Homework: Week 4

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1:

1. Find the expectation values of (x, x^2) and (p, p^2) in a harmonic oscillator energy eigenstate $|n\rangle$.

Also, verify that in general, for a eigenstate $|\Phi\rangle$ of a 1-dimensional system with the Hamiltonian:

$$H = K + V$$

where $K := \frac{P^2}{2m}$ is its Kinetic term and $V(X)$ is the potential, the following identity holds

$$2\langle\Phi|K|\Phi\rangle = \langle\Phi|X\frac{\partial V}{\partial X}|\Phi\rangle$$

2:

2. For the harmonic oscillator, one can define a special type of quantum state called a coherent state. Given any complex number z, the non-normalized coherent state $|z)$ (we use a right parenthesis instead of $\rangle$ to emphasize that this quantum state is not normalized) can be defined as:

$$|z) = e^{z\hat{a}^\dagger}|0\rangle$$

- Please verify $|z) = \sum_{n=0}^{+\infty} \frac{z^n}{\sqrt{n!}}|n\rangle$.
- Please prove that $|z)$ is an eigenstate of the annihilation operator $\hat{a}$ [1], satisfying:

[1] Note that since a is not a Hermitian operator, its eigenvalue can be a complex number z.

$$\hat{a}|z) = z|z)$$

- Please normalize $|z)$ and calculate the inner product between the coherent states $|z)$ and $|w)$, denoted as $(w|z)$.
- Please calculate the uncertainties δX and δP of the position operator X and the momentum operator P in the coherent state $|z)$ and prove that $\delta X \delta P$ attains the minimum value, that is:

$$\delta X \delta P = \frac{\hbar}{2}$$