Assignment 7

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1 Exercise 14.14: a, b, and c.

a. Which of the following are asserted by the network *structure*?

(i) $\mathbf{P}(B,I,M) = \mathbf{P}(B)\mathbf{P}(I)\mathbf{P}(M)$.

(ii) $\mathbf{P}(J|G) = \mathbf{P}(J|G,I)$.

(iii) $\mathbf{P}(M|G,B,I) = \mathbf{P}(M|G,B,I,J)$.

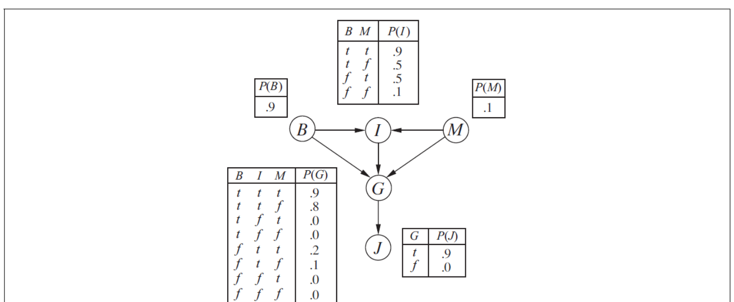


Figure 14.23 A simple Bayes net with Boolean variables $B = BrokeElectionLaw$, $I = Indicted$, $M = PoliticallyMotivatedProsecutor$, $G = FoundGuilty$, $J = Jailed$.

Answer: ii, iii

b. Calculate the value of $P(b,i,\neg m,g,j)$.

Answer: $P(b,i,\neg m,g,j) = \text{P(b)P(i|b,}\neg\text{m)P(}\neg\text{m)P(g|b,i,}\neg\text{m)P(j|g)} = 0.9*0.5*0.9*0.8*0.9=0.2916$

c. Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.

Answer: $\text{P(j|b,i,m)}=\frac{\sum_{g'} P(j,b,i,m,g')}{\sum_{j',g'} P(j',b,i,m,g')}$

$$=\frac{\sum_{g'} P(j|g')P(b)P(i|b,m)P(m)P(g'|b,i,m)}{\sum_{j',g'} P(j'|g')P(b)P(i|b,m)P(m)P(g'|b,i,m)}$$

$$=\frac{P(b)P(i|b,m)P(m)\sum_{g'} P(j|g')P(g'|b,i,m)}{P(b)P(i|b,m)P(m)\sum_{j',g'} P(j'|g')P(g'|b,i,m)}$$
$$=\frac{P(j|g)P(g|b,i,m)+P(j|\neg g)P(\neg g|b,i,m)}{P(j|g)P(g|b,i,m)+P(j|\neg g)P(\neg g|b,i,m)+P(\neg j|g)P(g|b,i,m)+P(\neg j|\neg g)P(\neg g|b,i,m)}$$
$$=\frac{0.9*0.9+0}{0.9*0.9+0+0.1*0.9+1*0.1}$$
$$=\frac{0.81}{0.81+0.09+0.1}$$
$$=\frac{0.81}{1}$$
$$=0.81$$

2

In Figure 1, suppose we observe an unending sequence of days on which the umbrella appears. As the days go by, the probability of rain on the current day increases toward a fixed point, we expect that $\vec{P}(R_t|u_{1:t}) = \vec{P}(R_{t-1}|u_{1:t-1}) = \langle \rho, 1-\rho \rangle$. Find ρ.

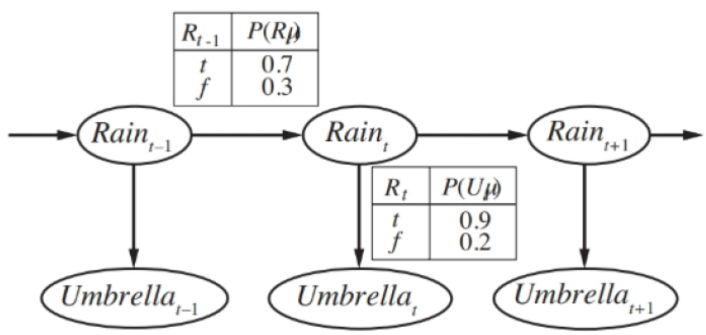


Figure 1: Bayesian network structure and conditional distributions describing the umbrella world. The transition model is $\vec{P}(R|R_{t-1})$ and the sensor model is $\vec{P}(U|R_t)$.

Answer: $\vec{P}(R_t|u_{1:t}) = \alpha \vec{P}(u_t|R_t) \sum_{R_{t-1}} \vec{P}(R_t|R_{t-1})\vec{P}(R_{t-1}|u_{1:t-1})$

$=\alpha \vec{P}(u_t|R_t)(\langle P(R_t|R_{t-1})P(R_{t-1}|u_{1:t-1}), P(\neg R_t|R_{t-1})P(R_{t-1}|u_{1:t-1})\rangle +$
$\langle P(R_t|\neg R_{t-1})P(\neg R_{t-1}|u_{1:t-1}), P(\neg R_t|\neg R_{t-1})P(\neg R_{t-1}|u_{1:t-1})\rangle)$
$=\alpha\langle 0.9, 0.2\rangle(\langle 0.7*\rho, 0.3*\rho\rangle + \langle 0.3*(1-\rho), 0.7*(1-\rho)\rangle)$
$=\alpha\langle 0.27+0.36\rho, 0.14-0.08\rho\rangle=\langle \rho, 1-\rho\rangle$
$\alpha = \frac{1}{0.27+0.36\rho+0.14-0.08\rho}=\frac{1}{0.41+0.28\rho}$
$\rho = \frac{0.27+0.36\rho}{0.41+0.28\rho}$
$\rho(0.41+0.28\rho) = 0.27+0.36\rho$
$28\rho^2 + 5\rho - 27 = 0$
$\rho = \frac{-5\pm\sqrt{25+4*28*27}}{56}$=0.8967 or -1.0753
ρ=0.8967

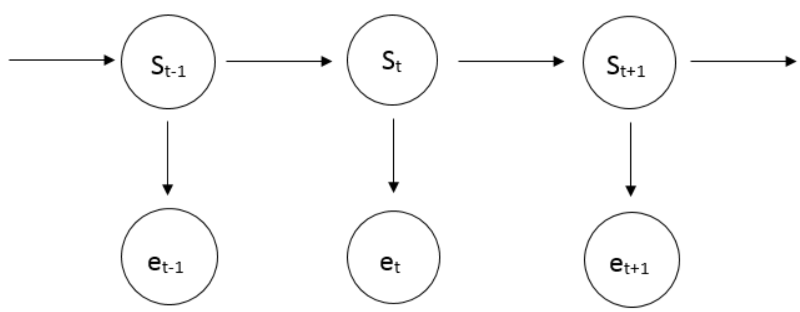
3

A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observations, is 0.7.
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

(a) Formulate this information as a hidden Markov model that has only a single observation variable. Give a bayesian network and conditional distributions.

Answer: as below



S_0	$P(S_0)$
S_0	0.7
$\neg S_0$	0.3

S_t	$P(S_{t+1})$
S_t	0.8
$\neg S_t$	0.3

S_t	e_t	$P(e_t)$
S_t	r, c	0.02
S_t	r, ¬c	0.18
S_t	¬r, c	0.08
S_t	¬r, ¬c	0.72
$\neg S_t$	r, c	0.21
$\neg S_t$	r, ¬c	0.49
$\neg S_t$	¬r, c	0.09
$\neg S_t$	¬r, ¬c	0.21

(b) Consider the following evidences, and compute $\vec{P}(ES_2|\vec{e}_{1:2})$ and $\vec{P}(ES_1|\vec{e}_{1:3})$.

- $\vec{e}_1$ = not red eyes, not sleeping in class
- $\vec{e}_2$ = red eyes, not sleeping in class
- $\vec{e}_3$ = red eyes, sleeping in class

Answer: $\vec{P}(ES_1|\vec{e}_1) = \alpha\vec{P}(\vec{e}_1|ES_1)\sum_{ES_0}\vec{P}(ES_1|ES_0)P(ES_0|\vec{e}_0)$
$=\alpha\vec{P}(\vec{e}_1|ES_1)(\langle P(ES_1|ES_0)P(ES_0|\vec{e}_0), P(\neg ES_1|ES_0)P(ES_0|\vec{e}_0)\rangle +$
$\langle P(ES_1|\neg ES_0)P(\neg ES_0|\vec{e}_0), P(\neg ES_1|\neg ES_0)P(\neg ES_0|\vec{e}_0)\rangle$
$=\alpha\langle 0.72, 0.21\rangle(\langle 0.8, 0.2\rangle * 0.7 + \langle 0.3, 0.7\rangle * 0.3)$
$=\alpha\langle 0.468, 0.0735\rangle$
$=\langle \frac{0.468}{0.468+0.0735}, \frac{0.0735}{0.468+0.0735}\rangle$
$=\langle 0.8643, 0.1357\rangle$
$\vec{P}(ES_2|\vec{e}_{1:2}) = \alpha\vec{P}(\vec{e}_1|ES_1)\sum_{ES_1}\vec{P}(ES_2|ES_1)P(ES_1|\vec{e}_1)$
$=\alpha\langle 0.18, 0.49\rangle(\langle 0.8, 0.2\rangle * 0.8643 + \langle 0.3, 0.7\rangle * 0.1357)$
$=\alpha\langle 0.18, 0.49\rangle\langle 0.73215, 0.26785\rangle$
$=\alpha\langle 0.131787, 0.131247\rangle$
$=\langle \frac{0.131787}{0.263034}, \frac{0.131247}{0.263034}\rangle$
$=\langle 0.5010, 0.4990\rangle$
$\vec{P}(ES_1|\vec{e}_{1:3}) = \alpha\vec{P}(ES_1|\vec{e}_1)\sum_{ES_2}P(\vec{e}_2|ES_2)P(\vec{e}_3|ES_2)\vec{P}(ES_2|ES_1)$
$=\alpha\vec{P}(ES_1|\vec{e}_1)P(\vec{e}_3)\sum_{ES_2}P(\vec{e}_2|ES_2)\vec{P}(ES_2|ES_1)$
$=\alpha\vec{P}(ES_1|\vec{e}_1)P(\vec{e}_3)(\langle P(\vec{e}_2|ES_2)P(ES_2|ES_1), P(\vec{e}_2|ES_2)P(ES_2|\neg ES_1)\rangle +$
$\langle P(\vec{e}_2|\neg ES_2)P(\neg ES_2|ES_1), P(\vec{e}_2|\neg ES_2)P(\neg ES_2|\neg ES_1)\rangle)$
$=\beta\langle 0.8643, 0.1357\rangle(\langle 0.8, 0.3\rangle * 0.18 + \langle 0.2, 0.7\rangle * 0.49)$
$=\beta\langle 0.8643, 0.1357\rangle\langle 0.242, 0.397\rangle$
$=\beta\langle 0.2091606, 0.0538729\rangle$
$=\langle \frac{0.2091606}{0.2630335}, \frac{0.0538729}{0.2630335}\rangle$
$=\langle 0.7952, 0.2048\rangle$