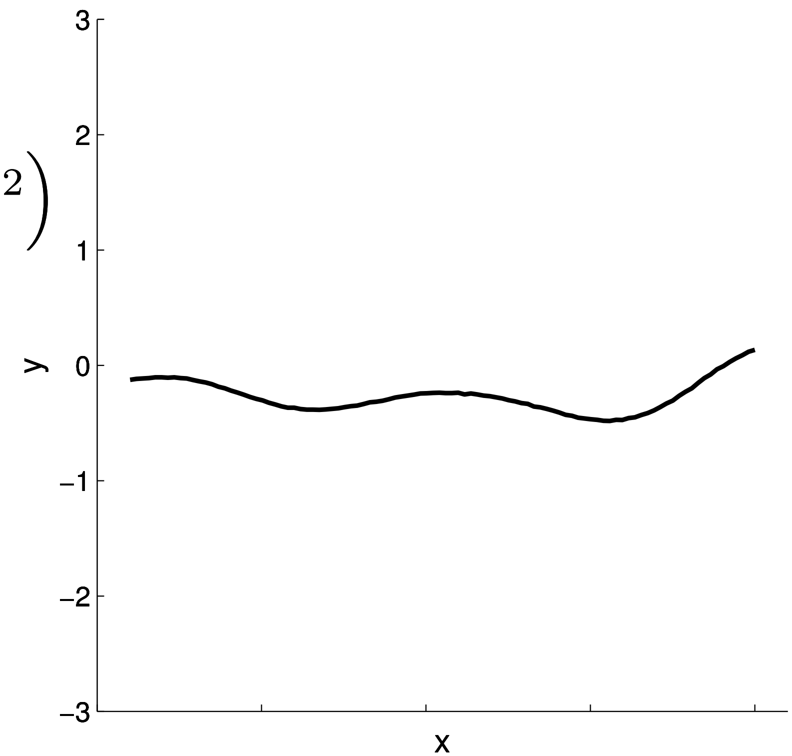
$\Sigma =$ 



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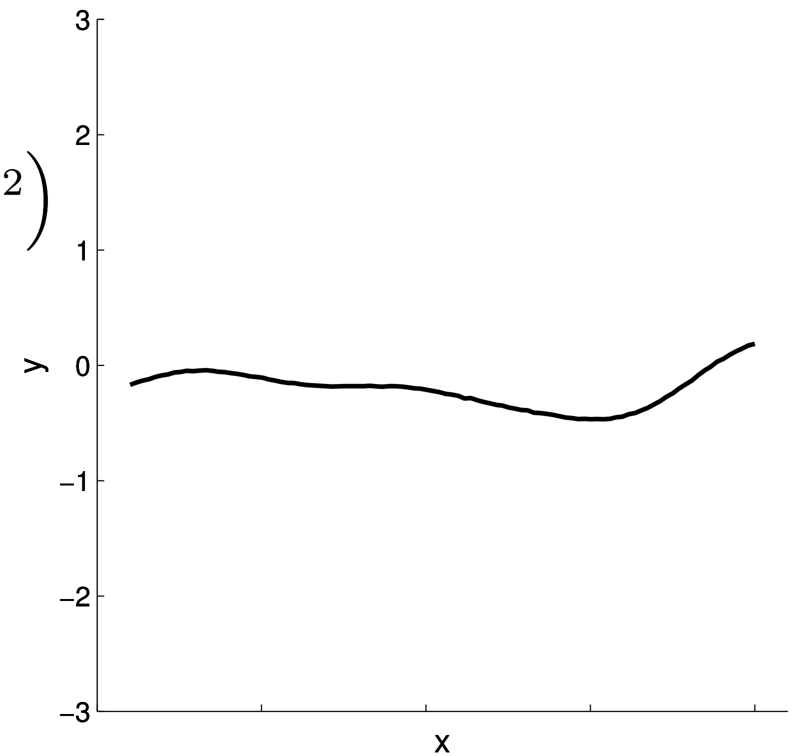

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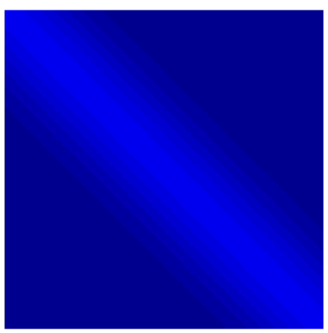
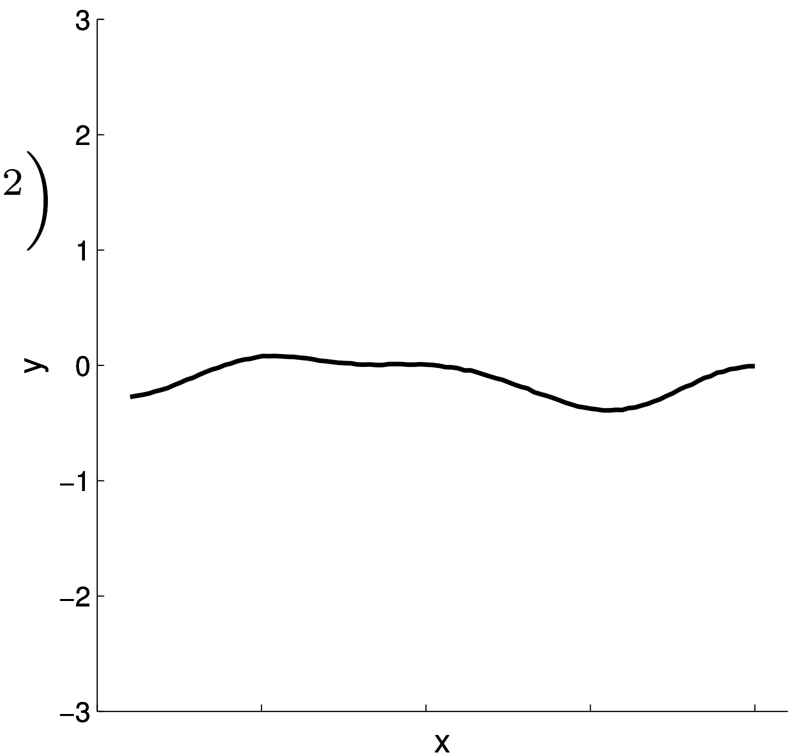
$\Sigma =$ 



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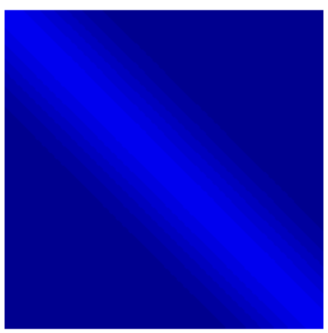
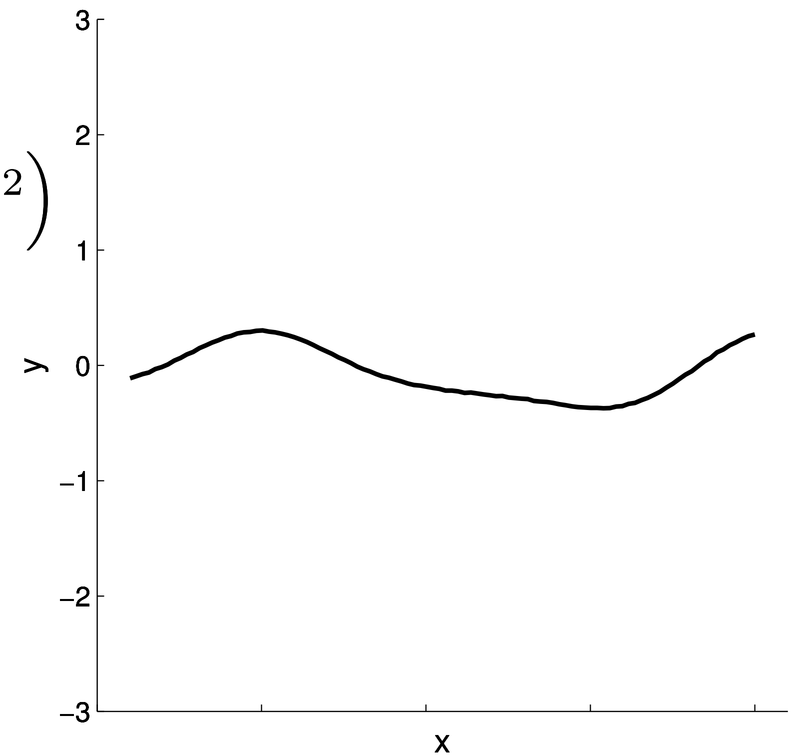
$\Sigma =$ 



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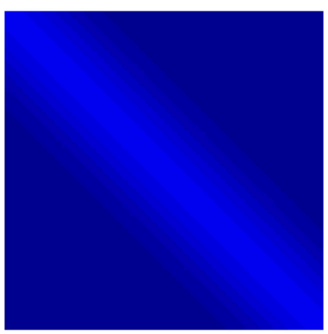

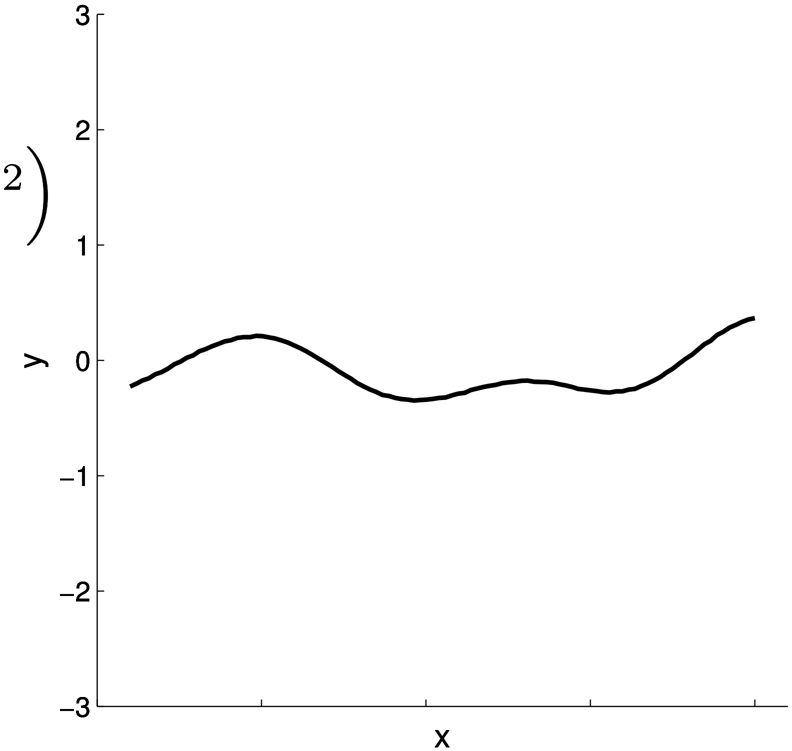

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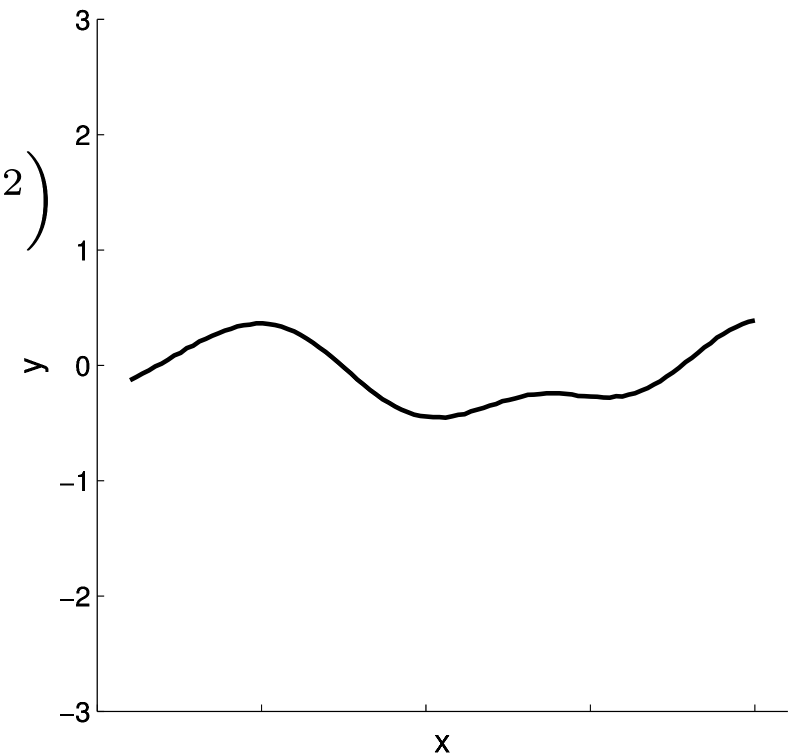

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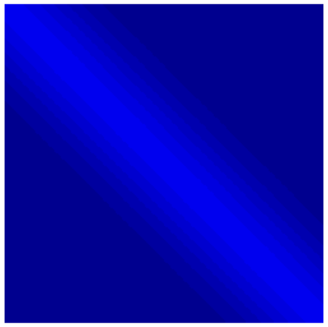
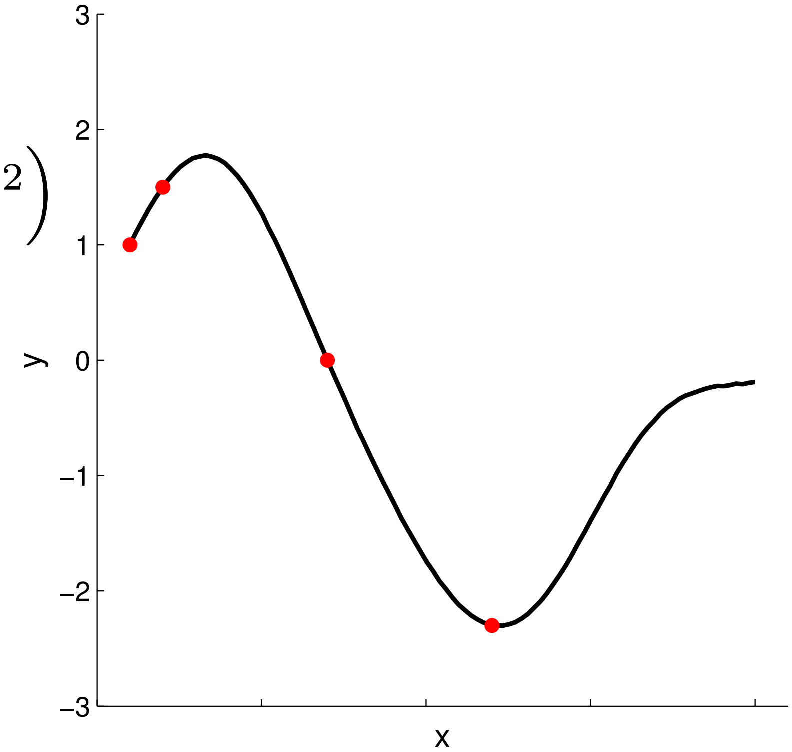
$\Sigma =$ 



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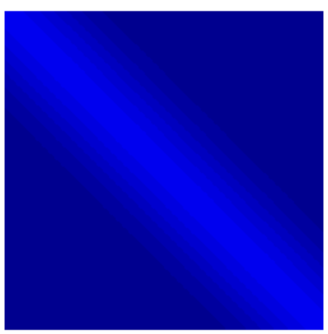
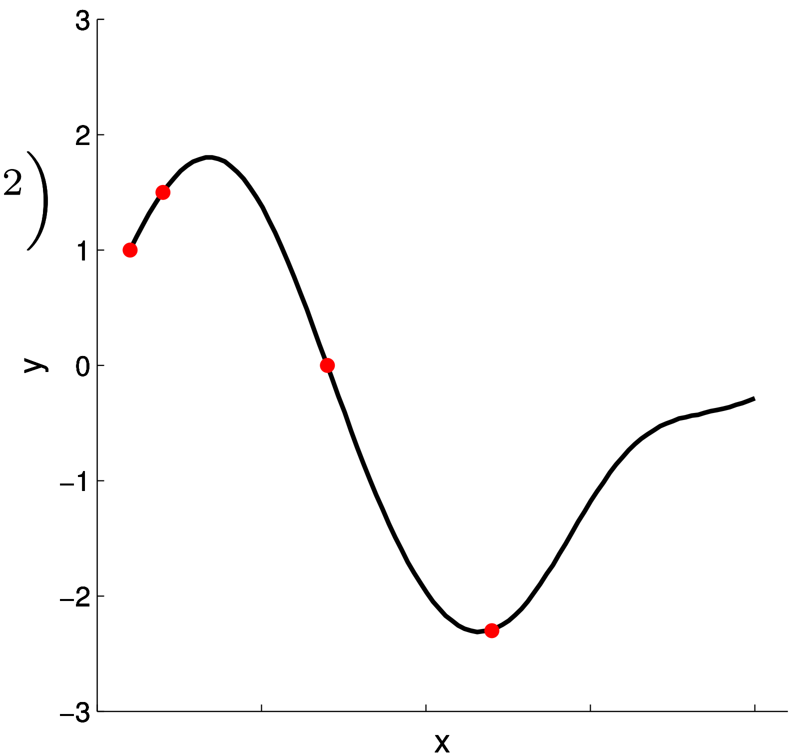
$\Sigma =$ 



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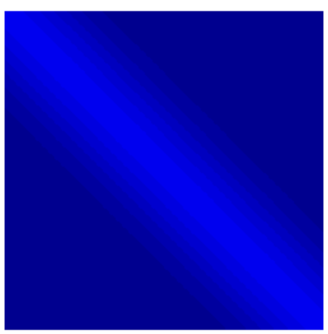
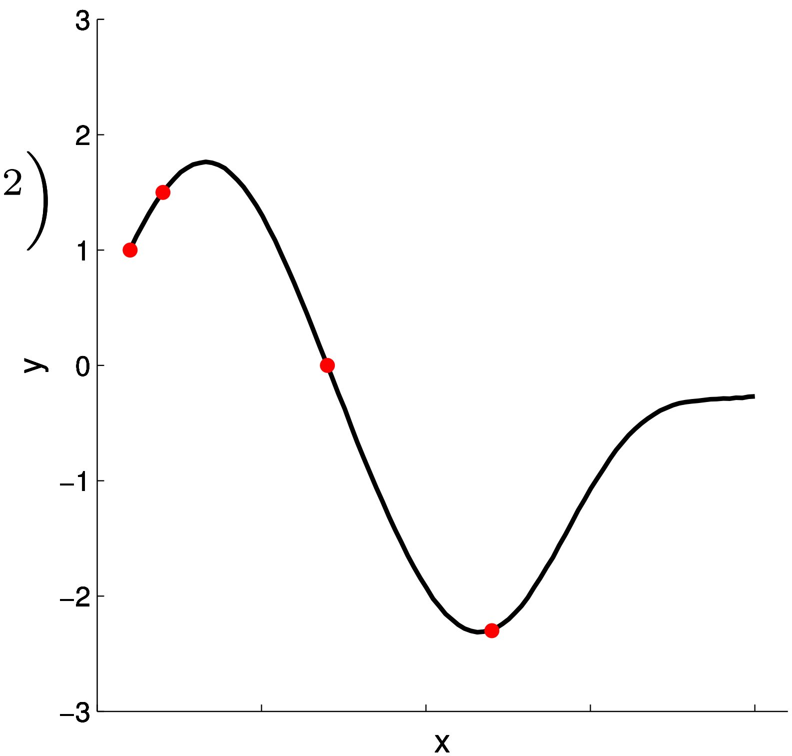
$\Sigma =$ 



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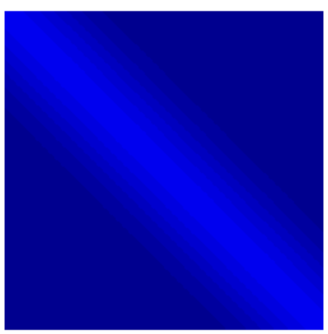

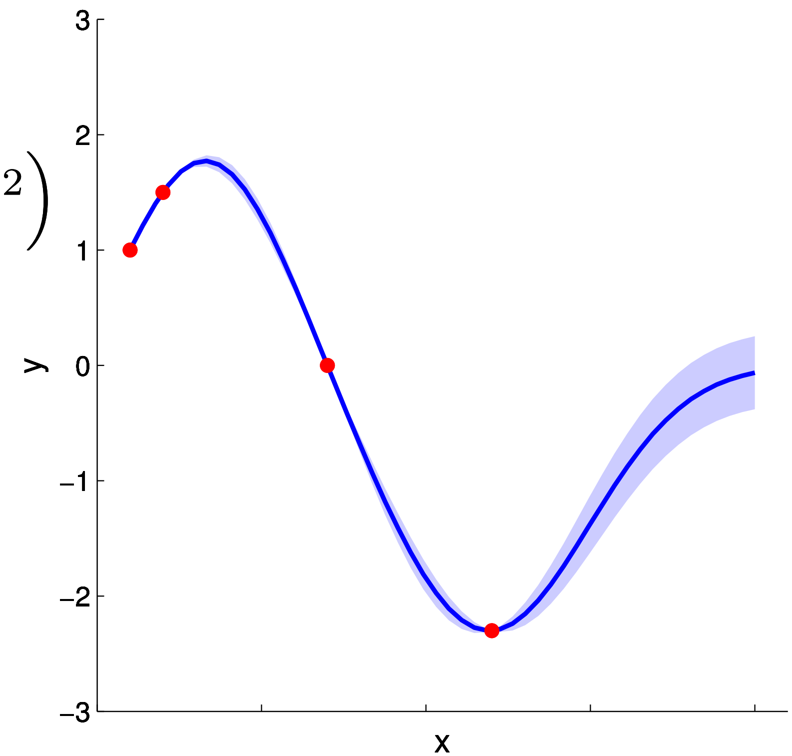

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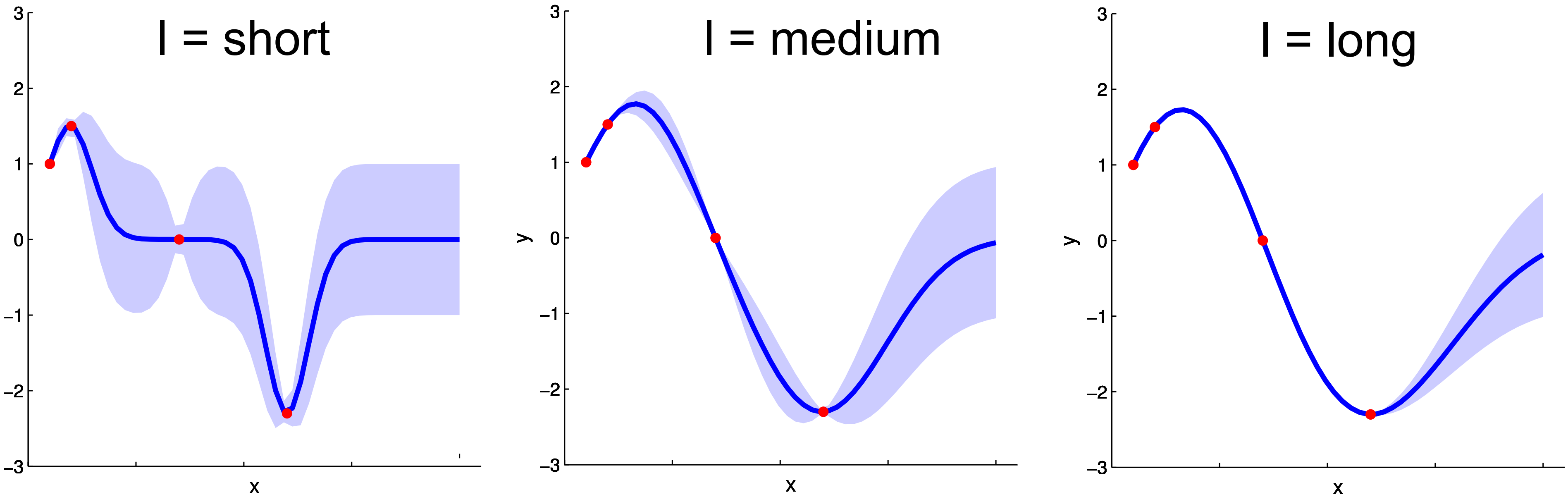
- Hyper-parameters have a strong effect
  - l controls the horizontal length-scale
  - σ^2 controls the vertical scale of the data
- $\Longrightarrow$ we need automatic ways of learning the hyper-parameters from data



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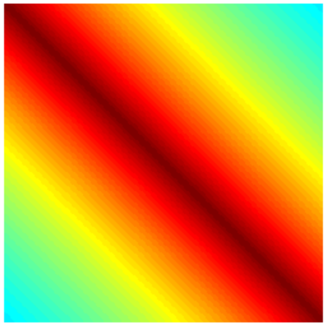
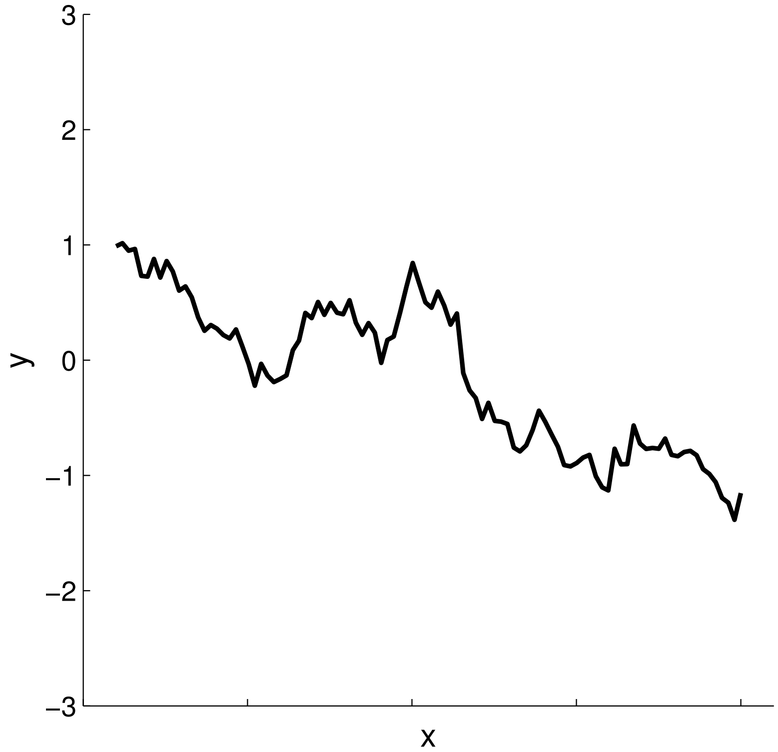
$$K(x_1, x_2) = \sigma^2 \exp\left(-\frac{1}{2l^2}|x_1 - x_2|\right)$$

Laplacian covariance function

Browninan motion

Ornstein-Uhlenbeck

$\Sigma =$ 



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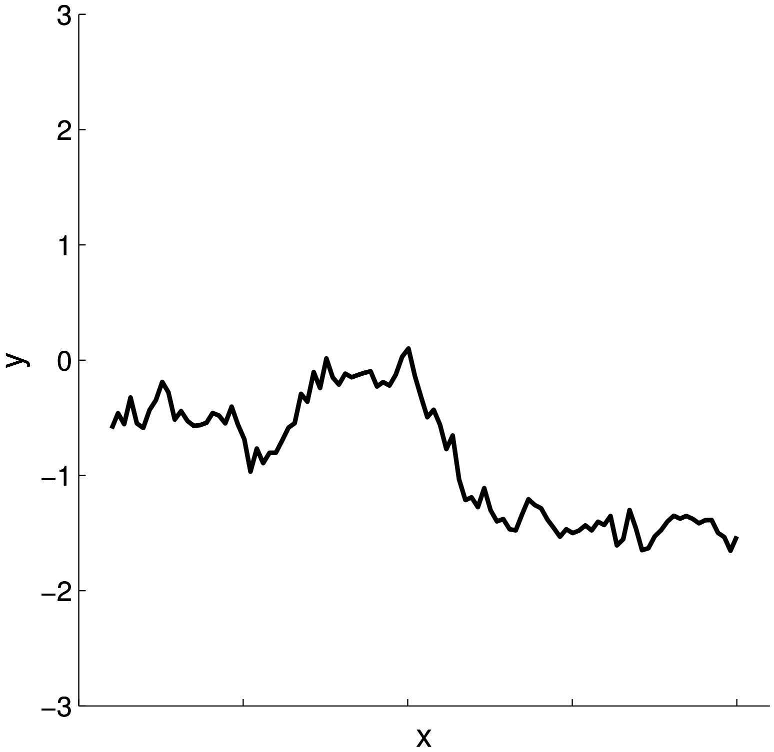
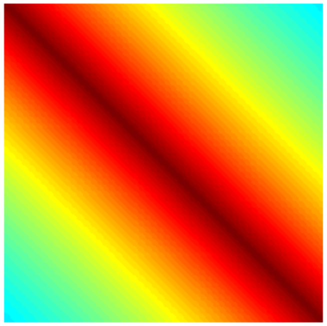


$\Sigma =$ 

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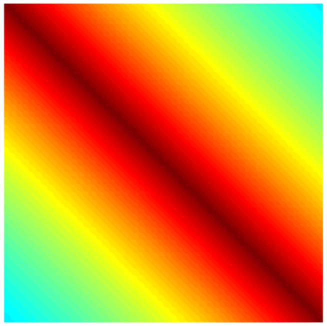
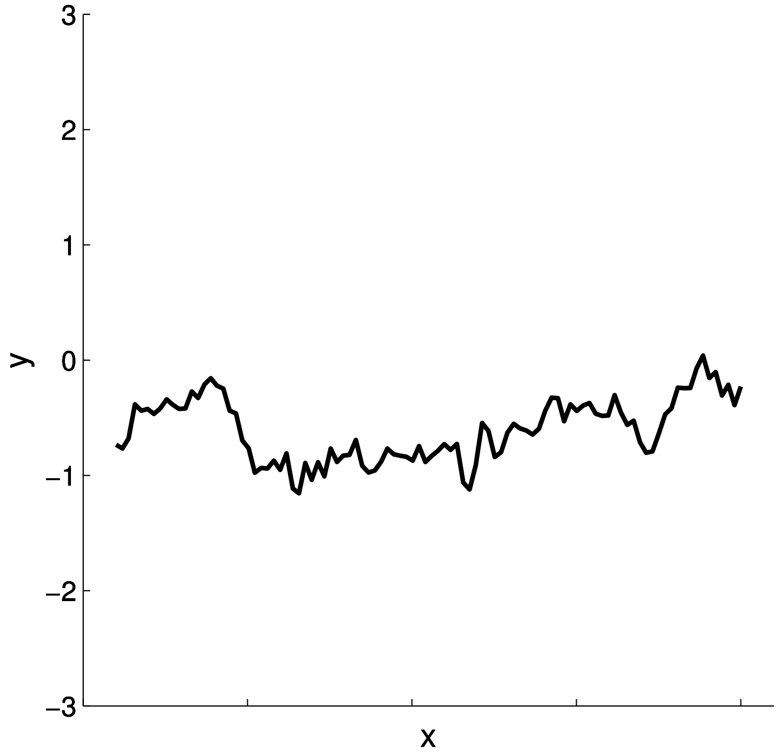
$\Sigma =$ 



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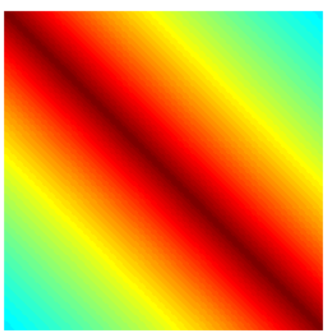
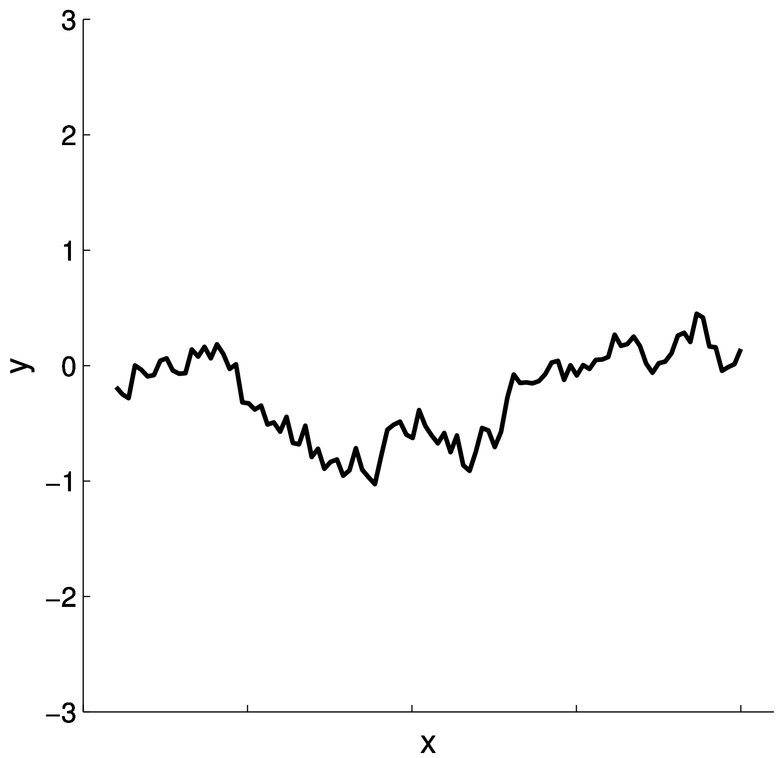
$\Sigma =$ 



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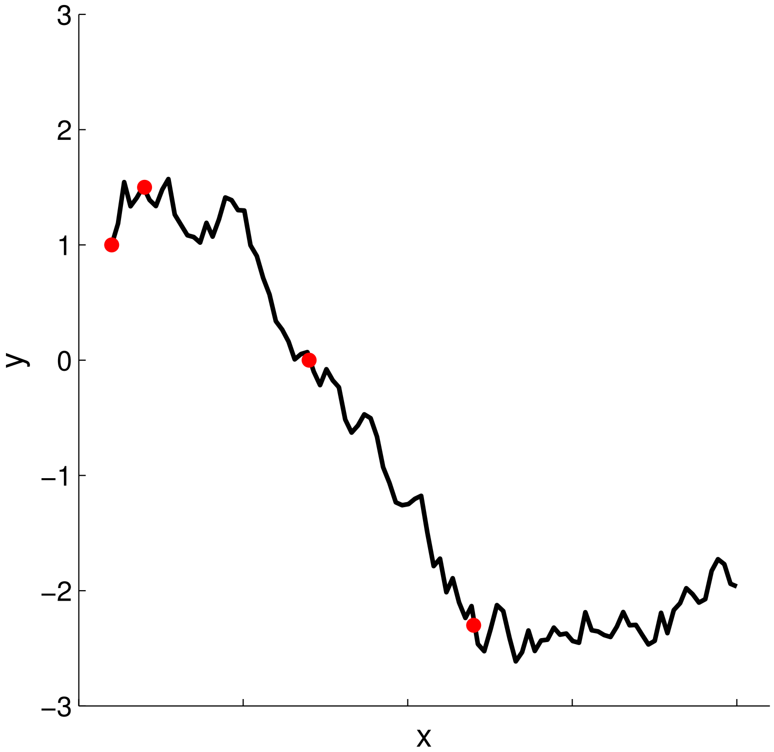

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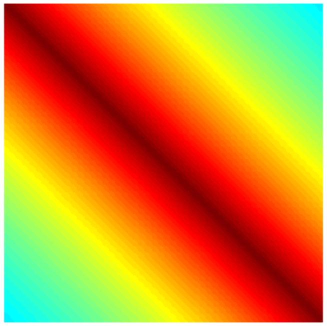

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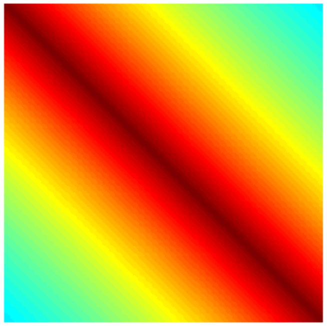
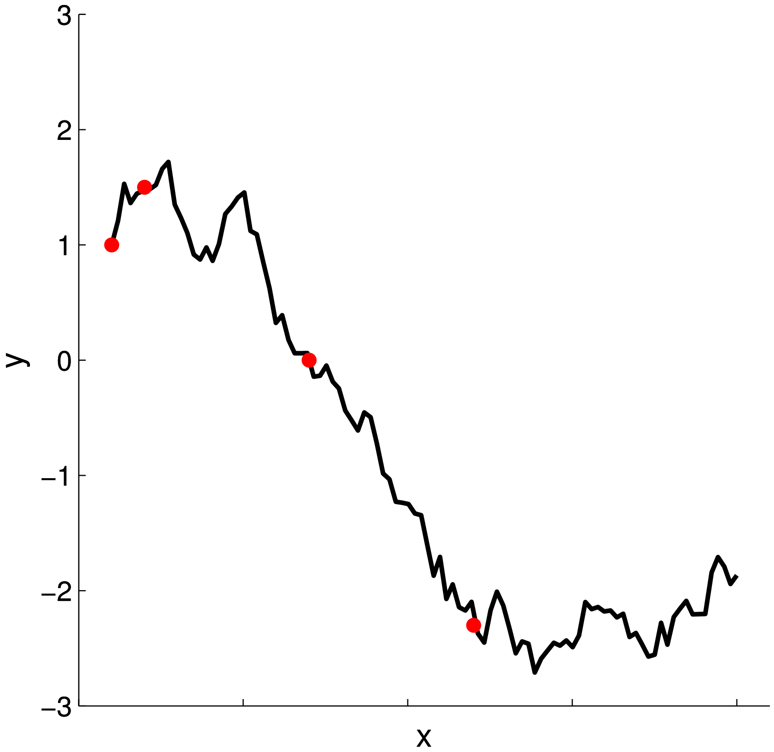
$\Sigma =$ 



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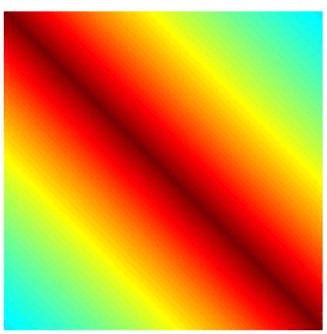
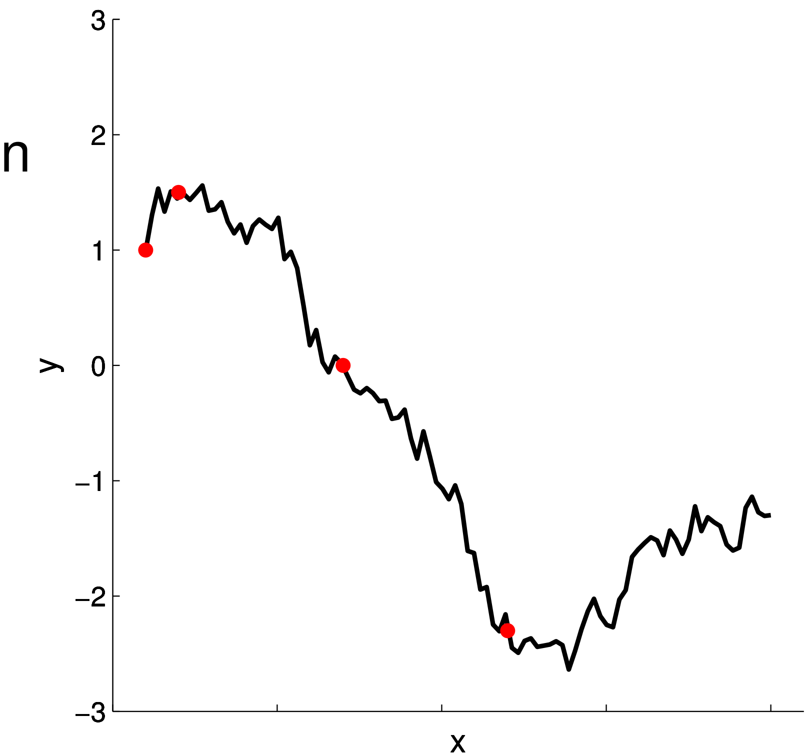
$\Sigma =$ 



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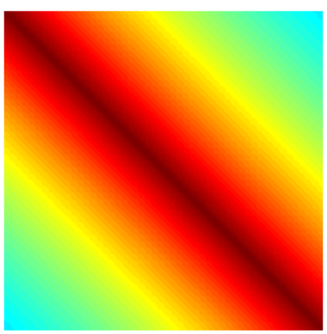

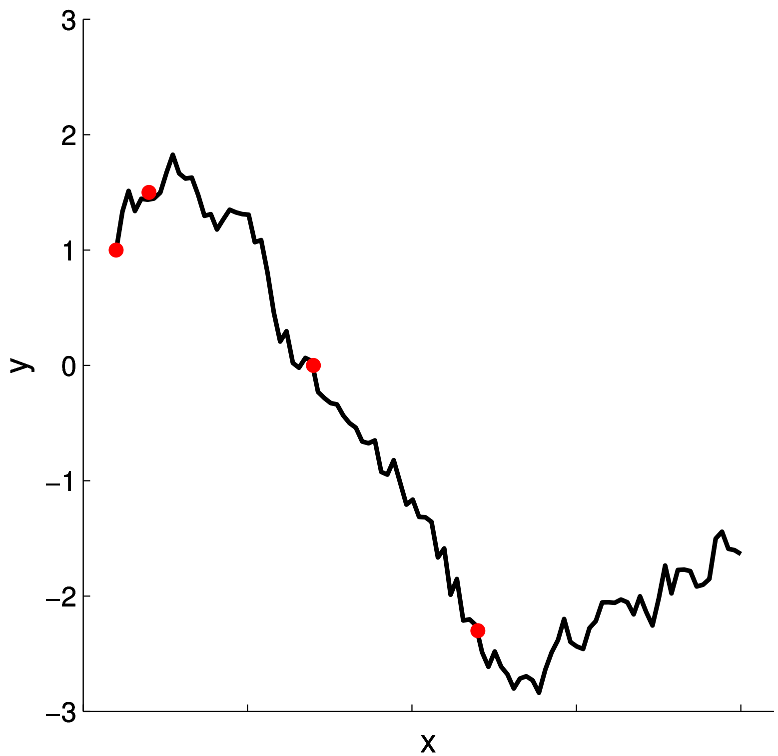

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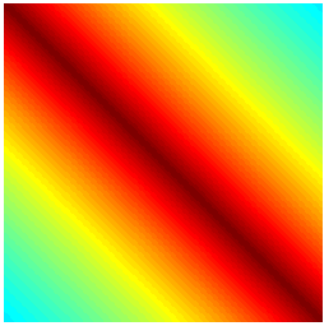
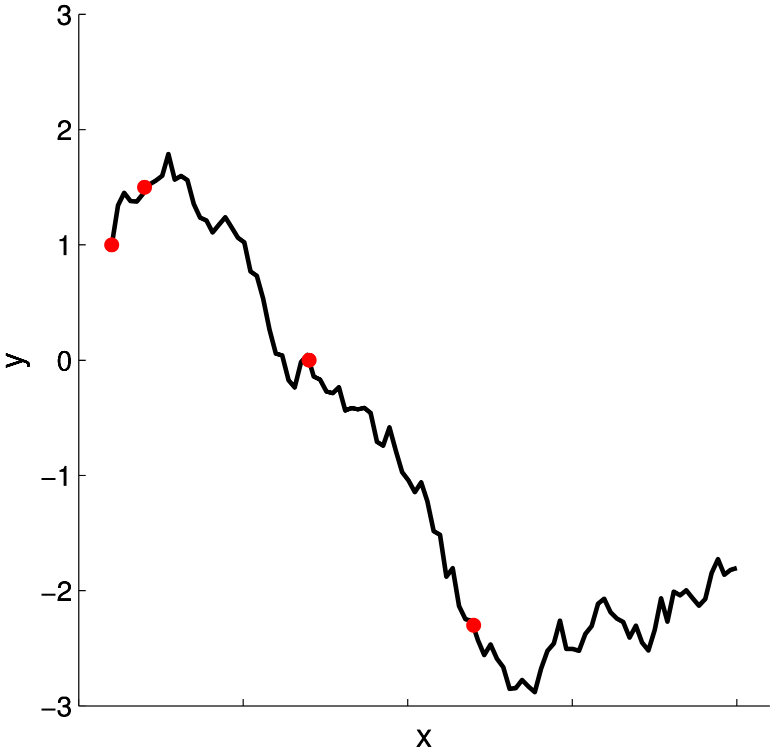
$\Sigma =$ 



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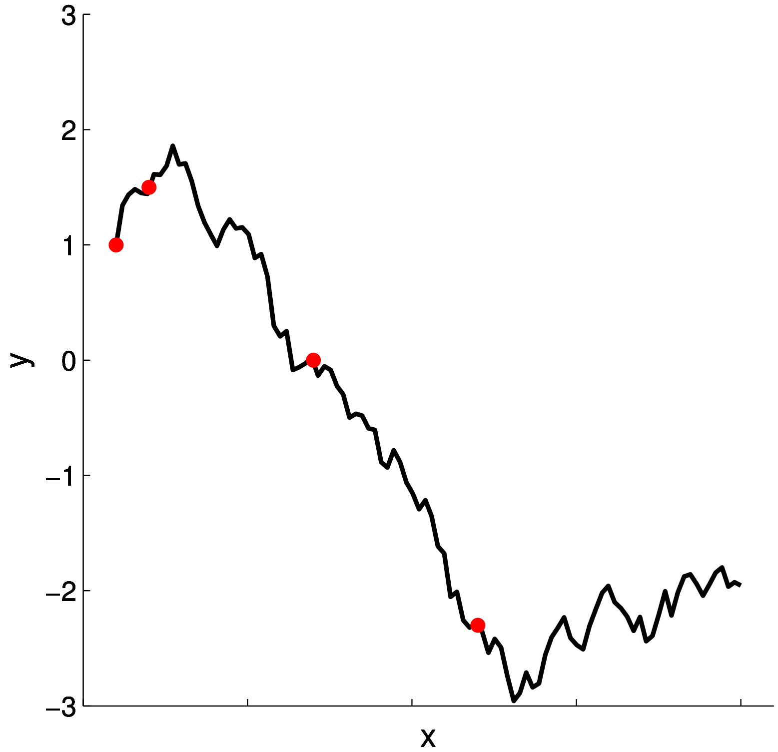
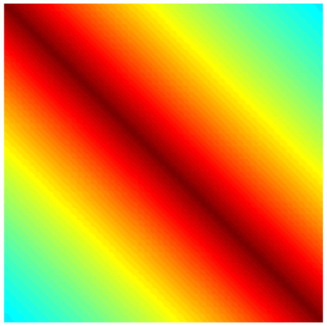


$\Sigma =$ 

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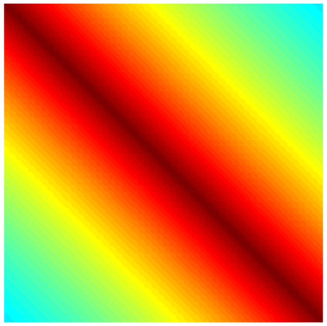

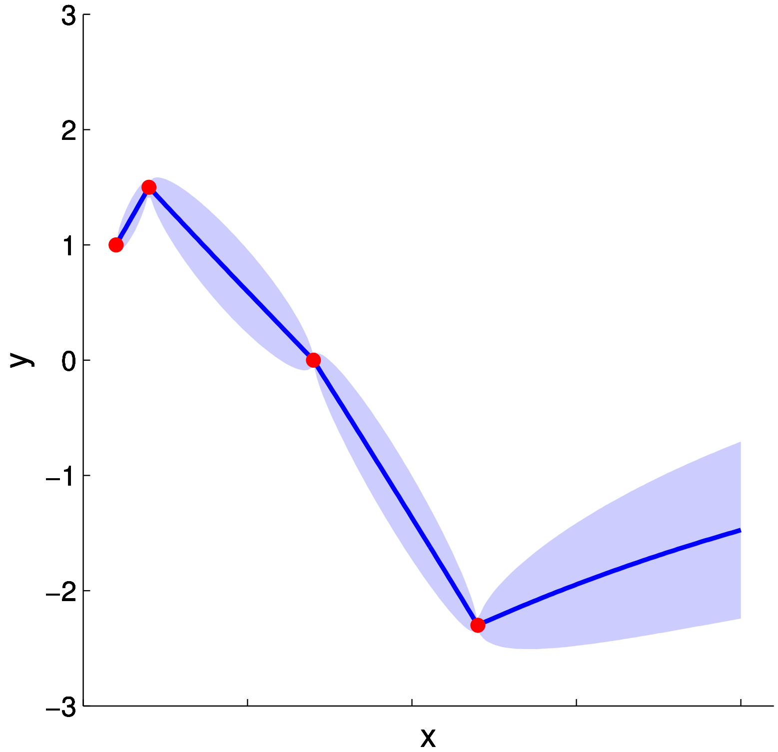

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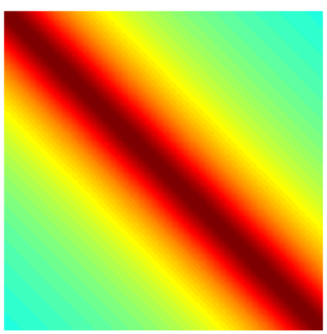

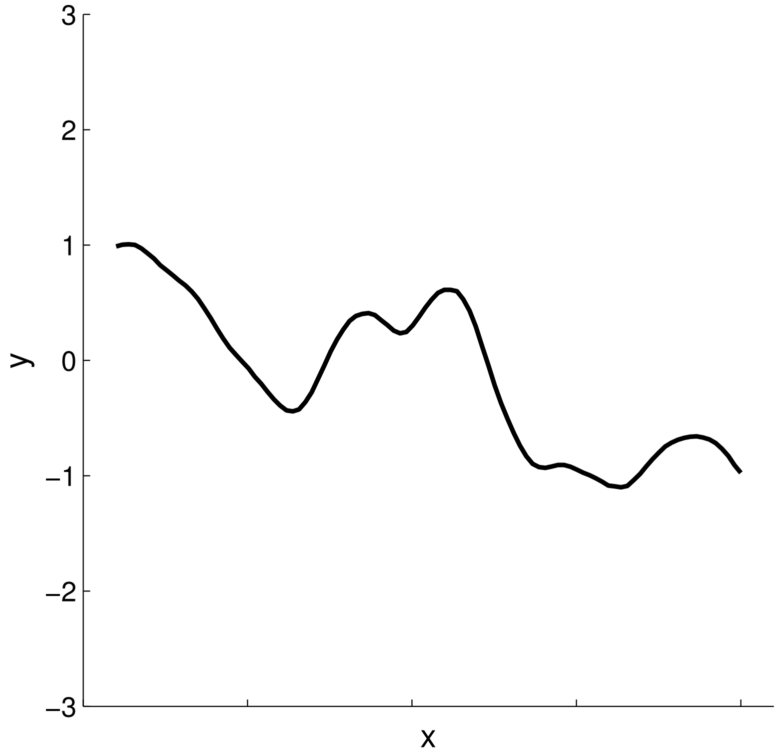

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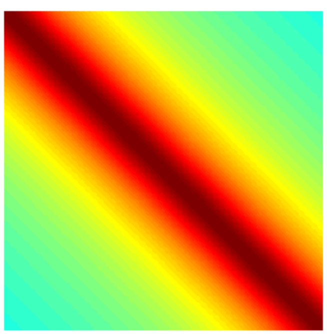
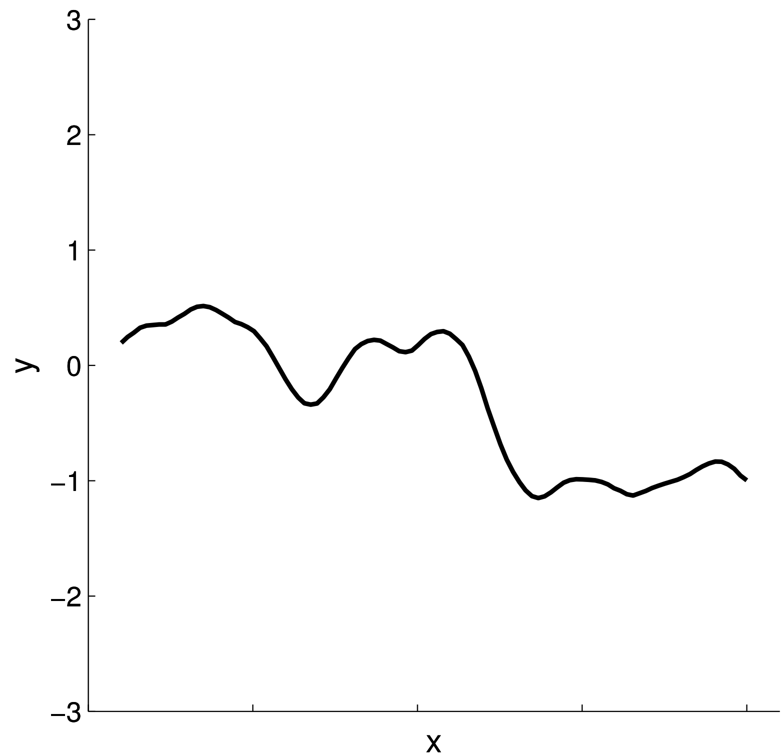
$\Sigma =$ 



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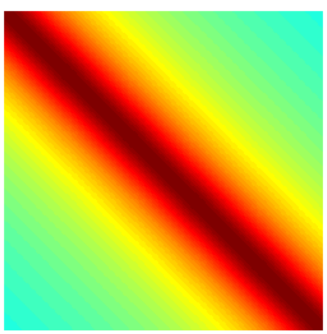

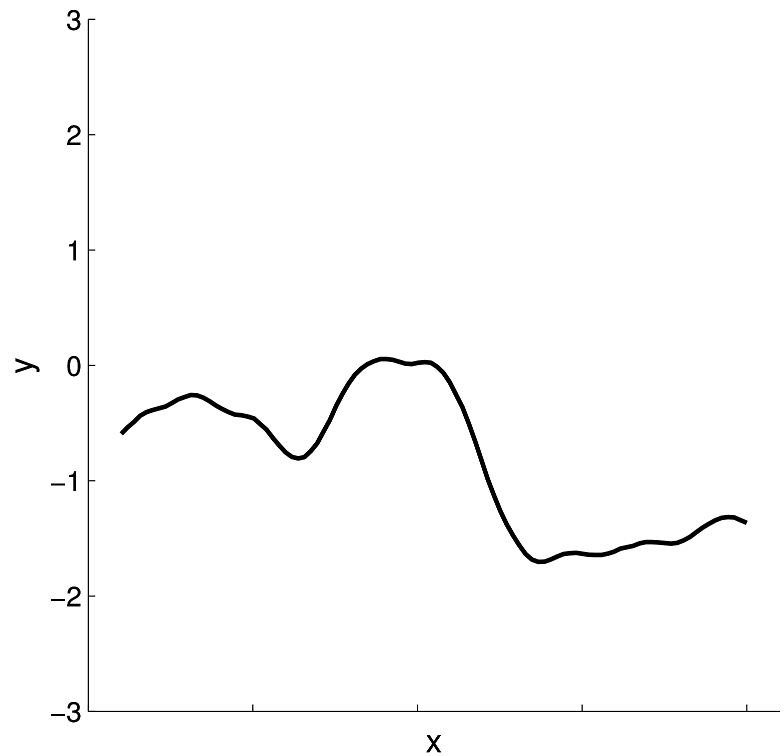

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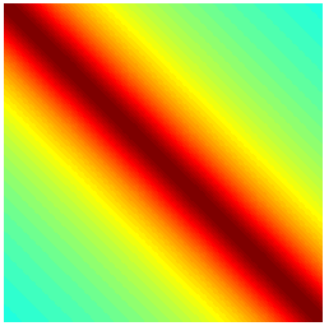

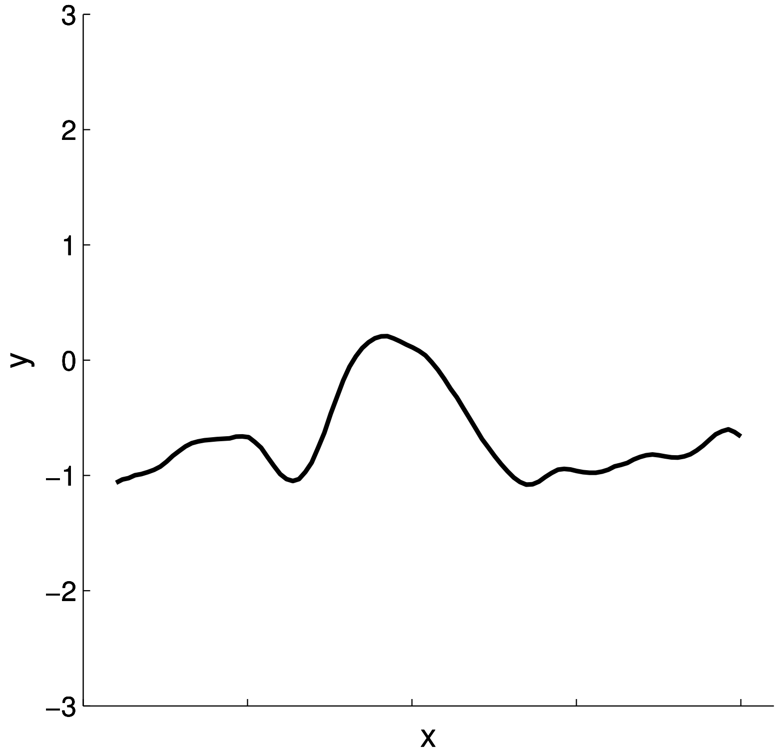

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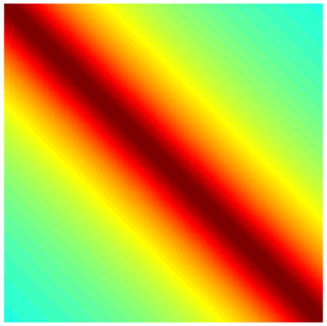
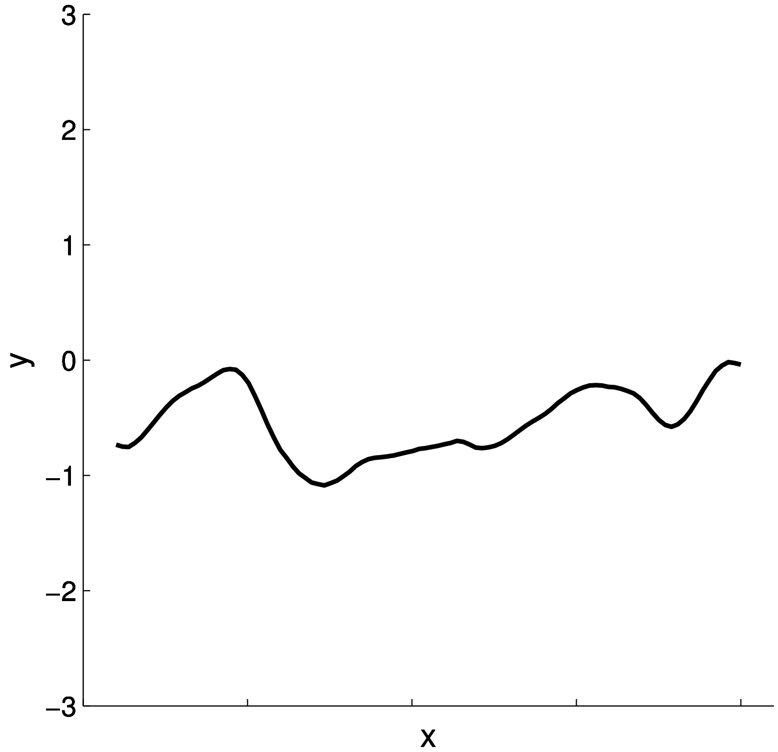
$\Sigma =$ 



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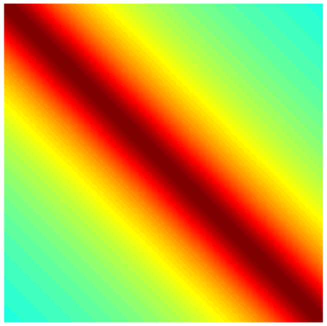
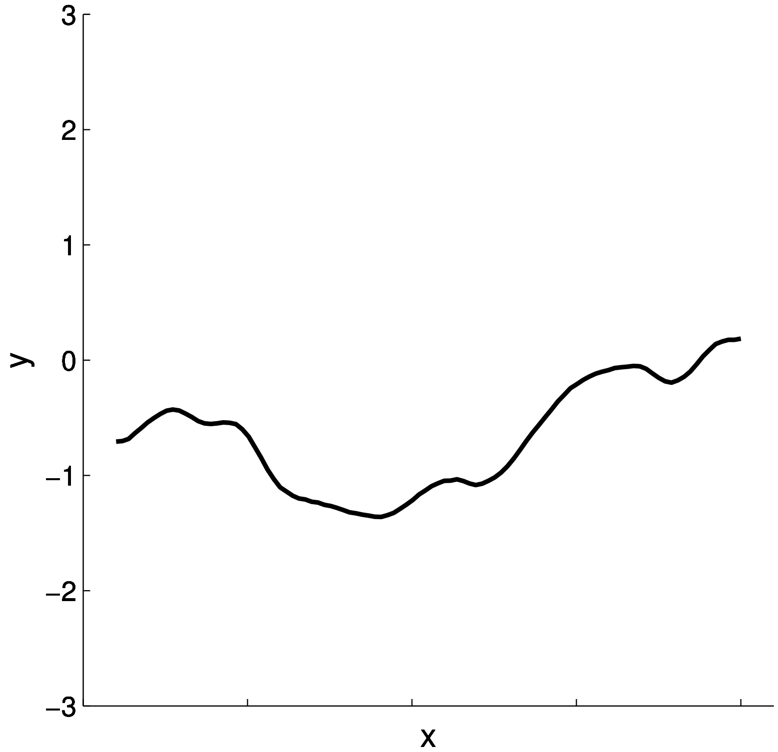
$\Sigma =$ 



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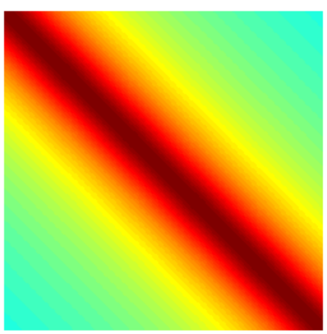

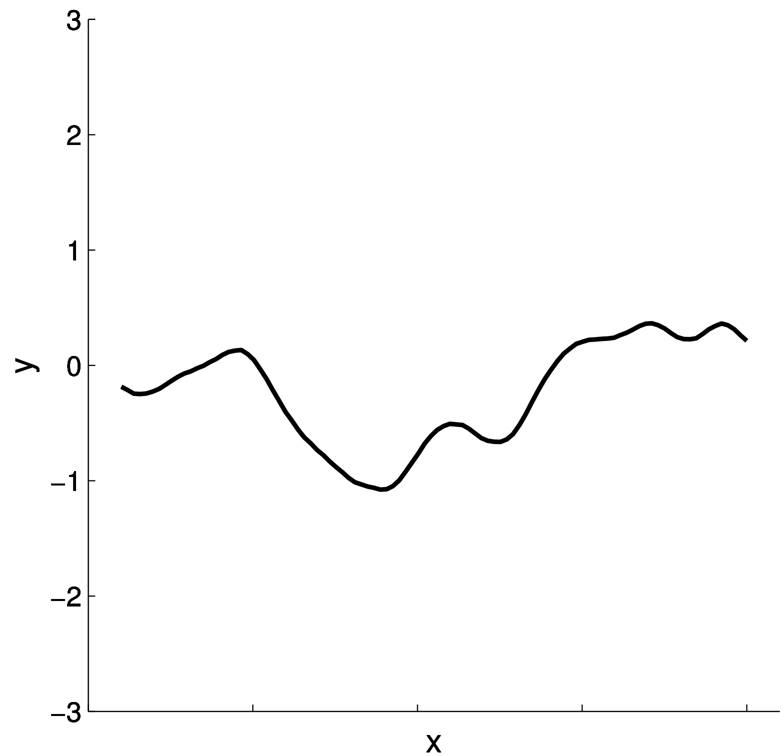

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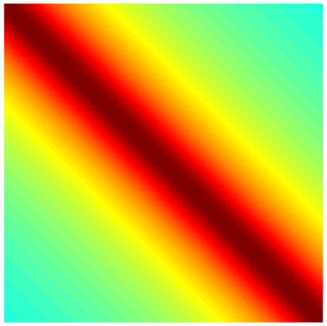
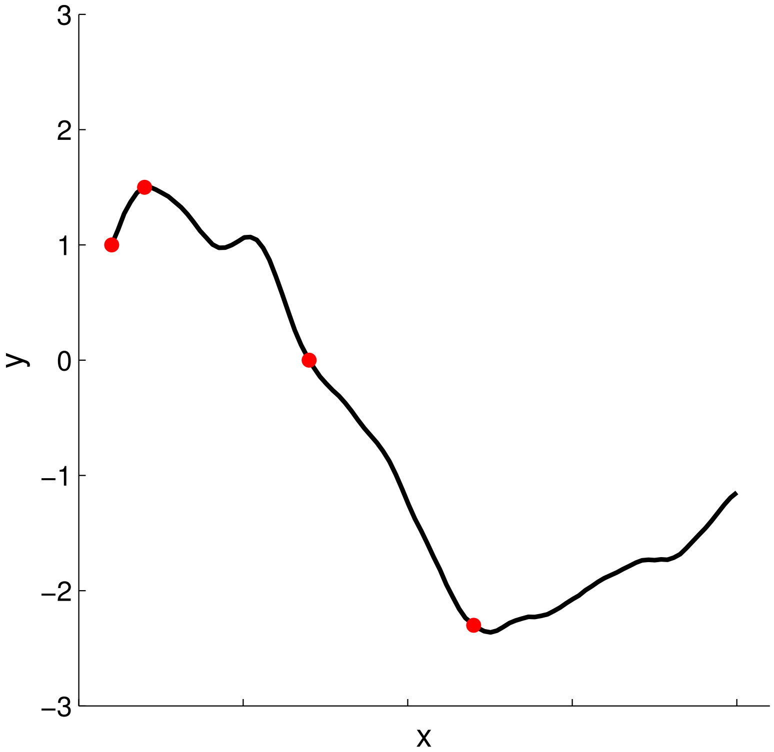
$\Sigma =$ 



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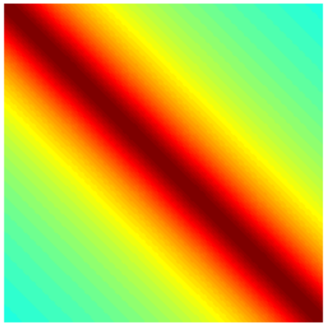
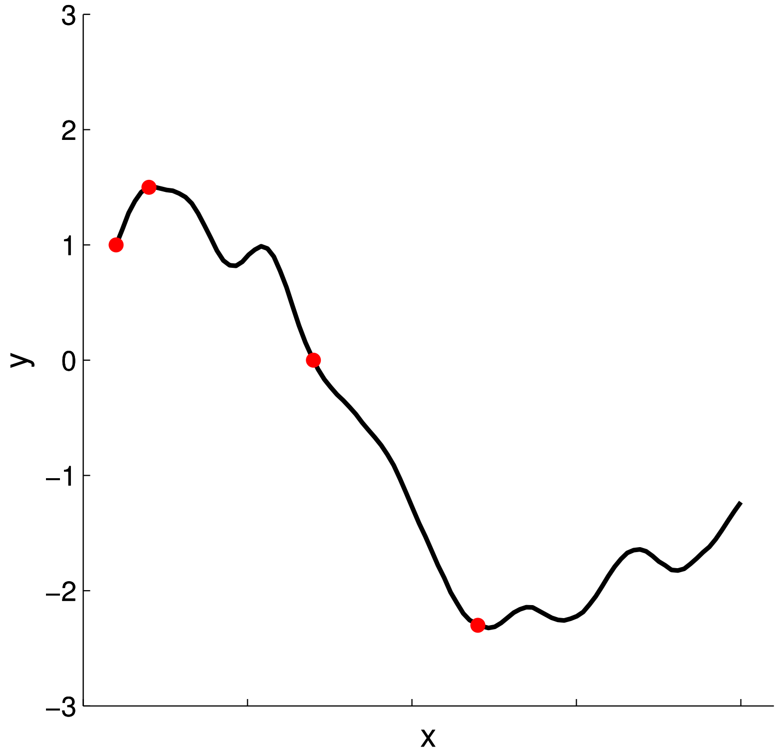
$\Sigma =$ 



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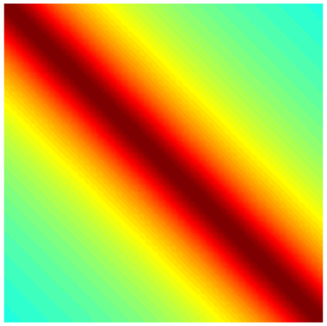
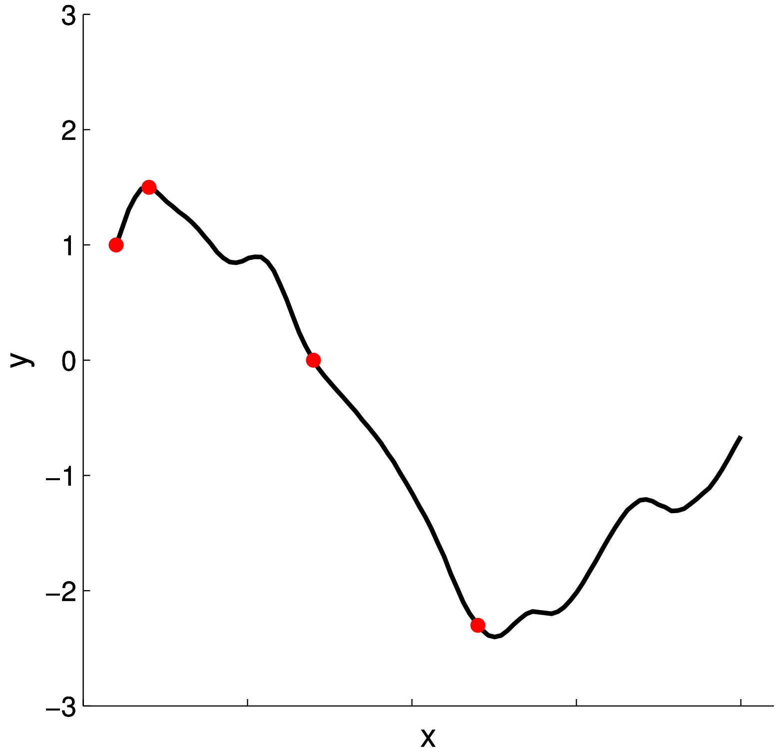
$\Sigma =$ 



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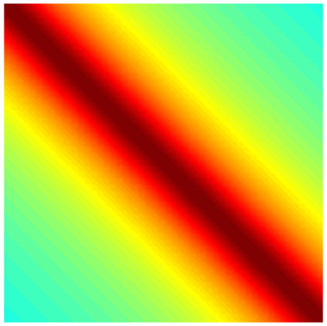
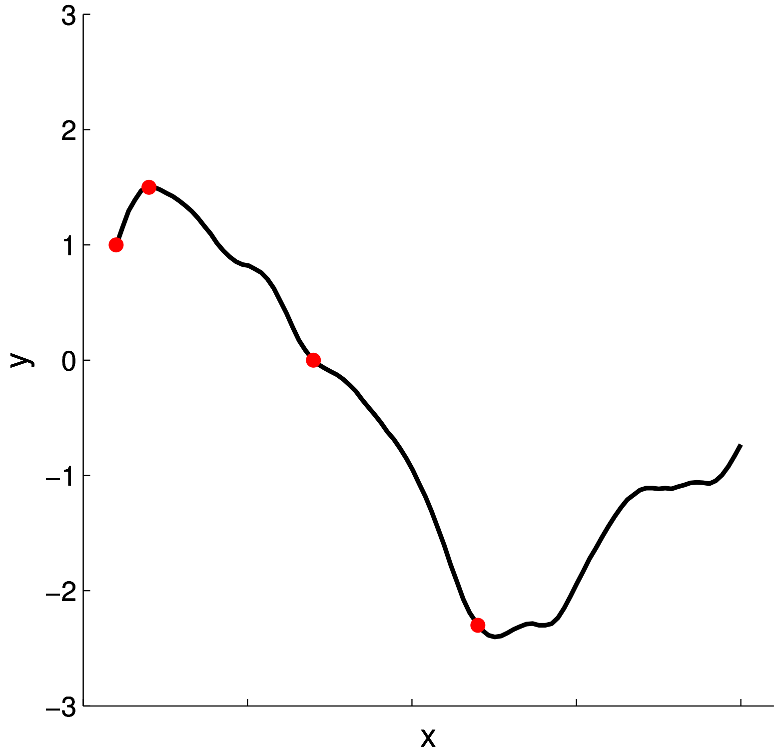
$\Sigma =$ 



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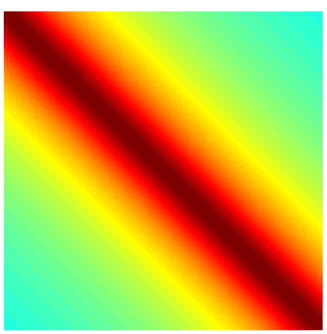
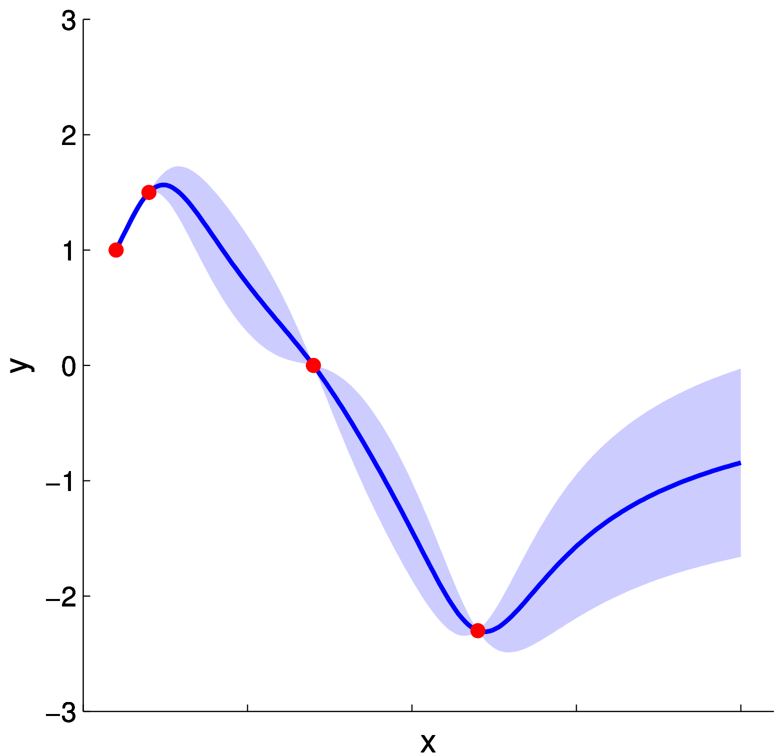
$\Sigma =$ 



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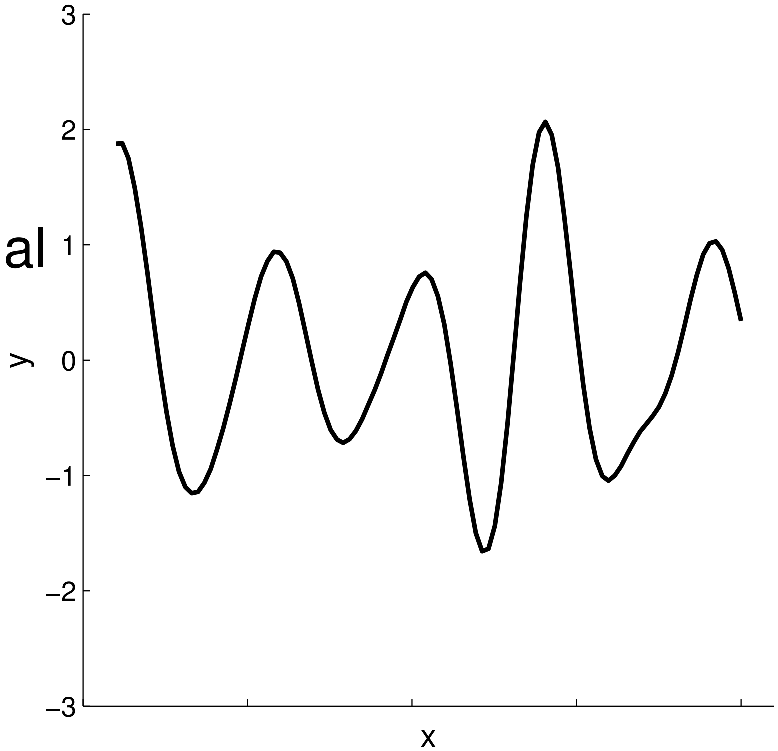
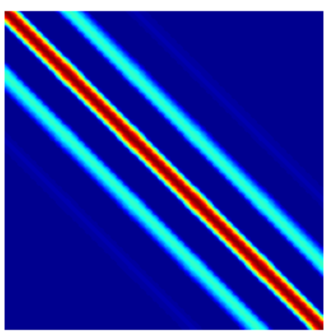
Periodic

sinusoid $\times$ squared exponential



$\Sigma =$ 

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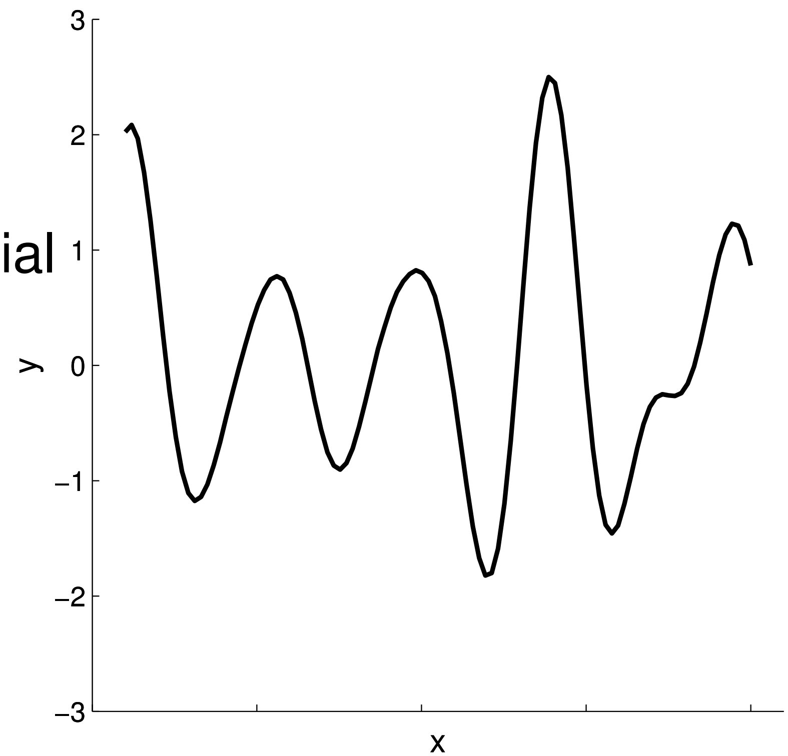
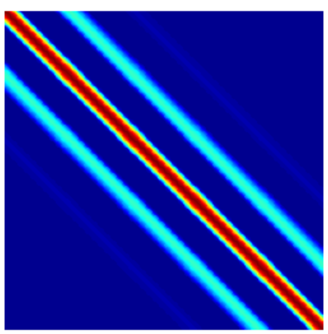


$\Sigma =$ 

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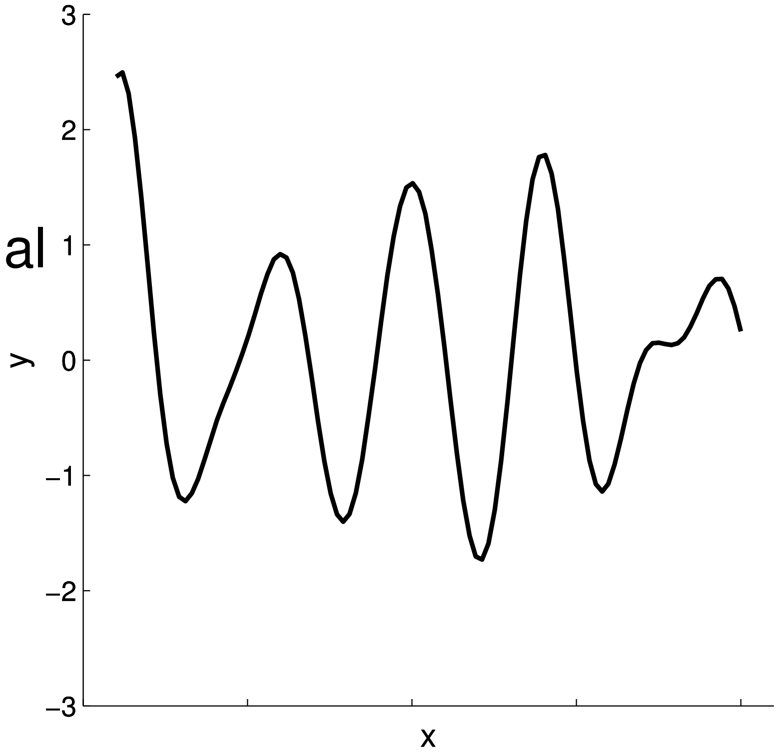
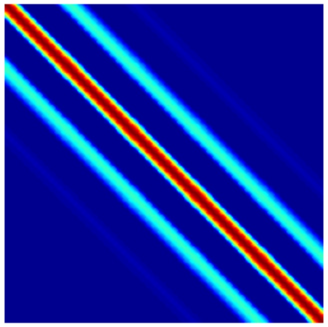


$\Sigma =$ 

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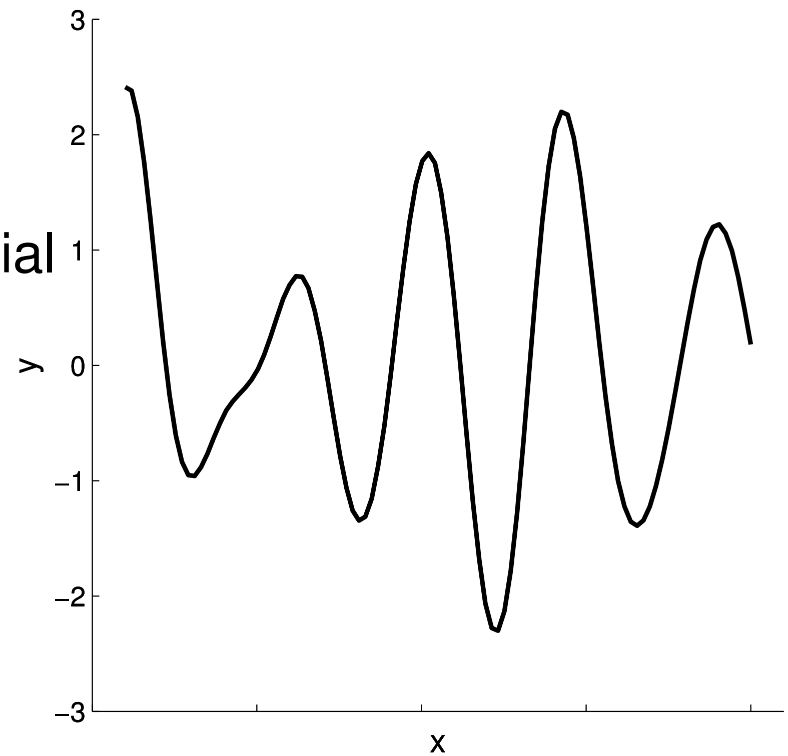
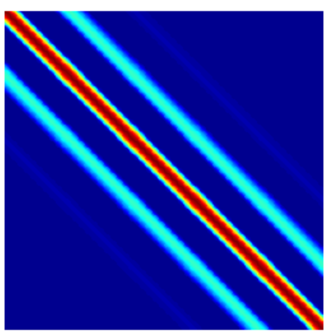


$\Sigma =$ 

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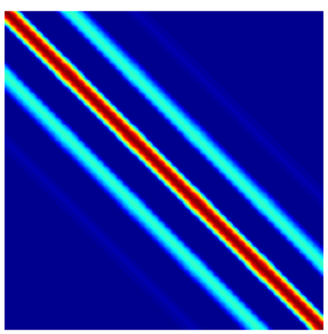
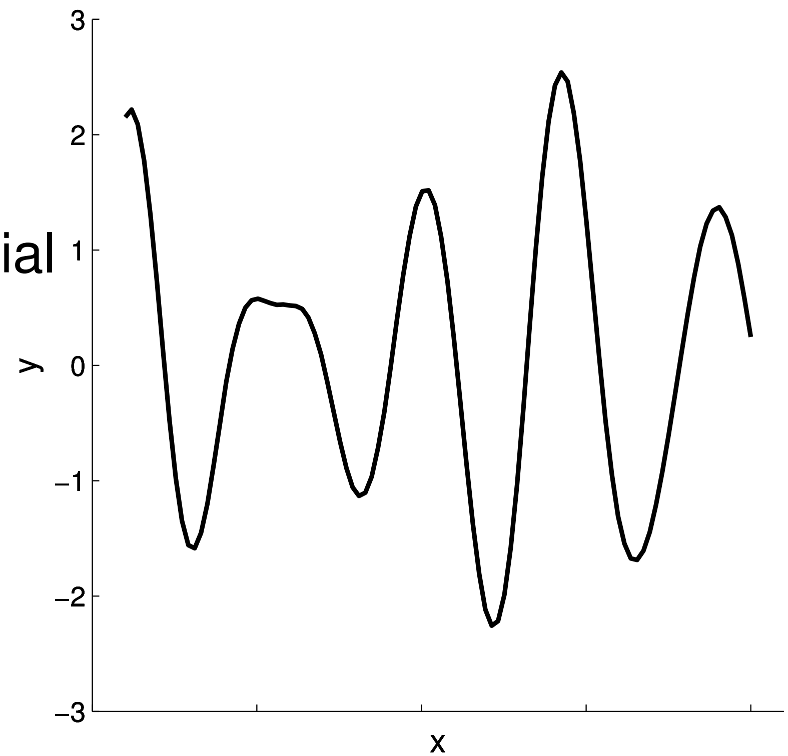
$\Sigma =$ 



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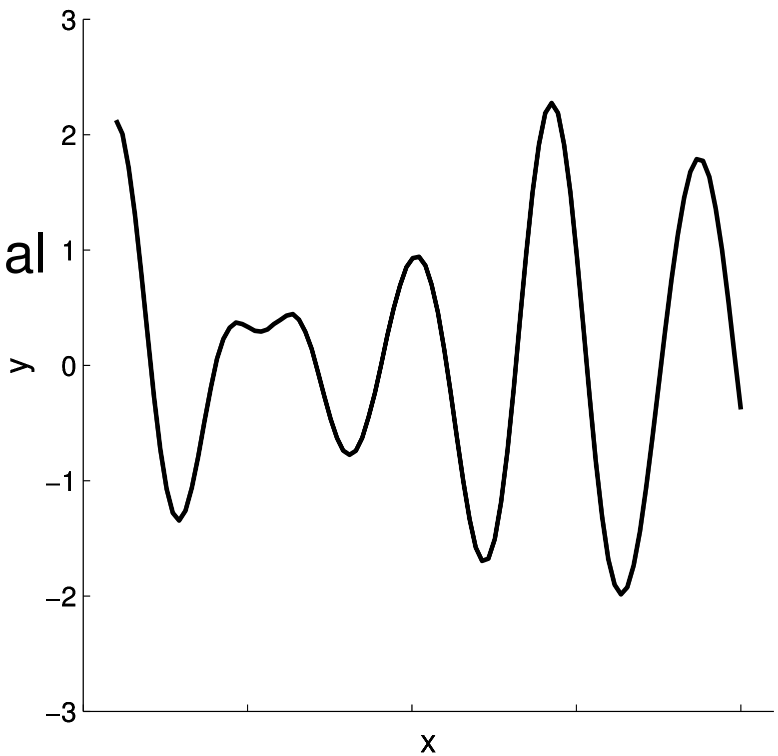
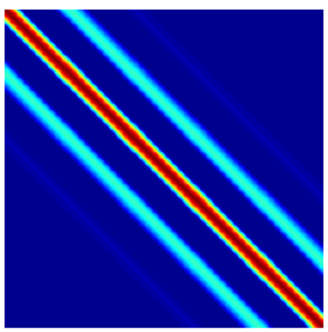


$\Sigma =$ 

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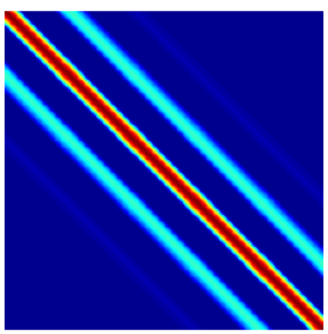
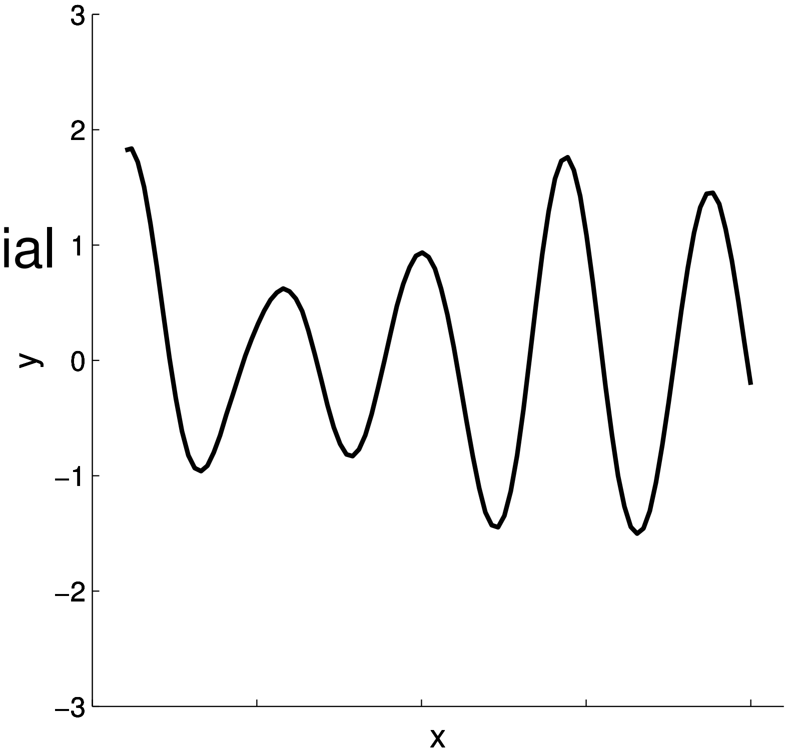
$\Sigma =$ 



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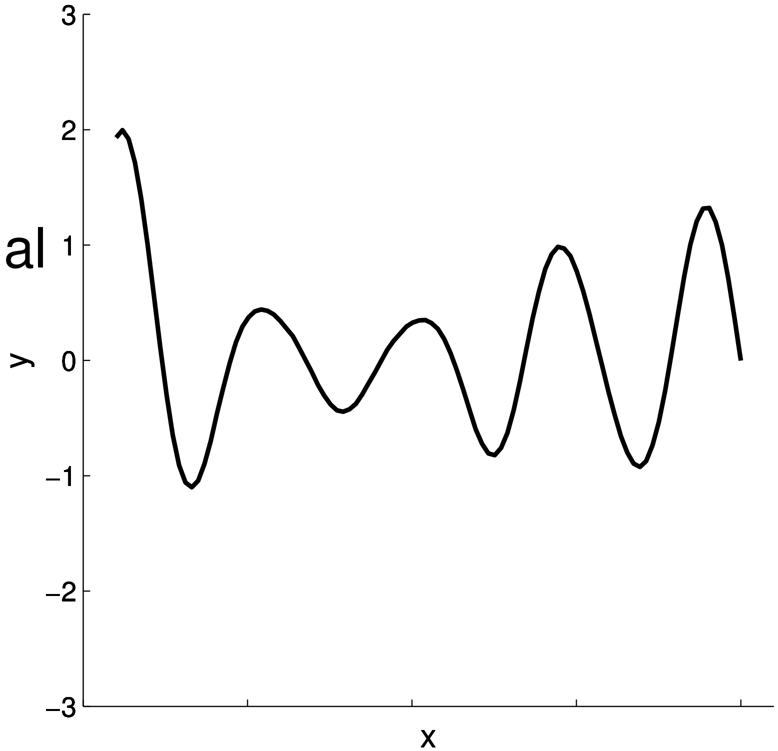
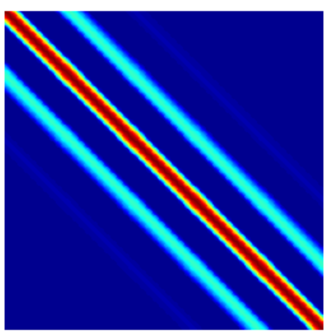


$\Sigma =$ 

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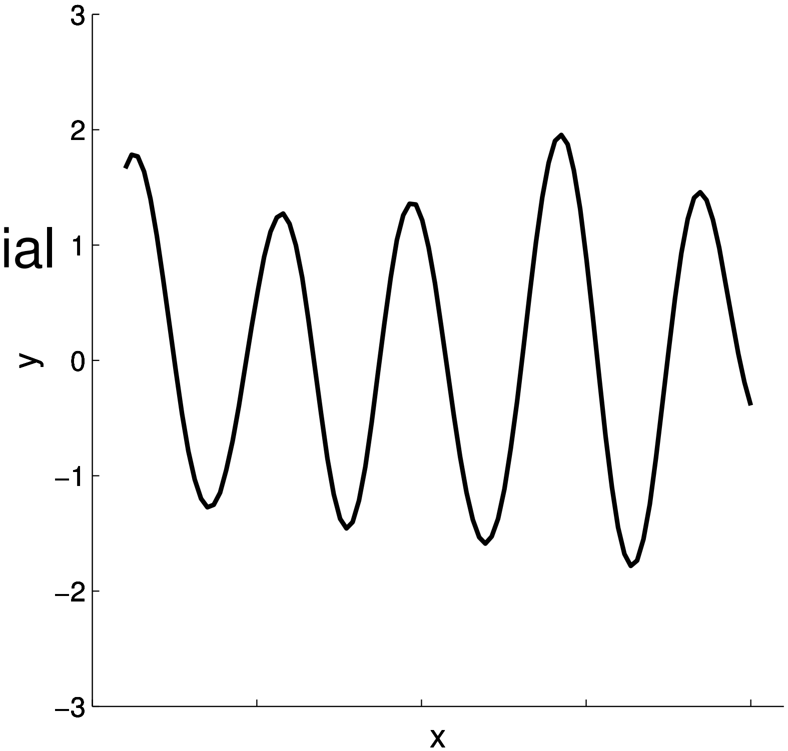
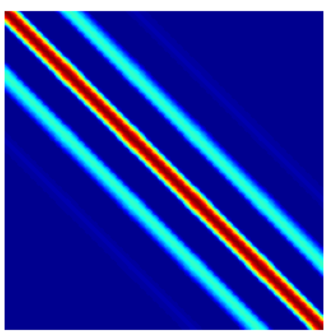


$\Sigma =$ 

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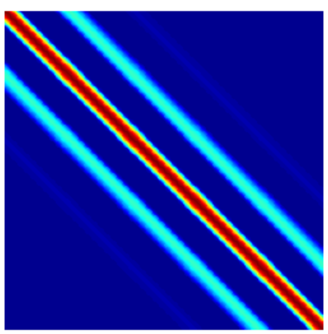
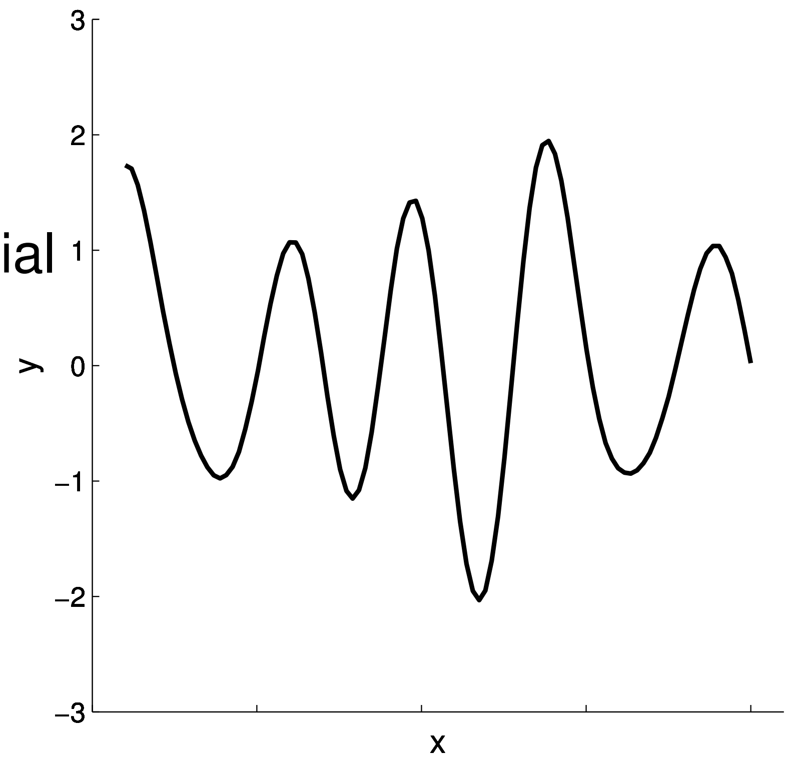
$\Sigma =$ 



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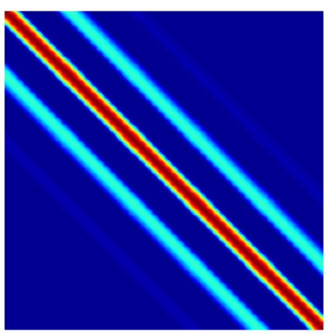
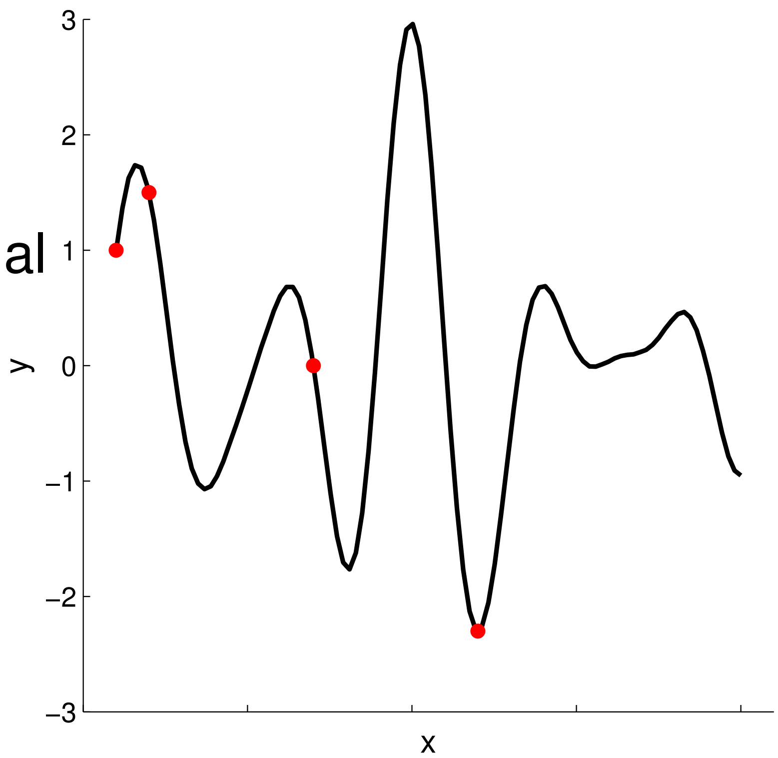
$\Sigma =$ 



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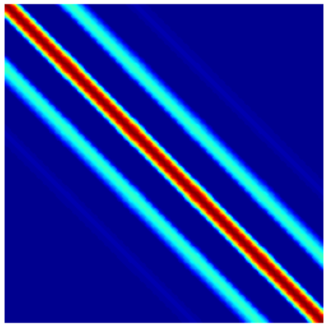
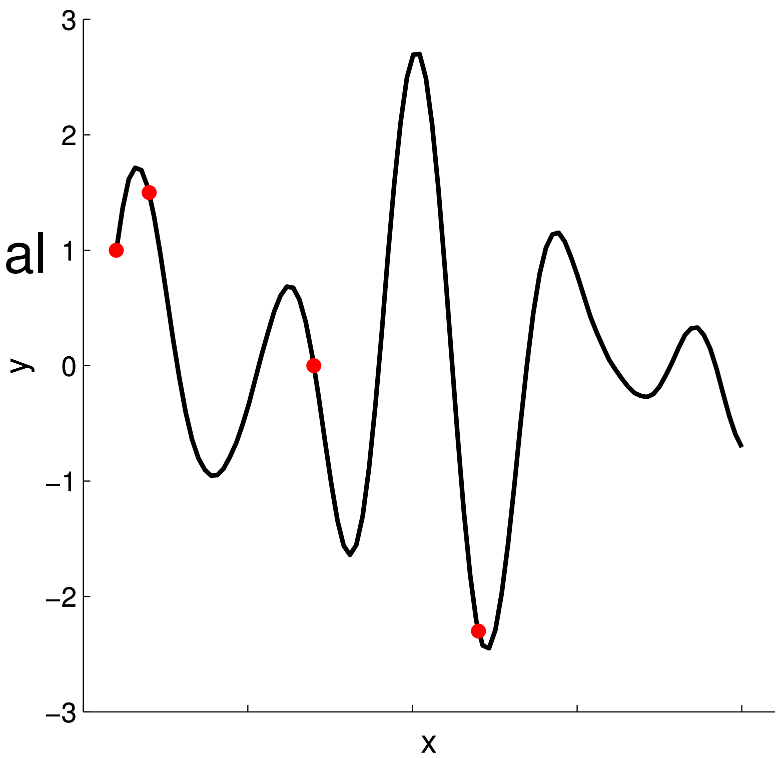
$\Sigma =$ 



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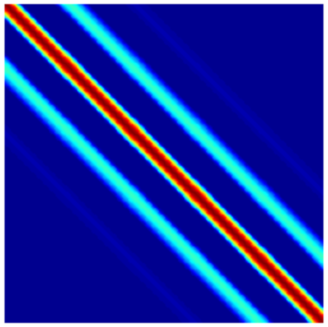
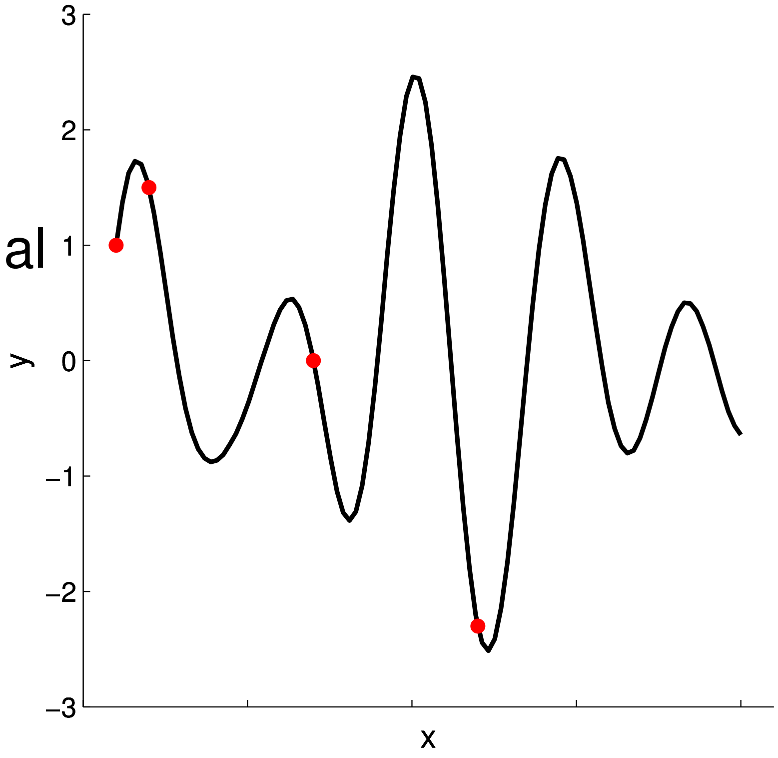
$\Sigma =$ 



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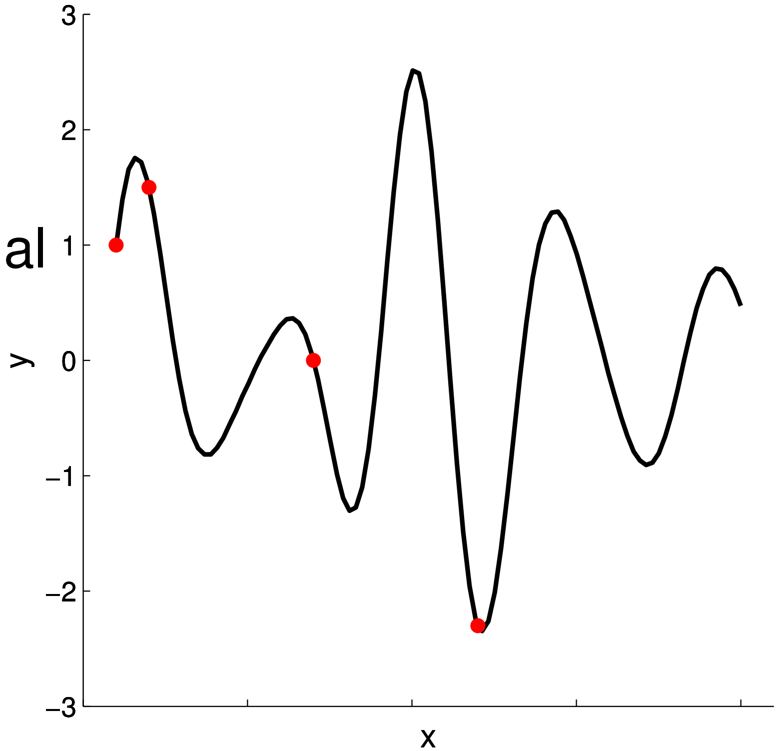
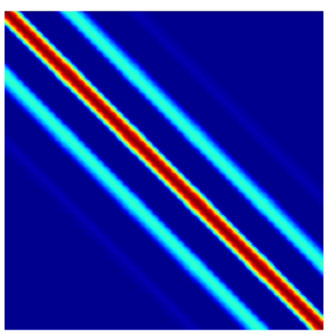


$\Sigma =$ 

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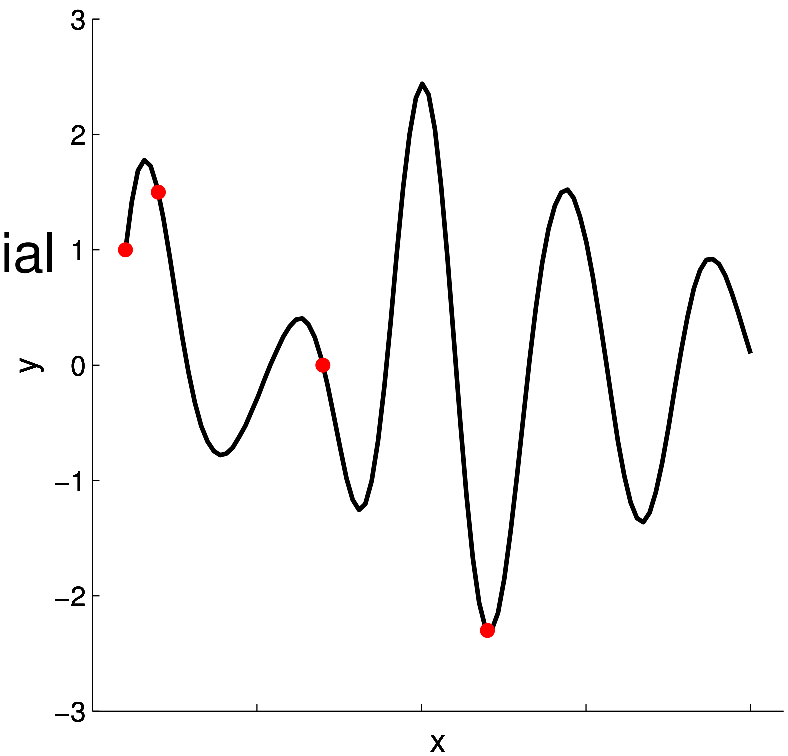
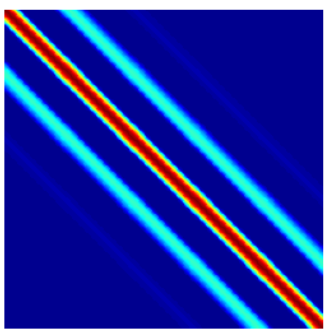


$\Sigma =$ 

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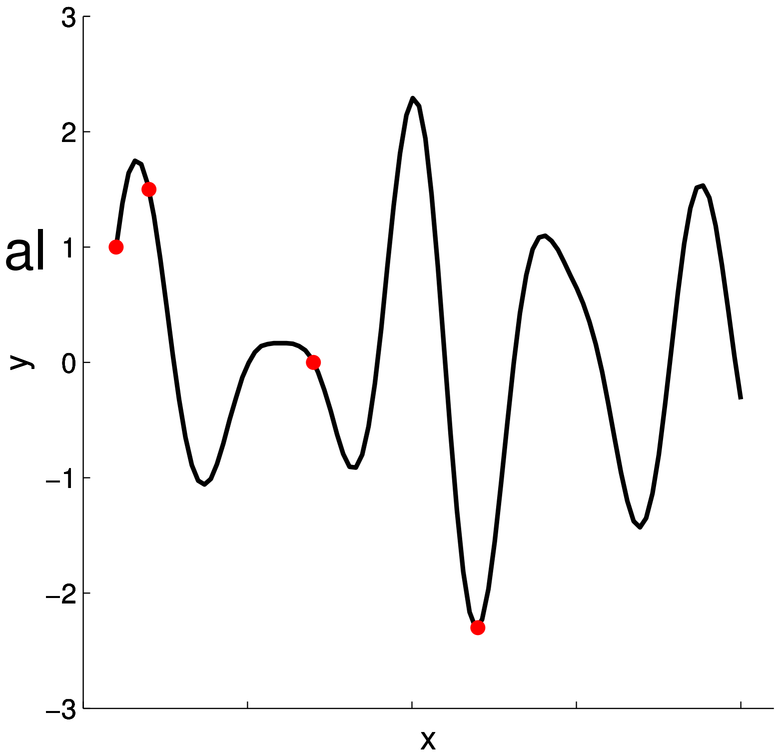

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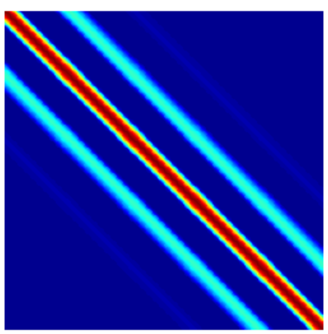

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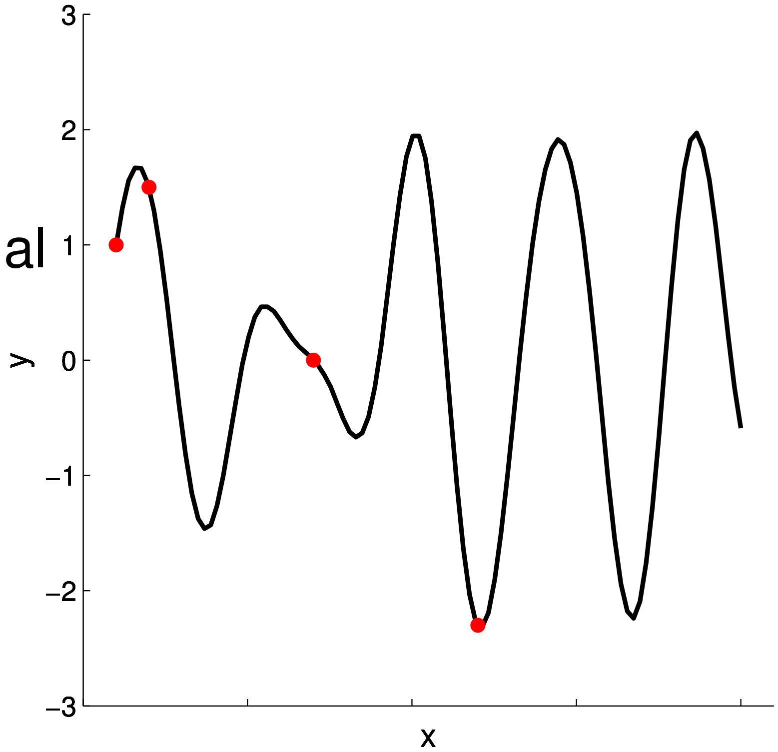
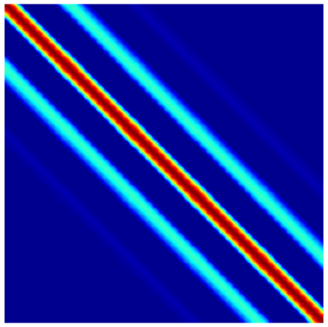


$\Sigma =$ 

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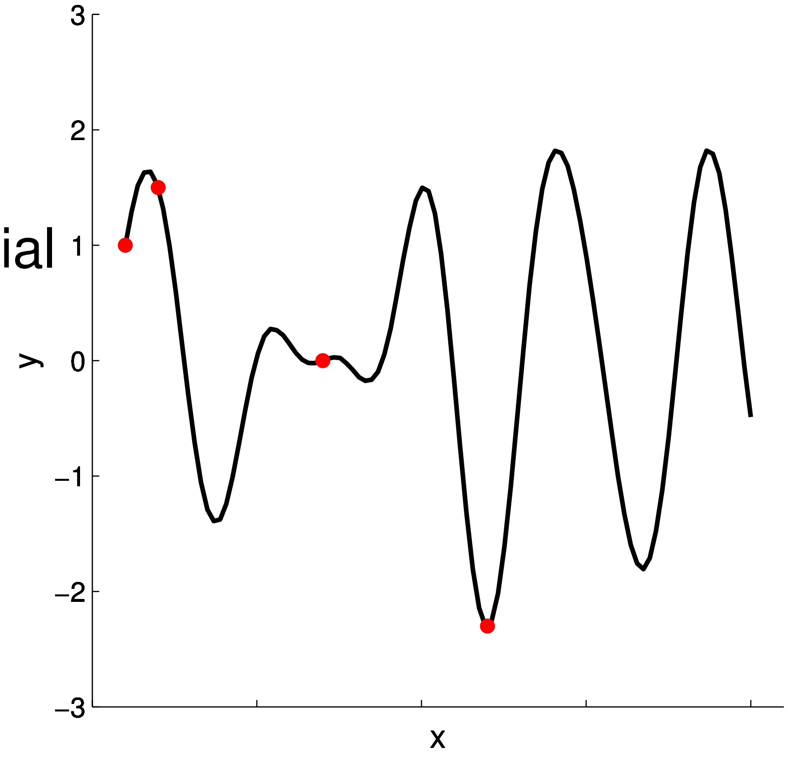
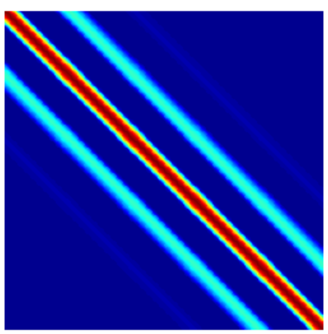


$\Sigma =$ 

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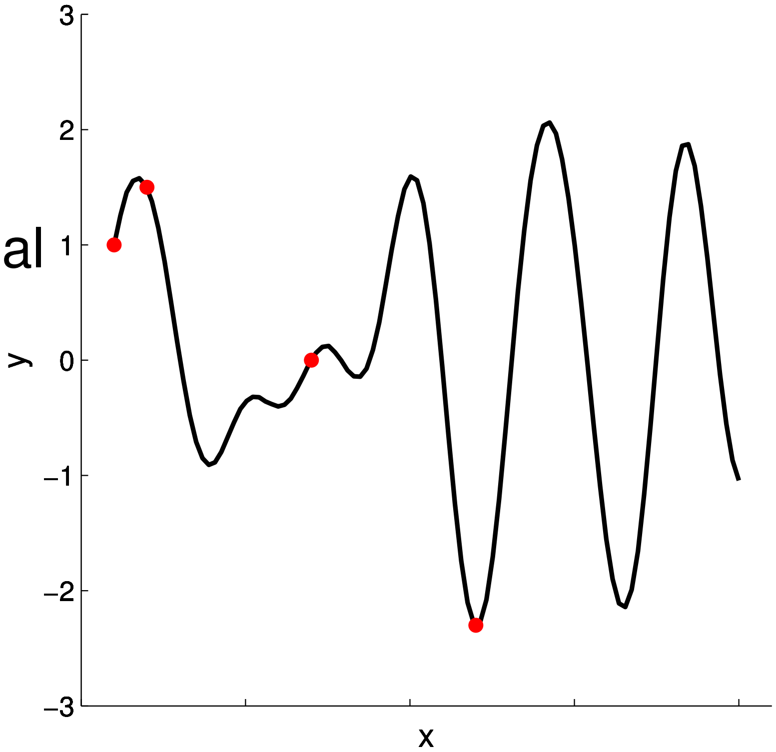
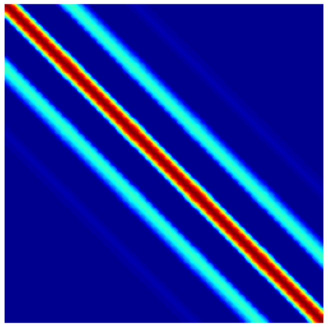


$\Sigma =$ 

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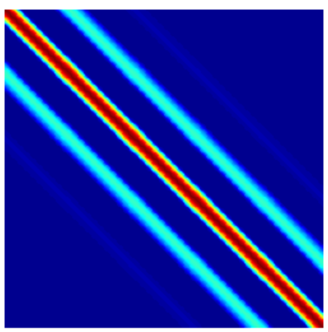
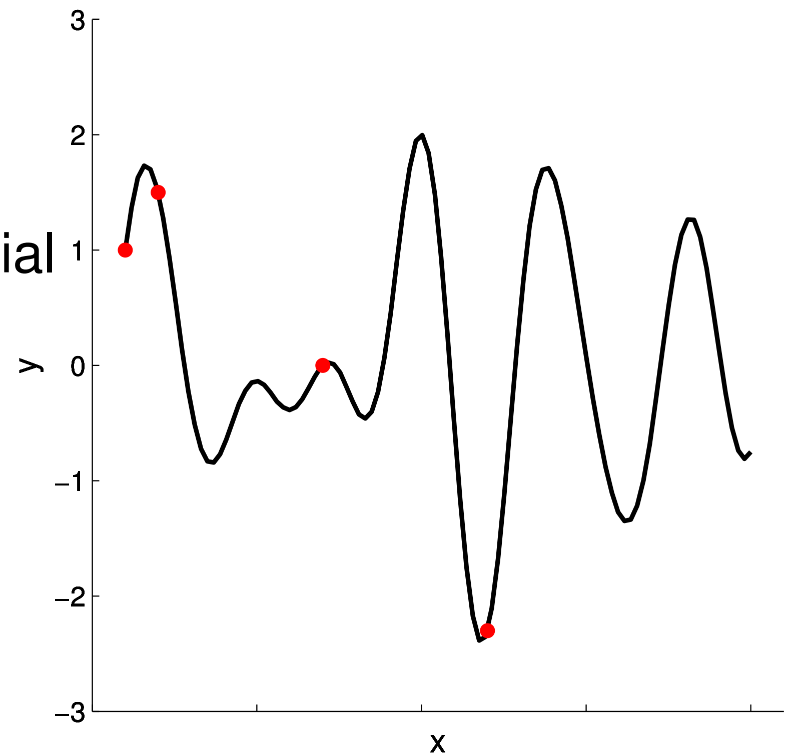
$\Sigma =$ 



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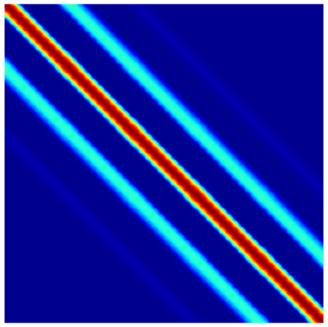
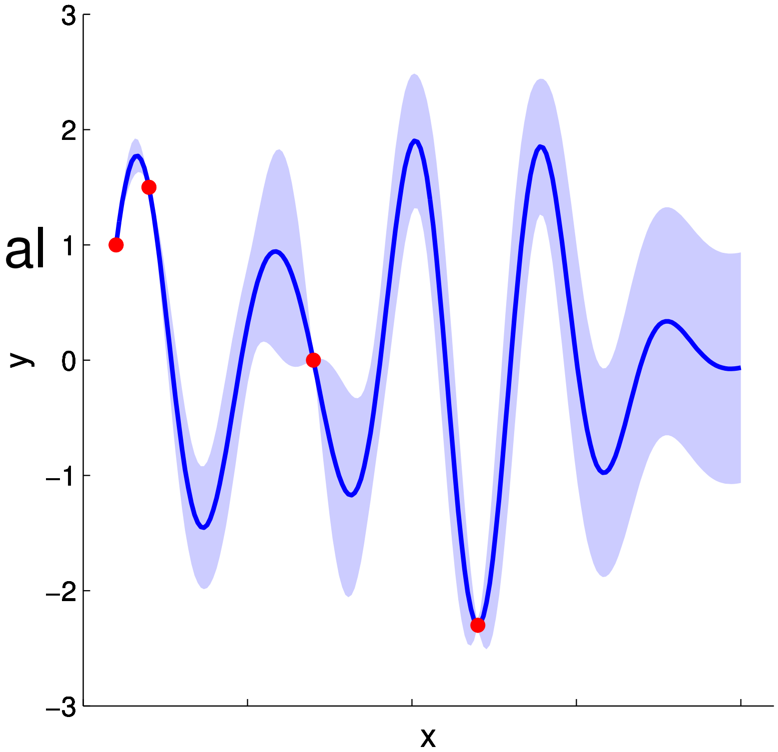
$\Sigma =$ 



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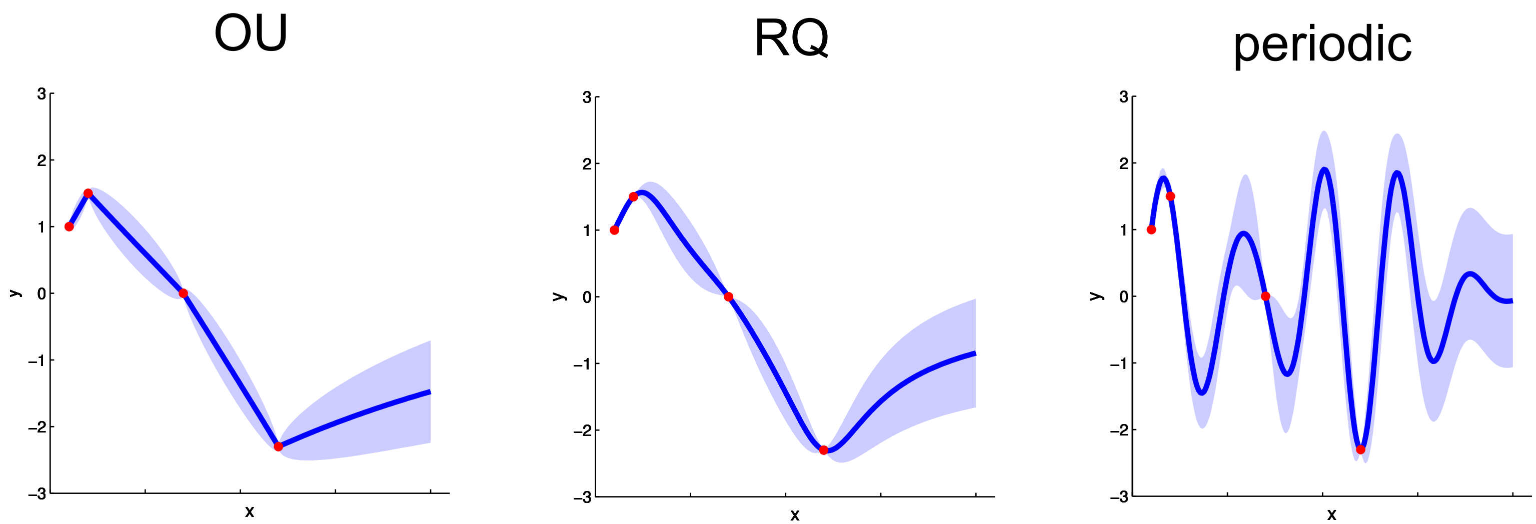

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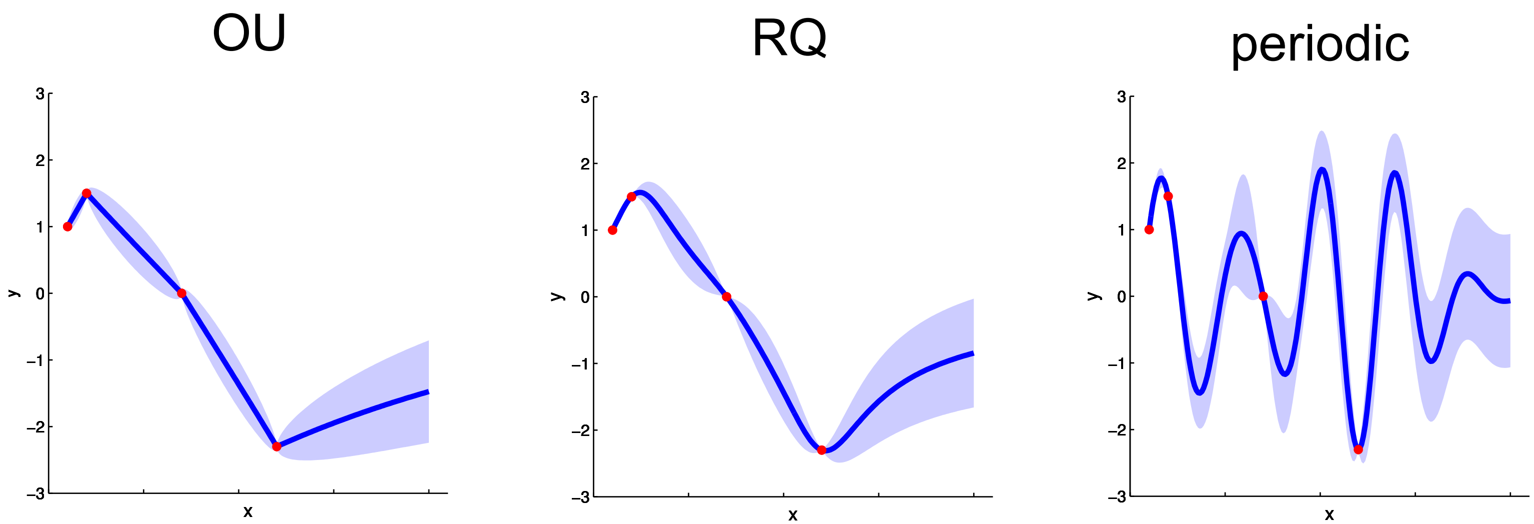

Bayesian model comparison:

$$p(M|\mathbf{y}_{1:N}) = \frac{p(\mathbf{y}_{1:N}|M)p(M)}{\sum_{M'} p(\mathbf{y}_{1:N}|M')p(M')}$$

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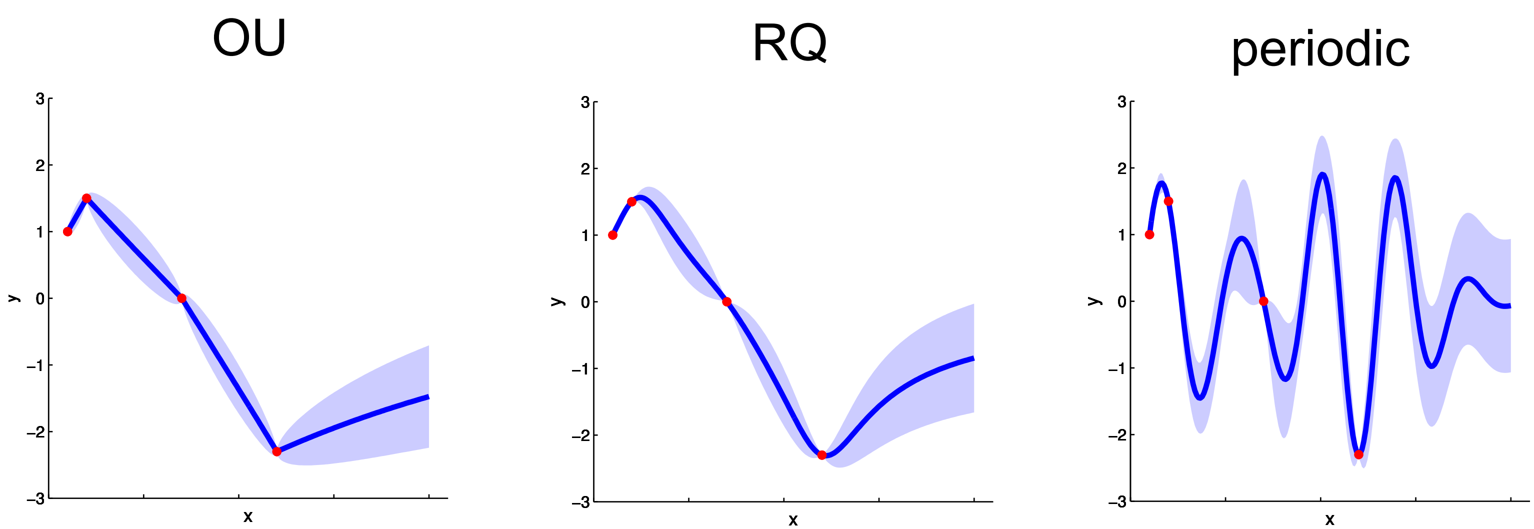

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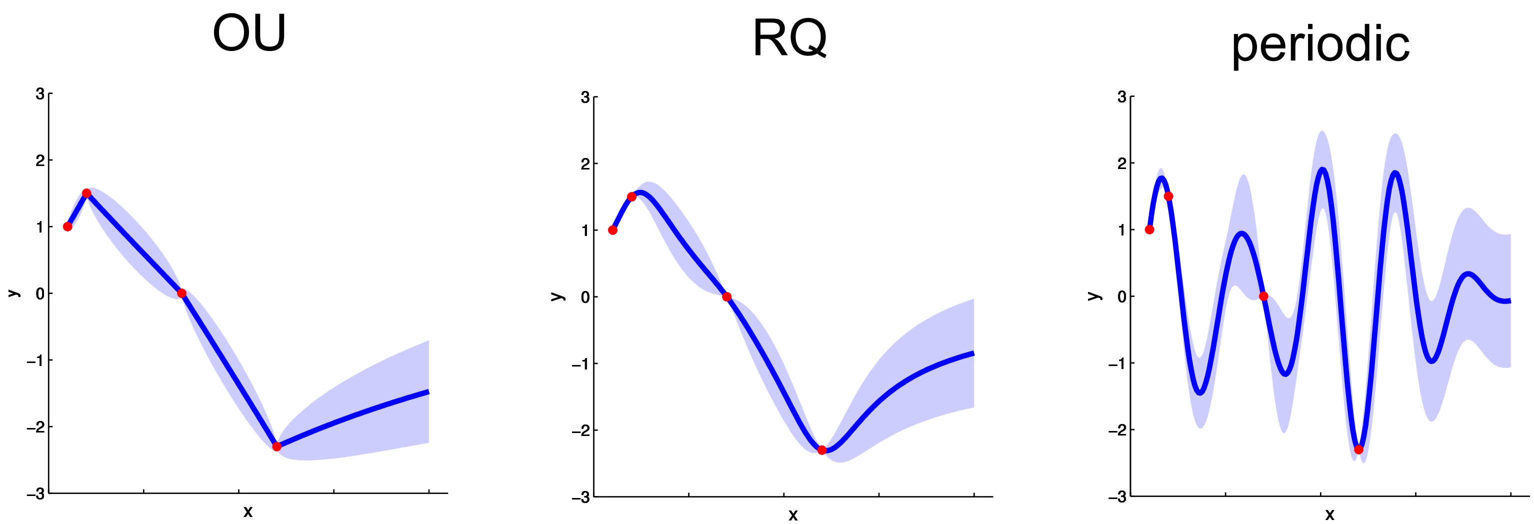

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$$\color{red}{p(M|\mathbf{y}_{1:N})} = \frac{\color{green}{p(\mathbf{y}_{1:N}|M)}\color{blue}{p(M)}}{\sum_{M'} p(\mathbf{y}_{1:N}|M')p(M')}$$

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marginal likelihood $\color{green}{p(\mathbf{y}_{1:N}|M) = \int \mathrm{d}\theta \; p(\mathbf{y}_{1:N}|\theta, M)p(\theta|M)}$

Making new covariance functions from old

(positive) linear combinations
of covariance functions

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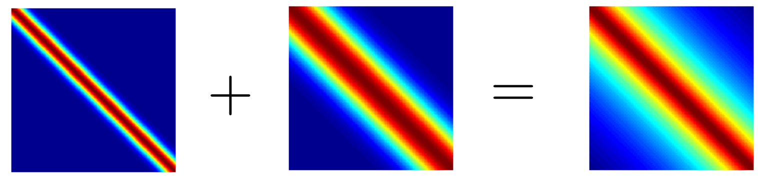

e.g. scale mixture of SE = rational quadratic

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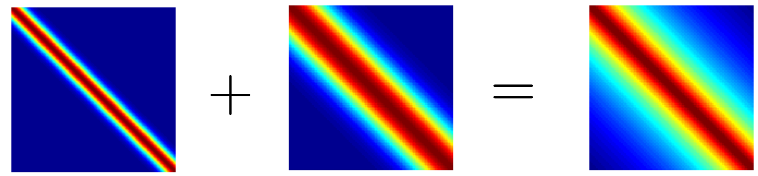

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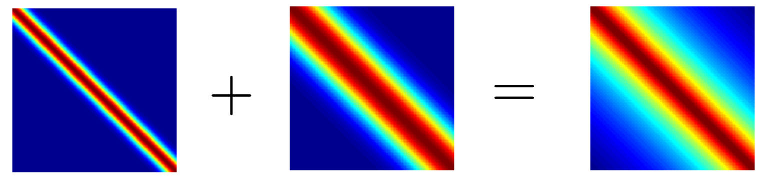

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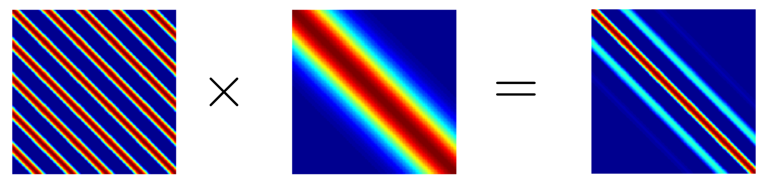

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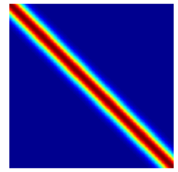
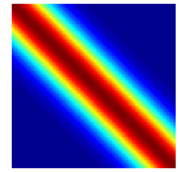
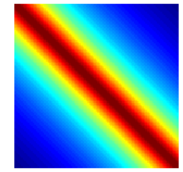
 $+$  $=$ 

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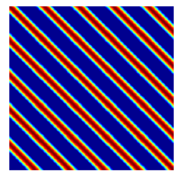
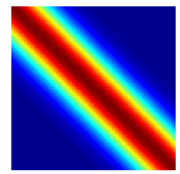
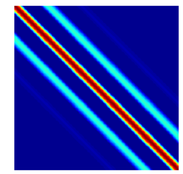
 $\times$  $=$ 

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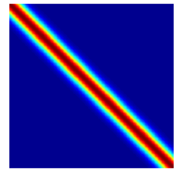
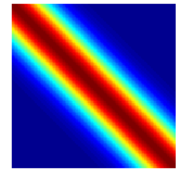
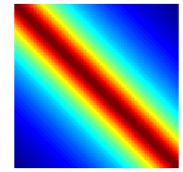
 $+$  $=$ 

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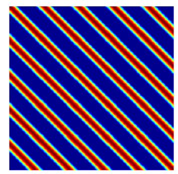
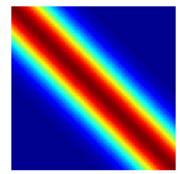
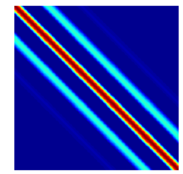
 $\times$  $=$ 

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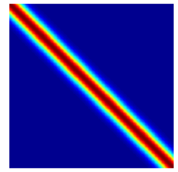
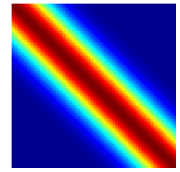
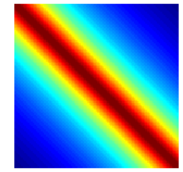
 $+$  $=$ 

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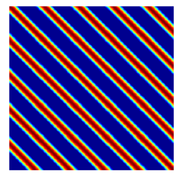
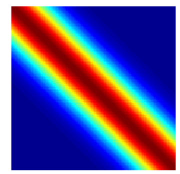
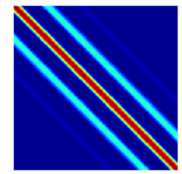
 $\times$  $=$ 

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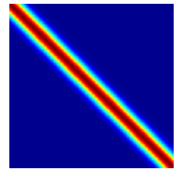
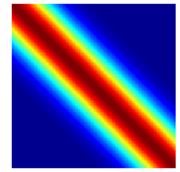
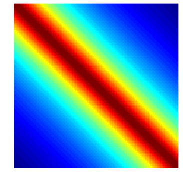
 $+$  $=$ 

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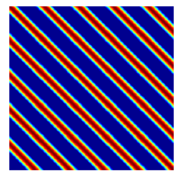
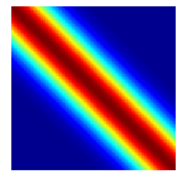
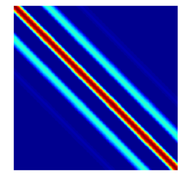
 $\times$  $=$ 

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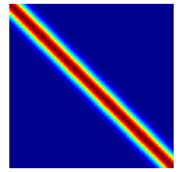
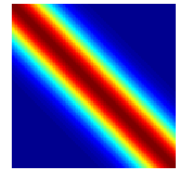
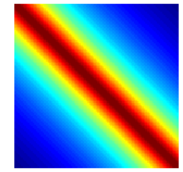
 $+$  $=$ 

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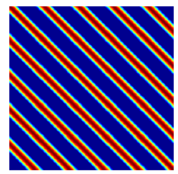
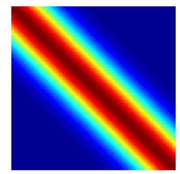
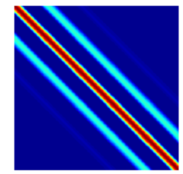
 $\times$  $=$ 

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new basis: $g'_k(x) = \frac{d}{dx} g_k(x)$

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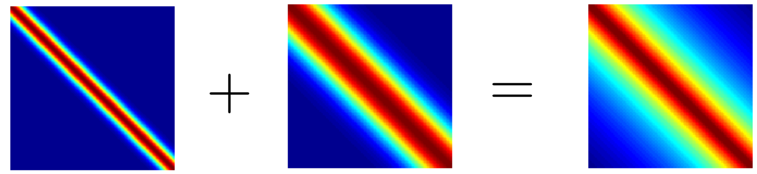

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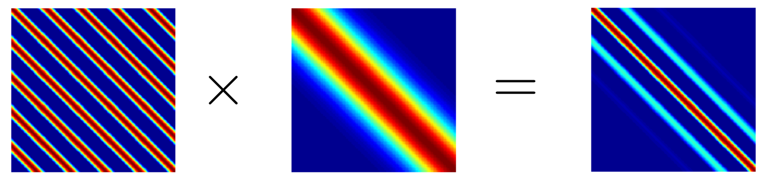

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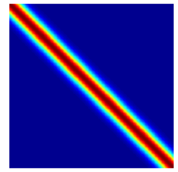
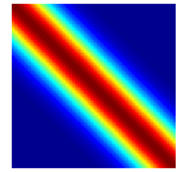
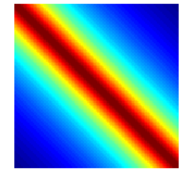
 $+$  $=$ 

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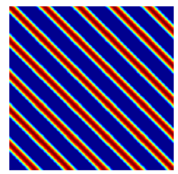
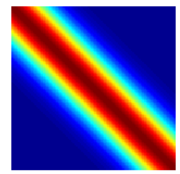
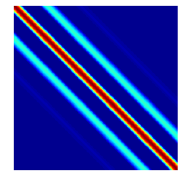
 $\times$  $=$ 

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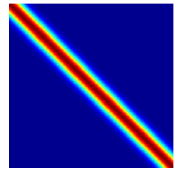
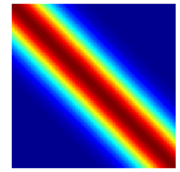
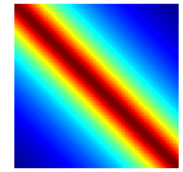
 $+$  $=$ 

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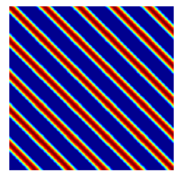
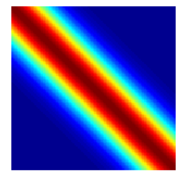
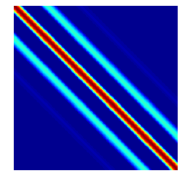
 $\times$  $=$ 

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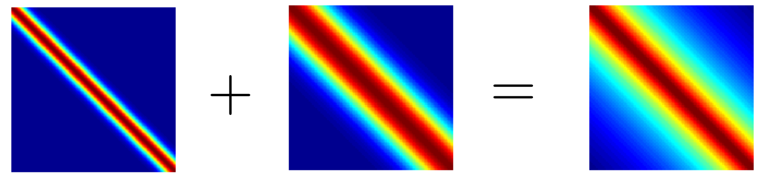

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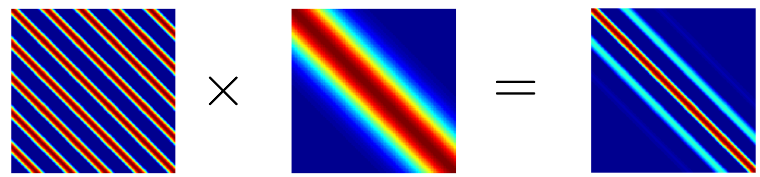

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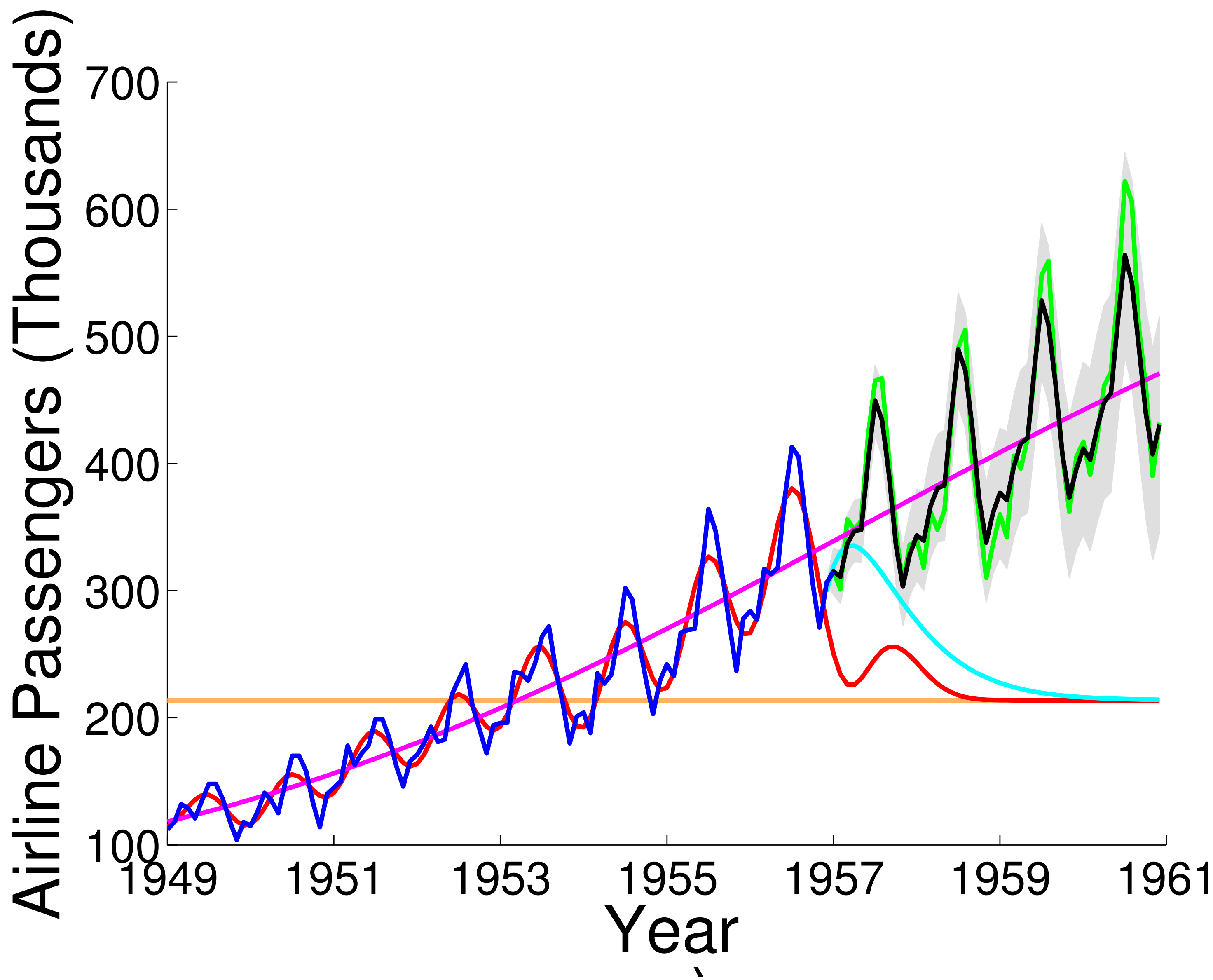
Meta-kernel, learn which is right directly

$$k(\tau) = \sum_{q=1}^{Q} w_q \prod_{p=1}^{P} \exp\{-2\pi^2 \tau_p^2 v_q^{(p)}\} \cos(2\pi \tau_p \mu_q^{(p)}).$$



Deep GPs

$$y(x) = f(g(x)) + \sigma_{\mathrm{y}}\epsilon$$

$$f(x) = \mathcal{GP}(0, K_f(x, x'))$$
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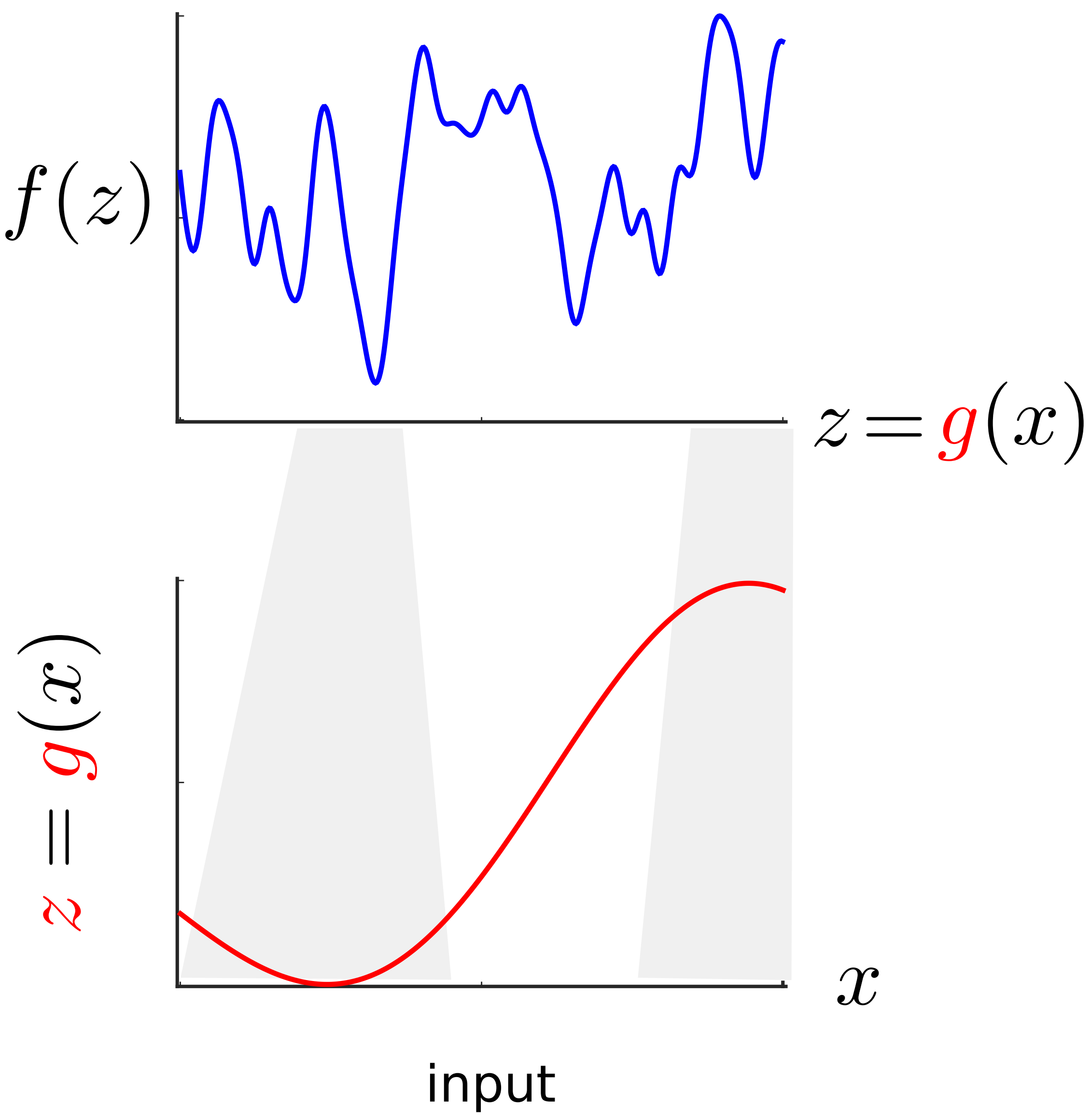

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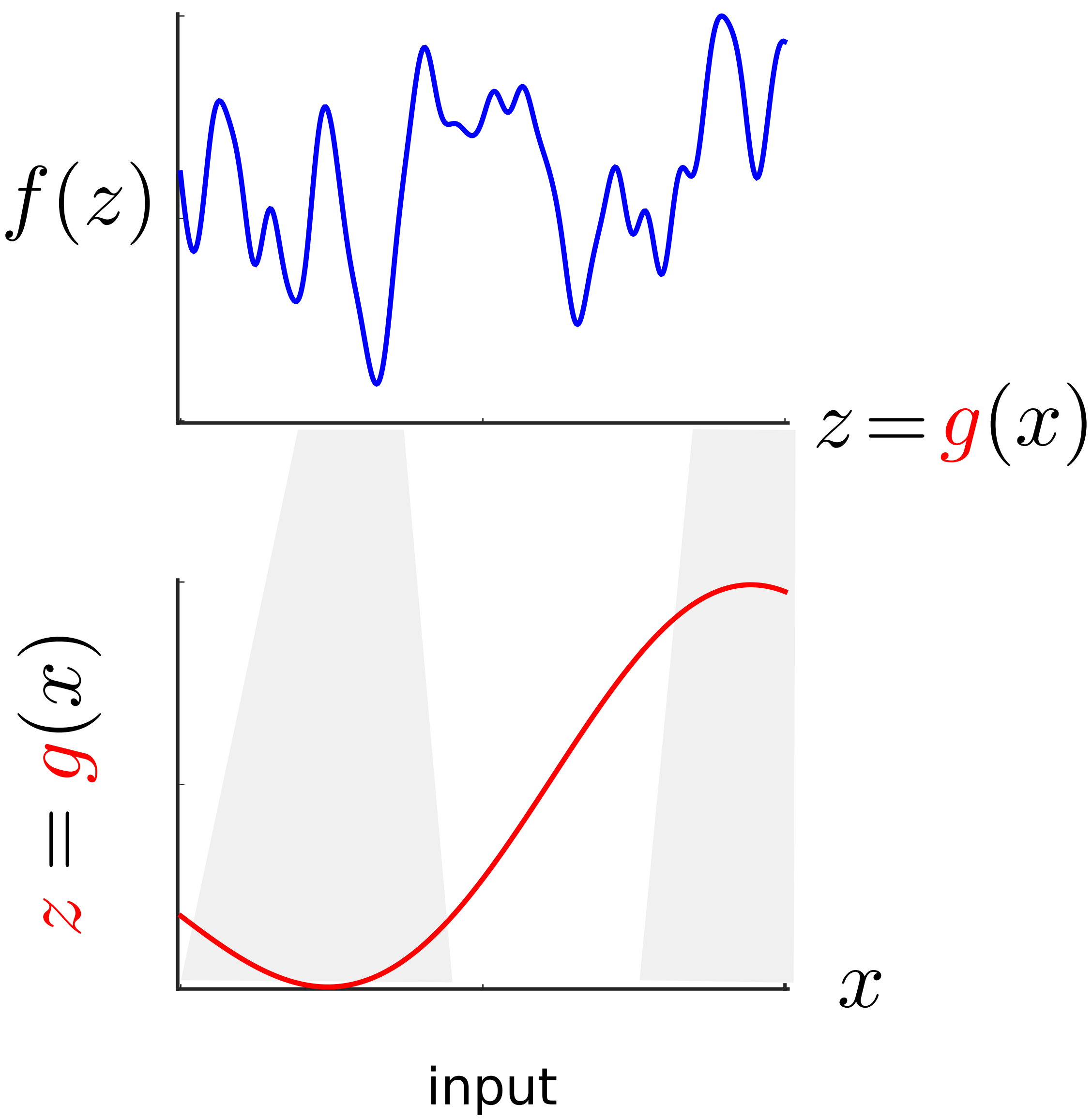

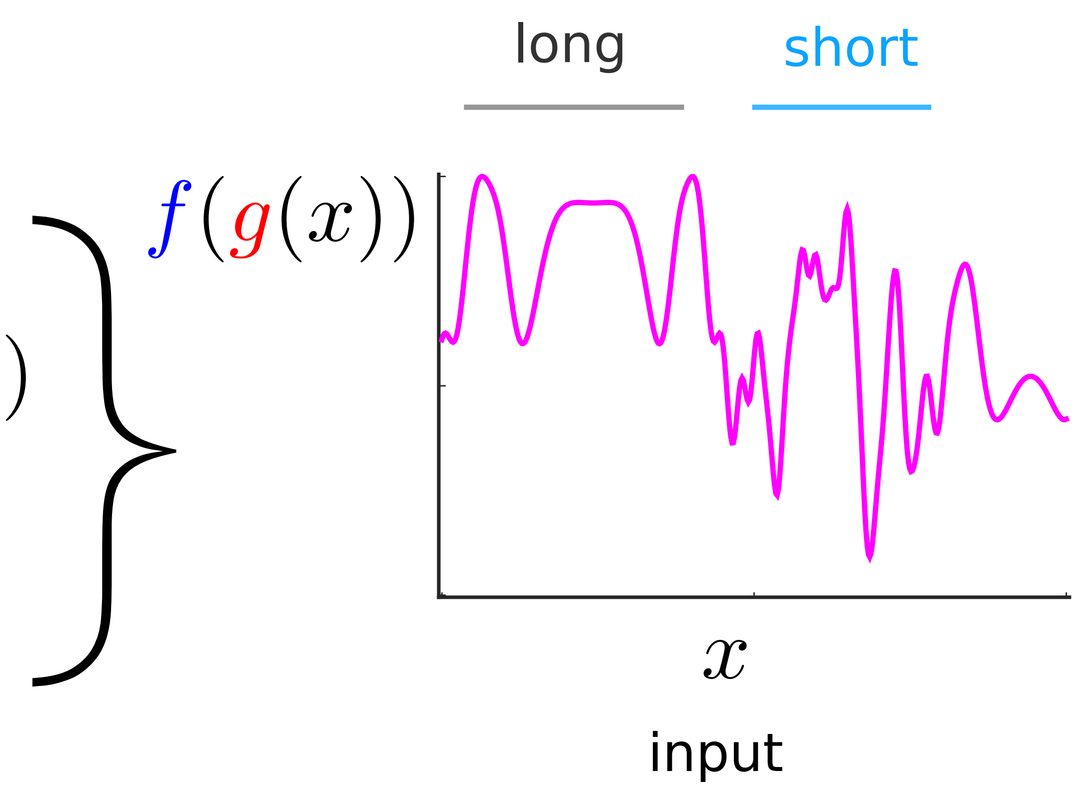

DGP = multi-layer neural network,
infinitely wide hidden layer

How do we choose the hyper-parameters?

idea: use probability distributions to represent plausibility of hyper-parameters (uncertainty) given the data

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$p(\mathbf{y}_{1:N}|\theta)$ = likelihood of the parameters

= how well did θ predict the data we observed

$$p(\mathbf{y}_{1:N}|\theta) = \frac{1}{\det(2\pi\Sigma(\theta))^{-1/2}} \exp\left(-\frac{1}{2}\mathbf{y}_{1:N}^{\mathsf{T}}\Sigma^{-1}(\theta)\mathbf{y}_{1:N}\right)$$

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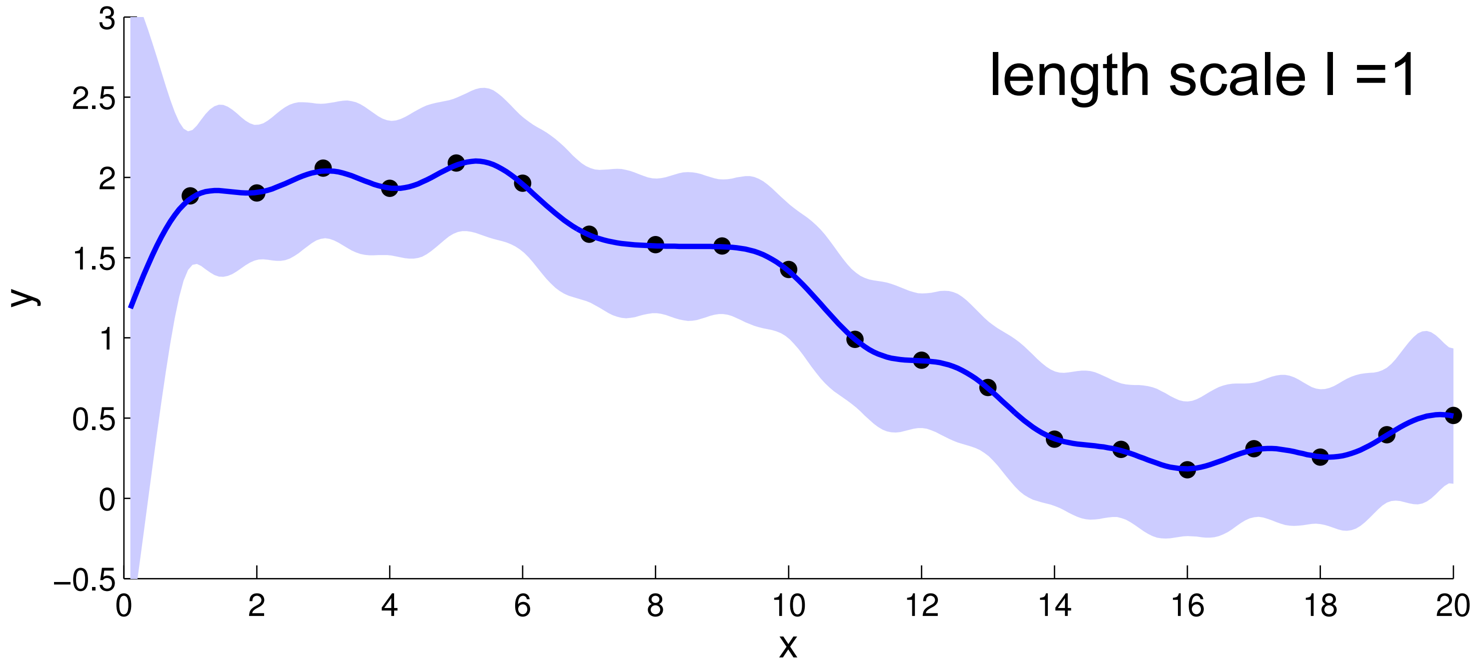

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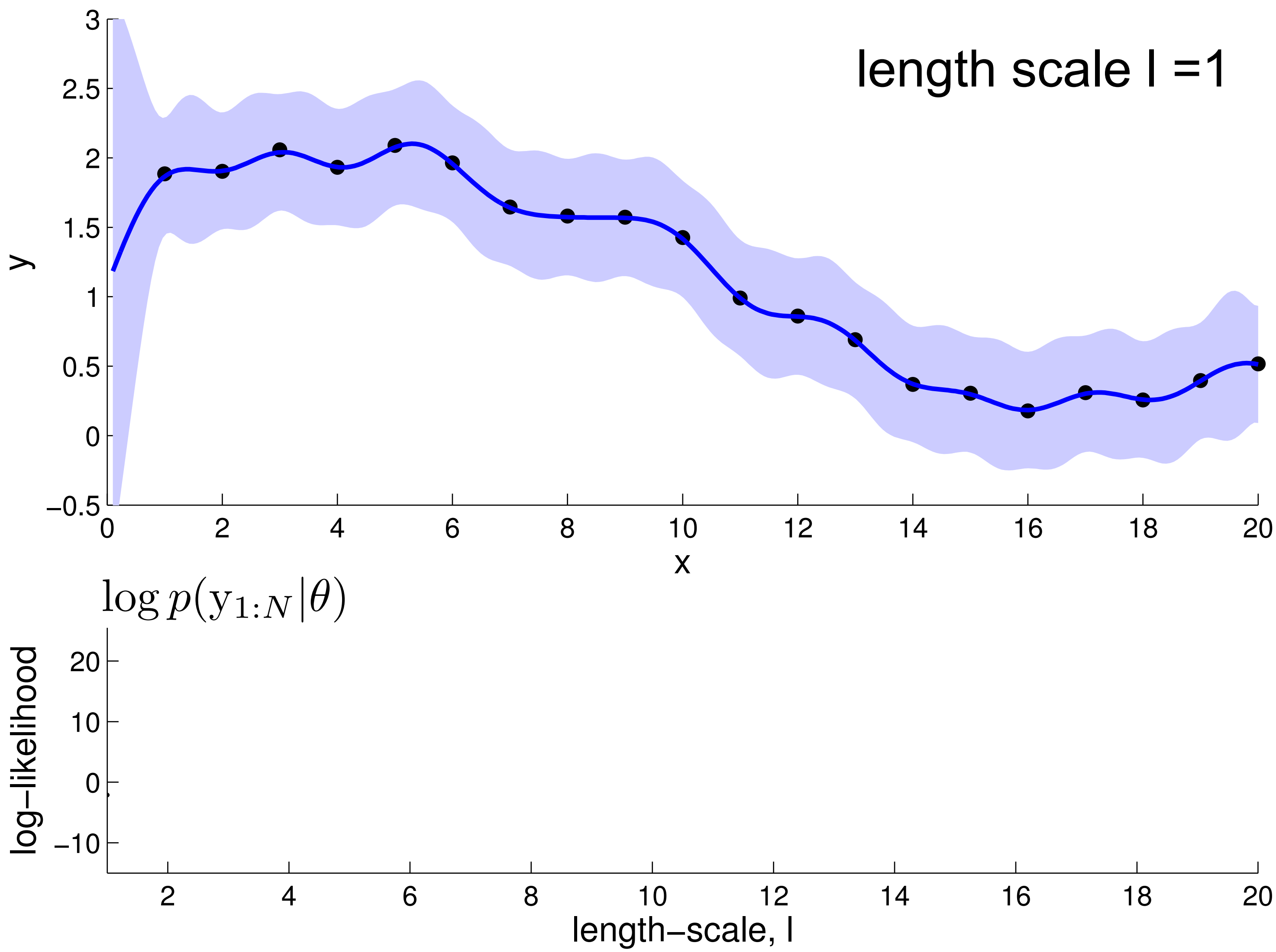

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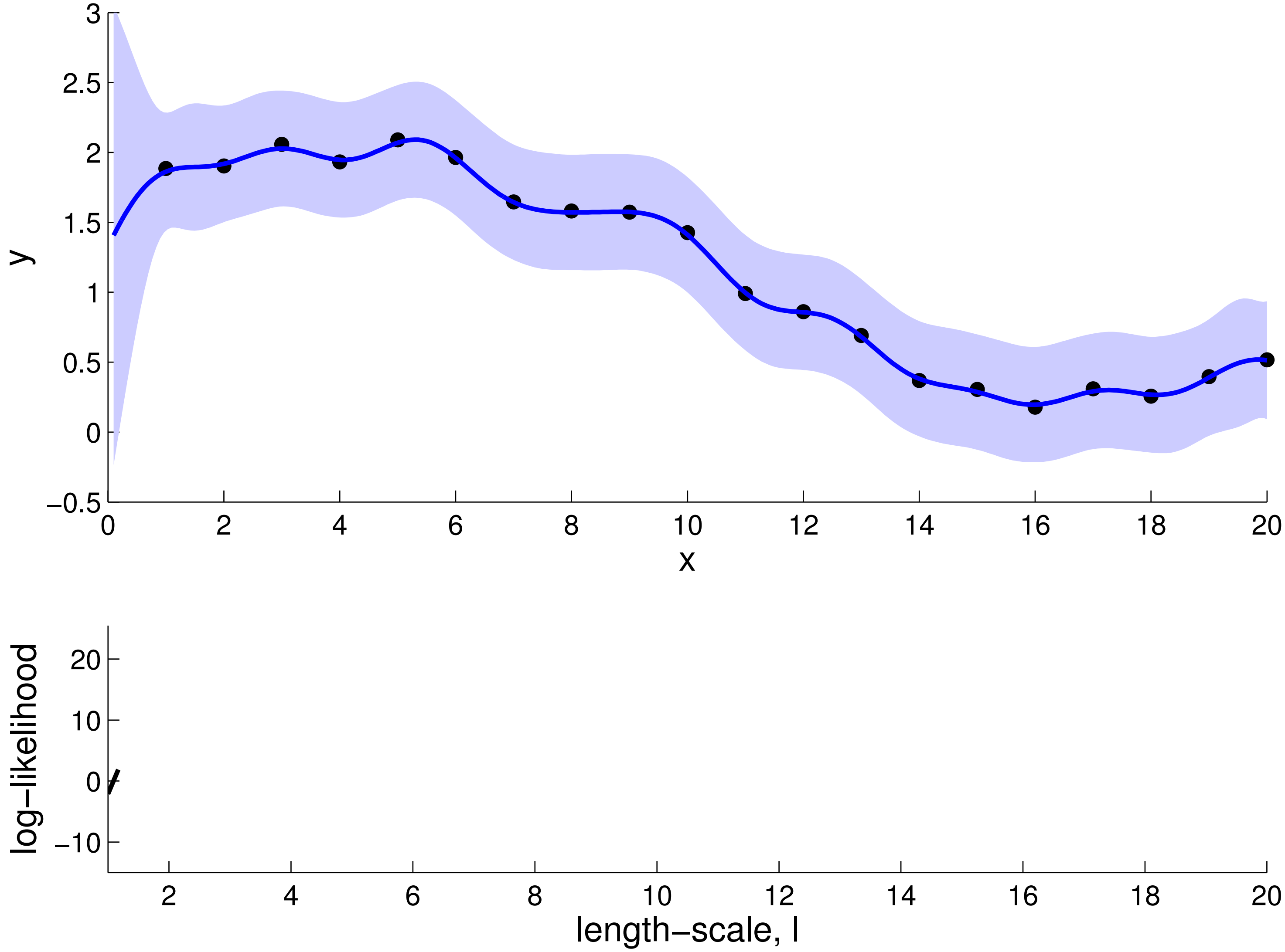

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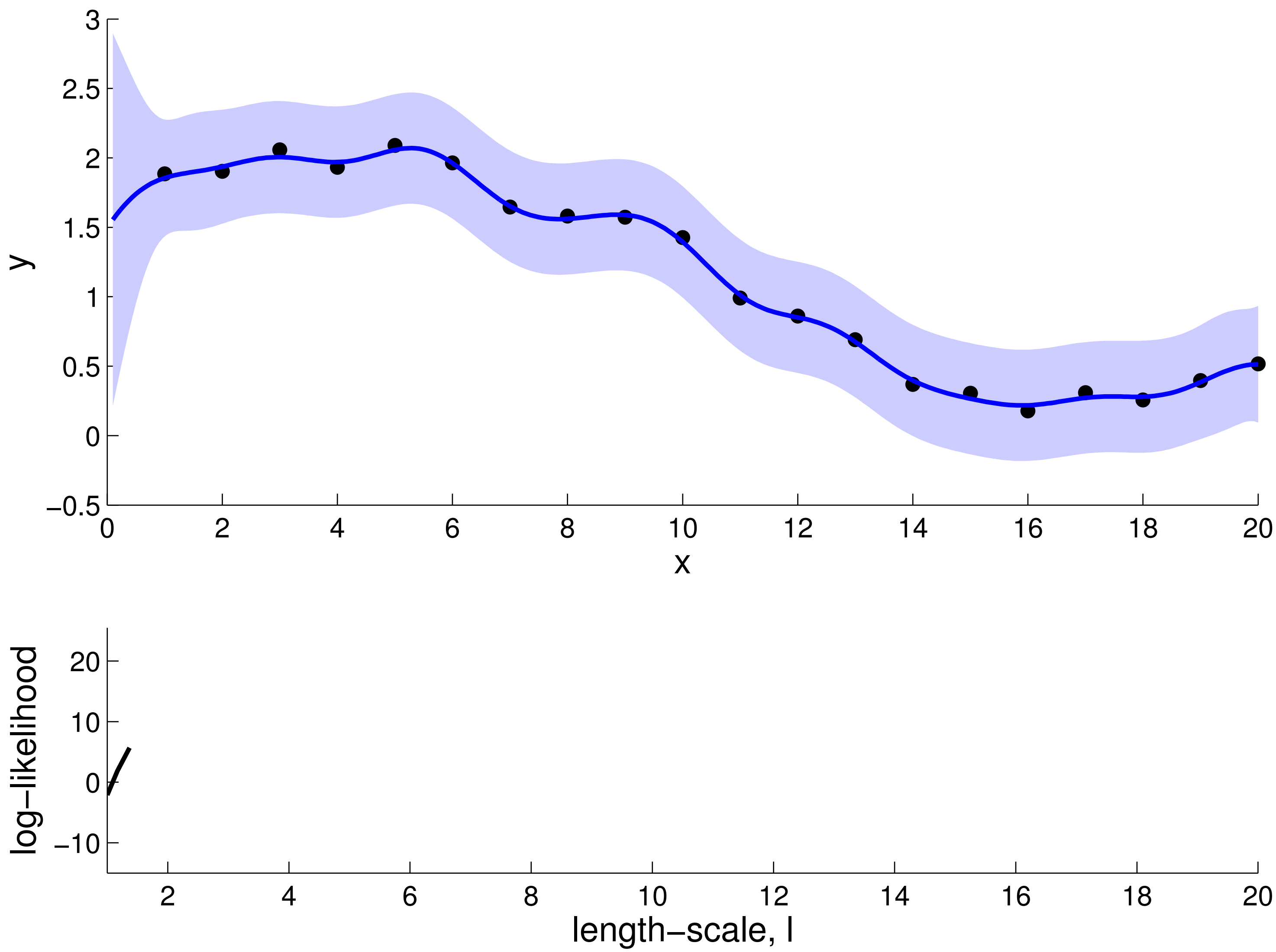

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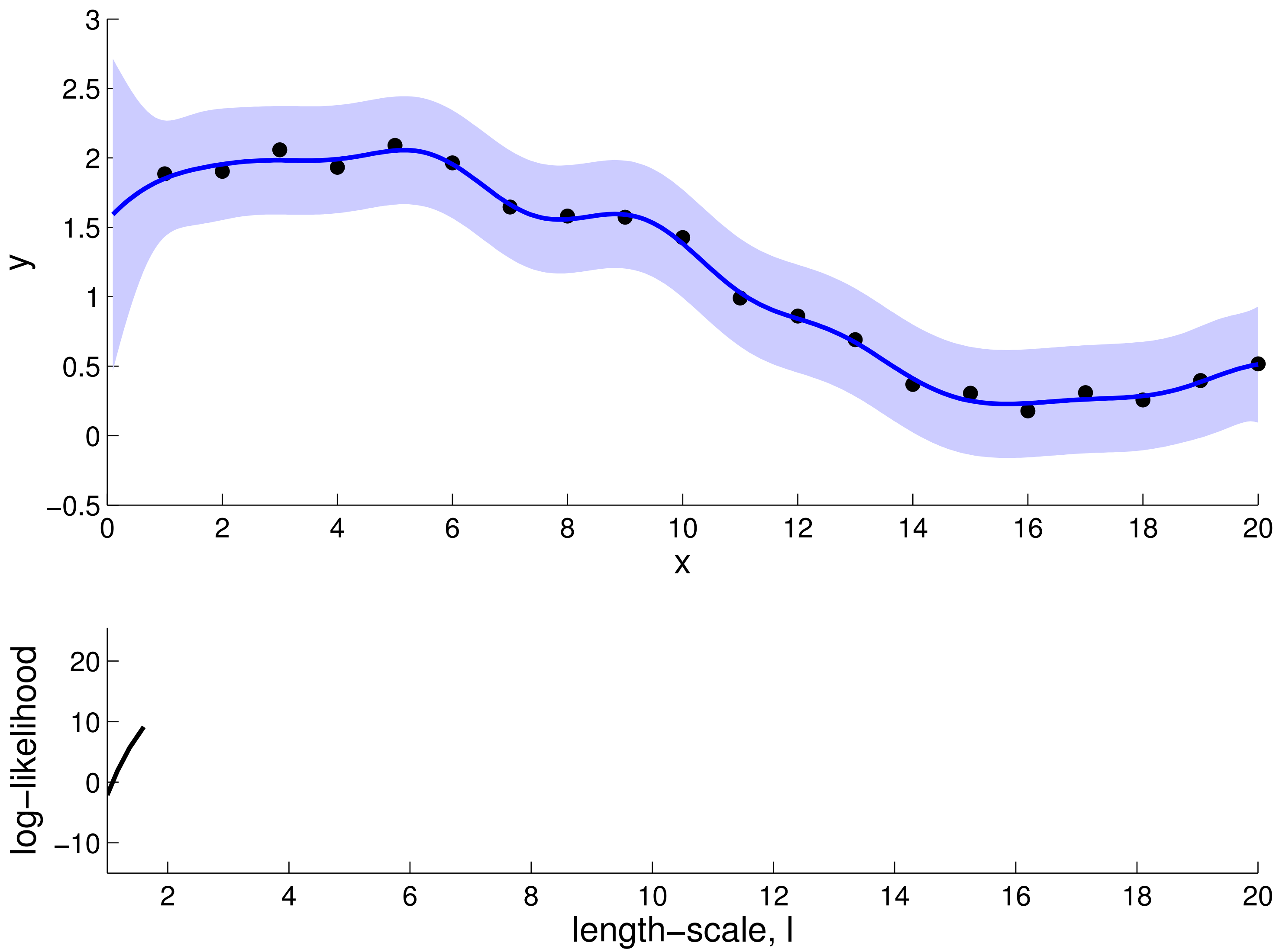

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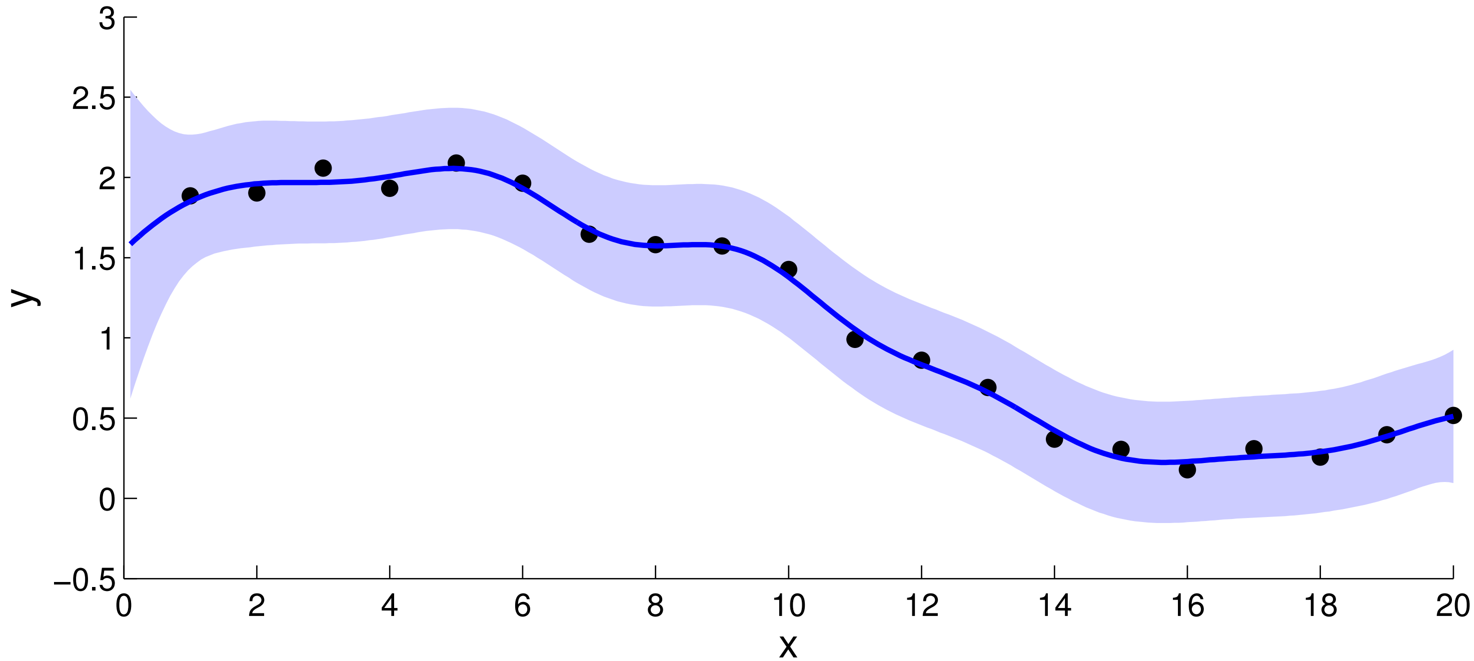

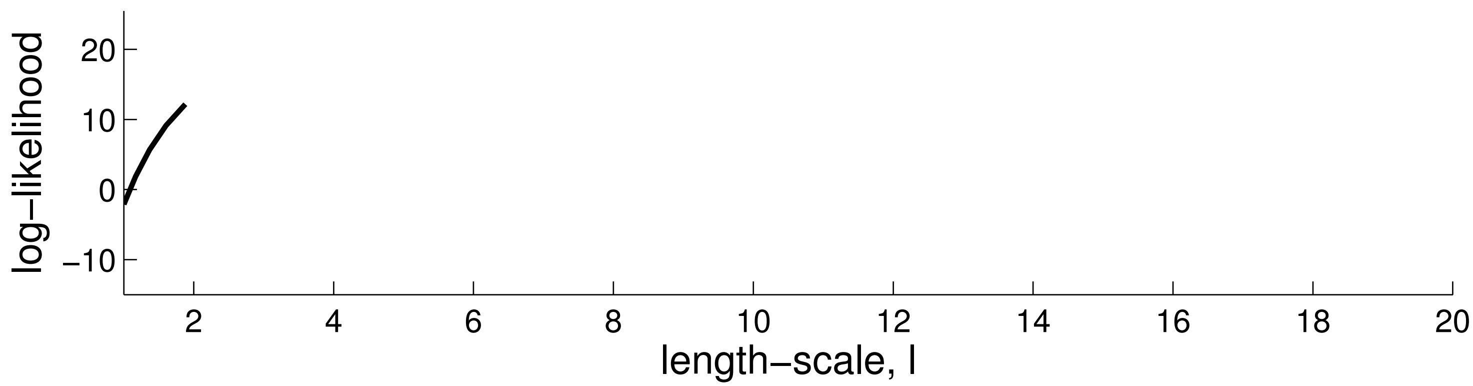

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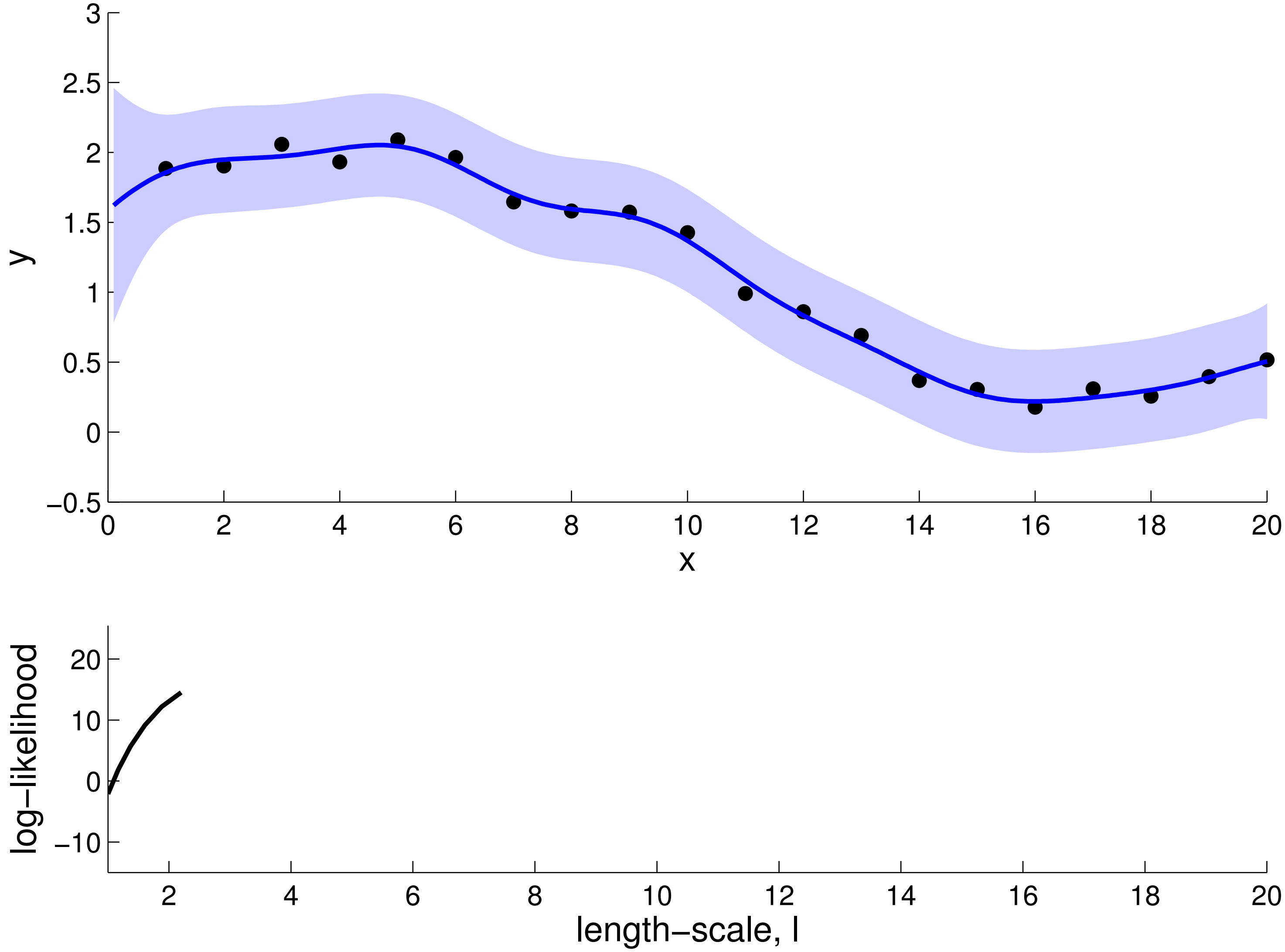

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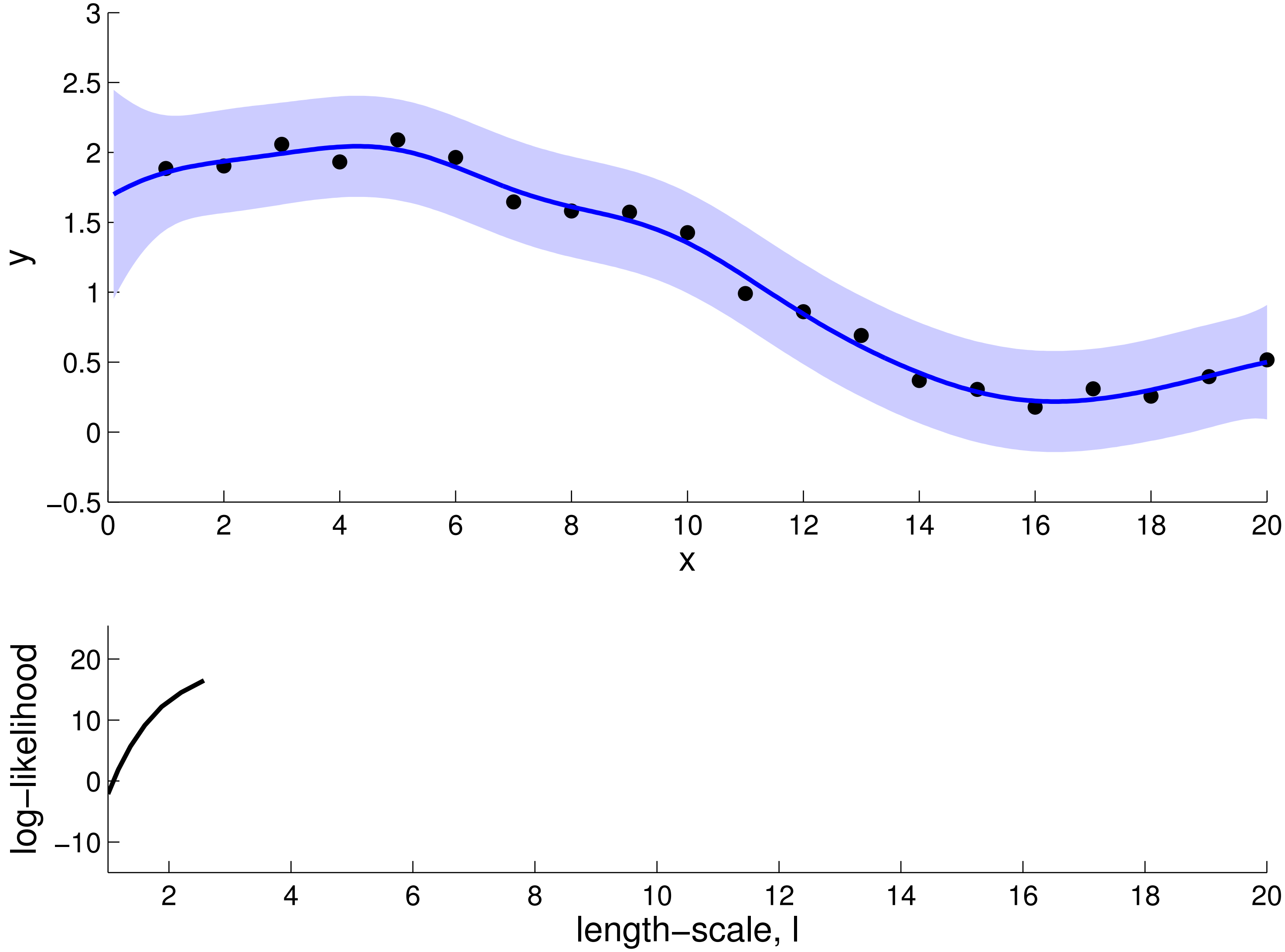

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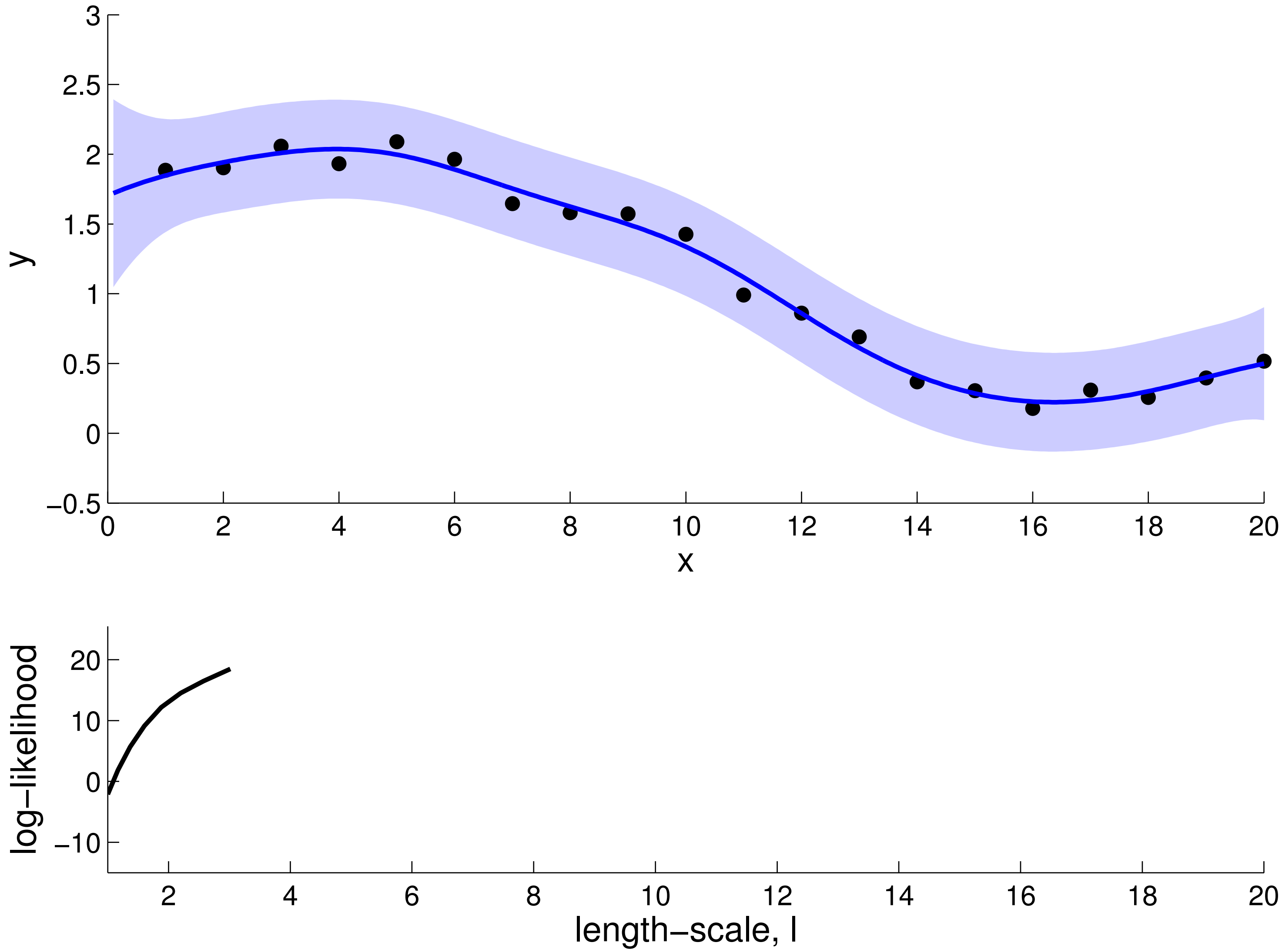

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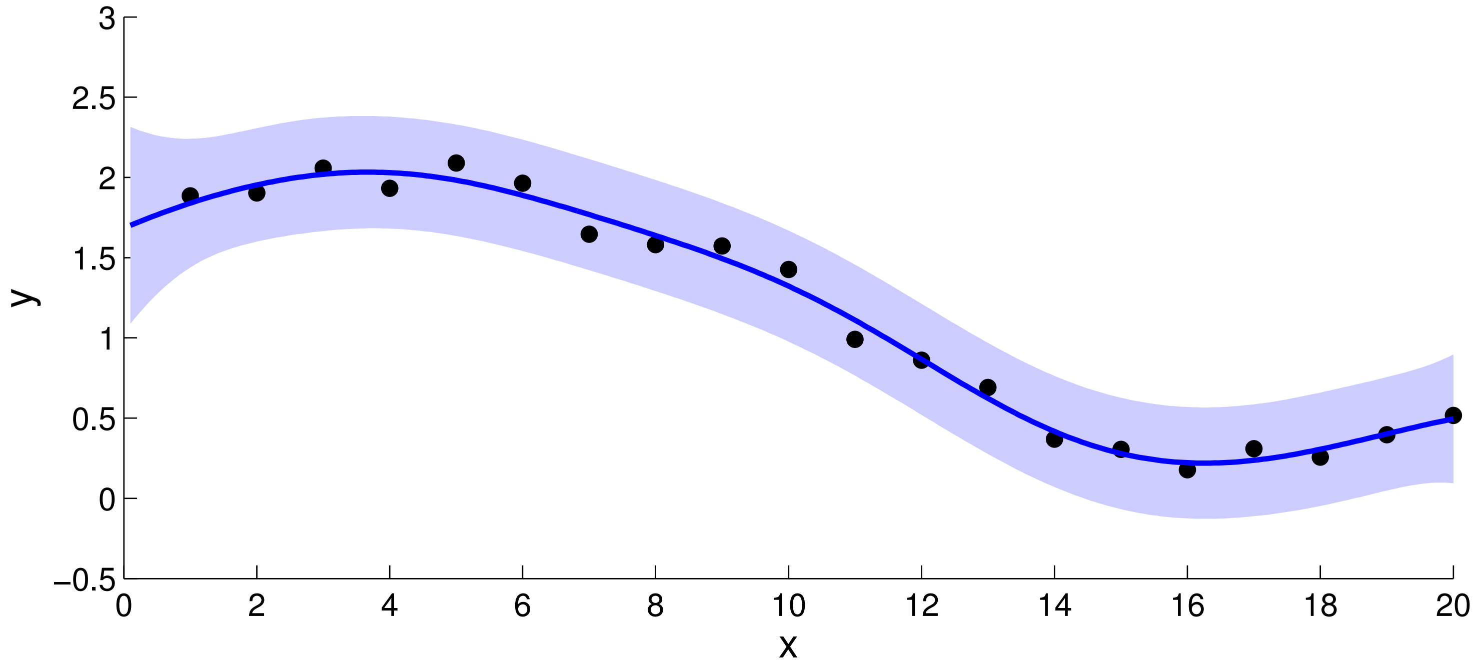

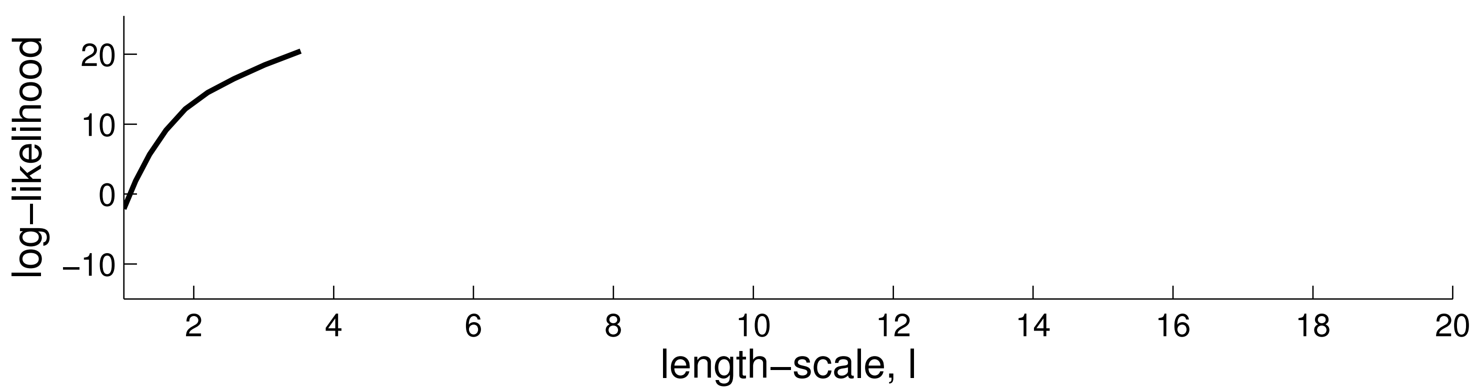

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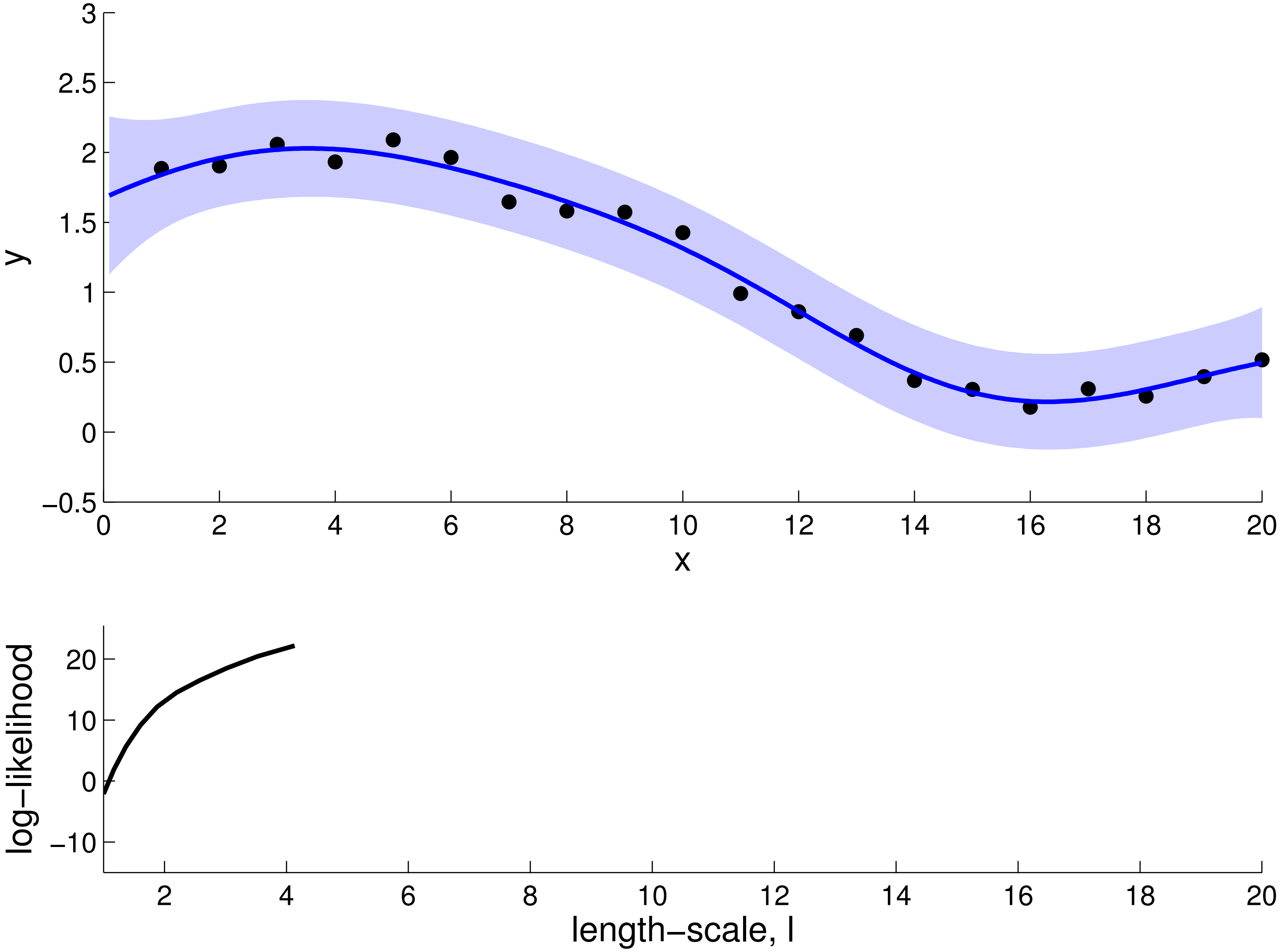

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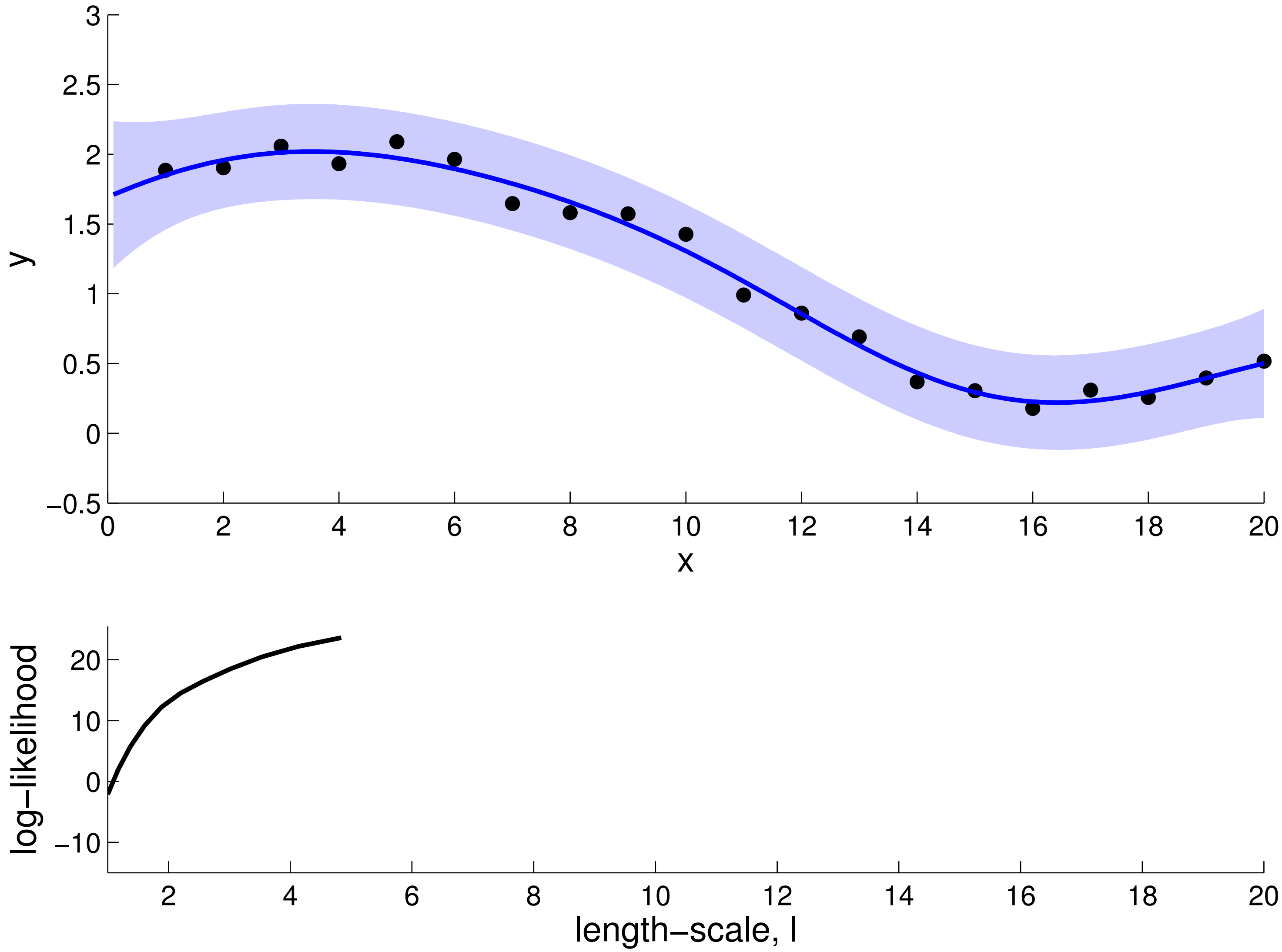

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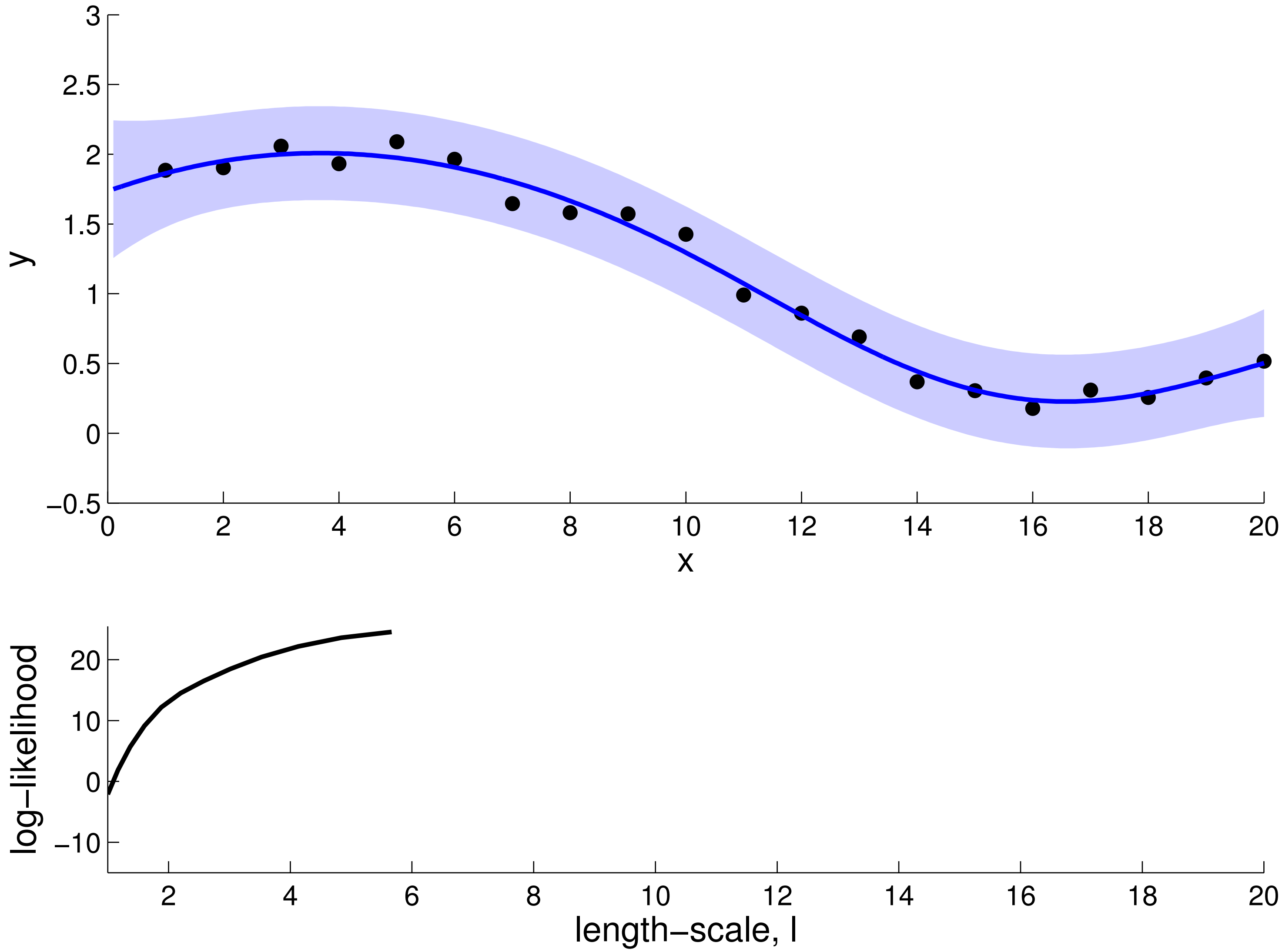

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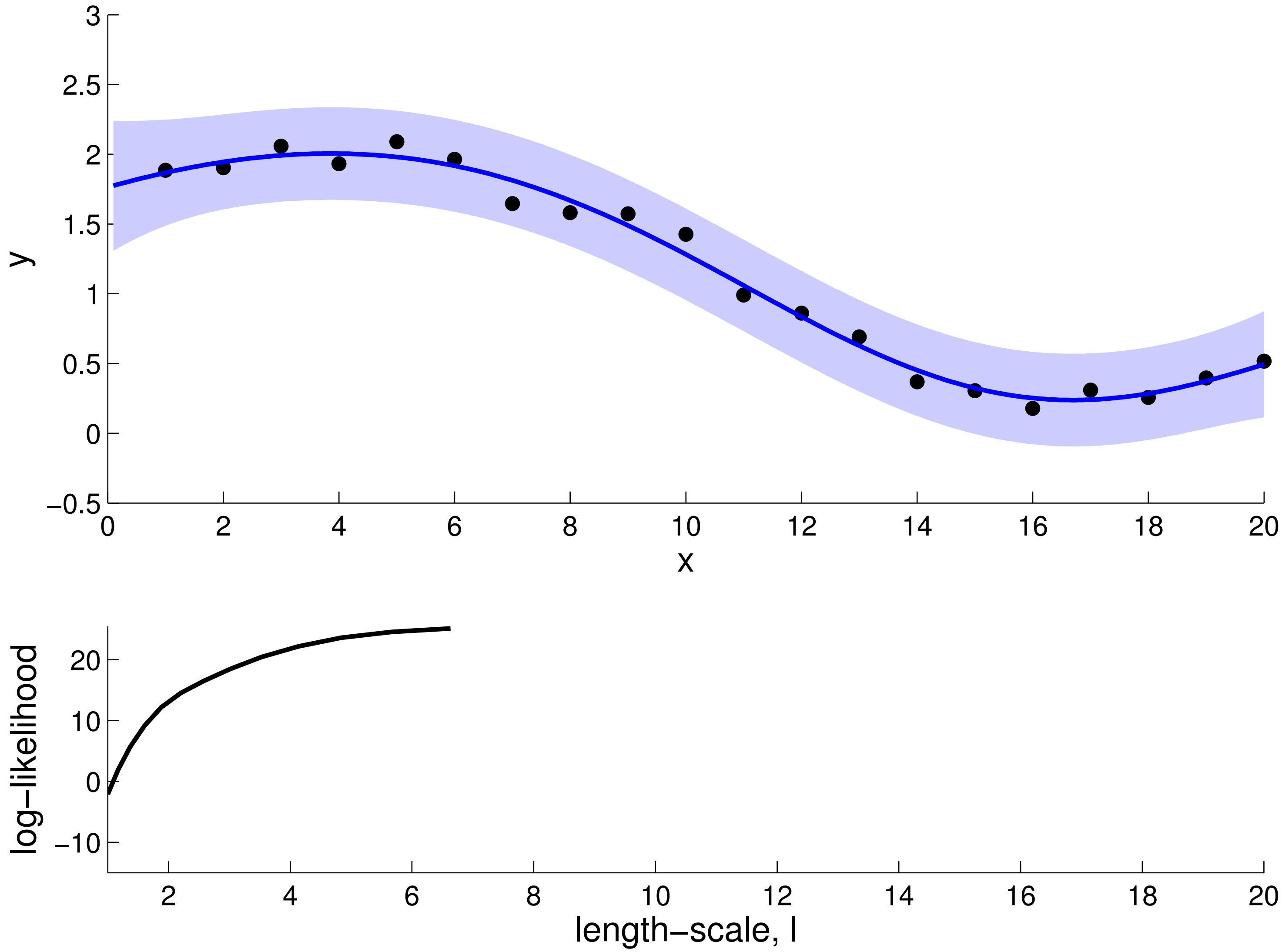

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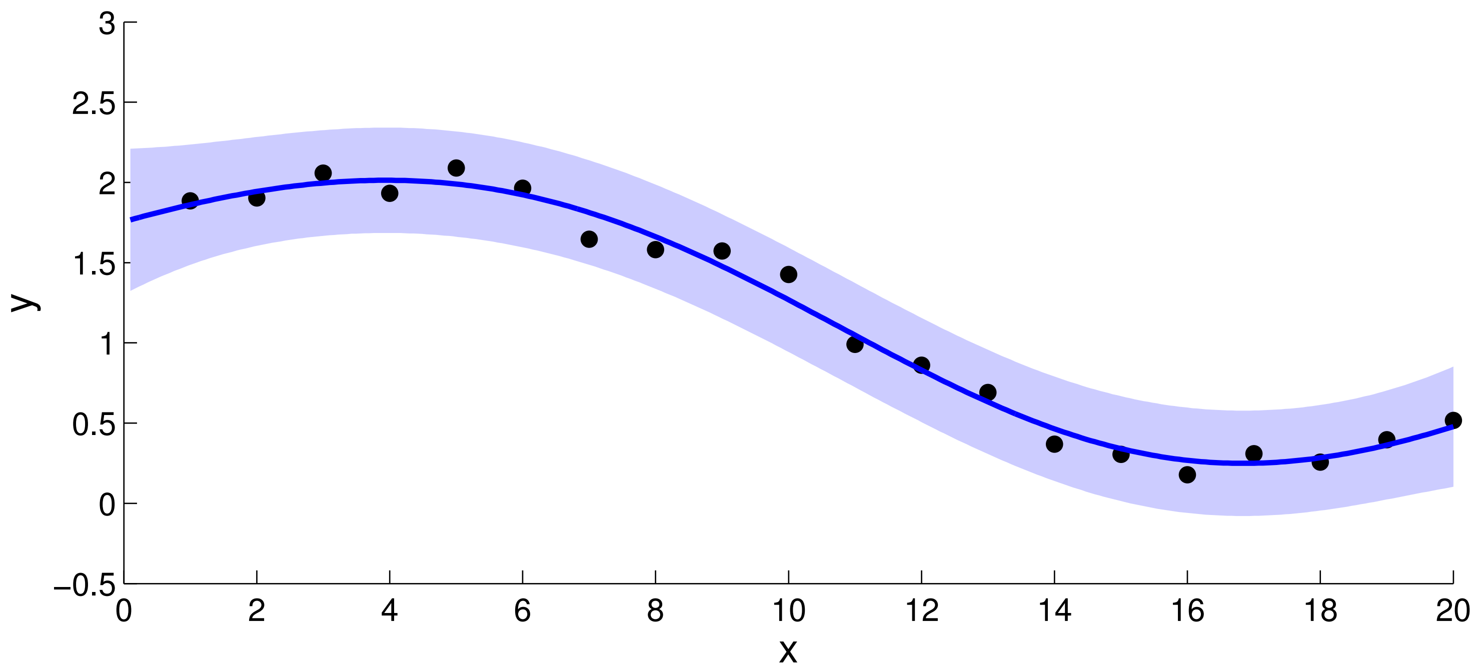

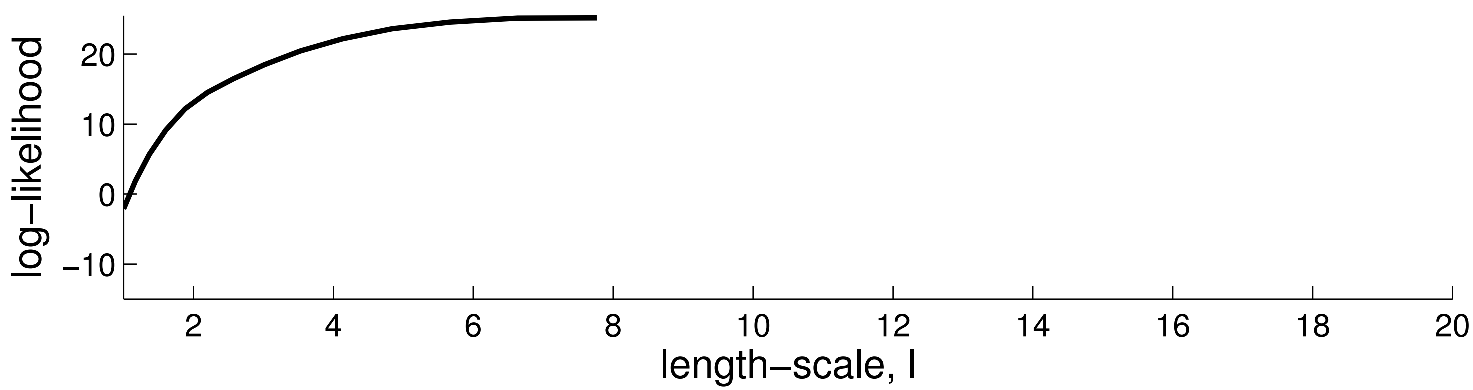

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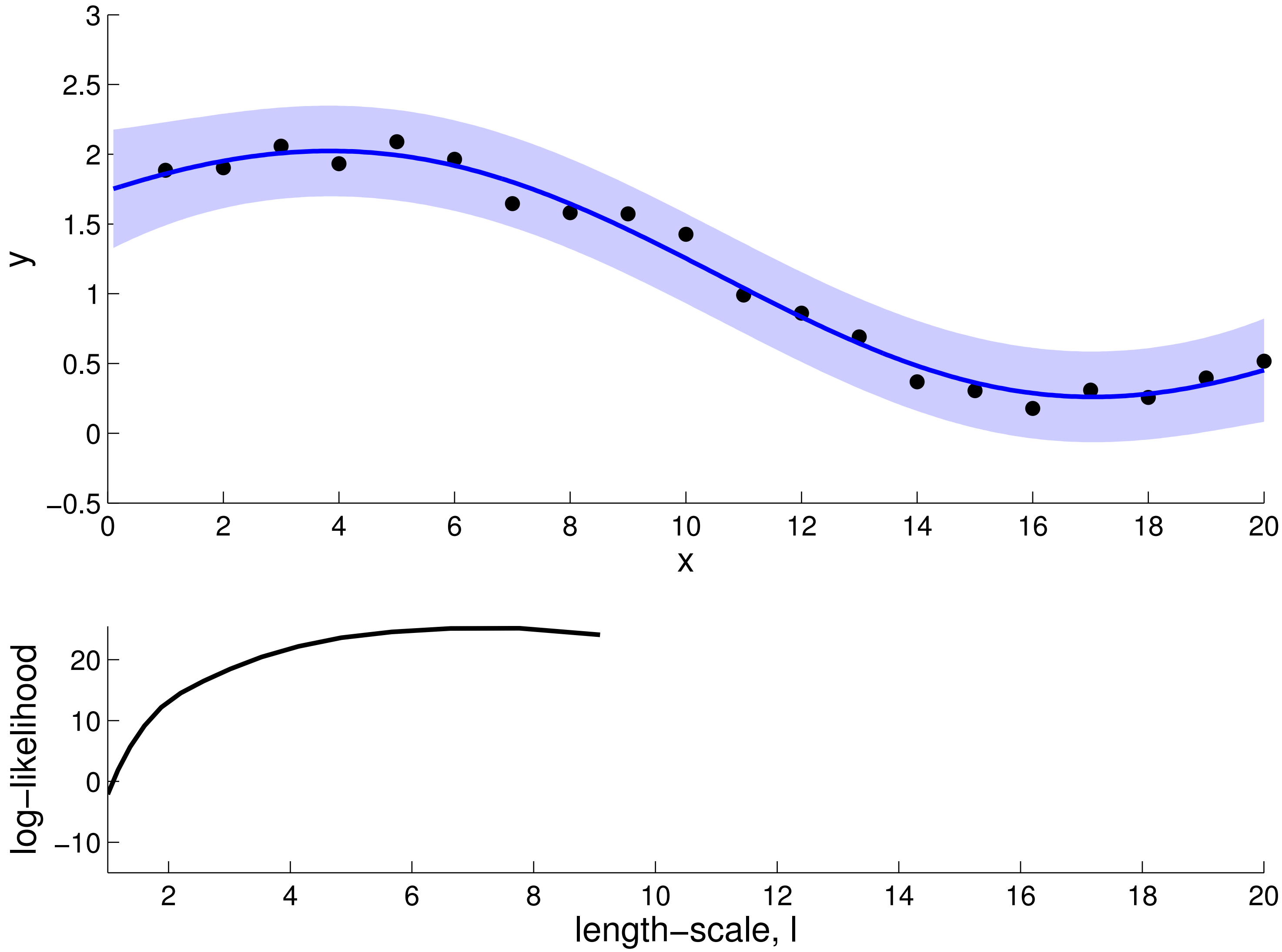

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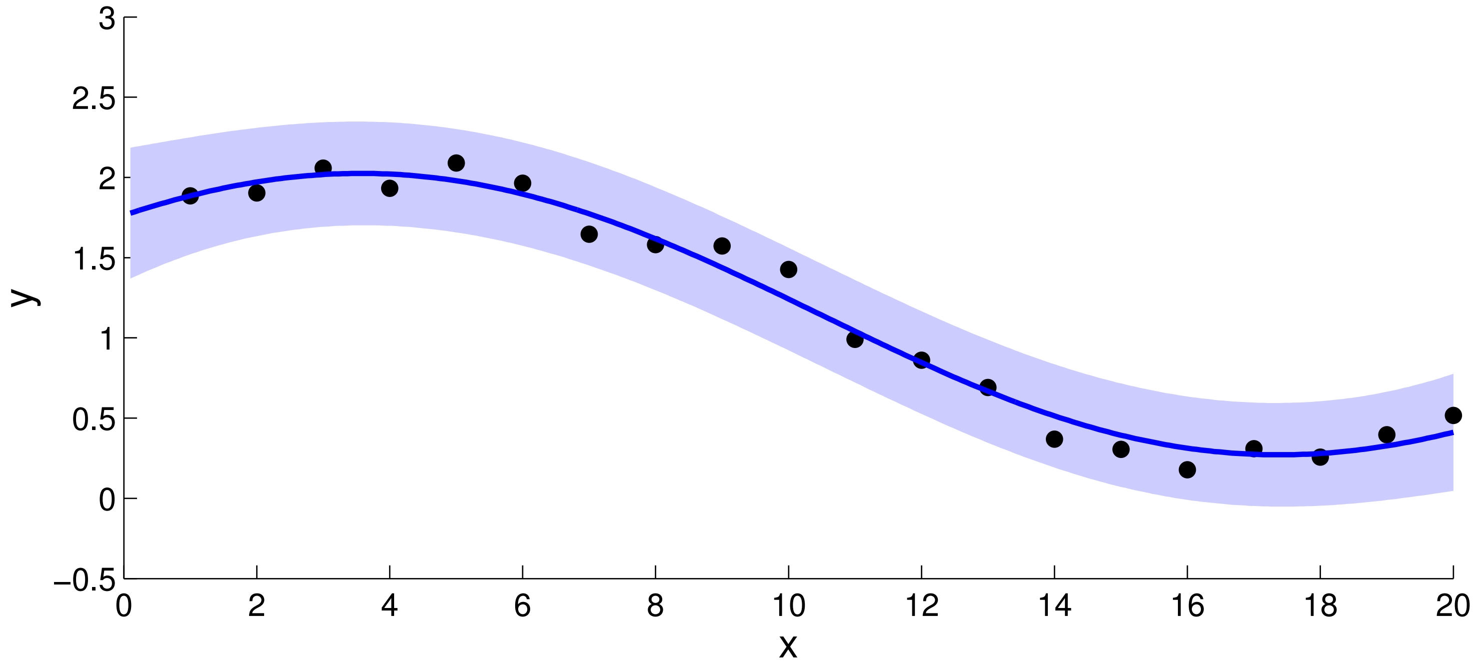

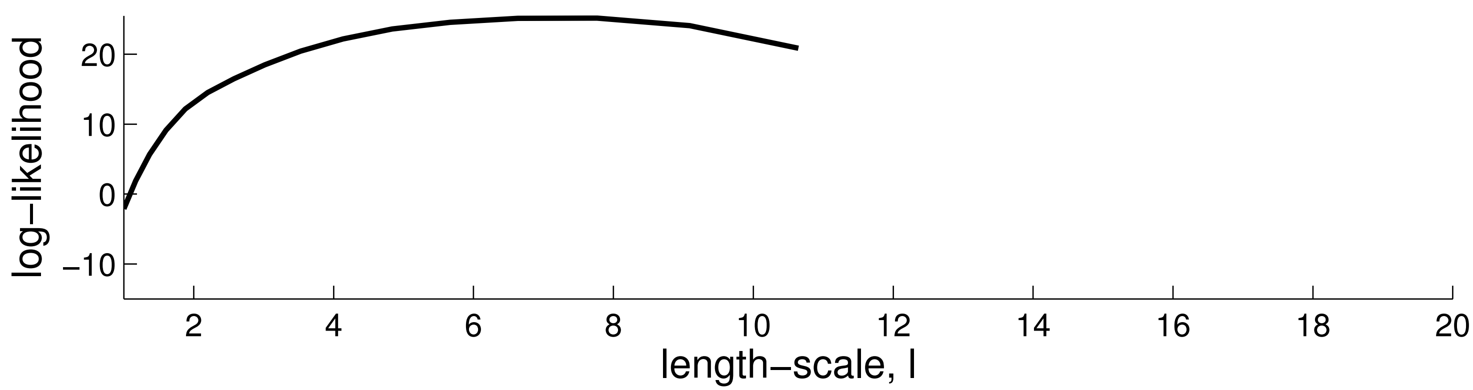

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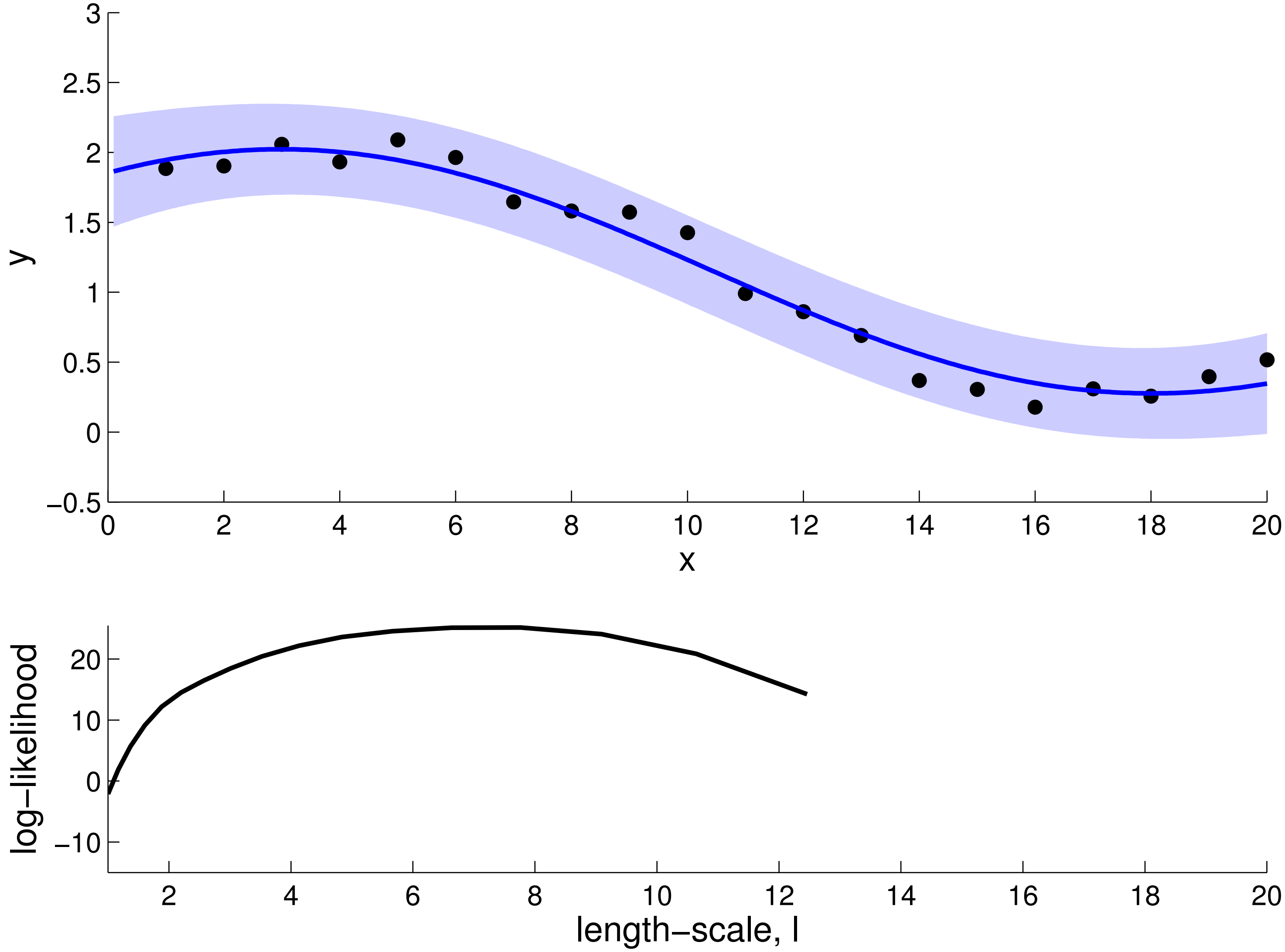

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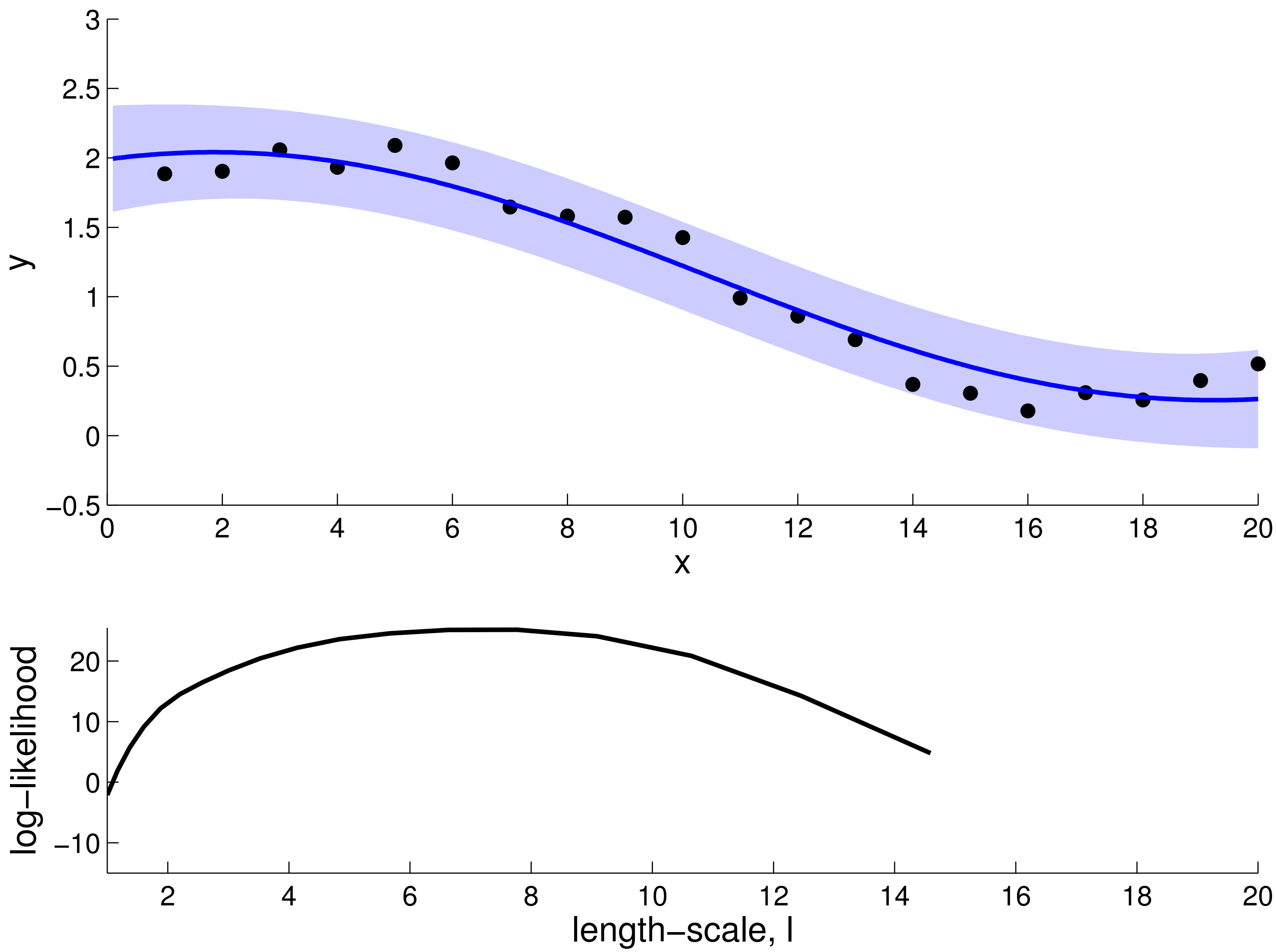

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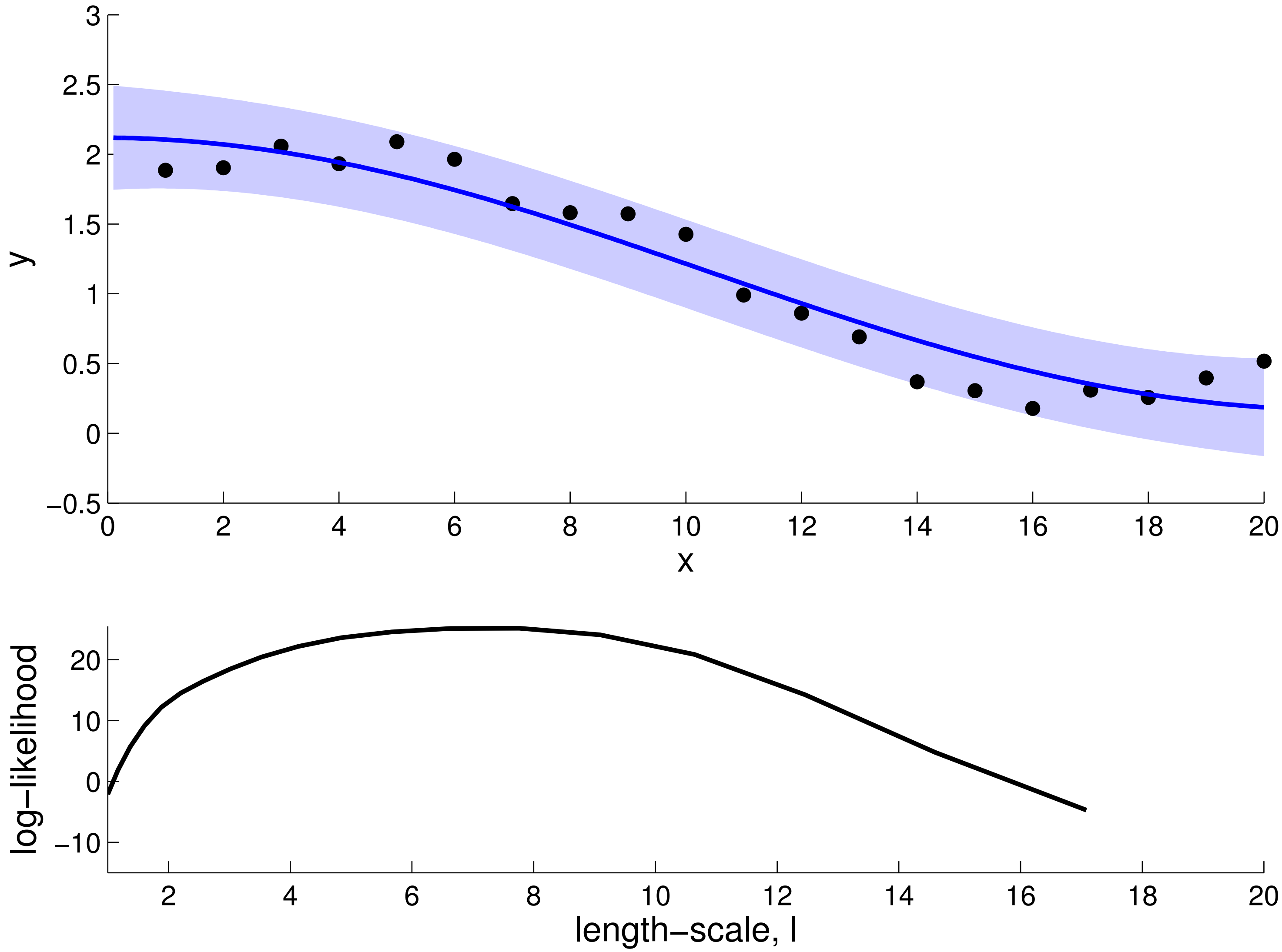

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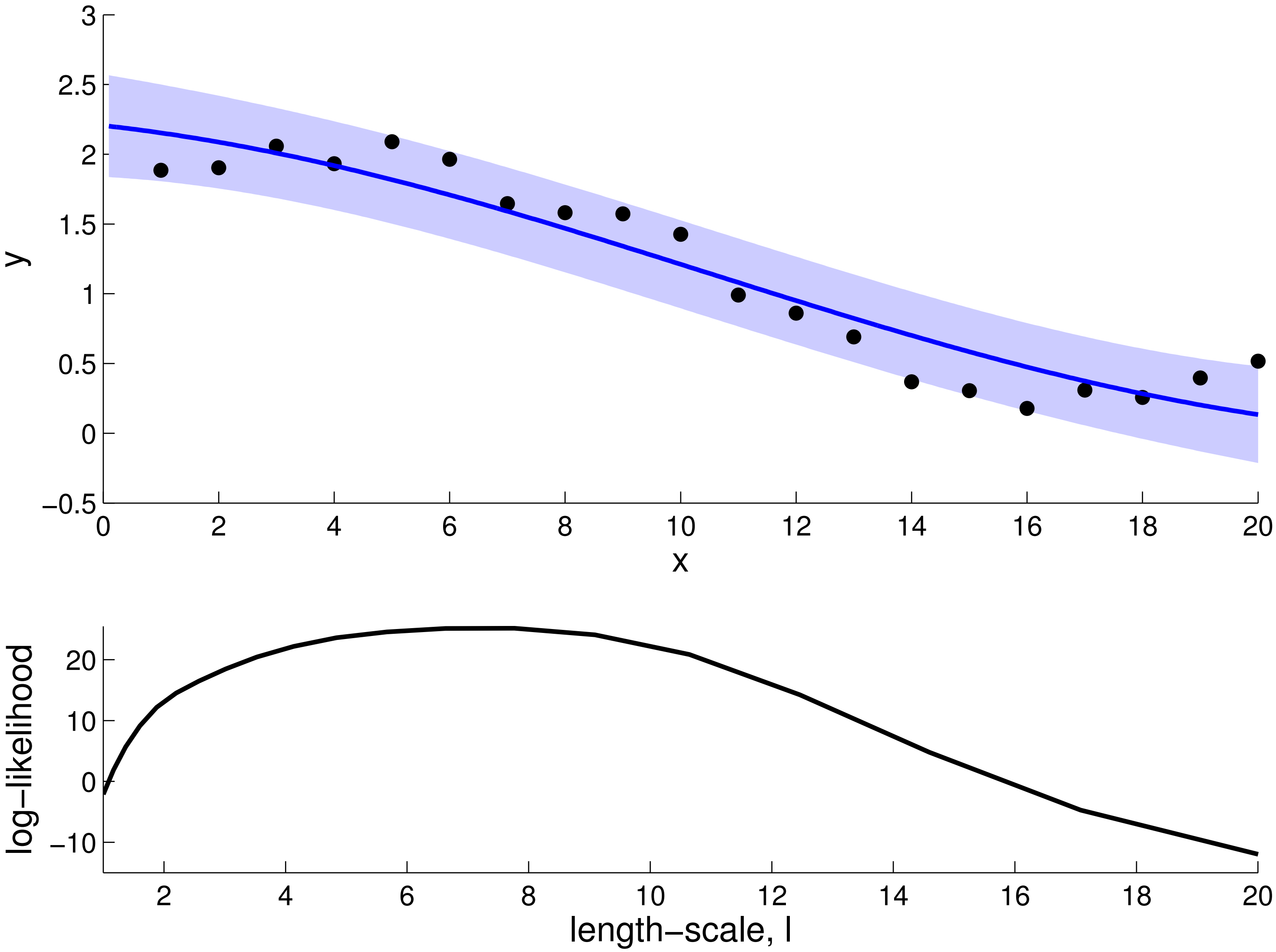

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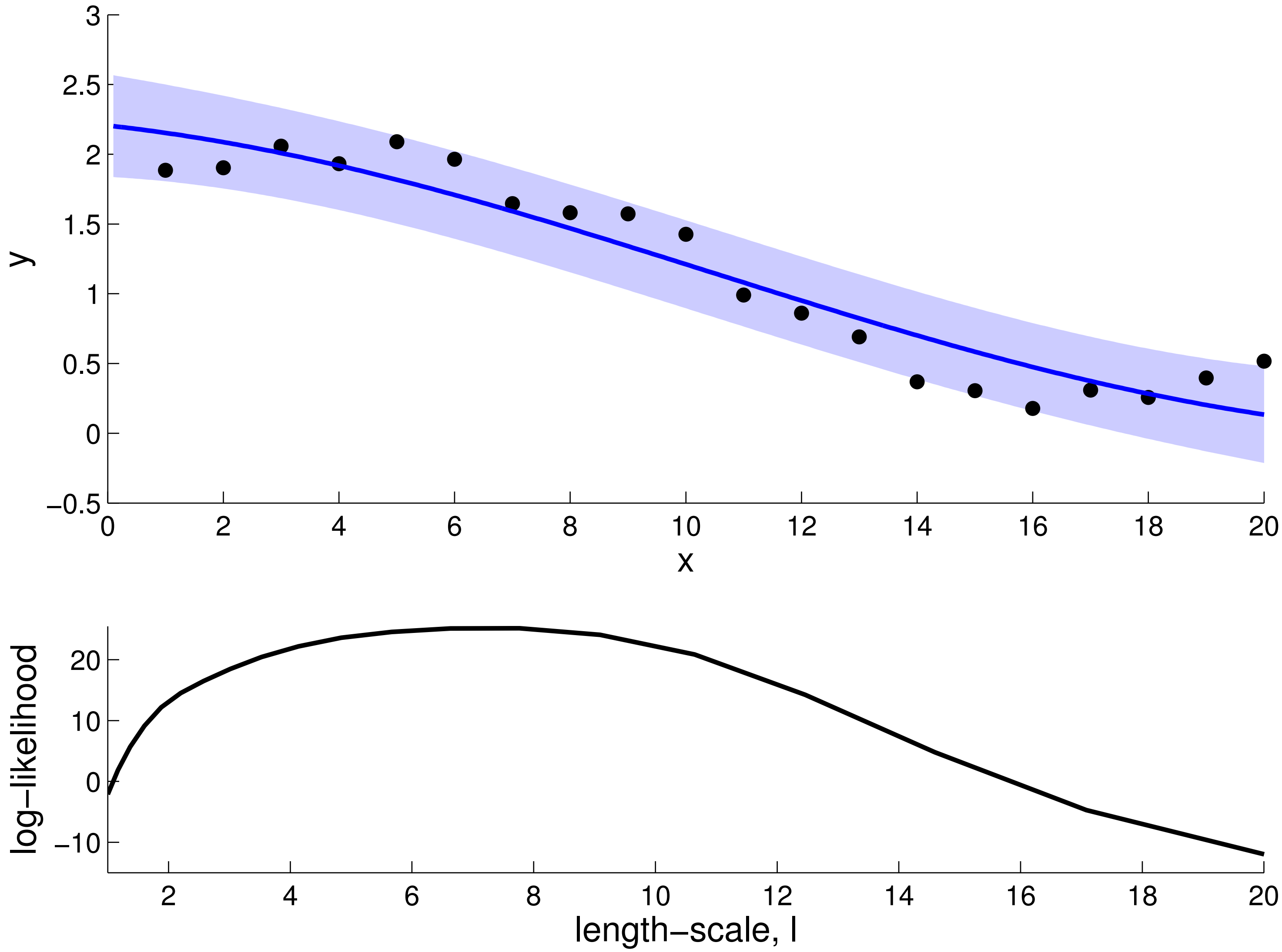

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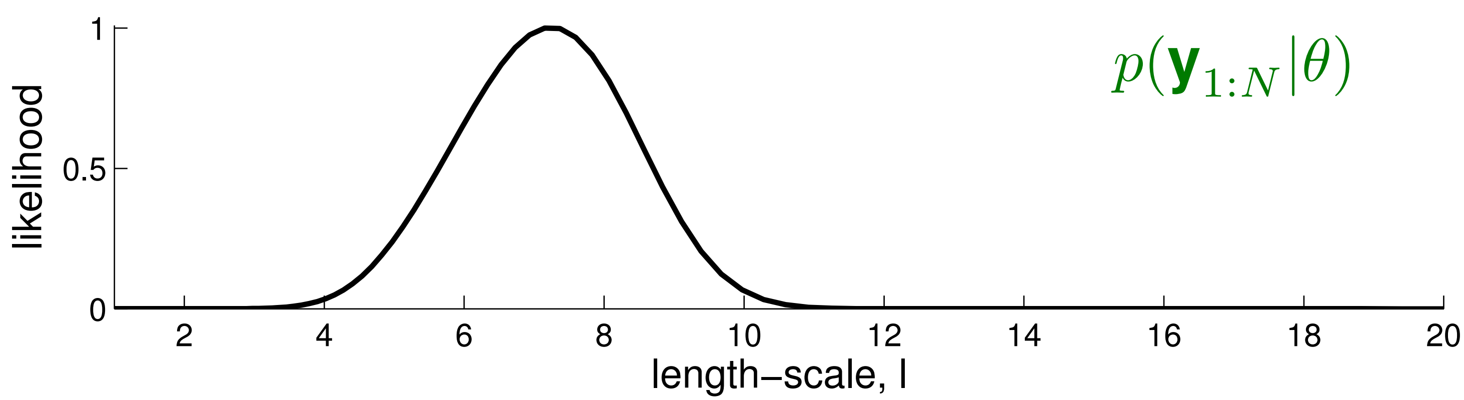

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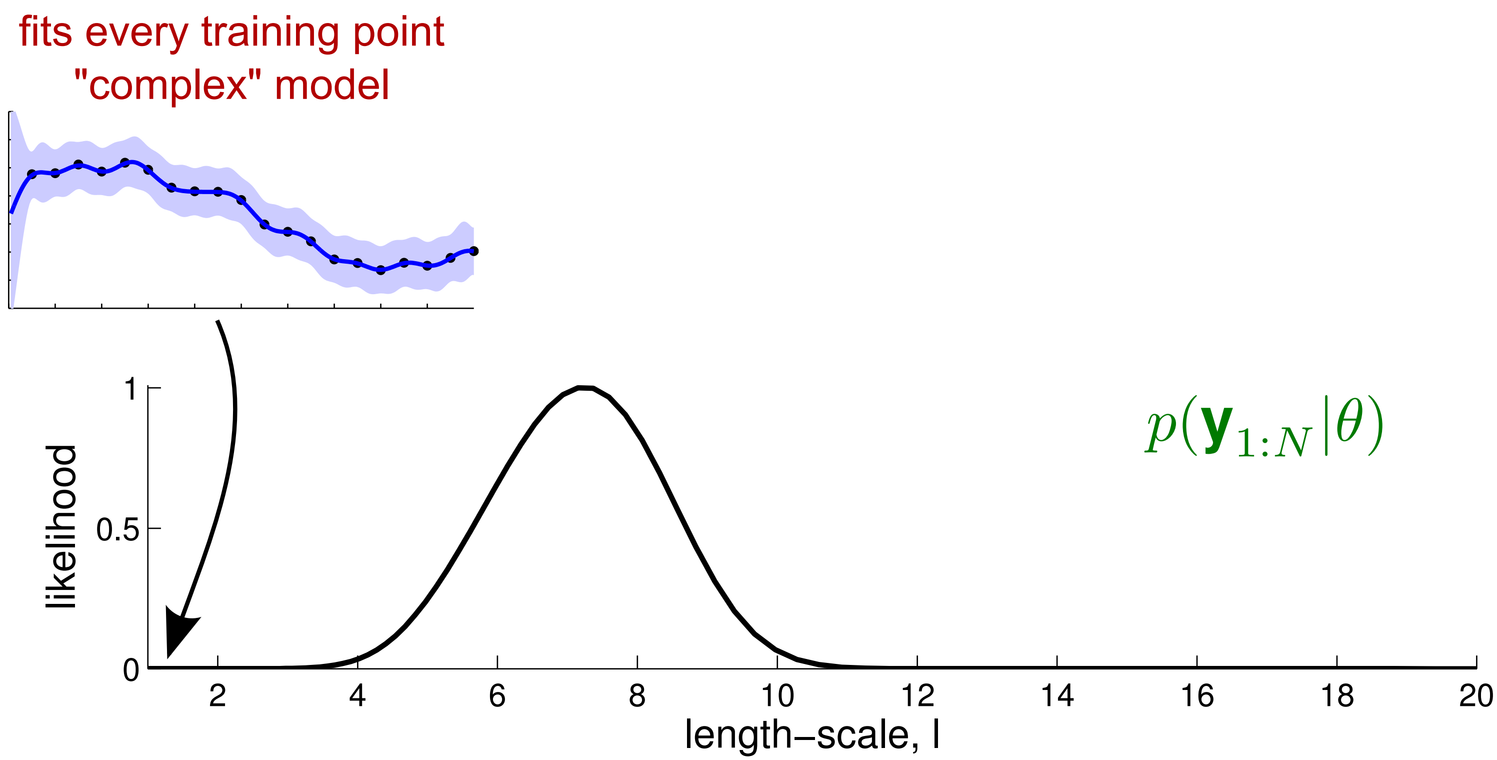

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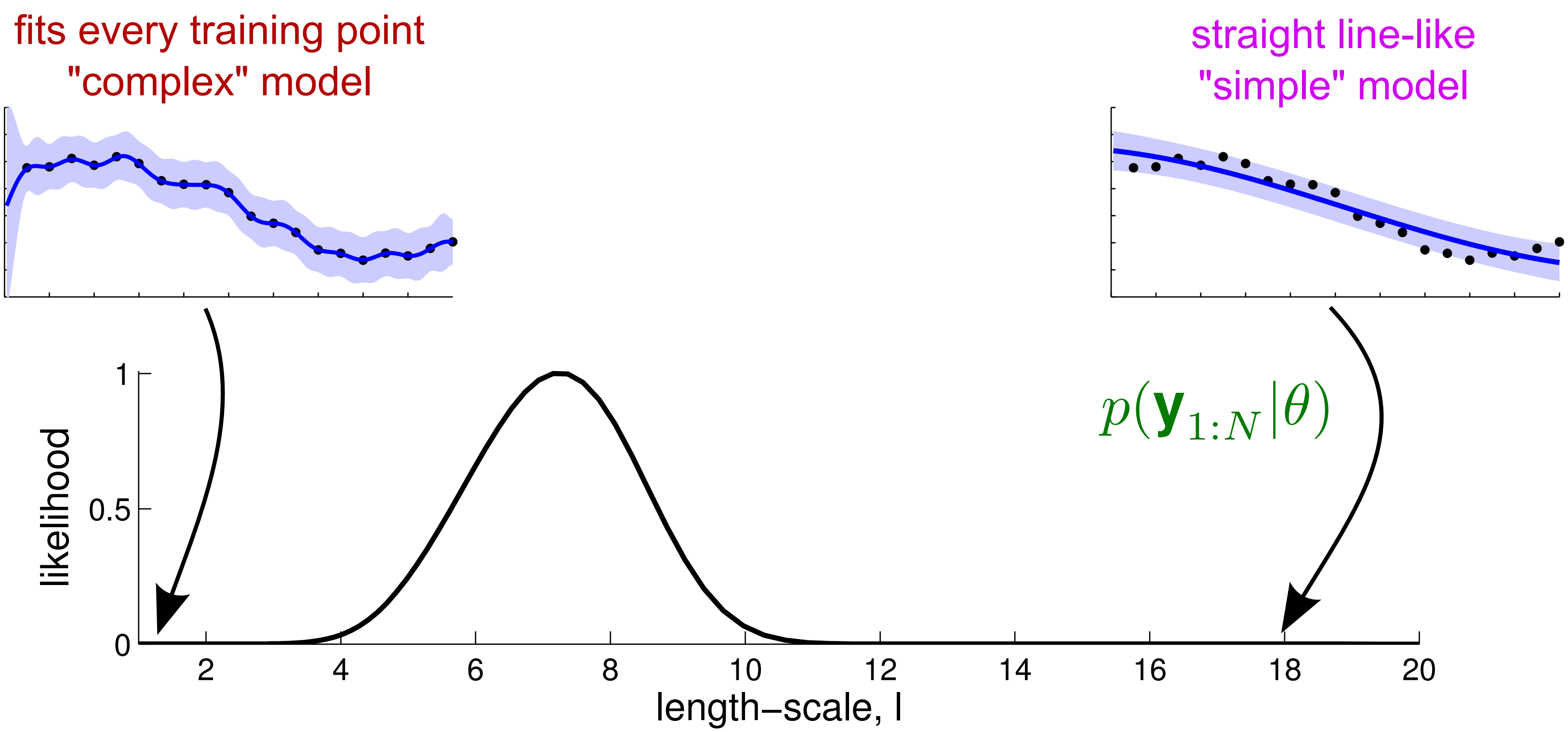

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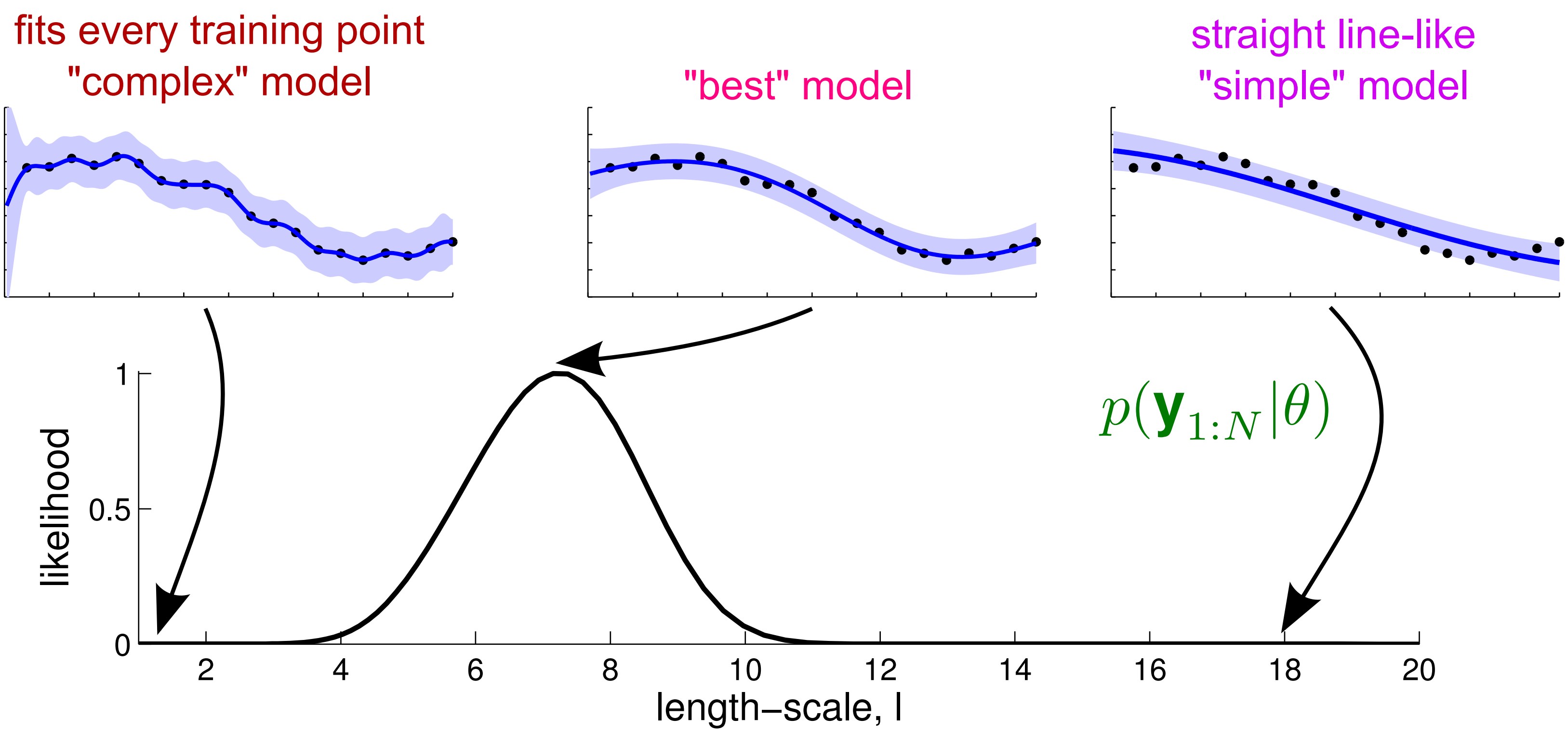

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