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List of selected exercises

1 Recommended Exercises from JR

408, 409, 411, 412, 413, 414, 415, 416, 423

2 In-class Exercises on Continuous Random Variables

1. (191030.1)

1. Den kontinuerliga slumpvariabeln X har täthetsfunktion

$$f_X(x) = cx^3, \quad 0 \le x \le 2.$$

(Om $x < 0$ eller $x > 2$ så är $f_X(x) = 0$.) (5p)

(a) Bestäm c. (1p)
(b) Beräkna $P(X \le 1)$. (1p)
(c) Beräkna $P(X > 3)$. (1p)
(d) Beräkna $E(X)$. (1p)
(e) Beräkna $V(X)$. (1p)

Answer: a) $\frac{1}{4}$, b) $\frac{1}{16}$, c) 0, d) 1.6, e) 0.11.

2. (180828.2)

2. En kontinuerlig slumpvariabel X har fördelningsfunktion (5p)

$$F(x) = \begin{cases} 1 & \text{om } x > 2, \\ cx & \text{om } 0 \leq x \leq 2, \\ 0 & \text{annars.} \end{cases}$$

Antag att $F(x)$ är kontinuerlig för alla x.

(a) Bestäm c. (1p)

(b) Beräkna $P(X \leq 1)$. (2p)

(c) Beräkna väntevärdet, $E(X)$. (2p)

Answer: a) $\frac{1}{2}$, b) $\frac{1}{2}$, c) 1.

3. Match the examples with one of the distributions from {Poisson, normal, binomial, exponential}:

   (a) The frequency of people with blood group A is 40%. Fifty people are picked at random and their blood group is determined. Define X = "Number of people with blood group A". What is the distribution of X?

   (b) Define X = "Number of phone calls received by a call center per hour". What is the distribution of X?

   (c) Define X = "Waiting time for the next phone call". What is the distribution of X?

   (d) Suppose we measure the length of a table. Define X = "Deviation of the measurement from the true length". What is the distribution of X?

Answer: a) binomial, b) Poisson, c) exponential, d) normal.

4. A street has length 13 meters. A car with length 5 meters parks uniformly at random in this street. What is the probability that another car with the same length can also park here after?

Answer: $\frac{3}{4}$.

5. 411@JR (Property of exponential distribution)