Name:____________________ PSU ID: ____________________

Final project: Two detention ponds in series
Total: 100 points
Due 5 pm, Tuesday, December 17, 2019

Detention ponds are used to mitigate excessively high flood discharge into rivers and streams. You can find detention ponds in your neighborhood or around campus. For this final project, a schematic diagram of the problem is shown in Figure 1. The bottom of each pond is a rectangle with an area of A_1 and A_2, respectively. The maximum depth is H_1 and H_2, respectively. The water depth in each pond is denoted as h_1 and h_2, respectively. Each pond has an inlet and an outlet. Pond 1 discharges into Pond 2 through a connection pipe. The connection pipe has a diameter of D_1. The outlet pipe for Pond 2 has a diameter of D_2.

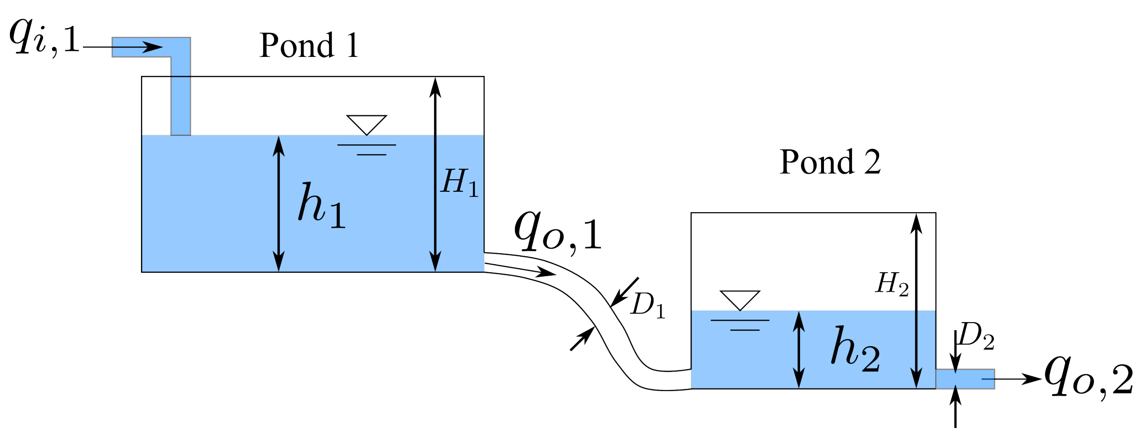

Figure 1: Two detention ponds in series.

The inflow into Pond 1, $q_{i,1}(t)$, can be simplified as a triangular hydrograph as shown in Figure 2. The flood starts at time t_0, peaks at t_1, and stops at t_2.

During a storm, if too much water enters a pond and the water depth h reaches its maximum depth H, water will overflow and be lost. In other words, the water depth h should not exceed H in each pond.

Based on the conservation of mass, the water depth in the two ponds can be described by the follow ODEs:

$$A_1 \frac{dh_1}{dt} = q_{i,1} - q_{o,1} \tag{1}$$

$$A_2 \frac{dh_2}{dt} = q_{o,1} - q_{o,2} \tag{2}$$

where the left hand side of the equations denotes the change rate of water volume in each

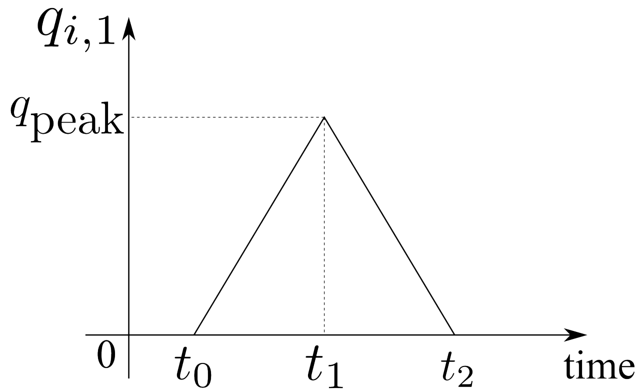

Figure 2: Triangular inflow hydrograph $q_{i,1}(t)$.

pond, and the right hand side denotes the net flux (inflow minus outflow, $q_i - q_o$) into the pond. Please note that the outflow of Pond 1 equals the inflow into Pond 2.

Using the conservation of energy (Bernoulli's equation) and the cross-sectional area of a circular pipe, the outflow discharge rate can be calculated as area times velocity:

$$q_{o,1} = \frac{\pi D_1^2}{4}\sqrt{2gh_1} \tag{3}$$

$$q_{o,2} = \frac{\pi D_2^2}{4}\sqrt{2gh_2} \tag{4}$$

where g is the gravitational acceleration constant (=9.8 m/s^2).

Thus, the water depth in both ponds can be described by the following ODE system made of two coupled first-order, nonlinear ODEs:

$$\frac{dh_1}{dt} = \frac{q_{i,1} - \frac{\pi D_1^2}{4}\sqrt{2gh_1}}{A_1} \tag{5}$$

$$\frac{dh_2}{dt} = \frac{\frac{\pi D_1^2}{4}\sqrt{2gh_1} - \frac{\pi D_2^2}{4}\sqrt{2gh_2}}{A_2} \tag{6}$$

Assume the two ponds initially are both empty, i.e., the initial condition is

$$h_1(t=0) = 0, h_2(t=0) = 0 \tag{7}$$

Other parameters for this problem are given as follows:

- Pond geometry: bottom area A_1 = 2,000 m^2 and A_2 = 1,000 m^2, pond maximum depth $H_1 = 5\ m$ and $H_2 = 4\ m$.
- Outlet pipe diameters: $D_1 = D_2 = 0.2\ m$.

- Inflow hydrograph parameters: $q_{\text{peak}} = 2\ m^3/s$, $t_0 = 1$ hour, $t_1 = 3.5$ hour, and $t_2 = 6$ hour.

In this project, you are required to do the following:

- (60 points) Write a Python program to numerically solve the ODE system. You are free to use any scheme or call any Python library functions. Please note that in your code, you should check at each time step whether the water depth h in each pond exceeds the pond maximum depth H. If the water depth exceeds the maximum (overflows), cap the water depth as the pond maximum depth.
- (40 points) Run your code for time $t \in [0, 80]$ hours. With the computed results, do the following:
  - (20 points) Plot the time history of water depth $h_1(t)$ and $h_2(t)$. What do you see? Briefly describe the process. Any overflow happens?
  - (20 points) Plot the time history of outflow discharge $q_{o,2}$ on the same plot with the inflow discharge $q_{i,1}$. What do you see? How do the two discharges compare in terms of peaks and their timing?

  Your plots should have the basic elements,such as title, ticks, axis labels, legend, etc.

A few notes:

- The ODEs involve the square root of water depth. Physically, water depth can not be negative. However, numerically, water depth can be negative but with a very small magnitude, which will throw Python off. To avoid numerical problem, you can check your water depth with the following code before calculate the square root:

```
if h1<0:
  h1 = 0
if h2 <0:
  h2 = 0
```

- For your report, you are free to use Jupyter Notebook for both code and documentation, or use other Word processing software of your choice.