Homework: Week 9

Fengjun Xu

Nov. 13, 2024

1:

Find, by explicit construction using Pauli matrices, the eigenvalues for the Hamiltonian

$$H = -\frac{2\mu}{\hbar}\mathbf{S}\cdot\mathbf{B}$$

for a spin $\frac{1}{2}$ in the presence of a magnetic field $\mathbf{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$

2:

Construct the matrix representations of the operators J_x and J_y for a spin 1 system, in the J_z basis, spanned by the kets $|+\rangle := |1,1\rangle, |0\rangle := |1,0\rangle, |-\rangle := |1,-1\rangle$. Use these matrices to find the three analogous eigenstates for each of the two operators J_x and J_y in terms of $|+\rangle, |0\rangle, |-\rangle$

3:

A particle in a spherically symmetrical potential is known to be in an eigenstate of $\mathbf{L}^2$ and L_z with eigenvalues $\hbar^2 l(l+1)$ and $m\hbar$, respectively. Prove that the expectation values between $|lm\rangle$ states satisfy

$$\langle L_x\rangle = \langle L_y\rangle = 0, \qquad \langle L_x^2\rangle = \langle L_y^2\rangle = \frac{\hbar^2 l(l+1) - m^2\hbar^2}{2}$$

Interpret this result semi-classically.

4:

The goal of this problem is to determine degenerate eigenstates of the three-dimensional isotropic harmonic oscillator (see sec.3.7.3 in **J.J.Sakurai .et.al** "Modern Quantum Mechanics") written as eigenstates of $\mathbf{L}^2$ and $\mathbf{L}_z$, in terms of the Cartesian eigenstates $|n_x n_y n_z\rangle$.

a). Show that the angular-momentum operators are given by

$$\begin{aligned}\mathbf{L}_i =& i\hbar\epsilon_{ijk} a_j a_k^\dagger \\ \mathbf{L}^2 =& \hbar^2[N(N+1) - a_k^\dagger a_k^\dagger a_j a_j]\end{aligned}$$

where summation is implied over repeated indices, ϵ_{ijk} is the totally anti-symmetric symbol, and $N = a_j^\dagger a_j$ is the number operator counting the total number of quanta.

b).Use these relations to express the states $|qlm\rangle = |01m\rangle, m = 0, \pm 1$, in terms of the three eigenstates $|n_x n_y n_z\rangle$ that are degenerate in energy. Write down the representation of your answer in coordinate space, and check that the angular and radial dependences are correct.

c). Repeat for $|qlm\rangle = |100\rangle$.