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Written assignment 3

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3) The given theta values were not clearly assigned, and are therefore assumed as follows:
$\theta_{11}^{(1)} = 0.5$
$\theta_{21}^{(1)} = 0.1$
$\theta_{12}^{(1)} = 0.5$
$\theta_{22}^{(1)} = 0.7$
$\theta_{11}^{(2)} = 1$
$\theta_{12}^{(2)} = 2$

3.1 $x1 = 0.5, x2 = 0.9$ Using forward propagation this gives:
$g(x)$, the Sigmoid function $= \frac{1}{1+e^{-\theta}}$.
$a_1^{(2)} = g(\theta_{10}^{(1)} * x_0^{(1)} + \theta_{11}^{(1)} * x_1 + \theta_{12}^{(1)} * x_2) = 0.711$
$a_2^{(2)} = g(\theta_{20}^{(1)} * x_0^{(1)} + \theta_{21}^{(1)} * x_1 + \theta_{22}^{(1)} * x_2) = 0.707$
$a_1^{(3)} = g(\theta_{10}^{(2)} * x_0^{(2)} + \theta_{11}^{(2)} * a_1^{(2)} + \theta_{21}^{(2)} * a_2^{(2)}) = 0.912$

3.2 The error is calculated using back propagation, the correct output, y, is given as 1. Therefore:
$\delta_1^{(3)} = y - a_1^{(3)} = 1 - 0.912 = 0.088$
$\delta_1^{(2)} = \theta_{11}^{(2)} * \delta_1^{(3)} = 1 * 0.088 = 0.088$
$\delta_2^{(2)} = \theta_{12}^{(2)} * \delta_1^{(3)} = 2 * 0.088 = 0.176$
$x_1^{(1)} = \theta_{11}^{(1)} * \delta_1^{(2)} + \theta_2^{(1)} 1 * \delta_2^{(2)} = 0.0352$
$x_2^{(1)} = \theta_{12}^{(1)} * \delta_1^{(2)} + \theta_2^{(1)} 2 * \delta_2^{(2)} = 0.1672$

4.1)
The boundary can be described as the line x2 = x1 + 2. This can be used to find the weights: $w_0 2, w_1 = 2, w_2 = 1$.
$4.2a)$
The boolean A AND (NOT B) can be written as a perceptron in the following way. The perceptron has two inputs and a bias unit, which is has the input of 1. The thetas for the perceptron are:

$\theta_{10}^{(1)} = -15$
$\theta_{11}^{(1)} = 11$
$\theta_{12}^{(1)} = -6$
Instead of the sigmoid function the following function is used. g(x) = [if w0*x0 + w1*x1 + w2*x2 >0 g(x) = 1, else g(x) = 0]

The perceptron, given x1 = [1 1 0 0] and x2 = [1 0 1 0], returns the truth table [0 1 0 0]. which is the same as the truth table of x1 AND NOT x2.

4.2b)
The perceptron for A XOR B is logically the same as the perceptron for ((A OR B) AND NOT(A AND B)). This makes it easier to create the perceptron using two layers. The layers both have two inputs and a bias unit. The bias unit has input 1. The thetas or weights are as follows.

$\theta_{10}^{(1)} = -5$
$\theta_{20}^{(1)} = 20$
$\theta_{11}^{(1)} = 10$
$\theta_{21}^{(1)} = -11$
$\theta_{12}^{(1)} = 10$
$\theta_{22}^{(1)} = -11$

$\theta_{10}^{(2)} = -11$
$\theta_{11}^{(2)} = 10$
$\theta_{12}^{(2)} = 10$

The same function is used as in the question above.
This perceptron give the truth table [0 1 1 0] when given the input as in the question above. This is the same as the truth table for A XOR B.