

A PROBLEM ABOUT PERMISSION*

1. THE GAME

Consider a little language game that is played as follows.

(1) There are three players, called the *Master*, the *Slave*, and the *Kibitzer*. It would change nothing to have more than one slave, or more than one kibitzer, but let us put aside the complications that arise if a slave must serve two masters. (They say it can't be done.)

(2) There is a certain set of strings of symbols, called the set of *sentences*. A player may at any time make the move of *saying* any sentence *to* any other player within earshot.

(3) There is a certain function that assigns to any sentence ϕ , at any pair $\langle t, w \rangle$ of a time t during the game and a suitable possible world w , a value 1 or 0 called the *truth value* of ϕ at t at w . (We leave off the 'at w ' when w is the actual world.) ϕ is called *true* or *false* at t at w according as the truth value is 1 or 0.

(4) There is another function that assigns to any such pair $\langle t, w \rangle$ a set of worlds called the *sphere of permissibility* at t at w . Worlds in this set are said to be *permissible* at t at w .

(5) There is another function that assigns to any such pair $\langle t, w \rangle$ a set of worlds called the *sphere of accessibility* at t at w . Worlds in this set are said to be *accessible* at t at w . These worlds are the alternatives, including always w itself, that are left open by the past history of w up to t . They share that history, but they continue it in divergent ways. Spheres of accessibility are always contracting (except in trivial cases) and the contraction is irreversible: once a world has become inaccessible, it remains so forevermore. (I am not sure, but perhaps we should impose another condition: if one world is accessible at t at another, then the two worlds have exactly the same sphere of accessibility at t .)

(6) The $\langle t, w \rangle$ pairs on which the functions listed in (3)–(5) are defined include all of those such that t is a time during the game and w is accessible (at the actual world) at the time when the game begins. Let us henceforth tacitly omit from consideration all times and worlds but these.

(7) There is a certain symbol ! that may be prefixed to any sentence \varnothing to make a new sentence ! \varnothing , called an *imperative* sentence, that is true at t at w iff \varnothing is true at t at every world that is both accessible and permissible at t at w .

(8) There is a certain symbol ; that may be prefixed to any sentence \varnothing to make a new sentence ; \varnothing , called a *permissive* sentence, that is true at t at w iff \varnothing is true at t at some world that is both accessible and permissible at t at w .

(9) The sphere of permissibility at any time (at any world) depends as follows on the past history of the world. When the game begins, it is the set of all worlds. Thereafter it remains unchanged except when the Master says to the Slave an imperative or permissive sentence that would be false, when said, if the sphere remained unchanged. Then the sphere adjusts itself, if possible, to make the Master's sentence true. Suppose that at t the Master says to the Slave ! \varnothing ; and suppose that the sphere of permissibility just before t contains some worlds, accessible at t , where \varnothing is false at t . Then the sphere must contract to cut those worlds out: at t , and thereafter at least until the next change, none of those worlds are permissible. If the Master changes the sphere in this way by saying ! \varnothing , we say that the Master *commands* that \varnothing . Or suppose that at t the Master says to the Slave ; \varnothing ; and suppose that the sphere of permissibility just before t contains no worlds, accessible at t , where \varnothing is true at t ; and suppose that there do exist some such worlds outside the sphere. Then the sphere must expand to take in some of those worlds: at t , and thereafter at least until the next change, some of those worlds are permissible. If the Master changes the sphere in this way by saying ; \varnothing , we say that the Master *permits* that \varnothing .

(10) The Slave tries to see to it that the actual world is within the sphere of permissibility at all times. If the Slave knows, at a time t , that he acts in a certain way at t throughout the worlds that are permissible and accessible at t – for instance, if he knows that at all such worlds he begins a certain task at t – then he tries to act in that way at the actual world.

(11) Each player tries to see to it that he never says a sentence to another player unless that sentence is true at the time when he says it. The Master, when he commands or permits, is automatically truthful since the sphere adjusts to make him so; other players, and even the Master when he is not commanding or permitting, are truthful by choosing sentences to say that are true at the worlds that conform to their beliefs.

2. COMMENTS

The point of the game, as regards commanding and permitting, is to enable the Master to control the actions of the Slave. What the Slave does depends on the present sphere of permissibility, which depends in turn on the Master's previous commands and permissions. We need not ask why the Slave is willing to play his part. Perhaps he does so by habit; perhaps he is coerced; perhaps he is obligated; or perhaps he hopes that the Master's control over him will be used to his benefit as well as to the Master's. In any case, the game is played. And we may suppose it to be common knowledge that the game is played: each player expects the others to play their parts, expects the others so to expect, and so on.

In this simple example, I have tried to merge two complementary approaches to the study of imperatives. The semantic analysis of ! and ; given in (3)–(8) is taken, with slight changes, from Chellas [1] and [2]. The treatment of commanding and permitting as part of a social practice for enabling one person to control another is taken from Stenius [5] and Lewis [4].

If there were only commanding and no permitting, the language game could be described more simply. We could drop (4)–(10) and replace them as follows. If at any time t the imperative sentences said by the Master to the Slave before t are given by the list

$$(L) \quad !\varnothing_1 \text{ at } t_1, \dots, !\varnothing_n \text{ at } t_n,$$

then the Slave tries at t to see to it that \varnothing_1 is true at t_1, \dots , and that \varnothing_n is true at t_n . On this account, the only truth value that we need to associate with an imperative sentence ! \varnothing is the truth value of the content sentence \varnothing (at the time when it was commanded). We could call *this* the truth value of the imperative ! \varnothing , and say that the Slave tries to see to it that the Master's previous imperatives to him are made true. That was my account of imperatives in [4]. But then what do we make of permission? It is easy enough to provide for annulment of commands: the Master may at any time remove an item from the list (L), after which the Slave acts as if that command never had been given. But permissions are not, in general, annulments of particular past commands. A permission may partly undo several past commands, without fully undoing any of them. We need a device for integrating a succession of commands and permissions. A list with additions and deletions is one such device, but it is not flexible enough. The sphere of permissibility is meant to be a better device to serve the same purpose.

Commanding and permitting are not the whole of our language game. As regards all other sentence-saying, the point of the game is to enable the players to impart information to one another. Whenever truthfulness is not automatic, the hearer who expects the speaker to be truthful can infer something about the speaker's beliefs from the sentences that the speaker is willing to say; and often the hearer can go on to infer conclusions about the world, premised on confidence that the speaker's beliefs about certain topics tend to be correct. To the extent that the speaker can anticipate these inferences, he can control the hearer's beliefs by what he says. In particular, one player may wish to inform another about the present state of the sphere of permissibility – that is, about the integrated effect of the Master's commands and permissions up to now. There is nothing to keep him from doing so, given the way we have set up the language game, by using the same imperative and permissive sentences that the Master himself uses to change the sphere. One and the same sentence '*! the Slave carries rocks all day*' may be said by the Master to the Slave to reshape the sphere of permissibility and cause the Slave to carry the rocks; by the Slave to the Kibitzer to elicit sympathy; by the Master to the Kibitzer to explain why the Slave is not working on his usual chores; by the Slave or the Kibitzer to the forgetful Master to remind him what the Slave is supposed to be doing; and so on. It may even be used by the Master to the forgetful Slave as a reminder, with no further adjustment of the sphere of permissibility. Likewise '*! the Slave does no work tomorrow*' may be said by the Master to the Slave to grant a holiday; by the Master to the Kibitzer to point out that the Slave's lot is not so very bad after all; and so on.

While I admit to an inclination to play Old Harry with the performative/constative fetish, I insist that I have not erased the distinction between different speech-acts that may be performed by saying an imperative sentence. The sentence may be used to command: the Master says it to the Slave, his purpose is to control the Slave's actions by changing the sphere of permissibility, and truthfulness is automatic because the sphere adjusts so that saying so makes it so. The sentence may be used to inform: either the speaker is not the Master or the hearer is not the Slave, the speaker's purpose is to impart information to one who does not yet possess it, and truthfulness is not automatic. Or the sentence may be used to remind (an intermediate case): again the Master says it to the Slave, but this time his purpose is to impart (or re-impart) information, and although truthfulness would be automatic the Master intends the sentence to be true even with-

out any adjustment of the sphere of permissibility. Likewise for permissive sentences, except that truthfulness is never quite automatic since the Master cannot truly permit what is impossible. These are perfectly good distinctions; my point is only that they need not be part of semantics, insofar as semantics deals with truth conditions. In fact, they must not be. Only if the truth conditions are uniform from one use to another can we use the given formulation of (9).

I have no real dispute, however, with anyone who finds it intolerable to say that an imperative sentence, when used to command, has a truth value. In describing the language game I did not really use any semantic terms as primitives. I could have; but the description I actually gave is related to a description using semantic primitives as the Ramsey sentence of a term-introducing scientific theory is related to the theory itself. For instance, 'truth value' serves only as a mnemonic label for the values of the function introduced in (3) by existential quantification. If you dislike that label – or any other – feel free to substitute the euphemism of your choice.

3. PERMISSIBILITY KINEMATICS

I said that the changing sphere of permissibility integrates the effect of the Master's successive commands and permissions, but I did not say exactly how. The requirements in (9) constrain, but do not determine, the evolution of the sphere. When the Master says to the Slave an imperative or permissive sentence that would be false if the sphere remained unchanged, there will ordinarily be infinitely many alternative adjustments that would make his sentence true.

For commanding, at least, it is easy enough to say precisely how the sphere should change. Suppose that at time t (at a given world) the Master says to the Slave $!\varnothing$, and suppose that a change in the sphere of permissibility is needed to make $!\varnothing$ true at t . Let P be the old sphere just before t , and let $[\varnothing \text{ at } t]$ be the set of all worlds where \varnothing is true at t . Then the new sphere at t , and thereafter until the next change, should be the intersection $P \cap [\varnothing \text{ at } t]$. All worlds accessible at t where \varnothing is false at t must be removed from the sphere, according to (9); but it would be gratuitous to remove any further accessible worlds, since the Master has commanded that \varnothing and nothing further, and it would be gratuitous to add any accessible worlds that were not permissible before, since the Master has not permitted anything but only commanded something. As

for inaccessible worlds, it makes no difference which are removed or added so I have made the most convenient arbitrary stipulation.

If the sphere's evolution under the impact of commands does go by intersection in the way just proposed, then we have the proper result for the special case that there is only commanding and no permitting. Let the Master's commands before t be: $!\varnothing_1$ at $t_1, \dots, !\varnothing_n$ at t_n . Then by successive intersections the sphere of permissibility at t is $P_0 \cap [\varnothing_1 \text{ at } t_1] \cap \dots \cap [\varnothing_n \text{ at } t_n]$, where the initial sphere P_0 is the set of all worlds. The Slave, according to (10), tries at t to see to it that the actual world is within the sphere of permissibility at t . That is to say that he tries at t to see to it that the actual world is in all of the sets $[\varnothing_1 \text{ at } t_1], \dots, [\varnothing_n \text{ at } t_n]$. And that is to say exactly what we said before about this special case: that he tries at t to see to it that \varnothing_1 is true at t_1, \dots , and \varnothing_n is true at t_n .

One sort of commanding may seem to require special treatment: commanding the impermissible. Suppose that $[\varnothing \text{ at } t]$ contains no worlds that are both accessible and permissible at t , so that $\neg \varnothing$ is false at t . The Master may nevertheless wish to command at t that \varnothing . For instance, he may have changed his mind. Having commanded at dawn that the Slave devote his energies all day to carrying rocks, the Master may decide at noon that it would be better to have the Slave spend the afternoon on some lighter or more urgent task. If the Master simply commands at t that \varnothing , and if the sphere evolves by intersection, then *no* world accessible at t remains permissible; the Slave, through no fault of his own, has no way to play his part by trying to see to it that the world remains permissible. We have no idea what the Slave may do to make the best of an impermissible situation. Should we therefore say that in this case the sphere evolves not by intersection but in some more complicated way? I think not. The resources of the language game are not to blame if the Master removes all accessible worlds from the sphere of permissibility by commanding the impermissible. Rather the Master is to blame for misusing those resources. What he should have done was first to permit and then to command that \varnothing . He should say to the Slave, in quick succession, first $\neg \varnothing$ and then $!\varnothing$; that way, he would be commanding not the impermissible but the newly permissible. We could indeed have equipped the language game with a labor-saving device: whenever $\neg \varnothing$ is false, a command that \varnothing is deemed to be preceded by a tacit permission that \varnothing , and the sphere of permissibility evolves accordingly. But this is a frill that we can well afford to ignore, since it does not enable the Master to do anything more than he can do in the original, simpler language game.

Turning now to the evolution of the sphere under the impact of permissions, we reach the problem announced in my title. The natural parallel to evolution by intersection in the case of commands would be evolution by union, as follows: if at t the Master says to the Slave $\neg \varnothing$, and if a change in the sphere of permissibility is needed to make $\neg \varnothing$ true at t , and if P and $[\varnothing \text{ at } t]$ are as before, then the new sphere at t , and thereafter until the next change, is the union $P \cup [\varnothing \text{ at } t]$. But this sort of evolution by union, unlike evolution by intersection in the case of commands, is far from realistic. There could be a language game that did work that way – the rules are up to the players – but it would lack one salient and problematic feature of permission as we know it.

The problem is this. When the Master permits something, he does not thereby permit that thing to come about in whatever way the Slave pleases – not if the game is to be realistic. Suppose the Slave has been commanded to carry rocks every day of the week, but on Thursday the Master relents and says to the Slave ' \neg the Slave does no work tomorrow'. That is all he says. He has thereby permitted a holiday, but not just any possible sort of holiday. He has presumably not thereby permitted a holiday that starts on Friday and goes on through Saturday, or a holiday spent guzzling in his wine cellar. *Some* of the accessible worlds where the Slave does no work on Friday have been brought into permissibility, but not all of them. The Master has not said which ones. He did not need to; somehow, that is understood.

Perhaps the incorrect principle of evolution by union in the case of permissions has some correct consequences, as follows. First, the new sphere at t should contain some world in $[\varnothing \text{ at } t]$ that is accessible at t , if there exists some such world; that much is required by (9). Second, it should be included in $P \cup [\varnothing \text{ at } t]$; since the Master has permitted that \varnothing and nothing further, it would be gratuitous to bring worlds into permissibility where \varnothing is false at t . Third, it should include all of P ; since the Master has not commanded anything but only permitted something, it would be gratuitous to remove any worlds from permissibility. In short, the new sphere at t is the union of the old sphere P and some subset or other of $[\varnothing \text{ at } t]$, where all we know yet about this subset of $[\varnothing \text{ at } t]$ is that it must, if possible, contain some world that is accessible at t .

Let us return to our example. Hitherto the Slave has been commanded to carry rocks every day of the week, to abstain from the Master's wine, and perhaps other things besides. Now he has been permitted (on Thursday) to do no work on Friday. So the newly permissible worlds are all of the

worlds that were permissible hitherto, along with some of the accessible worlds, formerly impermissible, where the Slave does no work on Friday. (If such there be; but in this case there are.) But only some, not all. The worlds brought newly into permissibility include none of those where the Slave does no work on Friday or on Saturday either; nor any of those where he does no work on Friday and drinks the Master's wine.

Why not? Various answers might be given. But though they seem sensible in this case, I do not think any of them lead to any simple and definite general principle of evolution.

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Answer 1. To enlarge the sphere of permissibility so that it includes worlds where the Slave does no work on Saturday, or worlds where he drinks the Master's wine, would be a gratuitous enlargement. It would be more of an enlargement than is needed to make it permissible not to work on Friday.

I reply that the same is true of any reasonable enlargement. If the game is to be at all true to life, there will be more than one permissible way for the Slave to spend his holiday. (Even if he is required to spend the day at prayer, still he is no doubt free to choose the points in his prayers at which to take a breath.) Then more than the least possible number of worlds – more than one – must have been brought into permissibility.

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Answer 2. To include worlds in the enlarged sphere of permissibility where the Slave does no work on Saturday, or where he drinks the Master's wine, would be gratuitous change, not in a quantitative but in a qualitative way. The newly permissible worlds should be selected to resemble (as closely as possible) the worlds that were permissible before.

I reply that according to my offhand judgments of similarity, that principle instructs us to select worlds where the Slave spends Friday in the gymnasium lifting weights. Among worlds where the Slave does not work on Friday, are not these the worlds most similar to the previously permissible worlds – worlds where he spends Friday carrying rocks? But surely a weight-lifting holiday is not the only sort of holiday that has been made permissible.

To be sure, the outcome depends on the relation of comparative similarity that guides the selection. Offhand judgments are no safe guide. Not every similarity relation worthy of the name gives significant weight to

the obvious similarity between rock-carrying and weight-lifting. So perhaps it is true, *under the right similarity relation*, that the worlds that become permissible are those of the worlds where the Slave does no work on Friday that most resemble the previously permissible worlds. But which similarity relation is the right one for our present purpose? This is just a restatement of our original problem, and seems to me unhelpful.

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Answer 3. Before the Master's permission, all worlds where the Slave did no work on Friday were impermissible; but they were not equally impermissible. Those where he also failed to work on Saturday, or where he drank the Master's wine, were more impermissible – or more remote from permissibility – than some of the others. (Whether or not they were also more dissimilar from the permissible worlds in other respects, at least they were more dissimilar in respect of their degree of permissibility.) If the Slave cannot (or will not) see to it that the actual world is within the sphere of permissibility, he may at least try for second best and keep the world as nearly permissible as he can. The relation of comparative near-permissibility determines what is second best. Perhaps it is this same relation that selects the newly permissible worlds when the Master enlarges the sphere of permissibility: the worlds that become permissible are those of the worlds where the Slave does no work on Friday that were most nearly permissible before.

I reply that this may be; and that it seems right to connect the problem of evolution under permissions with the problem of second-best courses of action for the Slave. (I am grateful to Robert Stalnaker for pointing out this connection.) Still it seems to me that again the problem has been restated rather than solved. Given the relation of comparative near-permissibility at every stage of the game, we may have a complete principle governing the evolution of the sphere of permissibility; but how does the comparative relation evolve from stage to stage? Is it so that the spheres of permissibility and accessibility at any stage suffice somehow to determine the comparative near-permissibility of worlds-at that stage? If so, how?

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Answer 4. Perhaps we should look outside the game to the goals it serves. It is to serve some purposes that the Master controls the Slave by commanding and permitting. The Slave either shares these purposes or at least acquiesces enough that he continues to play his part in the game.

When the Slave is permitted to do no work on Friday, some worlds remain impermissible because if they were to become permissible and the Slave were to actualize one of them, that would not serve the purposes for which the game is played. It is understood that these purposes require the Slave to work hard and to keep away from the Master's wine. Therefore worlds where the Slave does no work on Saturday, or where he drinks the Master's wine, are not readily brought into permissibility when the Master permits a holiday on Friday.

I reply that either the Slave does know what would serve the purposes in question, or he does not. If he does, then what is the point of a game of commanding and permitting? The Slave might as well simply ignore what the Master says and do whatever he judges to serve the purposes. The game is played exactly because the Slave needs guidance in serving those purposes. But if the Slave does not know what would serve the purposes, and if the evolution of the sphere of permissibility depends on what would serve the purposes, then the Slave is not in a good position to figure out how the sphere has evolved, and hence is not in a good position to figure out what is permissible. For the Slave to suffer this difficulty will itself interfere with the success of the game of commanding and permitting in serving those purposes for the sake of which it is played.

The best that might be done along these lines, I suppose, is as follows. It might be that the Slave knows just enough, and not too much, about what would serve the purposes. Since he knows enough, he is in a position to figure out how the sphere of permissibility evolves when the Master enlarges it, as by permitting a holiday on Friday. Since he does not know too much he remains in need of guidance if the purposes are to be served, and the game does not become pointless. This might be so. But I find it hard to believe that only when a delicate balance has been struck does the game I have described both retain its point and become playable.

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Answer 5. At any stage, the sphere of permissibility may be specified by a list of requirements. (The list may or may not match the list of commands by which the sphere was shaped.) Each requirement on the list is satisfied at every permissible world; the worlds that are permissible are exactly those that satisfy every requirement on the list. The list might be as follows:

The Slave carries rocks all day on Sunday.

The Slave carries rocks all day on Friday.

The Slave carries rocks all day on Saturday.

The Slave never drinks the Master's wine.

Find those entries on the list that conflict with the Master's permission that the Slave do no work on Friday. There is one and only one; strike it out. The new sphere of permissibility consists of exactly those worlds that satisfy the remaining requirements.

I reply that it all depends on how you encode the sphere of permissibility by a list of requirements. If you do it the right way, as above, the technique of striking out requirements that conflict with the Master's permission will give the right answer. Unfortunately, there are also wrong ways. The same sphere could have been encoded by another list:

The Slave carries rocks every morning of the week.

The Slave carries rocks every afternoon of the week.

The Slave never drinks the Master's wine.

Now we cannot strike out the one and only requirement that conflicts with the Master's permission; the first two both conflict. We could strike out both of them; but that will make it permissible to do no work on Saturday. Or take this list, another that encodes the sphere:

The Slave carries rocks all day on Sunday or drinks the Master's wine.

The Slave carries rocks all day on Friday or drinks the Master's wine.

The Slave carries rocks all day on Saturday or drinks the

Master's wine.

The Slave never drinks the Master's wine.

Now there is no one requirement which conflicts, all by itself, with the Master's permission; but there are two that jointly conflict with it. Strike out the right one of the two, and all is well. Strike out the wrong one (or strike out both) and the results are not at all as we would wish. Strike out the requirement that the Slave never drinks the Master's wine and take the new sphere of permissibility to consist of those worlds that satisfy the remaining requirements on the list. This enlargement brings into permissibility worlds where the Slave does no work on Friday, does no work on Saturday either, and spends both days drinking the Master's wine. (It would also bring in worlds where the Slave does no work earlier in the week, except that by Thursday these worlds are inaccessible.)

So the method of listing and striking out will not work unless we choose the right one of the lists of requirements that encode the original sphere of permissibility. Which one is that? Again we have a restatement of our original problem, not a solution.

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How much of a solution is it reasonable to expect? There are cases where it is really unclear which worlds have been brought into permissibility. That means that no principle can be both as definite as we might hope and clearly correct. One such case is given by Thomas Cornides in a discussion of our problem [3]. (He defends a version of Answer 5, but is well aware of the reasons why the procedure of listing and striking out will not always give a determinate answer, even if the correct list is somehow given us.) His example is as follows. First comes the command '! you play only if you do your homework.' Second comes the command '! you watch television only if you play.' And third comes the permission '¡ you watch television and you do not do your homework.' Is it now permissible to watch television, not do the homework, and not play? That is unclear; and I think it might be left unclear even if we knew all that was relevant about the players and about their reasons for playing a game of commanding and permitting. So a principle governing the evolution of permissibility cannot settle this case in a way that is clearly correct.

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NOTES

* Thanks are due to audiences on several occasions, and especially to Robert Martin and Robert Stalnaker, for comments on a previous version of this paper.

I am told that Thomas Ballmer has developed a theory similar to that presented here. However, I have not seen any details of his work.

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