Note

iMPS to quantum circuit

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Outline

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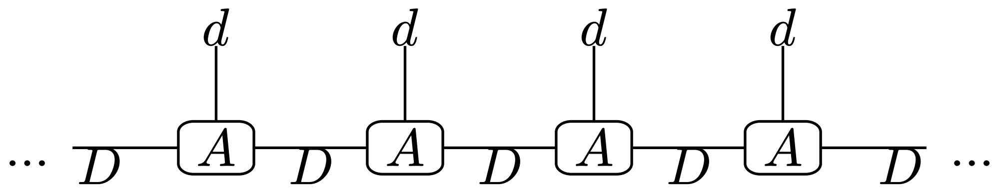
Summary

We translate infinite, translationally invariant matrix product states (iMPS) into finite-depth quantum circuits.

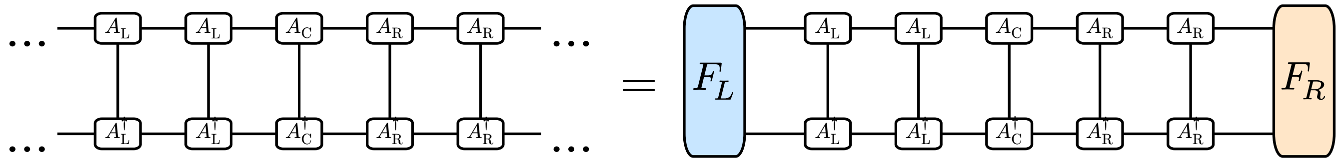
The ground state of the transverse field Ising model is obtained through variational optimization of circuit parameters.

iMPS

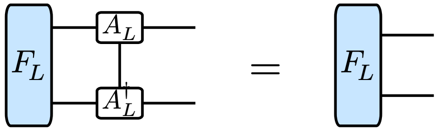
- an infinite, translationally invariant quantum spin chain (1-site unit cell):



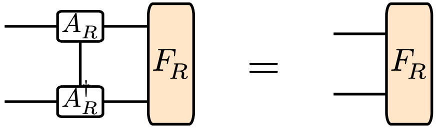
- left and right environment representation of overlap (in mixed canonical form):



- F_L and F_R satisfy fixed point equation:



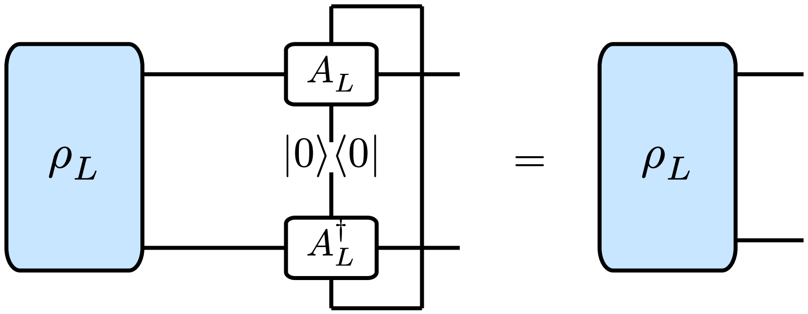
iMPS



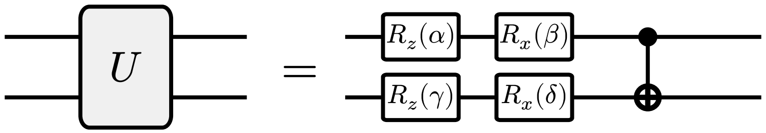
- Power method: in each iteration, contract the environment and transfer matrix until they follow the fixed point equation.

Quantum Channel

- Quantum channel representation:

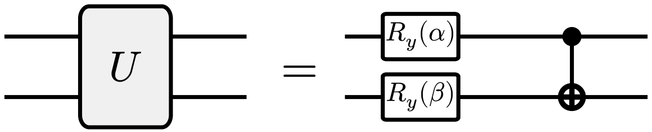


- Suppose:
  - $d = D = 2$
  - the iMPS are in right canonical form, $\Rightarrow \rho_R = \mathbb{I}$.
- Compile to circuit:

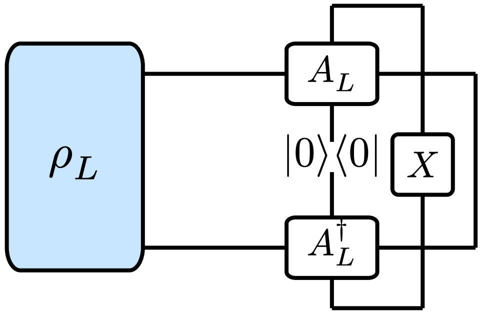


Quantum Channel

or

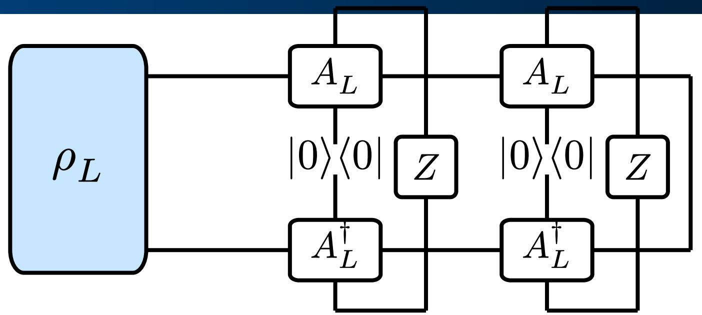


- methods to get local expectation value:
  - contract directly (exact result of the ansatz):
    - $\langle X_i \rangle$

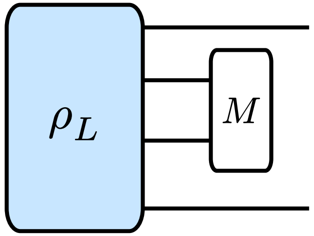


    - $\langle Z_i Z_{i+1} \rangle$

Quantum Channel

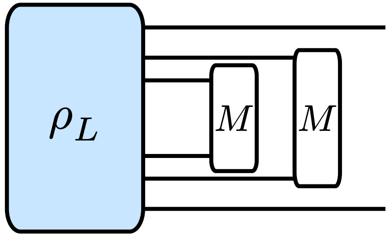


- Measure:
  - $\langle X_i \rangle$



- $\langle Z_i Z_{i+1} \rangle$

Quantum Channel



Outline

Introduction

Algorithm Box

Results

Algorithm 1: Variational iMPS Ground State Optimization

procedure VARIATIONAL-iMPS(θ_0, g, J, maxiter)
    ▷ Initialize parameters and convergence criteria
    $\theta \leftarrow \theta_0$
    iter $\leftarrow$ 0

    while iter < maxiter && g_tol > 1e-10 **do**
        ▷ Construct parameterized quantum circuit
        $U(\theta) \leftarrow \mathrm{ConstructCircuit}(\theta)$

        ▷ Iterate quantum channel to fixed point
        $\rho_L \leftarrow \mathrm{IterateChannel}(U(\theta))$

        ▷ Evaluate energy expectation
        $\langle X \rangle \leftarrow \mathrm{Expectation}(\rho_L, X)$
        $\langle ZZ \rangle \leftarrow \mathrm{Expectation}(\rho_L, Z \otimes Z)$

$E \leftarrow -g \cdot \langle X \rangle - J \cdot \langle ZZ \rangle$

$\triangleright$ Update parameters

$\theta \leftarrow \mathrm{NelderMead}(\theta, E)$

iter $\leftarrow$ iter + 1

end

return θ, E

end

Outline

Exact contraction

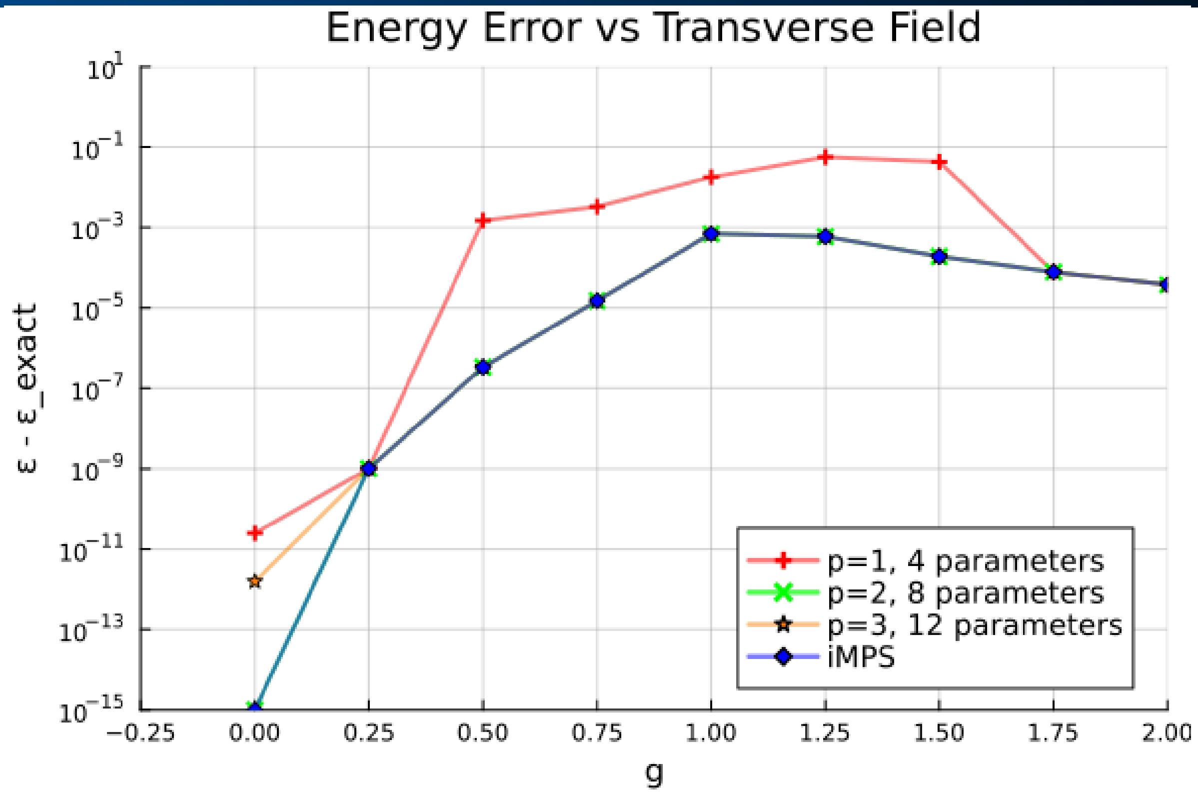


Fig. 1 Energy error v.s. transverse field strength for different circuit depths. iMPS: analytical results of $d = 2$, $D = 2$ infinite MPS from MPSKit.jl.

Measurement

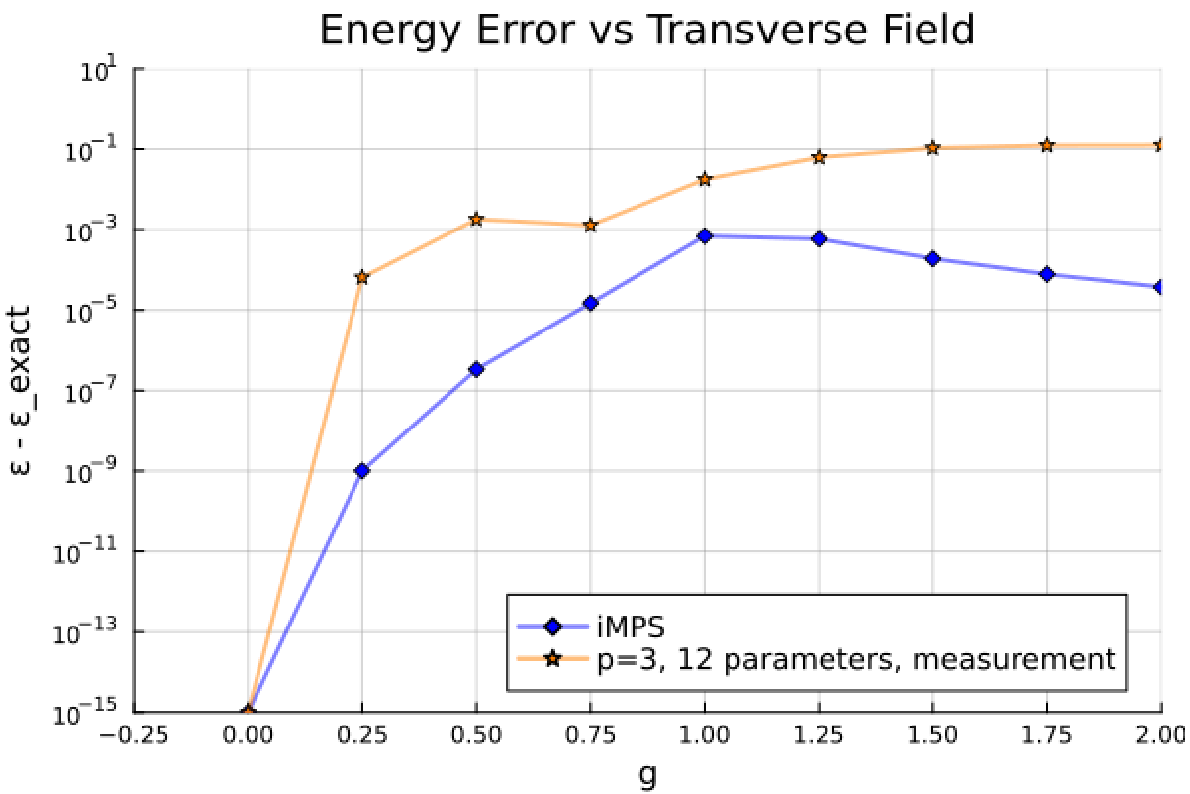


Fig. 2 Energy error v.s. transverse field strength for p=3 through measurement