You submitted this quiz on **Wed 11 Nov 2015 7:59 AM CST**. You got a score of **375.00** out of **400.00**. You can attempt again, if you'd like.

### **Question 1**

Overfitting and Deterministic Noise

Deterministic noise depends on  $\mathcal{H}$ , as some models approximate f better than others. Assume  $\mathcal{H}' \subset \mathcal{H}$  and that f is fixed. **In general** (but not necessarily in all cases), if we use  $\mathcal{H}'$  instead of  $\mathcal{H}$ , how does deterministic noise behave?

	Score	Explanation
~	20.00	
	20.00 /	
	20.00	
	•	

### **Question 2**

**Regularization With Polynomials** Polynomial models can be viewed as linear models in a space  $\mathcal{Z}$ , under a nonlinear transform  $\Phi: \mathcal{X} \to \mathcal{Z}$ . Here,  $\Phi$  transforms the scalar x into a vector  $\mathbf{z}$  of Legendre polynomials,  $\mathbf{z} = (1, L_1(x), L_2(x), \dots, L_Q(x))$ . Our hypothesis set will be expressed as a linear combination of these polynomials,

$$\mathcal{H}_Q = \left\{ h \mid h(x) = \mathbf{w}^{\mathrm{T}} \mathbf{z} = \sum_{q=0}^{Q} w_q L_q(x) \right\}, \text{ where } L_0(x) = 1.$$

Consider the following hypothesis set defined by the constraint:

$$\mathcal{H}(Q, c, Q_o) = \{h \mid h(x) = \mathbf{w}^T \mathbf{z} \in \mathcal{H}_Q; w_q = c \text{ for } q \geq Q_o\}, \text{ which of the following } \mathbf{z} \in \mathcal{H}_Q$$

statements is correct?		
Your Answer	Score	Explanation
$\bigcirc \mathcal{H}(10,1,3) \cap \mathcal{H}(10,1,4) = \mathcal{H}_1$		
$\bigcirc \mathcal{H}(10,0,3) \cup \mathcal{H}(10,0,4) = \mathcal{H}_4$		
	<b>✓</b> 20.00	
$\bigcirc \mathcal{H}(10,0,3) \cup \mathcal{H}(10,1,4) = \mathcal{H}_3$		
onone of the other choices		
Total	20.00 / 20.00	

Regularization and Weight Decay Consider the augmented error

$$E_{\text{aug}}(\mathbf{w}) = E_{\text{in}}(\mathbf{w}) + \frac{\lambda}{N} \mathbf{w}^T \mathbf{w}$$

with some  $\lambda > 0$ .

If we want to minimize the augmented error  $E_{\rm aug}(\mathbf{w})$  by gradient descent, with  $\eta$  as learning rate, which of the followings are the correct update rules?

Your Answer	Score	Explanation
• $\mathbf{w}(t+1) \longleftarrow (1 - \frac{2\eta\lambda}{N})\mathbf{w}(t) - \eta\nabla E_{\text{in}}(\mathbf{w}(t)).$	<b>✓</b> 20.00	
$\bigcirc \mathbf{w}(t+1) \longleftarrow \mathbf{w}(t) - \eta \nabla E_{\mathrm{in}}(\mathbf{w}(t)).$		
$\bigcirc$ $\mathbf{w}(t+1) \longleftarrow (1 - \frac{\eta \lambda}{N}) \mathbf{w}(t) - \eta \nabla E_{\text{in}}(\mathbf{w}(t)).$		
$\bigcirc$ <b>w</b> $(t+1) \longleftarrow (1 + \frac{2\eta\lambda}{N})$ <b>w</b> $(t) - \eta\nabla E_{\rm in}($ <b>w</b> $(t)).$		
Total	20.00 / 20	0.00

## **Question 4**

Let  $\mathbf{w}_{lin}$  be the optimal solution for the plain-vanilla linear regression and  $\mathbf{w}_{reg}(\lambda)$  be the optimal solution for the formulation above. Select all the correct statements below:

Your Answer		Score	Explanation
onone of the other choices			
$\ \mathbf{w}_{\mathrm{reg}}(\lambda)\ $ is a non-decreasing function of $\lambda$ for $\lambda \geq 0$			
$\ \mathbf{w}_{\mathrm{reg}}(\lambda)\ $ is a non-increasing function of $\lambda$ for $\lambda \geq 0$	<b>~</b>	20.00	
$\ \mathbf{w}_{\mathrm{reg}}(\lambda)\  > \ \mathbf{w}_{\mathrm{lin}}\ $ for any $\lambda > 0$			
$\ \mathbf{w}_{\mathrm{reg}}(\lambda)\  = \ \mathbf{w}_{\mathrm{lin}}\ $ for any $\lambda > 0$			
Total		20.00 / 20.00	

## **Question 5**

#### **Leave-One-Out Cross-Validation**

You are given the data points: (-1,0),  $(\rho,1)$ , (1,0),  $\rho \geq 0$ , and a choice between two models: constant [ $h_0(x) = b_0$ ] and linear [ $h_1(x) = a_1x + b_1$ ]. For which value of  $\rho$  would the two models be tied using leave-one-out cross-validation with the squared error measure?

Your Answer		Score	Explanation
$\bigcirc\sqrt{\sqrt{3}+4}$			
$\bigcirc\sqrt{\sqrt{3}-1}$			
• $\sqrt{9 + 4\sqrt{6}}$	<b>~</b>	20.00	
$\bigcirc\sqrt{9}-\sqrt{6}$			
onone of the other choices			
Total		20.00 / 20.00	

### **Question 6**

**Learning Principles** In Problems 6-7, suppose that for 5 weeks in a row, a letter arrives in the mail that predicts the outcome of the upcoming Monday night baseball game. (Assume there are

no tie.) You keenly watch each Monday and to your surprise, the prediction is correct each time. On the day after the fifth game, a letter arrives, stating that if you wish to see next week's prediction, a payment of NTD 1,000 is required.

Which of the following statement is true?

Score	Explanation
✔ 20.00	
20.00 / 20.00	
	20.00 /

## **Question 7**

If the cost of printing and mailing out each letter is NTD 10. If the sender sends the minimum number of letters out, how much money can be made for the above 'fraud' to succeed once? That is, one of the recipients does send him NTD 1,000 to receive the prediction of the 6-th game?

Your Answer		Score	Explanation
<ul><li>NTD 370</li></ul>	<b>~</b>	20.00	
O NTD 340			
O NTD 430			
O NTD 460			
NTD 400			
Total		20.00 / 20.00	

For Problems 8-10, we consider the following. In our credit card example, the bank starts with some vague idea of what constitutes a good credit risk. So, as customers  $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$  arrive, the bank applies its vague idea to approve credit cards for some of these customers based on a formula  $a(\mathbf{x})$ . Then, only those who get credit cards are monitored to see if they default or not. For simplicity, suppose that the first N=10,000 customers were given credit cards by the credit approval function  $a(\mathbf{x})$ . Now that the bank knows the behavior of these customers, it comes to you to improve their algorithm for approving credit. The bank gives you the data  $(\mathbf{x}_1,y_1),\ldots,(\mathbf{x}_N,y_N)$ . Before you look at the data, you do mathematical derivations and come up with a credit approval function. You now test it on the data and, to your delight, obtain perfect prediction.

What is M, the size of your hypothesis set?

Your Answer		Score	Explanation
$\odot 2^N$	×	-5.00	
$\bigcirc N$			
<b>)</b> 1			
$\mathcal{N}^2$			
We have no idea about it.			
Total		-5.00 / 20.00	
10101		3.00 / 20.00	

## **Question 9**

With such an M, what does the Hoeffding bound say about the probability that the true average error rate of g is worse than 1% for N=10,000?

Your Answer		Score	Explanation
$\bigcirc \leq 0.171$			
○ ≤ 0.221			
<b>●</b> ≤ 0.271	~	20.00	
○ ≤ 0.321			

$\bigcirc \leq 0.371$	
Total	20.00 / 20.00

You assure the bank that you have a got a system g for approving credit cards for new customers, which is nearly error-free. Your confidence is given by your answer to the previous question. The bank is thrilled and uses your g to approve credit for new customers. To their dismay, more than half their credit cards are being defaulted on. Assume that the customers that were sent to the old credit approval function and the customers that were sent to your g are indeed i.i.d. from the same distribution, and the bank is lucky enough (so the `bad luck' that ``the true error of g is worse than 1%'' does not happen). Select all the true claims for this situation.

Your Answer	Score	Explanation
O By applying $a(\mathbf{x})$ XNOR $g(\mathbf{x})$ to approve credit for new customers, the performance of the overall credit approval system can be improved with guarantee provided by the previous problem.		
onone of the other choices		
O By applying $a(\mathbf{x})$ XOR $g(\mathbf{x})$ to approve credit for new customers, the performance of the overall credit approval system can be improved with guarantee provided by the previous problem.		
O By applying $a(\mathbf{x})$ OR $g(\mathbf{x})$ to approve credit for new customers, the performance of the overall credit approval system can be improved with guarantee provided by the previous problem.		
• By applying $a(\mathbf{x})$ AND $g(\mathbf{x})$ to approve credit for new customers, the performance of the overall credit approval system can be improved with guarantee provided by the previous problem.	<b>✓</b> 20.00	
Total	20.00 / 20.00	

#### Virtual Examples and Regularization

Consider linear regression with virtual examples. That is, we add K virtual examples  $(\tilde{\mathbf{x}}_1, \tilde{\mathbf{y}}_1), (\tilde{\mathbf{x}}_2, \tilde{\mathbf{y}}_2), \dots, (\tilde{\mathbf{x}}_K, \tilde{\mathbf{y}}_K)$  to the training data set, and solve

$$\min_{\mathbf{w}} \frac{1}{N+K} \left( \sum_{n=1}^{N} (y_n - \mathbf{w}^T \mathbf{x}_n)^2 + \sum_{k=1}^{K} (\widetilde{y}_k - \mathbf{w}^T \widetilde{\mathbf{x}}_k)^2 \right).$$

We will show that using some 'special' virtual examples, which were claimed to be a possible way to combat overfitting in Lecture 9, is related to regularization, another possible way to combat overfitting discussed in Lecture 10. Let  $\widetilde{\mathbf{X}} = [\widetilde{\mathbf{x}}_1 \widetilde{\mathbf{x}}_2 \dots \widetilde{\mathbf{x}}_K]^T$ , and  $\widetilde{\mathbf{y}} = [\widetilde{y}_1, \widetilde{y}_2, \dots, \widetilde{y}_K]^T$ .

What is the optimal w to the optimization problem above, assuming that all the inversions exist?

Your Answer	Score	Explanation

20.00

- none of the other choices
- $(\mathbf{X}^T\mathbf{X})^{-1}(\mathbf{X}^T\mathbf{y} + \widetilde{\mathbf{X}}^T\widetilde{\mathbf{y}})$

$$(\mathbf{X}^T\mathbf{X} + \widetilde{\mathbf{X}}^T\widetilde{\mathbf{X}})^{-1}(\mathbf{X}^T\mathbf{y} + \widetilde{\mathbf{X}}^T\widetilde{\mathbf{y}})$$

$$(\mathbf{X}^T\mathbf{X})^{-1}(\widetilde{\mathbf{X}}^T\widetilde{\mathbf{y}})$$

$$(\mathbf{X}^T\mathbf{X} + \widetilde{\mathbf{X}}^T\widetilde{\mathbf{X}})^{-1}(\widetilde{\mathbf{X}}^T\widetilde{\mathbf{y}})$$

Total 20.00 / 20.00

### **Question 12**

For what  $\widetilde{X}$  and  $\widetilde{y}$  will the solution of this linear regression equal to

$$\mathbf{w}_{\text{reg}} = \operatorname{argmin}_{\mathbf{w}} \frac{\lambda}{N} \|\mathbf{w}\|^2 + \frac{1}{N} \|\mathbf{X}\mathbf{w} - \mathbf{y}\|^2?$$

Your Answer Score Explanation

$$\bigcirc \widetilde{\mathbf{X}} = \lambda \mathbf{I}, \widetilde{\mathbf{y}} = \mathbf{1}$$

none of the other choices

$$\bigcirc \widetilde{\mathbf{X}} = \mathbf{I}, \, \widetilde{\mathbf{y}} = \mathbf{0}$$

$\bigcirc \widetilde{\mathbf{X}} = \sqrt{\lambda} \mathbf{X},  \widetilde{\mathbf{y}} = \mathbf{y}$		
$\odot \widetilde{\mathbf{X}} = \sqrt{\lambda} \mathbf{I}, \widetilde{\mathbf{y}} = 0$	<b>~</b>	20.00
Total		20.00 / 20.00

#### **Experiment with Regularized Linear Regression and Validation**

Consider regularized linear regression (also called ridge regression) for classification.

$$\mathbf{w}_{\text{reg}} = \operatorname{argmin}_{\mathbf{w}} \frac{\lambda}{N} \|\mathbf{w}\|^2 + \frac{1}{N} \|\mathbf{X}\mathbf{w} - \mathbf{y}\|^2,$$

Run the algorithm on the following data set as  ${\cal D}$ 

https://d396qusza40orc.cloudfront.net/ntumlone%2Fhw4%2Fhw4\_train.dat

and the following set for evaulating  $E_{\it out}$ 

https://d396qusza40orc.cloudfront.net/ntumlone%2Fhw4%2Fhw4\_test.dat

Because the data sets are for classification, please consider only the 0/1 error for all the problems below.

Let  $\lambda = 10$ , which of the followings is the corresponding  $E_{in}$  and  $E_{out}$ ?

<b>✓</b> 20.00	
20.00 / 20.00	

### **Question 14**

Among  $\log_{10} \lambda = \{2, 1, 0, -1, \dots, -8, -9, -10\}$ . What is the  $\lambda$  with the minimum  $E_{in}$ ? Compute  $\lambda$  and its corresponding  $E_{in}$  and  $E_{out}$  then select the closest answer. Break the tie by selecting the largest  $\lambda$ .

Your Answer	Score	Explanation
	<b>✓</b> 20.00	
$O\log_{10} \lambda = -10, E_{in} = 0.030, E_{out} = 0.040$		
$O\log_{10} \lambda = -6, E_{in} = 0.030, E_{out} = 0.040$		
$O\log_{10} \lambda = -2, E_{in} = 0.030, E_{out} = 0.040$		
$O\log_{10} \lambda = -4, E_{in} = 0.015, E_{out} = 0.020$		
Total	20.00 / 20.00	

## **Question 15**

Among  $\log_{10}\lambda=\{2,1,0,-1,\ldots,-8,-9,-10\}$ . What is the  $\lambda$  with the minimum  $E_{out}$ ? Compute  $\lambda$  and the corresponding  $E_{in}$  and  $E_{out}$  then select the closest answer. Break the tie by selecting the largest  $\lambda$ .

Your Answer	Score	Explanation
$\bigcirc \log_{10} \lambda = -3, E_{in} = 0.015, E_{out} = 0.015$		
$\bigcap \log_{10} \lambda = -5, E_{in} = 0.015, E_{out} = 0.030$		
$\bigcap \log_{10} \lambda = -9, E_{in} = 0.030, E_{out} = 0.030$		
$\bigcirc \log_{10} \lambda = -1, E_{in} = 0.015, E_{out} = 0.015$		
	<b>✓</b> 20.00	
Total	20.00 / 20.00	)

## **Question 16**

Now split the given training examples in  $\mathcal D$  to the first 120 examples for  $\mathcal D_{train}$  and 80 for  $\mathcal D_{val}$ .

Ideally, you should randomly do the 120/80 split. Because the given examples are already randomly permuted, however, we would use a fixed split for the purpose of this problem.

Run the algorithm on  $\mathcal{D}_{train}$  to get  $g_{\lambda}^-$  , and validate  $g_{\lambda}^-$  with  $\mathcal{D}_{val}$  .

Among  $\log_{10} \lambda = \{2, 1, 0, -1, \dots, -8, -9, -10\}$ . What is the  $\lambda$  with the minimum  $E_{train}(g_{\lambda}^-)$ ?

Compute  $\lambda$  and the corresponding  $E_{train}(g_{\lambda}^{-})$ ,  $E_{val}(g_{\lambda}^{-})$  and  $E_{out}(g_{\lambda}^{-})$  then select the closet answer. Break the tie by selecting the largest  $\lambda$ .

Your Answer	Score	Explanation
$\log_{10} \lambda = -6, E_{train}(g_{\lambda}^{-}) = 0.010, E_{val}(g_{\lambda}^{-}) = 0.010, E_{out}(g_{\lambda}^{-}) = 0.025$		
$\log_{10} \lambda = -8, E_{train}(g_{\lambda}^{-}) = 0.000, E_{val}(g_{\lambda}^{-}) = 0.050, E_{out}(g_{\lambda}^{-}) = 0.025$	<b>✓</b> 20.00	
$\log_{10} \lambda = -4, E_{train}(g_{\lambda}^{-}) = 0.000, E_{val}(g_{\lambda}^{-}) = 0.010, E_{out}(g_{\lambda}^{-}) = 0.035$		
$\log_{10} \lambda = -0, E_{train}(g_{\lambda}^{-}) = 0.000, E_{val}(g_{\lambda}^{-}) = 0.050, E_{out}(g_{\lambda}^{-}) = 0.025$		
$\log_{10} \lambda = -2, E_{train}(g_{\lambda}^{-}) = 0.010, E_{val}(g_{\lambda}^{-}) = 0.050, E_{out}(g_{\lambda}^{-}) = 0.035$		
Total	20.00 / 20.00	

## **Question 17**

Among  $\log_{10} \lambda = \{2, 1, 0, -1, \dots, -8, -9, -10\}$ . What is the  $\lambda$  with the minimum  $E_{val}(g_{\lambda}^{-})$ ? Compute  $\lambda$  and the corresponding  $E_{train}(g_{\lambda}^{-})$ ,  $E_{val}(g_{\lambda}^{-})$  and  $E_{out}(g_{\lambda}^{-})$  then select the closet answer. Break the tie by selecting the largest  $\lambda$ .

Your Answer		Score	Explana	tion
$\log_{10} \lambda = -6, E_{train}(g_{\lambda}^{-}) = 0.066, E_{val}(g_{\lambda}^{-}) = 0.038, E_{out}(g_{\lambda}^{-}) = 0.038$				
$\log_{10} \lambda = 0, E_{train}(g_{\lambda}^{-}) = 0.033, E_{val}(g_{\lambda}^{-}) = 0.038, E_{out}(g_{\lambda}^{-}) = 0.028$	<b>~</b>	20.00		
$\log_{10} \lambda = -3, E_{train}(g_{\lambda}^{-}) = 0.000, E_{val}(g_{\lambda}^{-}) = 0.028, E_{out}(g_{\lambda}^{-}) = 0.038$				

$$\log_{10} \lambda = -9, E_{train}(g_{\lambda}^{-}) = 0.033, E_{val}(g_{\lambda}^{-}) = 0.028, E_{out}(g_{\lambda}^{-}) = 0.028$$

$$\log_{10} \lambda = -8, E_{train}(g_{\lambda}^{-}) = 0.066, E_{val}(g_{\lambda}^{-}) = 0.028, E_{out}(g_{\lambda}^{-}) = 0.028$$

$$20.00$$
/
20.00

Run the algorithm with the optimal  $\lambda$  of the previous problem on the whole  $\mathcal D$  to get  $g_\lambda$ . Compute  $E_{in}(g_\lambda)$  and  $E_{out}(g_\lambda)$  then select the closet answer.

Your Answer	Score	Explanation
$\bigcirc E_{in}(g_{\lambda}) = 0.025, E_{out}(g_{\lambda}) = 0.030$		
$\bullet E_{in}(g_{\lambda}) = 0.035, E_{out}(g_{\lambda}) = 0.020$	<b>✓</b> 20.00	
$\bigcirc E_{in}(g_{\lambda}) = 0.055, E_{out}(g_{\lambda}) = 0.020$		
$\bigcirc E_{in}(g_{\lambda}) = 0.045, E_{out}(g_{\lambda}) = 0.030$		
$\bigcirc E_{in}(g_{\lambda}) = 0.015, E_{out}(g_{\lambda}) = 0.020$		
Total	20.00 / 20.00	

## **Question 19**

Now split the given training examples in  $\mathcal{D}$  to five folds, the first 40 being fold 1, the next 40 being fold 2, and so on. Again, we take a fixed split because the given examples are already randomly permuted.

Among  $\log_{10}\lambda=\{2,1,0,-1,\ldots,-8,-9,-10\}$ . What is the  $\lambda$  with the minimum  $E_{cv}$ , where  $E_{cv}$  comes from the five folds defined above? Compute  $\lambda$  and the corresponding  $E_{cv}$  then select the closet answer. Break the tie by selecting the largest  $\lambda$ .

Your Answer	Score	Explanation

$\log_{10} \lambda = -2, E_{cv} = 0.020$	
$\bigcap \log_{10} \lambda = -4, E_{cv} = 0.030$	
$\bigcap \log_{10} \lambda = -6, E_{cv} = 0.020$	
$\bigcap \log_{10} \lambda = 0, E_{cv} = 0.030$	
	<b>2</b> 0.00
Total	20.00 / 20.00

Run the algorithm with the optimal  $\lambda$  of the previous problem on the whole  $\mathcal D$  to get  $g_\lambda$ . Compute  $E_{in}(g_\lambda)$  and  $E_{out}(g_\lambda)$  then select the closet answer.

Your Answer		Score	Explanation
$\bullet E_{in}(g_{\lambda}) = 0.015, E_{out}(g_{\lambda}) = 0.020$	<b>~</b>	20.00	
$\bigcirc E_{in}(g_{\lambda}) = 0.005, E_{out}(g_{\lambda}) = 0.010$			
$\bigcirc E_{in}(g_{\lambda}) = 0.045, E_{out}(g_{\lambda}) = 0.020$			
$\bigcirc E_{in}(g_{\lambda}) = 0.035, E_{out}(g_{\lambda}) = 0.030$			
$\bigcirc E_{in}(g_{\lambda}) = 0.025, E_{out}(g_{\lambda}) = 0.020$			
Total		20.00 / 20.00	