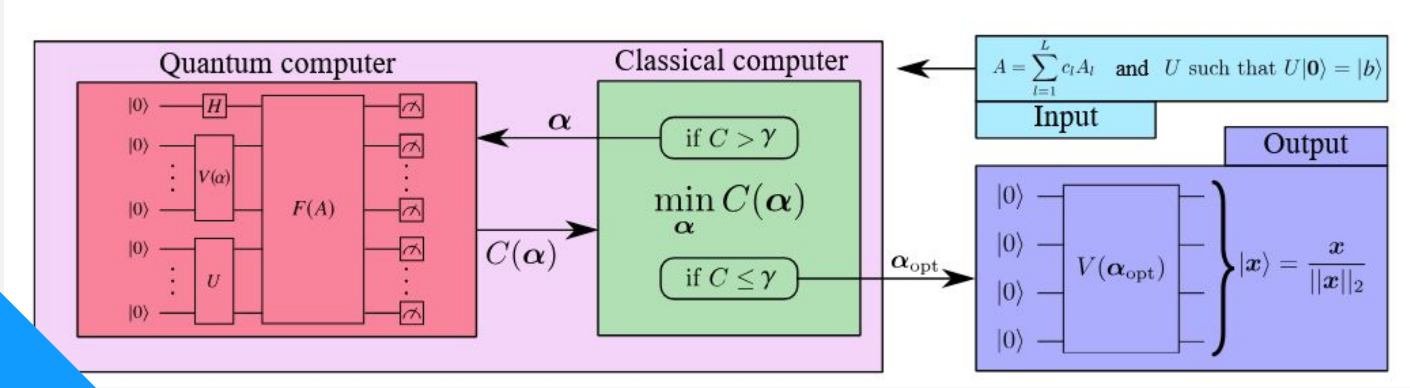
Quantum Algorithms for Linear System Problem



Cre: arXiv:1909.05820

Linear Problem

HHL

HHL-Cutoff

VQLS

Linear System Problem

$$A.x = b$$

Linear equations play an important role in virtually all fields of science and engineering

Many practical problems are, or approximately, encoded in linear equations

However, the sizes of the data sets growing rapidly over space-time, so that terabytes and even petabytes of data may need to be processed to obtain a solution

Frequently, one is interested not in the full solution to the equations, but rather in computing some function of that solution

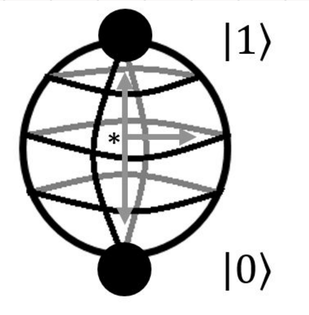
Complexity

For a classical method in general requires computational overhead that scales at least as N

HHL algorithm

(Harrow-Hassidim-Lloyd)

Hermitian matrix, sparse matrix



$$A.|x> = |b>$$

Complexity

Quantum method approximate the solution in time which scales logarithmically in N

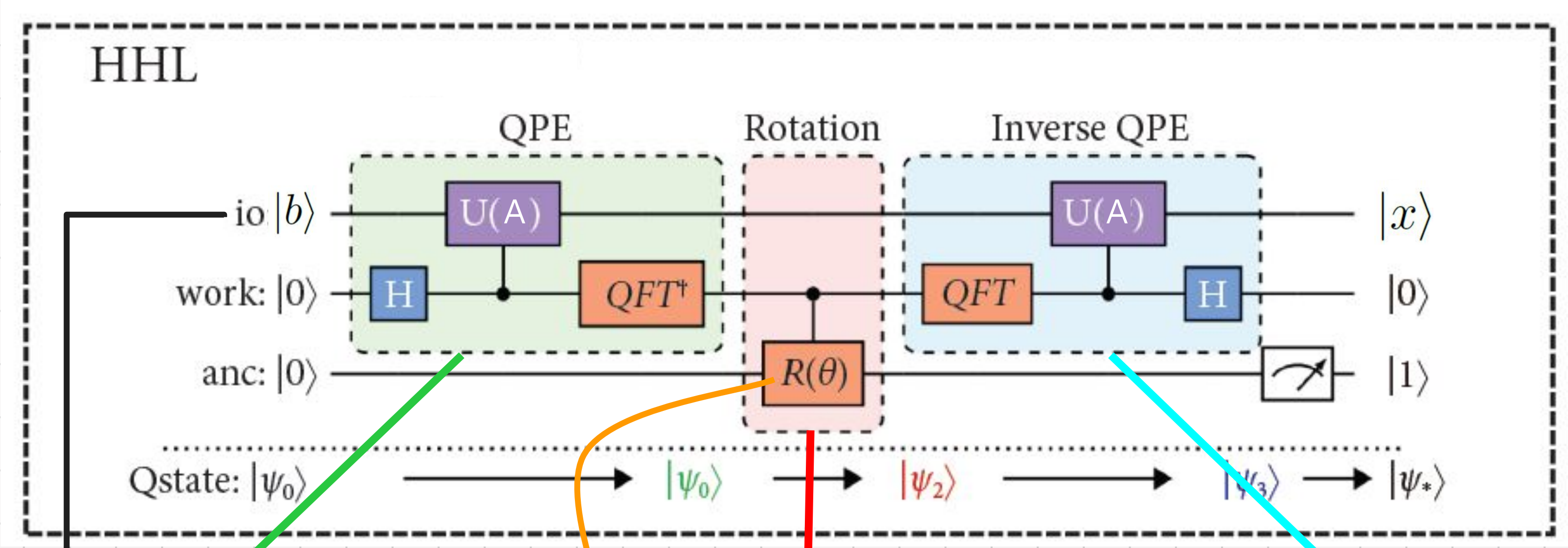
Encode data in quantum states and operators

Apply quantum circuit operation to generate a quantum state corresponding to solution

Works well in Fault-Tolerance Quantum Computers, adverse to Near-term Noisy Quantum Computer devices though

$$\sum_{j=1}^{N} \beta_j \lambda_j^{-1} |u_j\rangle = A^{-1}|b\rangle = |x\rangle$$

Cre: 10.23919/IEN.2022.0021



$$\sum_i \boxed{\beta_i|u_i\rangle}|\lambda_i t/2\pi\rangle$$

$$\sum_i \lambda_i^{-1}\beta_i|u_i\rangle|\lambda_i t/2\pi\rangle$$

$$\sum_i \lambda_i^{-1}\beta_i|u_i\rangle = |x\rangle$$

Other method (no Θ-classical calculation cost):
arXiv:1807.03206

HHL-Cutoff algorithm

$$x = |QPE(A)|b\rangle|$$

Only interested in compute solution x

More noise-resilient approach for QLS study compared to full HHL

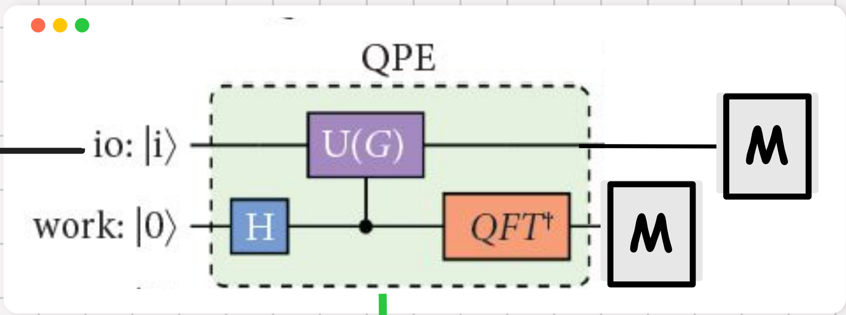
Classically, post-processing measured data from quantum circuit to construct solution x

Complexity

Reduce the quantum circuits depth by half of the full HHL algorithm

Cons

The classically post-processing data is ineffective in full noiseless QC devices, may suit for the mid-term QC devices



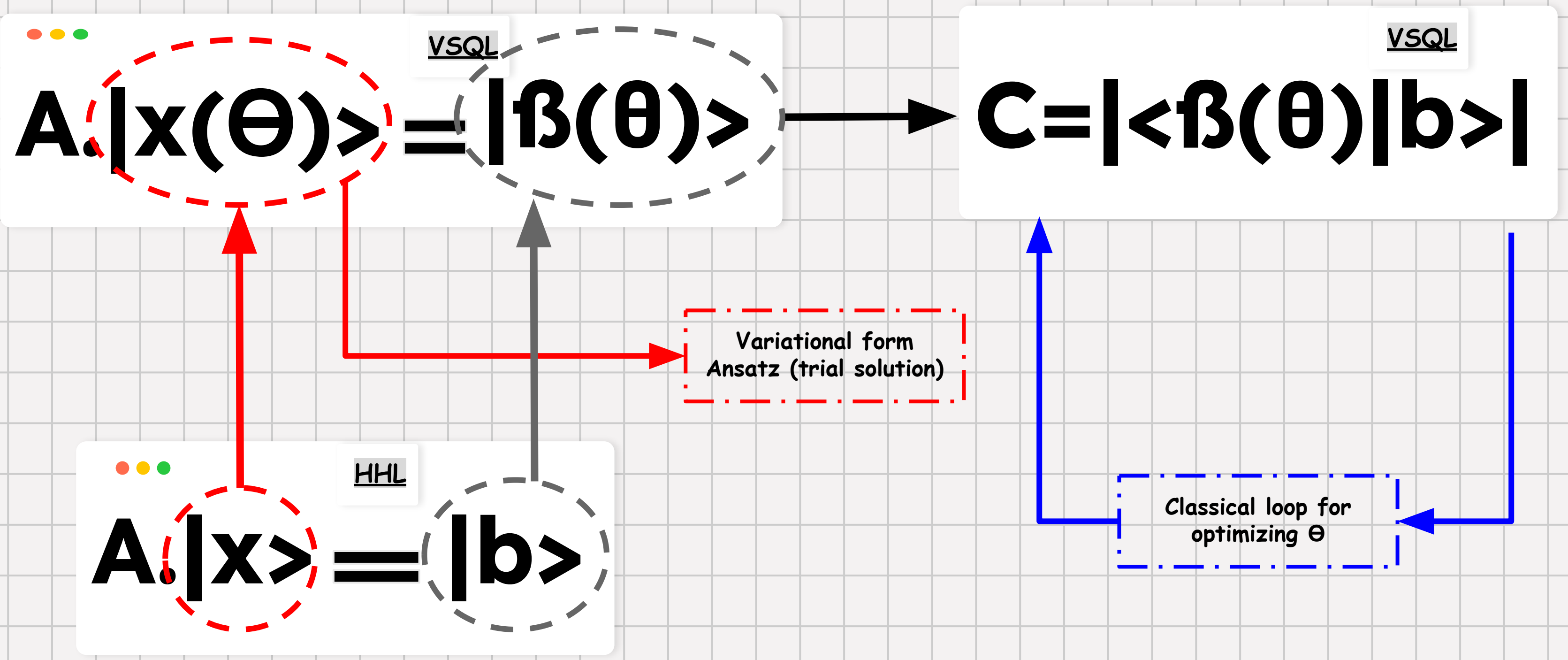
$$\sum_{j=1}^{N} \beta_j \left| u_j \right\rangle$$

$$\sum_i \beta_i |u_i\rangle |\lambda_i t/2\pi\rangle$$

$$\sum_i \lambda_i^{-1} \beta_i |u_i\rangle = |x\rangle$$

Name Department Course Number Section Time Location DATE

VSQL algorithm



Variational Quantum Linear Solver

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