Boosting LLM Reasoning: Push the Limits of Few-shot Learning with Reinforced In-Context Pruning

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Abstract

Large language models (LLMs) have shown impressive capabilities in various tasks, yet they still struggle with math reasoning. Despite efforts to optimize Chain-of-Thoughts (CoT) prompts and fine-tune LLMs, the potential of few-shot learning remains unexplored. In this work, we propose CoT-Influx, a novel approach pushing the boundaries of few-shot CoT learning to improve LLM math reasoning capabilities. CoT-Influx addresses the challenges of the selection of useful examples and limited number of examples due to restricted context window length. Inspired by our observation that natural language inputs contain many redundancy, we propose a coarse-to-fine pruner as a plug-and-play module for LLMs, which first identifies as many crucial CoT examples as possible and then further prunes unimportant tokens within the context window. To train the pruner, we collect a math reasoning dataset with diverse difficulty and steps, introduce a reward to measure both the input's effectiveness for math reasoning and token length constraints, and propose a novel training approach with reinforcement learning. As a result, CoT-Influx significantly outperforms CoT and few-shot prompting baselines across various LLMs (LLaMA2-7B, 13B, 70B) and 5 math datasets, achieving up to 4.55% absolute improvements. Remarkably, without any fine-tuning, LLaMA2-70B with CoT-Influx surpasses GPT-3.5 and a wide range of larger LLMs (PaLM, Minerva, etc.) on the GSM8K.

1 Introduction

Large Language Models (LLMs) have demonstrated remarkable capabilities across a range of tasks (Brown et al., 2020; OpenAI, 2023). However, when it comes to reasoning tasks, particularly math reasoning, it remains a significant challenge to improve LLM performance, particularly smaller ones like LLaMA (Touvron et al., 2023).

While existing efforts focus on optimizing Chain-of-Thought (CoT) prompts (Wei et al., 2022; Wang et al., 2023d; Yao et al., 2023; Chen et al., 2023c) and fine-tuning LLMs (Luo et al., 2023; Yue et al., 2023) under the zero-shot setting, our complementary approach explores the untapped potential of few-shot learning in improving math reasoning. Inspired by the human problem-solving process, if LLMs are shown more examples of stepby-step math problem solving strategies (i.e., CoTs) before answering questions, it could potentially unlock LLMs reasoning capability to generate a correct solution. This leads to our question: what's the upper limit of LLM performance gain in math reasoning achievable through inputting as many few-shot CoTs as possible?

However, we face two challenges. First, LLMs context window has a limited token length, which severely restricts the number of few-shot examples that can be inputted. Extending context window is one solution, but despite numerous efforts, it requires expensive LLM fine-tuning and increases inference overhead (Chen et al., 2023b; Peng et al., 2023a). While prompt compression (Li et al., 2023b; Jiang et al., 2023) is another approach, it underperforms in math reasoning. Tokens like numerical and format ones, though identified redundant, are crucial for few-shot math problem-solving.

Second, it's challenging to select helpful CoT examples. Random choices can harm reasoning capabilities (Chen et al., 2023a). Despite various methods like heuristic-based (Liu et al., 2021; Robertson et al., 2009) and retrieval-model based methods (Scarlatos and Lan, 2023; Wang et al., 2023b), they are not specifically tailored for math reasoning, making them suboptimal. For example, these retrieved examples are model-agnostic. However, we found that LLMs with different capabilities favor CoT examples of varying difficulties.

In this work, we propose **CoT-Influx**, which addresses all the above challenges and pushes the

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boundaries of utilizing few-shot learning to improve LLM math reasoning capability. CoT-Influx is motivated by the observation that current LLM context window has not been fully utilized due to redundancy at both the example and token levels in natural language input. As such, these redundant inputs can be pruned to free up space for more informative context. The central idea of CoT-Influx is to input long lengthy CoT examples, identify the crucial examples for the target LLM, and then prune redundant tokens to fit within the original LLM context window. As a result, by inputting much more helpful CoT examples, each composed solely of informative tokens and of a shorter length, we greatly improve LLM ability to solve math problems. Moreover, as all these inputs remain within the context window, we do not increase the inference overhead. This stands in stark contrast to other methods (Hao et al., 2022; Chen et al., 2023b).

CoT-Influx treats the target LLM as a black-box, and serves as a plug-and-play module for LLMs as shown in Fig. 3. The key module is a coarse-to-grained pruner involves two steps: (i) a first shot pruner selects the most helpful CoT examples from a large batch of shots, and (ii) a second token pruner removes unimportant tokens from these selected CoT examples. To effectively train the pruner module tailored for math reasoning, CoT-Influx is built upon the following novel techniques.

First, CoT-Influx requires a CoT dataset for training and inference. Existing CoT examples, heavily reliant on costly human engineering, often struggle with diversity and quality. To address this, we employ GPT-4 (OpenAI, 2023) and Evol-Instruct (Xu et al., 2023) to create a math reasoning dataset, called *MRD*³. With problems of varying difficulty and reasoning steps, MRD³ enables CoT-Influx to generalize across a wide range of math problems.

Second, training the pruner presents challenges due to two factors: (1) since we identify discrete tokens before LLM tokenizer, LLM loss gradient cannot be backpropagated through the tokenizer to update the pruner; (2) The high difficulty of many math problems, which consistently yield incorrect answers regardless of the quality of compressed few-shot examples, poses a challenge to the effective training of our pruner module. To this end, we introduce a novel training approach with reinforcement learning to mitigate the gradient issue. We design a reward function to measure both the few-shot effectiveness for math reasoning

and token length constraints. Then, we design a difficulty-aware dataloader that filtering appropriate problems and employ REINFORCE (Williams, 1992) to maximize the reward.

Extensive experiments on various LLMs and five math datasets demonstrate the effectiveness of CoT-Influx. CoT-Influx significantly boosts LLM reasoning capability, achieving 1.36%-14.09% absolute improvements over SOTA baselines, and establishes a new prompting-based benchmark in math reasoning accuracy without any fine-tuning or additional inference costs. Remarkably, LLaMA2-70B with CoT-Influx outperforms a broad range of larger LLMs and surpasses GPT-3.5 by 2.5% on GSM8K. Moreover, CoT-Influx excels over retrieval and prompt compression baselines in example selection and identifying crucial tokens.

2 Related Works

LLMs for Math Reasoning. Drawing from the Chain-of-Thought (CoT) (Wei et al., 2022), recent research has greatly improve the reasoning capabilities of LLMs by providing step-by-step reasoning paths. The main efforts are twofold: enhancing CoT prompts, such as Program-of-Thoughts (Chen et al., 2023c), Tree-of-Thoughts (Yao et al., 2023), and Everything-of-Thoughts (Ding et al., 2023), and innovating CoT-based training data for fine-tuning LLMs, like WizardMath (Luo et al., 2023) and ToRA (Gou et al., 2023).

However, most works focus on the zero-shot setting, where only task instruction and CoT prompt are provided for math problem-solving, leaving the potential of few-shot CoT largely untapped. This work explores the use of few-shot CoT examples to improve LLMs math reasoning capabilities.

Prompt Compression. To address the challenge of limited few-shot examples due to restricted context window length, one related works involves prompt compression. Key approaches include: (1) token pruning (Kim et al., 2022; Li et al., 2023a), requiring model intermediate results during inference; (2) soft prompt compression methods (Wingate et al., 2022; Mu et al., 2023; Chevalier et al., 2023; Ge et al., 2023), requiring fine-tuning LLMs; and (3) information-entropy-based approaches (Li et al., 2023b; Jiang et al., 2023), relying on extra language models to calculate each token's importance.

However, they do not effectively solve our problem for two reasons. First, our redundant tokens include both coarse-grained entire CoT examples and fine-grained tokens. Second, some tokens such

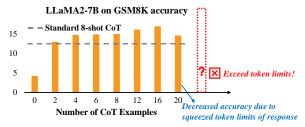


Figure 1: LLaMA2-7B reasoning accuracy under a increasing number of TopK retrieved CoT examples.

as numerical and format tokens, although redundant, are crucial for math reasoning.

Prompt Retrieval optimizes task performance by selecting high-quality few-shot examples using either heuristics or a supervised retriever model. Heuristic methods, such as the widely used TopK retrieval (Liu et al., 2021; Gao et al., 2021), BM25 (Robertson et al., 2009), VoteK (Hongjin et al., 2022), and entropy (Lu et al., 2022), select examples based on semantic similarity. Recently, supervised-based methods like EPR (Rubin et al., 2021), LLM-R (Wang et al., 2023b), and IDS (Qin et al., 2023) have been proposed, which train a retrieval model to learn better example selection.

However, these methods are sub-optimal for math reasoning. For example, the retrieved examples are model-agnostics, while LLMs with different capabilities favor CoT examples of varying complexities. Moreover, these methods don't account for token redundancy, which restricts the number of retrieved examples.

3 Pilot Study

This section presents our key observations of fewshot learning in improving LLMs math reasoning, upon which the CoT-Influx design is based. Note that experiments are done with our proposed CoT dataset, MRD³, as introduced in Sec. 4.1.

Observation 1: LLMs can improve reasoning with more helpful CoT examples, but the current context window restricts the number of CoT examples.

A standard practice for evaluating LLMs' math reasoning capability is the use of 8-shot manually-designed CoTs (Wei et al., 2022). We increase the number of CoT shots to see if reasoning accuracy improves. To avoid poor-quality examples, we use TopK method (Liu et al., 2021) to select the k most relevant CoT examples for each question. Given LLaMA2's context window limit of 4096 tokens, we could only input up to 20 CoT examples [‡]. As

Table 1: The selection of CoT examples heavily impacts LLM reasoning performance.

Model	Manual 8 Shots	Method (16 Shots)			
1,10001		Random 1	Random 2		
LLaMA2-7B LLaMA2-13B	14.60 27.82	12.36 23.04	13.27 23.28		

Fig. 1 shows, increasing CoT examples improves LLaMA2-7B's reasoning accuracy on the GSM8K dataset, significantly outperforming the standard 8-shot setting. However, the limited LLM context window hinders the full potential of few-shot CoT learning for improving math reasoning. For instance, even with only 20 examples, the accuracy declines as the lengthy input context begins to constrain LLM's response space.

Observation 2: CoT example selection is crucial for math reasoning. Simply adding CoT examples randomly doesn't boost performance.

The prior study suggests that more CoT examples can improve LLM reasoning performance. However, the quality of CoT examples is crucial for fully utilizing the context window. As shown in Table 1, even with up to 16 CoT shots, random selection underperforms the standard 8-shot setting, which is manually curated for quality.

A compressed CoT example by LLMLingua:

Q: There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?

A: Let's think step by step. There are 15-trees originally. Then there were 21 trees after some more were planted. So there must have been 21 - 15 = 6. The answer is 6.

Figure 2: A compressed CoT example using the prompt compression tool of LLMLingua (Jiang et al., 2023). The pruned tokens contain truly redundant tokens (colored in blue) and crucial tokens (colored in red).

Observation 3: A CoT example contains redundant tokens for math reasoning, which can be pruned to free up space for more informative content.

Observation 2 indicates that few-shot CoT examples contain non-useful or even harmful examples that can be pruned. We further observe that a CoT example often has redundant tokens. For instance, the blue tokens in Fig. 2 can be removed without affecting LLM performance. However, identifying redundant tokens for math reasoning is a challenge. Simply using existing prompt compression methods (Jiang et al., 2023; Li et al., 2023b) can lead to a significant performance decline. Fig. 2 shows a compressed example using LLMLingua (Jiang et al., 2023). Some numerical and format tokens

[‡]The input context token length is less than context window token limit, as the answer generation also share this limit.

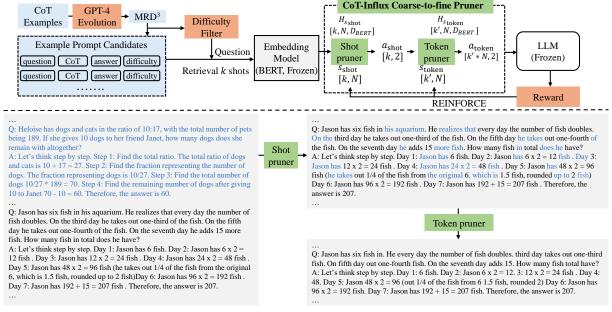


Figure 3: *Above*: The overview procedure of CoT-Influx; *Below*: an example illustrating the use of CoT-Influx to first prune entire CoT examples and then prune tokens.

(colored in red), while identified as redundant, are crucial for LLM to comprehend the context for solving a math problem.

Our opportunity. These observations motivate us to introduce CoT-Influx, a plug-and-play pruner module. For a target LLM, it removes unimportant CoT examples and tokens from long lengthy fewshot inputs. This results in denser, more useful content within the context window length, improving the target LLM capability to solve math problems.

4 CoT-Influx Methodology

4.1 CoT Dataset Collection

We start by collecting a high-quality math reasoning dataset, comprising diverse CoT examples with varying steps and difficulties. We merge the training set of GSM8K (Cobbe et al., 2021), MAWPS, MAWPS-single (Koncel-Kedziorski et al., 2016), and 1000 random examples from AQuA (Ling et al., 2017) to create an initial dataset of 9.7K questionanswer pairs. We then prompt GPT-4 to generate formatted CoT reasoning steps. Notably, it's crucial to maintain a consistent format for each example in few-shot learning. Our dataset also assigns a difficulty score from 1 to 10 for each question, based on GPT-4's evaluation, where 1 signifies the easiest questions and 10 the most difficult.

We observe most questions in this initial dataset score between 2-4. To improve the difficulty diversity, we draw inspiration from (Luo et al., 2023) and use GPT-4 to mutate questions, generating corresponding CoTs with varied difficulty levels. We

apply 5 mutation schemes, three to increase reasoning difficulty and two to simply questions. The final dataset is referred to as Math Reasoning Dataset with Diverse Difficulty (MRD^3) .

4.2 Problem Formulation

Let \mathcal{D} denote our proposed CoT dataset, the MRD³, and $\hat{\mathcal{D}} = \{x_i^{\text{cot}}\}_{i=1}^k$ be a sampled subset of k CoT examples, each composed of a question, reasoning steps, and an answer. The total number of tokens in these k CoT examples far exceeds the LLM context window length limit of T. CoT-Influx is designed to perform a two-step pruning process:

$$\hat{\mathcal{D}} = \{x_i^{\text{cot}}\}_{i=1}^k \xrightarrow{\text{Shot Pruner}} \{x_j^{\text{cot}}\}_{j=1}^{k'} \xrightarrow{\text{Token Pruner}} \{\hat{x}_j^{\text{cot}}\}_{j=1}^{k'} \quad (1)$$

Initially, non-useful CoT examples are pruned from $\hat{\mathcal{D}}$, resulting in a reduced set of k' examples. Then, for each retained CoT example x^{cot} , redundant tokens are pruned, yielding a shorter example, $x^{\hat{\text{cot}}}$.

Let $x^{\rm question}$ denote the math question that the LLM is tasked to solve. In the final input $x^{\rm input}$ for the LLM, we concatenate all tokens from $\{\hat{x}_j^{\rm cot}\}_{j=1}^{k'}$ and place them before $x^{\rm question}$. Our goal is to optimize the input context $x^{\rm input}$, so that the LLM can correctly answer the question under $x^{\rm input}$. Meanwhile, the token count of $x^{\rm input}$, t, must be less than LLM context window limit T. Formally, we aim to optimize the following:

$$\begin{split} & \min_{\hat{\mathcal{D}} \subseteq \mathcal{D}} L_{\text{LLM}}\left(x^{\text{input}}\right), \max_{\hat{\mathcal{D}} \subseteq \mathcal{D}} R_{\text{Acc}}\left(y_{\text{LLM}}\left(x^{\text{input}}\right), y^{\text{answer}}\right), \\ & s.t. \quad t\left(x^{\text{input}}\right) = \sum_{1}^{k'} |\hat{x}^{\text{cot}}| + |x^{\text{question}}| \leq T \end{split} \tag{2}$$

where $L_{\rm LLM}$ is LLM loss, and $R_{\rm Acc}$ evaluates if LLM's answer $y_{\rm LLM}(x^{\rm input})$ matches the correct answer $y^{\rm answer}$, this will be elaborated in Sec. 4.4. **Overview**. Fig. 3 illustrates our proposed approach. The core component is a lightweight, plug-and-play module (Sec. 4.3), which includes a small text embedding extractor and a coarse-to-fine pruner. To optimize the pruner, we address the gradient issue that arises from pruning discrete tokens before the LLM tokenizer, as the LLM loss gradient cannot be backpropagated through the tokenizer. To tackle this, we propose a multi-objective reward and leverage reinforcement learning for training (Sec. 4.4).

4.3 Coarse-to-fine Pruner Design

Text embedding extractor. As CoT-Influx serves as an external plug-and-play module for LLM, we extract text embeddings as prediction features. However, it's non-trivial to extract features for long inputs that exceeds LLM context window. To address this, we use a small encoder model, BERT-Large (Devlin et al., 2018), to extract sentence-level (i.e., a CoT example) embeddings instead of extracting token embedding from the entire long context. For a batch of k CoT examples, each example is padded to N=512 tokens. BERT then inferences these examples to obtain the final layer of text embedding, denoted as $H_{s_{\rm shot}} \in \mathbb{R}^{k \times N \times D_{BERT}}$, where D_{BERT} is BERT's hidden dimension size.

State. As shown in Fig. 3, we define state $s_{\text{shot}} \in \mathbb{R}^{k \times N}$ for the first shot pruner, representing the input batch of k CoT examples $\in \hat{\mathcal{D}}$. For the second token pruner, we define state $s_{\text{token}} \in \mathbb{R}^{k' \times N}$, which represents all remaining tokens after the shot pruner. k' is the number of retained examples.

Action. Let $a_{\rm shot}$ and $a_{\rm token}$ denote the actions predicted by the shot and token pruner, respectively. The action space is defined as $\{0,1\}$, where 1 signifies retention and 0 indicates pruning. Specifically, $a_{\rm shot}$ determines whether each CoT example should be pruned, while $a_{\rm token}$ predicts the pruning of each token in the retained CoT examples.

Two-stage policy network. Our pruner module is a two-stage policy network, each stage is a two-layer feed-forward network (MLP) with GELU activation. This module outputs a continuous categorical distribution π , used for sampling discrete actions (i.e., $\{0, 1\}$). Let θ denote the MLP's trainable parameters and $\sigma(\cdot)$ the sigmoid function. Based on the current states $\{s_{\text{shot}}, s_{\text{shot}}\}$ and the hidden states $\{H_{s_{\text{shot}}}, H_{s_{\text{token}}}\}$, the policy network sequen-

tially make two action predictions as follows:

$$\pi(a_{\rm shot}|s_{\rm shot};\theta) = \sigma\left(\text{MLP}\left(H_{s_{\rm shot}}\right)\right) \tag{3}$$

$$\pi(a_{\rm token}|s_{\rm token};\theta) = \sigma\left({\rm MLP}\left(H_{s_{\rm token}}\right)\right), \tag{4}$$

where $a_{\rm shot}$ and $a_{\rm token}$ are the predicted actions, sequentially predicting whether to prune each of the k CoT examples and each token within the retained examples, respectively. Specifically, we predict the discrete action by sampling from the categorical distribution π with a softmax function.

4.4 End-to-end Optimization

Multi-objective Reward. Our objective is to train the pruner module to effectively identify the most crucial CoT examples and useful tokens for math problem solving. Meanwhile, the final token count should within the original LLM context window. Therefore, we design a multi-objective reward.

Let x^{input} be the final input for the LLM, which includes the retained CoT tokens from the policy network and the target question. t represents the token count of x^{input} , and T is the token count limit. The reward R is defined as follows:

$$R\left(x^{\text{input}}\right) = \left(\frac{1}{1 + L_{\text{LLM}}} + R_{\text{Acc}}\right) \times \left[\frac{t}{T}\right]^{w}$$
 (5)

where the first term evaluates the effectiveness of inputted CoT tokens, and the second term ensures they are within the LLM context window. $L_{\rm LLM}$ is LLM's prediction loss under the input of $x^{\rm input}$, $R_{\rm Acc}$ evaluates the reasoning accuracy (to be discussed later). w is a hyperparameter that penalizes the token count t for being too short (i.e., w<0) or exceeding (i.e., w>0) the token limit T.

In Eq. 5, we introduce $R_{\rm Acc}$ to evaluate math reasoning accuracy under the input x^{input} , in addition to L_{LLM} . This is because L_{LLM} , the average loss of generated tokens, doesn't fully capture the LLM's ability to provide correct answers. Specifically, R_{Acc} is set to 1 for a correct answer and 0 for an incorrect one. Notably, we found that if format or crucial tokens are pruned, LLM struggles to interpret the input context correctly, leading to irrelevant answers for math problem solving. In such cases, we penalize R_{Acc} with a value of -0.1. **Optimization with REINFORCE**. We employ reinforcement learning to maximize the reward and train the two-stage policy network. According to REINFORCE (Williams, 1992), the network parameters are updated by the gradients:

$$R \cdot \nabla_{\theta} \log \pi (a_{\text{shot}} | s_{\text{shot}}) \pi (a_{\text{token}} | s_{\text{token}})$$
 (6)

Notably, as shown in Fig. 3, only the parameters of the policy network require training. The embedding extractor and LLM are frozen, thus, the overall training overhead is lightweight.

Difficulty-aware data filter. We divide dataset \mathcal{D} into two subsets: a math question set $\mathcal{D}_{question}$ sampled using the proposed difficulty filter and a CoT example set \mathcal{D}_{cot} . During the training, each question in $\mathcal{D}_{\text{question}}$ samples a batch of k CoT examples from \mathcal{D}_{cot} . Notably, the questions in $\mathcal{D}_{question}$ are filtered to include only easy questions with a difficulty score below a threshold d_{thr} . This is due to the current underperformance of LLMs, particularly smaller ones, on math reasoning. If the question is too challenging for LLMs, the answer will always be incorrect, regardless of the quality of compressed few-shot CoT examples, making it challenging to effectively train our pruner module. Stabilize the training. Another obstacle is that pruning CoT and tokens during training introduces instability, posing challenges to effective training.

First, despite the optimization of question set $\mathcal{D}_{\text{question}}$ through the filter, LLM performance for a randomly sampled question under different fewshot prompts can still be unpredictable. This unpredictability, where a simple question might yield correct results under low-quality pruned prompts and a complex question might fail under carefully pruned prompts, can affect the pruner's training effectiveness. To address this, we continuously repeat a sampled question multiple times, t_{repeat} , each time with a different pruned few-shot prompt from the pruner. Moreover, we use exponential moving average (EMA) to smooth reward R_{Acc} in Eq. 5.

Second, during the early training, our pruner module makes random decisions, leading to arbitrary removal of CoT examples and tokens. This randomly pruned few-shot prompts can cause instability in RL training. Empirically, we append the manually-designed 8 shot CoTs (Wei et al., 2022) to the pruned prompts. This ensures a good lower bound and stabilizes the training.

Algorithm 1 outlines the overall procedure of our method. The detail algorithm of Phase 1: Data preparation is enumerated in Appendix C.

5 Evaluation

Models, datasets and metric. We evaluate CoT-Influx on LLaMA2-7B, LLaMA2-13B, and LLaMA2-70B. The mathematical datasets for evaluation include GSM8K (Cobbe et al., 2021),

Algorithm 1 Pruner Training and Inference

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Input: target LLM, dataset \mathcal{D}, number of CoTs k, token limit T, manual few-shot cot x^{\text{few-shot}}, repeat t_{\text{repeat}}
  1: ► Phase 1: MRD<sup>3</sup> preperation
  2: Perform evolution and difficulty evaluation to get \mathcal{D};
  3: Use the difficulty filter and split \mathcal{D} into questions set
         \mathcal{D}_{\text{question}} and CoT candidates set \mathcal{D}_{\text{cot}}

    4: ► Phase 2: Training the two-stage pruner (1 epoch)
    5: for (x<sup>question</sup>, y<sup>answer</sup>) in D<sub>question</sub> do

              Retrieve Top-k examples \hat{\mathcal{D}} = \{x^{\text{cot}}\}_{i=1}^k from \mathcal{D}_{\text{cot}}
  6:
               H_{s_{\text{shot}}} = \text{BERT}(\{x^{\text{cot}}\}_{i=1}^k)
  7:
  8:
              for j=1 to t_{\text{repeat}} do
                    Get \pi (a_{\text{shot}}|s_{\text{shot}};\theta) with Eq. 3, sample a_{\text{shot}}
  9:
                     \{x^{\text{cot}}\}_{i=1}^k \to \{x^{\text{cot}}\}_{i=1}^{k'}
10:
                     H_{s_{\text{token}}} = \text{BERT}\Big(\{\boldsymbol{x}^{\text{cot}}\}_{i=1}^{k'}\Big)
11:
12:
                     Get \pi (a_{\text{token}}|s_{\text{token}};\theta) with Eq. 4, sample a_{\text{token}}
                     \begin{cases} x^{\text{cot}} \}_{i=1}^{k'} \rightarrow \{\hat{x}^{\text{cot}}\}_{i=1}^{k'} \\ x^{\text{input}} = \left( \{\hat{x}^{\text{cot}}\}_{i=1}^{k'}, x^{\text{few-shot}}, x^{\text{question}} \right) 
13:
14:
                     Output LLM(x^{input}); Compute R with Eq. 5
15:
16:
               end for
               Compute policy gradient using Eq. 6, update \theta
17:
18: end for
         ► Phase 3: LLM reasoning with pruner and MRD<sup>3</sup>
20: Retrieve Top-k shots \left\{x_q^{\text{cot}}\right\}^k \in \mathcal{D} for target question q
21: Do pruning: \{x_q^{\text{cot}}\}^k \xrightarrow{\theta} \{\hat{x}_q^{\text{cot}}\}^{k'}, reconstruct \{\hat{x}_q^{\text{cot}}\}^{k'}
22: x_q^{\text{input}} = \left(\{\hat{x}_q^{\text{cot}}\}^{k'}, x_q^{\text{few-shot}}, x_q^{\text{question}}\right)
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AddSub (Hosseini et al., 2014), Multiarith (Roy and Roth, 2015), Svamp (Patel et al., 2021), and Singleeq (Koncel-Kedziorski et al., 2015). For evaluation metric, we report Exact Match (EM) accuracy of the predicted answers.

23: Get LLM reasoning output LLM(x_a^{input})

Baselines We evaluate CoT-Influx against three types of baselines:

- CoT and few-shot CoT prompting: We compare with widely-used prompts for LLM reasoning, including zero-shot, zero-shot-CoT (Kojima et al., 2022), and the standard few-shot-CoT (Wei et al., 2022) with 8 shots.
- *Prompt retrieval*: we also compare with retrieval baselines, specifically using random, TopK (Liu et al., 2021), and BM25 (Robertson et al., 2009) methods. We select as many CoT examples as possible using each method, without exceeding LLM context window. Random retrieval is to reflect the average quality of our CoT dataset.
- *Prompt compression*: To evaluate the effectiveness of identifying crucial tokens, we compare the resulting compressed prompts from the same batch of CoT shots with state-of-the-art prompt compression baselines: Selective Context (Li et al., 2023b), LLMLingua (Jiang et al., 2023), and compression through GPT-4.

Table 2: Comparison of EM (%) accuracy on GSM8K with state-of-the-art baselines. Note that the 20 CoT shots of retrieval baselines are the max number, given that the context window limit of LLaMA2 is 4096 tokens.

Method	#Input CoT shots	#Average tokens	LLaMA2-7B	LLaMA2-13B	LLaMA2-70B
Zero-shot	0	-	4.25	5.84	11.45
Zero-shot-CoT (Kojima et al., 2022)	0	-	1.74	12.28	21.91
Few-shot-CoT (Wei et al., 2022)	8	655	13.79	27.82	55.42
Random retrieval	20	3379.8	12.51	22.21	53.07
TopK retrieval (Liu et al., 2021)	20	3535.4	14.56	23.65	54.59
BM25 retrieval (Zhenyu et al., 2023)	20	3816.1	13.42	25.17	54.21
TopK+GPT4 Compression	40	1376.0	7.08	11.01	25.17
TopK+Selective Context (Li et al., 2023b)	40	2262.4	0.45	0.76	2.50
TopK+LLMLingua (Jiang et al., 2023)	40	2048.0	5.38	8.34	22.74
CoT-Influx	40	2037.0	15.92	32.37	59.59

Table 3: Comparison of EM (%) accuracy on Addsub, Multiarith, Svamp, and Singleeq math reasoning dataset

Model	Method	AddSub	Multiarith	Svamp	Singleeq	Avg.
	Zero-shot	58.73	5.50	32.2	62.79	39.81
	Few-shot-CoT	56.96	43.67	38.1	66.54	51.32
LLaMA2-7B	TopK retrieval	46.08	34.50	38.1	46.46	41.29
	TopK+LLMLingua	12.91	10.50	19.5	19.49	15.60
	CoT-Influx	62.28	47.00	40.2	72.05	55.38
	Zero-shot	70.13	6.50	43.8	71.07	47.88
	Few-shot-CoT	65.82	72.83	42.7	77.36	64.68
LLaMA2-13B	TopK retrieval	60.76	57.00	50.2	68.50	59.12
	TopK+LLMLingua	22.28	22.33	27.5	25.20	24.33
	CoT-Influx	69.62	73.87	50.5	83.07	69.26

5.1 Main Results

Effectiveness of enabling more few-shot CoTs.

We first evaluate how far the boundary of few-shot learning can be pushed using CoT-Influx. For comparison, we set up two baselines: (i) Few-shot CoT, inputting 8 manually designed CoT shots as the default setting for LLM math reasoning accuracy on GSM8K. (ii) TopK retrieves 20 CoT shots from our dataset, representing the maximum shot number within LLaMA2 context window.

For CoT-Influx, we test LLaMA2 7B and 13B on GSM8K, adjusting the number of CoT shots from 16 to 64 examples, which corresponds to $0.7 \times$ to 2.8× the token count of LLaMA2 context window. As shown in Fig. 4, we make two observations: (1) More CoT shots, facilitated by CoT-Influx, indeed boosts LLM math reasoning performance, particularly for larger LLMs. On LLaMA2-13B, by inputting 48 CoTs, we significantly outperform the standard few-shot CoT and TopK by 4.55% and 8.72%, respectively. (2) There is an optimal number of CoT shots for CoT-Influx. Its peak performance on LLaMA2 7B and 13B are at 40 and 48 shots, respectively. We attribute this to two potential reasons. First, an extremely large number of shots complicates CoT-Influx's optimization. Second, there may be an upper limit to improving LLM reasoning capability through few-shot learning.

Comparison with state-of-the-art baselines. Ta-

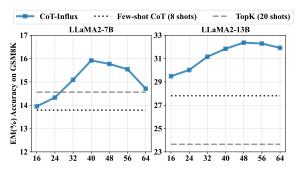


Figure 4: EM(%) accuracy on GSM8K with different input shot number of CoT-Influx pruner.

Table 4: Comparison of EM (%)accuracy on GSM8K with larger LLMs under the few-shot-CoT setting.

Model	Parameters	EM (%)
Finetuned GPT-3 (Wei et al., 2022)	175B	34.0
Chinchilla (Hoffmann et al., 2022)	70B	43.7
Text-davinci-002 (Kojima et al., 2022)	175B	51.5
PaLM (Chowdhery et al., 2022)	540B	56.5
GPT-3.5 (OpenAI, 2023)	-	57.1
Minerva (Lewkowycz et al., 2022)	540B	58.8
LLaMA2-70B+CoT-Influx	70B	59.6

ble 2 and Table 3 present the comparison results of CoT-Influx with state-of-the-art baselines across LLaMA2 family and 5 mathematical datasets, highlighting the following observations: (1) by utilizing more few-shot CoTs that are twice the LLM context window, CoT-Influx significantly outperforms all baselines, with 2.13% to 4.55% absolute improvements. (2) Despite using fewer input tokens, CoT-Influx consistently outperforms retrieval baselines by 1.36% to 14.09% absolute improvements. This is because our compressed tokens indicate more informational CoT examples without redundancy. In contrast, they select entire examples, which may contain redundant tokens, based on semantic similarity between the target question and CoT examples, without considering the different CoT preference of the target LLM. (3) CoT-Influx significantly outperforms prompt compression baselines in preserving the most crucial tokens

for math reasoning, while methods like Selective Context and LLMLingua suffer accuracy declines due to difficulties in maintaining few-shot prompt structure. GPT-4 tends to prune essential reasoning steps, which negatively impacts CoT effectiveness.

We further demonstrate the effectiveness of CoT-Influx by comparing LLaMA2-70B with larger size LLMs on GSM8K. As shown in Table 4, CoT-Influx significantly boosts LLM reasoning capabilities. Remarkably, without any fine-tuning, LLaMA2-70B with CoT-Influx outperform much larger LLMs. LLaMA2-70B surpasses GPT-3.5 with an absolute improvement of 2.5%.

5.2 Ablation Study and Analysis

The effectiveness of MRD³ dataset. Beyond our pruner, we introduce the CoT dataset MRD³, initially generated and evolved by GPT-4 for diverse reasoning steps and difficulties. We evaluate its effectiveness against two baselines: (1) MRD³ without evolution, excluding GPT-4 evolved examples, and (2) the human-labeled GSM8K training set. Testing CoT-Influx pruner on these datasets, as shown in Table 5, reveals that both GPT-4 generated and evolved CoT examples are vital for enhancing LLM reasoning performance.

Table 5: Comparison of EM(%) on GSM8K using CoT-Influx pruner across different CoT datasets.

CoT dataset	LLaMA2-7B	LLaMA2-13B	LLaMA2-70B
MRD^3	15.92	32.37	59.59
MRD3 w/o evolution	14.94	30.55	57.70
GSM8K training set	14.18	29.64	56.71

Ablation study on coarse-to-fine pruner. Our pruner operates at both shot and token levels to fully exploit redundancy within CoT examples. To verify the effectiveness, we conduct experiments with only shot or token pruner under the same training setting. As shown in Table 6, removing any pruning stage decreases reasoning performance. Notably, token-only pruning causes a larger accuracy drop than shot-only pruning, indicating shot-level is easier for the pruner to learn.

Table 6: Comparison of EM(%) on GSM8K with different pruning strategy.

Pruning Strategy	LLaMA2-7B	LLaMA2-13B	LLaMA2-70B
CoT-Influx (Prune shot and token)	15.92	32.37	59.59
Prune shot only	15.69	31.08	57.77
Prune token only	12.05	25.32	49.36

Token pruning ratios. We now investigate token pruning ratios by our pruner. Fig. 5 shows the remaining token length for LLaMA2-70B after our pruner. In total, we achieve a $4.28 \times$ pruning ratio, with shot pruner contributing a $3.87 \times$ ratio. The

results suggest that our pruner favors pruning more coarse-grained shots over fine-grained tokens.

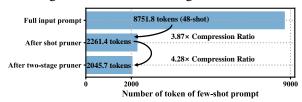


Figure 5: Token length after each stage of our pruner.

Inference cost. CoT-Influx is a lightweight plugand-play module, including a 336MB BERT-Large model and a small 4MB coarse-to-fine pruner. We now measure its additional inference cost. Table 7 shows the total inference latency and GPU memory required to run LLaMA2-7B with different methods on GSM8K, measured on a single NVIDIA A100 GPU. The results reveal that CoT-Influx introduces a negligible additional 1.4GB of memory and a 1.7% increase in inference latency. This is more cost-effective than prompt compression baselines, such as Selective Context and LLMLingua, which require significantly more latency and GPU memory, potentially hindering efficient deployment.

Table 7: The total inference costs on GSM8K.

Method	#Input-shot	#Token	Time	GPU Memory
LLaMA2-7B	12	2108.6	2.99h	19.7GB
Selective Context LLMLingua	40 40	2262.4 2048.0	4.38h 3.65h	23.5GB 33.0GB
CoT-Influx	40	2037.0	3.04h	21.1GB

Implications. Our analysis of retained CoT examples and tokens yields the following insights: (1) More capable LLMs favor harder CoT examples, while smaller LLMs opt for simpler ones. (2) Numerical and format tokens are essential for math reasoning. Function words like *with*, *the*, *then*, and irrelevant background context such as *theater* can be pruned without affecting reasoning capability.

6 Conclusion

In this work, we introduce CoT-Influx, a plugand-play module to improve LLM math reasoning. Based on the observation that more high-quality CoT examples can effectively unlock the reasoning potential of LLMs, CoT-Influx prunes few-shot examples at both the shot and token levels to push the limits of few-shot learning. To effective train the pruner module, we leverage reinforcement learning to optimize a math reasoning-specific reward function with the GPT-4 evolved CoT dataset MRD³. Extensive experiments on various datasets and LLMs compared with state-of-the-art baselines demonstrate the effectiveness of our method.

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Appendix

This appendix includes additional related works, prompt settings, evolution of MRD³, pruner training details, and additional case studies. These contents are organized in separate sections as follows:

- Sec. A elaborates previous LLM context window extension and LLM in-context learning methods, and analyzes the advantage of our proposed CoT-Influx compared with various previous methods.
- Sec. B introduces the prompt we used for evolution of the examples in our dataset. The examples of the evolution input and the evolution results are given. We also analyze the difficulty and reasoning step distribution of different evolution methods and derive a new observation regarding difficulty preference for different LLMs.
- Sec. C includes the algorithm for training data preparation as a supplement to Algorithm 1. The hyperparameter settings, the training dynamic of pruner, and the detailed introduction of evaluation dataset are also included.
- Sec. D provides comparison of CoT-Influx with context window extension methods. Additional case studies on math reasoning dataset GSM8K with different prompting methods are given to extensively prove the effectiveness of our method.
- Sec. E demonstrates the prompt we used in this work for difficulty and reasoning step evaluation, and GPT-4 based compression on input few-shot prompts.

A Additional Related Works

LLM In-Context Learning In-context learning (ICL) are one of the emerging abilities of LLMs that conduct various downstream tasks with provided few-shot demonstrations. To fully understand optimize the ICL paradigm, previous research mainly focus on the underlying mechanism of ICL or the proper application of ICL. Pioneering research (Von Oswald et al., 2023; Dai et al., 2023) empirically find the similarity between gradient-descent (GD) and ICL, which interprets the trained LLMs are meta-optimizer that can learn the examples in context in forward pass. More recently, Wang et al. (2023a) propose a hypothesis that label words in examples serve as anchors in ICL, and the anchors can help aggregate and distribute the task-relevant information flow. To better utilize ICL, previous research also research on the input format (Yoo et al., 2022) and order of examples (Min et al., 2022). Our work falls in the second category that shows the compressed examples are an optimal choice for the input of ICL.

LLM Context Window Extension Recently, there has been rising interests in extending the context window of existing pre-trained LLMs. Common approaches include augmenting external memory modules (Tworkowski et al., 2023; Wang et al., 2023c), which add extra modules to memorize long past contexts but requires complex training, manipulating attention mechanisms (Han et al., 2023; Xiao et al., 2023) or the positional encoding (Chen et al., 2023b; Peng et al., 2023b). However, these require LLM modifications. Our method, applicable to black-box LLMs and extendable context windows, is orthogonal to this direction.

Comparison of CoT-Influx with Previous Methods We summarize the advantage of our CoT-Influx compared with previous prompting strategies in Table 8. *Gradient-free* indicates the methods do not need to backward through LLMs. *Unlimited-token* represents the original input prompt for these methods are not limited by the context window length of LLMs. *Difficulty-aware* refers to whether the method take the difficulty of problems into the consideration of their prompt design. *Dynamic #Shots* means we do not need to setup a target shot number and the pruned input shot numbers are different across various questions. Our CoT-Influx demonstrate significant advantage over all previous methods.

Table 8: Comparison of the advantage of different prompting strategies.

Methods	Frozen LLMs	Automated	Gradient-free	Unlimited-token	Transferable	Interpretable	Difficulty-aware	Dynamic #Shots
Fine-Tuning	×	✓	×	X	Х	Х	X	Х
Manual Prompt	✓	×	✓	×	✓	✓	×	×
Soft Prompt Tuning	✓	✓	×	×	×	×	×	×
Prompt Retrieval	✓	✓	✓	×	✓	✓	×	×
AutoPrompt (Shin et al., 2020)	✓	✓	×	×	✓	✓	×	×
RLPrompt (Deng et al., 2022)	✓	✓	✓	×	✓	✓	×	×
Context Extension	✓	✓	✓	✓	✓	✓	×	×
LLMLingua (Jiang et al., 2023)	✓	\checkmark	✓	\checkmark	\checkmark	\checkmark	×	×
CoT-Influx(Ours)	✓	✓	✓	✓	✓	✓	✓	✓

B Evolution of MRD³

B.1 Prompt template for evolution

The prompt we used for evolution of the examples in our dataset are listed as follow:

Prompt for different evolution strategies

I want you to act as a Prompt Rewriter. Your objective is to rewrite a given prompt into a more complex version to make those famous AI systems (e.g., LLaMA, ChatGPT and GPT4) a bit harder to handle.

The prompt is made up of a math reasoning question and the corresponding answer.

The rewritten prompt must be reasonable and must be understood and responded by humans.

Your rewriting cannot omit or change the input and results in #Given Prompt#. Also, please retain the format of 'Question: ' and 'Answer: ' in your response.

You SHOULD complicate the given prompt using the following method:

{Evolution template}

You should try your best not to make the #Rewritten Prompt# become verbose, #Rewritten Prompt# can only add 10 to 20 words into #Given Prompt#.

The #Rewritten Prompt# should also follow the format that the rewritten question appears after 'Question: ' and the rewritten answer appears after 'Answer: '.

The rewritten answer should end up with 'The answer is [results]'.

#Given Prompt#:

Question: {Given question}
Answer: {Given answer}
#Rewritten Prompt#:

Evolution template for evolution strategy add_constraints:

Please add one more constraint/requirement to the question of #Given Prompt#

Evolution template for evolution strategy deepening:

Please increase the depth and breadth of the question and answer of #Given Prompt#

Evolution template for evolution strategy increase_reasoning:

If #Given Prompt# can be solved with just a few simple thinking processes, please rewrite it to explicitly request multiple-step reasoning.

Evolution template for evolution strategy revise_difficulty:

Please revise the high difficulty questions to lower difficulty.

Evolution template for evolution strategy produce_easier:

Please produce a new and easier question with another different topic.

Most part of the prompt of different evolution strategies are similar. Based on our quantitatively analysis on the difficulty and reasoning step distribution, GPT-4 can effectively follow our instruction to modify the constraints or difficulty level of input questions.

B.2 Evolution example

We give an example of a math questions and then show the evolved results of the questions and answers. The evolved results follow our instruction given in Sec. B.1

Example of prompt evolution

Input example for evolution:

Q: At the arcade Dave won 11 tickets. If he spent 5 tickets on a beanie and later won 10 more tickets, how many would he have?

A: Dave initially had 11 tickets, spent 5 on a beanie, and later won 10 more, resulting in a total of: 11 - 5 + 10 = 16 tickets. The answer is 16.

Evolution result of add constraints:

Q: At the arcade, Dave won 11 tickets. If he spent 5 tickets on a beanie, won 10 more tickets, and gave 4 tickets to his friend, how many would he have?

A: Dave initially had 11 tickets, spent 5 on a beanie, won 10 more, and gave 4 to his friend, resulting in a total of: 11 - 5 + 10 - 4 = 12 tickets. The answer is 12.

Evolution result of deepening:

Q: At the arcade, Dave triumphed in a game and earned 11 tickets. He redeemed 5 of those tickets for a beanie and subsequently won an additional 10 tickets. What is his final ticket count?

A: Dave started with 11 tickets, used 5 to acquire a beanie, and later added 10 more to his collection, resulting in: 11 - 5 + 10 = 16 tickets. The answer is 16.

Evolution result of increase_reasoning:

Q: Dave won 11 tickets at the arcade and spent 5 on a beanie. Afterward, he won 10 more tickets. Calculate his final ticket count by first finding the remaining tickets after his purchase and then adding the newly won tickets.

A: Dave had 11 tickets, spent 5, leaving him with 6. Then he won 10 more, resulting in: 6 + 10 = 16 tickets. The answer is 16.

Evolution result of revise_difficulty:

Q: Dave got 11 tickets at the arcade. He used 5 tickets to get a beanie and then won 10 more tickets. How many tickets does he have now?

A: Dave started with 11 tickets, used 5 for a beanie, and won 10 more, giving him: 11 - 5 + 10 = 16 tickets. The answer is 16.

Evolution result of produce_easier:

Q: In a game, Sarah scored 11 points. She lost 5 points in a penalty and then scored 10 more points. What is her final score?

A: Sarah initially scored 11 points, lost 5 due to a penalty, and scored 10 more, resulting in a total of: 11 - 5 + 10 = 16 points. The answer is 16.

As can be seen from the evolution results, the answer of input questions can be modified (*e.g.* ground truth answer change from 16 to 12 in add_constraints). The whole background of the questions can also be replaced (*e.g.* computation target of question change from current tickets at the arcade to final points of a game in produce_easier). These modification and variation improve the diversity of our prompt candidate sets, which are the building block for the training and reasoning with CoT-Influx.

B.3 Difficulty and Reasoning Steps Distribution of MRD³

Based on the GPT-4-based estimation, we are able to quantitatively look into the distribution of difficulty and reasoning step distribution in MRD³ without evolution and MRD³ with various evolution schemes. The results are shown in Figure 6. The original distribution of both difficulty level and reasoning steps of questions centralized between 2 to 4. More questions with higher difficulty using add_constraints, deepening, and increase_reasoning. As we discuss in the reward design of our RL pruner, easy questions are important for the stabilization of RL and can help effectively identify the quality of pruned prompt, more easier questions are generated with revise_difficulty and produce_easier evolution scheme.

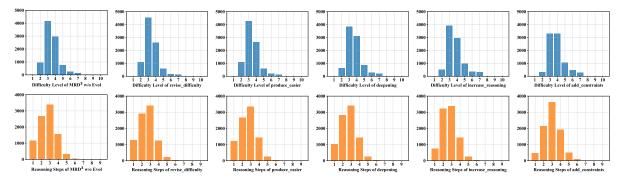


Figure 6: The difficulty distribution (first row) and the number of reasoning steps distribution (second row).

B.4 Additional observation on difficulty distribution

As shown in Figure 6, the difficulty diversity of examples in MRD³ are improved after prompt evolution. We then research into the difficulty distribution of the input examples for in-context learning. The observation is shown as follow in addition to the 3 main observations proposed in Sec. 3:

Observation 4: LLMs with different capabilities prefer CoT examples of varying difficulties.

In our further exploration of the optimal selection of CoT examples for improve mathematical reasoning, we observe that LLMs of different capabilities exhibit preferences for CoT examples of varying difficulty levels. As Table 9 shows, we categorize each CoT example in the MRD³-Evol dataset by difficulty level. We then select the top 16 CoT examples from different groups as few-shot examples for LLaMA2 models. Results show LLaMA2-7b prefers CoT examples with a difficulty level of 3-4, while LLaMA2-13b, more capable, prefers those with a difficulty level of 4 or above. This aligns with intuition: for instance, when assisting a middle school student with math problems, it is more beneficial to provide examples of moderate difficulty that they can comprehend, whereas for a high school student, examples with a higher level of difficulty are more useful.

Table 9: Smaller, less capable LLMs favor simpler CoT examples, while larger ones prefer more complex ones.

Model	Difficulty (≤ 3)	Difficulty (3-4)	Difficulty (≥ 4)
LLaMA2-7B	14.49	15.39	14.86
LLaMA2-13B	23.81	25.32	25.47

In our evaluation of CoT-Influx with various LLMs, we found that the shot selection results are consistent with our observation. The average difficulty score and number of reasoning steps for the examples selected by LLaMA2-70B pruner are 3.57 and 3.04, which are higher than the results of LLaMA2-13B are 3.51 and 2.98. The empirical results further support our assumption that LLMs with larger size prefers harder examples than smaller-scale LLMs.

C Pruner Training and Evaluation Details

C.1 Detailed algorithm for training data preparation

As a supplement to phase 1 in Algorithm 1, we show the algorithm for training data preparation in Algorithm 2. Both the difficulty level and number of reasoning step are involved in the GPT-4-based evaluation. However, we omit the reasoning step in this algorithm as we only use difficulty level in the training set split.

Algorithm 2 Training dataset preparation

```
Input: CoT dataset \{x_i^{\text{cot}}\}_{i=1}^L, difficulty threshold d_{\text{thr}},
Output: MRD<sup>3</sup> \mathcal{D} = \{x_j^{\text{cot}}, d_j\}_{j=1}^{L^{\text{MRD}^3}}, questions set \mathcal{D}_{\text{question}}, prompt set \mathcal{D}_{\text{cot}}
 1: ► Phase 1: MRD<sup>3</sup>-Evol Preparation
 2: MRD^3 dataset \mathcal{D} = \{\}
 3: for i = 1 to L do
          Perform GPT-4 based prompt evolution on x_i^{\text{cot}} to get \{x_{i,e}^{\text{cot-E}}\}_e
 4:
          Evaluate difficulty on \{x_{i,e}^{\text{cot-E}}\}_e to get score \{d_{i,e}\}_e using GPT-4
 5:
          Append examples \{x_{i,e}^{\text{cot-E}}, d_{i,e}\}_e to \mathcal{D}
 6:
 7: end for
 8: Prompt set \mathcal{D}_{cot} = \{\}, question set \mathcal{D}_{question} = \{\}
 9: for i = 1 to L^{\text{MRD}^3} do
10:
           if d_j \leq d_{\text{thr}} then
                Append example (x_j^{\text{cot}}, d_j) to \mathcal{D}_{\text{question}}
11:
12:
13:
                Append example (x_i^{\text{cot}}, d_j) to \mathcal{D}_{\text{cot}}
14:
15: end for
16: Return full dataset with evolution \mathcal{D}, questions set \mathcal{D}_{question}, prompt candidate set \mathcal{D}_{cot}
```

C.2 Detailed settings and hyperparameters

The detailed hyper-parameters setting of different LLMs' pruner are listed in Table 10. Majority of these hyperparameters are shared across different LLMs. The evolution subset as the prompt candidates for evaluation are determined by searching the performance of math reasoning on 100 random examples.

Model	LLaMA2-7B	LLaMA2-13B	LLaMA2-70B
Epoch	3	3	3
Batch Size	1	1	1
Pruner LLM Base	LLaMA2-13B	LLaMA2-13B	LLaMA2-70B
Input Shot	40	48	48
Input Shot (TopK)	32	32	32
Input Shot (Few-shot)	8	16	16
Optimizer	AdamW	AdamW	AdamW
Weight Decay	$1e^{-2}$	$1e^{-2}$	$1e^{-2}$
Learning Rate	$1e^{-5}$	$1e^{-5}$	$1e^{-5}$
Embedding Extractor	BERT-Large (cased)	BERT-Large (cased)	BERT-Large (cased)
Embedding Size	1024	1024	1024
Tokenizer Padding	512	512	512
Difficulty Threshold d_{thr}	2	2	2
Token Target T	2048	2048	2048
Token Penalty Coefficient w	(-1,1)	(-1,1)	(-1,1)
Selection Repeat t_{repeat}	10	10	5
Evol Subset	add_constraints	increase_reasoning	increase_reasoning
temperature	0.8	0.8	0.8
top_p	0.95	0.95	0.95
top_k	40	40	40
num_beams	1	1	1
max_new_tokens	1	1	1

Table 10: Detailed hyper-parameters for pruner training scheme of different LLMs.

C.3 Training dynamics

We visualize the RL training dynamics of the LLaMA2-13B pruner in Figure 7 including the LLM loss reward $\frac{1}{1+L_{\rm LLM}}$, prediction reward $R_{\rm Acc}$, moving average of the final pruner reward R, and remaining token count t. We can see from the results that reward increases steadily with the steps of RL training. The number of remaining tokens decreases rapidly in early steps and then oscillates around the token target. Since our prediction reward $R_{\rm Acc}$ are discrete value of $\{-0.1,0,1\}$, the oscillation phenomenon are more obvious compared with other reward term. This highlight the effectiveness of question repetition and using Exponential Moving Average (EMA) of final reward to suppress this oscillation.

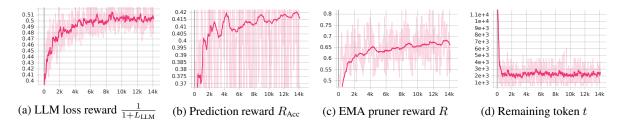


Figure 7: RL training dynamics of the LLaMA2-13B pruner.

C.4 Detailed introduction of dataset for evaluation

We introduce the details of the datasets we used for evaluation as follows:

• **GSM8K** (Cobbe et al., 2021) is a math reasoning dataset consisting high quality linguistically diverse grade school math word problems created by human problem writers. There are 7473 training examples

and 1319 validation examples in the dataset.

- SVAMP (Patel et al., 2021) representing Simple Variations on Arithmetic Math word Problems that conduct question sensitivity variation, reasoning ability variation, and structural variation on existing math datasets. There is a total of 1000 examples and all of them are used for evaluation in our settings.
- MultiArith (Roy and Roth, 2015) is a collection of multi-step arithmetic problems with 600 examples and all of them are used for evaluation in our settings.
- AddSub (Hosseini et al., 2014) is a dataset consisting of addition and subtraction problems with 395 examples and all of them are used for evaluation in our settings.
- SingleEq (Koncel-Kedziorski et al., 2015) consists grade-school algebra word problems that map to single equations with varying length. Every equation may involve multiple math operations including multiplication, division, subtraction, and addition over non-negative rational numbers and only one variable. There are 508 problems, 1117 sentences, and 15292 words in the dataset.

C.5 Rule-based prompt reconstruction

To make sure the input prompt for inference remain structurally intact, we apply a rule-based prompt reconstruction on the input. For example, "\n [question]" will be reconstructed to "\nQ: [question]" and "A: Let's step by step" will be repaired to "A: Let's think step by step". While our pruner has been trained to learn the importance of the structure integrity and consistency, there are still few cases when important tokens are pruned, leading to incorrect reasoning results. The rule-based reconstruction can effectively alleviate the influence of sub-optimal pruning strategy. The important tokens that should be reconstructed include 'Q:', 'A:', '\n', "Let's think step by step", and "The answer is".

D Additional Analysis and Case Study

D.1 Comparison with context window extension methods

While our work tackle the challenge of limited context window by pruning the redundant input few-shot prompts, another solution is to extend the context window of LLMs. We compare the math reasoning performance of LLaMA2-7B with CoT-Influx and LLaMA2-7B with 32K token context window extended with Positional Interpolation (PI) (Chen et al., 2023b). The results are listed in Table 11.

Table 11: Comparsion	of EM(%) on GS	SM8K of LLaMA2-7B v	with CoT-Influx a	and LLaMA2-7B-32K with PI.

Number of input shots	12	16	20	24	28	32	40
Average number of tokens	2108.6	2820.6	3535.4	4217.2	4929.1	5641.2	7070.8
LLaMA2-7B	13.87	15.08	14.02	-	-	-	-
LLaMA2-7B+CoT-Influx	-	-	-	14.33	15.09	15.92	15.77
LLaMA2-7B-32K	11.37	12.81	11.37	11.83	11.83	11.52	11.30

When the input prompt does not exceed the window token limit (the number of input shots is not more than 20), we compare the performance of LLaMA2-7B-32K with LLaMA2-7B. When the input prompt exceed the context window length, we apply our CoT-Influx to prune the prompts to make sure that they can be directly input to LLaMA2-7B without PI. The results show that the context window extension weaken the reasoning ability with the same input prompt. The limit of context window can be unlocked with our CoT-Influx. Moreover, our observation that LLMs can improve reasoning with more helpful CoT examples does not hold true for LLMs with extended context window.

D.2 Case Study on different prompt compression methods

To show how different prompt compression methods prune input few-shot prompts in different manners, we given an example of a 8-shot prompt selected using TopK retriever. The original full few-shot prompts are listed in the following box:

Original full few-shot prompt for math reasoning (1331 tokens):

- Q: Dave won 11 tickets at the arcade and spent 5 on a beanie. Afterward, he won 10 more tickets. Calculate his final ticket count by first finding the remaining tickets after his purchase and then adding the newly won tickets.
- A: Let's think step by step. Dave had 11 tickets, spent 5, leaving him with 6. Then he won 10 more, resulting in: 6 + 10 = 16 tickets. The answer is 16.
- Q: At the carnival, tickets for rides cost 0.75 dollars each, or you can buy a 15-dollar armband for unlimited rides for one night. To determine the number of rides where the armband's cost equals that of individual tickets, set up and solve an equation involving x, the number of rides.
- A: Let's think step by step. Let x be the number of rides. Equate the cost of x rides using individual tickets, 0.75x dollars, to the 15-dollar armband cost: 0.75x = 15. Solve for x: x = 15/0.75, which gives x = 20. The answer is 20.
- Q: Mitch, Jam, and Jay went out for a movie. Mitch paid \$7 per ticket for 3 friends, Jam purchased 2 popcorn boxes at \$1.5 each, and Jay got 3 milk teas for \$3 each. To equitably split the expenses, how much should each of them contribute?
- A: Let's think step by step. The total cost of 3 tickets at \$7 each, 2 popcorn boxes at \$1.5 each, and 3 milk teas at \$3 each is \$21 + \$3 + \$9 = \$33. Dividing the overall expenses among 3 friends results in a contribution of \$33/3 = \$11 per person. The answer is \$11.
- Q: Connor is taking his date to the movies, with tickets costing \$10.00 each. They opt for the large popcorn & 2 drink combo meal at \$11.00, and each choose a box of candy at \$2.50 per box. Determine the combined expenses for the movie tickets, combo meal, and candy to find the total amount Connor will spend on his date.
- A: Let's think step by step. Calculate the cost of two movie tickets $(2 \times \$10.00 = \$20.00)$, the combo meal (\\$11.00), and two boxes of candy $(2 \times \$2.50 = \$5.00)$, then sum them up (\\$20.00 + \\$11.00 + \\$5.00 = \\$36.00). The answer is \\$36.00.
- Q: Scott has 4 tickets. Ernest starts with 9 tickets and later discovers a stash of 72 more. Calculate the final number of tickets Ernest possesses.
- A: Let's think step by step. Ernest initially holds 9 tickets and acquires 72 additional ones, leading to a total of 9 + 72 = 81 tickets. The answer is 81.
- Q: Joseph and his friends watched two movies at his place. The first movie lasts 1 hour and 30 minutes, and the second is 30 minutes longer. They took 10 minutes for popcorn and double that for fries. Determine, in hours, the cumulative time spent cooking and watching movies by breaking down each component of time spent.
- A: Let's think step by step. First, find the second movie's length (1 hour and 30 minutes + 30 minutes = 2 hours). Then, sum both movies' lengths (1 hour and 30 minutes + 2 hours = 3 hours and 30 minutes). Next, calculate cooking time (10 minutes for popcorn + 20 minutes for fries = 30 minutes). Lastly, add movie and cooking times (3 hours and 30 minutes + 30 minutes = 4 hours). The answer is 4 hours.
- Q: The movie theater sold a number of tickets to the horror and romance movies. The horror movie ticket sales were 18 more than three times the romance movie ticket sales. If there were 25 romance movie tickets sold, how many tickets were sold for the horror movie, considering the given relationship?
- A: Let's think step by step. Let "h" represent the horror movie tickets sold. Given that h = 3(25) + 18, we can simplify the equation: h = 75 + 18, resulting in h = 93. The answer is 93.
- Q: On Saturday, Sara purchased 2 movie theater tickets at \$10.62 each, rented a movie for \$1.59, and bought another movie for \$13.95. Determine Sara's total expenditure on movies by performing a step-by-step calculation.
- A: Let's think step by step. Firstly, calculate the movie tickets' cost by multiplying the ticket price (\$10.62) by the quantity (2), resulting in \$21.24. Secondly, combine the rental (\$1.59) and purchase (\$13.95) costs, equaling \$15.54. Lastly, sum the ticket cost and rental/purchase cost: \$21.24 + \$15.54 = \$36.78. The answer is \$36.78.

Most of the examples above have similar background and target (tickets, movie, expense, etc.) but the difficulty and number of reasoning steps are various. In addition, the solution of most questions are highly redundant. When performing math reasoning with, it is important to select the most suitable and concise examples considering the characteristic of the current input question. In our evaluation across different methods shown in Sec. 5.1, we have empirically observe the previous methods fail to retain the structural integrity of prompt. We show the pruned prompt with different methods and similar token length in the following box. We can see that Selective Context and LLMLingua frequently discard the important part including 'Q:', 'A:', '\n', "Let's think step by step", and "The answer is" in these examples. Although GPT-4 can retain majority of these tokens, the reasoning steps are systematically removed because GPT-4 cannot be instructed to utilize the redundancy in both example-level and token-level. Our proposed CoT-Influx, however, select the most representative examples and only remove the redundant function words.

Pruned few-shot prompt of different methods:

Selective Context:

Q Dave won 11 tickets Afterward won: step Dave 11 tickets spent leaving Then won 10 resulting: 16 Q At tickets rides rides where set solve x: step Let x rides Equate x rides individual tickets dollars = x 20 Q Mitch Jam went paid per 3 friends Jam purchased equitably how: step 3 tickets + 3 friends results \$ Q Connor tickets They opt the large popcorn & 2 drink combo meal choose candy combo meal candy Connor: step combo boxes sum \$ Q Scott 4 tickets starts 9 tickets discovers 72 Ernest possesses: step initially holds 9 tickets 72 additional ones leading 81 Q Joseph watched lasts They popcorn double hours cooking breaking: step First find + Then sum both movies' lengths + Next, calculate cooking time popcorn + Lastly add movie cooking times + 4 hours Q sold 25 romance movie tickets considering the given relationship: step Let "h the horror movie tickets Given = 18 simplify 75 93 Q Sara purchased rented movies performing: step Firstly calculate resulting Secondly combine rental Lastly sum \$

LLMLingua:

: Dave won11ets the and5 a be. After he. his final count by first theets after the: Lets think. Daveets5,, in.

: the,ets 5, or a-ollarides for one. To theidesband cost equals of, equation involving r. A: think. Let.ides using individualets, the1ollar cost5 which. :, Jam and Jay a7 ticket3 Jam2orn5 Jay3 milk. To equ the.ets boxes53 milk each1. the overallenses3 friends a. The : Connor is his,... They theorn & drinkbo and0. theandy think. ofets0 theboal and two then :. Ernest and later a7. think. Ernest initially and, 9: friends at movie the minutes They and for. the spent by think, the, calculate The a the and ticket, think.:, bought.by-step calculation. A: Let's think step by step. Firstly, calculate the movie tickets' cost by multiplying the ticket price (\$10.62) by the quantity (2), resulting in \$21.24. Secondly, combine the rental (\$1.59) and purchase (\$13.95) costs, equaling \$15.54. Lastly, sum the ticket cost and rental/purchase cost: \$21.24 + \$15.54 = \$36.78. The answer is \$36.78.

GPT-4 Compression:

- Q: Dave won 11, spent 5 and won 10 more. Determine final count.
- A: The answer is 16.
- Q: Tickets cost 0.75 per ride, armband cost 15. Determine rides that armband's cost equals tickets.
- A: The answer is 20.
- Q: \$7 per ticket for 3, 2 popcorn boxes at \$1.5, 3 milk teas for \$3. Determine each contribute.
- A: The answer is \$11.
- Q: Tickets cost \$10.00 each, meal cost \$11.00, a box of candy at \$2.50. Determine the expenses.
- A: The answer is \$36.00.
- Q: Scott has 4. Ernest starts with 9 and discovers 72 more. Determine the final number.
- A: The answer is 81.
- Q: The first 1.5 hour, the second is 30 minutes longer. 10 minutes for popcorn. Determine the time.
- A: The answer is 4 hours.
- Q: Horror movie were 18 more than 3 times romance. 25 romance movie sold, Determine number of horror movie.
- A: The answer is 93.
- Q: Sara purchased 2 at \$10.62 each, a movie for \$1.59, and another \$13.95. Determine total expenditure.
- A: The answer is \$36.78.

CoT-Influx:

Q: Mitch, Jam, and went out a. Mitch paid \$7 per ticket for 3, Jam purchased 2 boxes at \$1.5 each, and got 3 for \$3 each. To equitably split, how much should each them contribute?

A: Let's think step by step. The total cost 3 tickets \$7 each, 2 popcorn boxes 1.5 each, and 3 milk \$3 each is 21 + 3 + 9 = 33. Dividing the overall expenses among 3 results of 33/3 = 11 per. The answer is 11.

Q: The theater sold number tickets to horror and romance movies. The horror movie ticket sales were 18 more than three times romance ticket. If there 25 romance sold, how many tickets were sold horror movie, considering?

A: Let's think step by step. Let "h" represent horror tickets sold. Given h = 3(25) + 18, we can simplify equation: h = 75 + 18, resulting h = 1. The answer is 93.

Q: On, Sara purchased 2 theater tickets \$10.62 each, rented movie \$1.59, and bought movie \$13.95. Determine Sara's total expenditure movies performing a calculation.

A: Let's think step by step. , calculate tickets' cost price (\$10.62) by quantity (2), resulting \$21.24. Secondly, combine rental (\$1.59) purchase (\$13.95), equaling. Lastly, sum ticket rental/purchase: \$21.24 + \$15.54. The answer is \$36.78.

E Prompt Settings

In this section, we show the prompt we used in this work for reproducibility. The prompt for evaluating the difficulty and reasoning steps of each examples are:

Prompt for difficulty and reasoning steps estimation:

We would like you to evaluate and rate the difficulty and complexity of the following question. You should first give an overall score on a scale of 1 to 10, where a higher score indicates higher difficulty and complexity. You should then evaluate the answer and give how many reasoning steps are in the answer. You must just give the score and the number of reasoning steps without any other reasons. The reply format should be 'Score': [score], 'Steps: [#steps]' ## Question: {Given question}

Answer: {Given answer}

Evaluation:

The prompt for GPT-4 Compression on prompts are shown as follow. Note that we encode the restriction of token limits in both the prompt and API by setting the *max_new_token*. However, the prompt compression results still fail to follow the restrictions for most cases. This disadvantages of uncontrollable token length is also discussed in previous work (Jiang et al., 2023).

Prompt for GPT-4-based compression:

Please compress the following examplars for few-shot in-context learning on math reasoning. The complete examplars could be removed if they are redundant and the tokens within each examplars can also be pruned. 'The answer is' in each examplar should be retained and please keep less than {Given token} tokens in total: {Given examplars}