Tutorial: An Information Theoretical View of Information Elicitation Mechanisms

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Peer prediction (PP) and prediction markets (PM) are two popular and important information aggregation tools. This tutorial will employ information theory to provide an information theoretical understanding for both PP and PM, including recasting several important prior results into this information theory framework. This new approach promises to provide a systematic theoretical understanding for both PP and PM, and we believe the entire EC community could benefit from better understanding these information theory tools.

Information Theory In the tutorial, we will review four important facts of Shannon information theory [7]—(1) entropy monotonicity (2) chain rule (3) sub(super)-additivity (4) information monotonicity/data processing inequality—which are all closely related to PP and PM (and proper scoring rules, which are a key tool in both of these settings).

Peer prediction Peer prediction mechanisms help elicit information which cannot (or cannot cheaply) be verified (e.g. restaurant reviews). Peer prediction mechanisms are more powerful than a simple majority vote, and can motivate agents who believe they are in the minority to report truthfully. Since Miller et al. [16] introduced the original peer prediction mechanism, several works follow the peer prediction framework and design information elicitation mechanisms without verification in different settings. The tutorial will recast two important works in the PP literature—(1) Bayesian Truth Serum [18] and (2) Dasgupta and Ghosh [8]—into the information theoretical framework. In the mechanisms designed in these two works, truth-telling is not only an equilibrium but also the best equilibrium. This tutorial shows that this "truth-telling is the best equilibrium" property is closely related to the *information monotonicity* tool. Moreover, this tutorial will also introduce the authors' several new PP mechanisms ([14, 15]) which are designed with the help of the *information monotoncity* tool.

Prediction market Although prediction markets are a quite strong information elicitation tool in real life, large holes still remain in the theory of prediction markets. For example, two basic questions—(1) how agents should reveal their information to maximize their payment (2) when does the market accurately aggregate information—have not been fully understood. Prior work [5, 6, 4, 12, 13, 19, 9, 10, 3, 1, 2, 11, 17] have provided important progresses. Hanson [12, 13] proposes the market scoring rule (MSR) model which is used broadly in the theoretical work of prediction market. For the information revelation problem, Chen et al. [5, 6], Chen and Waggoner [4] propose and study a three stages game called the Alice Bob Alice (A-B-A) game—Alice participates in the market first, then Bob joins, and then Alice has an opportunity to participate again. They also define two special information structures—"substitutes" and "compliments"—and show that when traders' information are substitutes (compliments), Alice should reveal her information as soon (late) as possible. For the information aggregation problem, Aaronson [1] cleverly shows that it takes only $O(\frac{1}{\epsilon})$ time for traders' beliefs being ϵ-"close".

In the tutorial, we will recast Chen et al. [5, 6], Chen and Waggoner [4] and Aaronson [1] into the information theoretical framework. We will show the previous information revelation results about "substitutes" and "compliments" are closely related to the *sub(super)-addictivity* property in information theory; the previous information aggregation result is closely related to the *entropy monotonicity* and *chain rule* property in information theory. Moreover, we will generalize the classical Shannon information theory concepts to provide a systematic information theoretical understanding for the prediction markets.

Biographies

Yuqing Kong is a PhD candidate in University of Michigan and interested in the intersection of theoretical computer science and the areas of game theory, information elicitation, prediction market, mechanism design, and social network analysis.

Grant Schoenebeck is an assistant professor at the University of Michigan in the Computer Science and Engineering division. His work spans diverse areas in theoretical computer science but has recently focused on applying ideas from theoretical computer science to the study of social networks and mechanism design for information elicitation. His research is supported by the NSF, Facebook, and Google including an NSF CAREER award. Before coming to the University of Michigan in 2012, he was a Postdoctoral Research Fellow at Princeton. Grant received his PhD at UC Berkeley, studied theology at Oxford University, and received his BA in mathematics and computer science from Harvard.

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