

# MOEAs on Problems with Difficult Pareto Set Topologies

Yuri Marca<sup>1</sup> Hernán Aguirre<sup>1</sup> Saúl Zapotecas<sup>2</sup>  
Arnaud Liefooghe<sup>3</sup> Bilel Derbel<sup>3</sup> Sébastien Verel<sup>4</sup>  
Kiyoshi Tanaka<sup>1</sup>

<sup>1</sup>Shinshu University

<sup>2</sup>Univ. Autónoma Metropolitana

<sup>3</sup>Univ. Lille, Inria Lille - Nord Europe

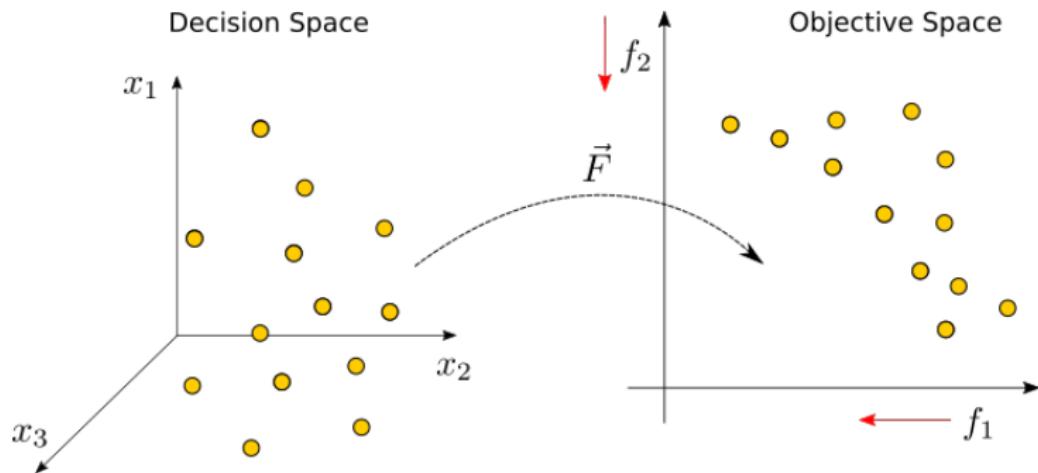
<sup>4</sup>Univ. Littoral Cote d'Opale

September 29, 2018



# Background

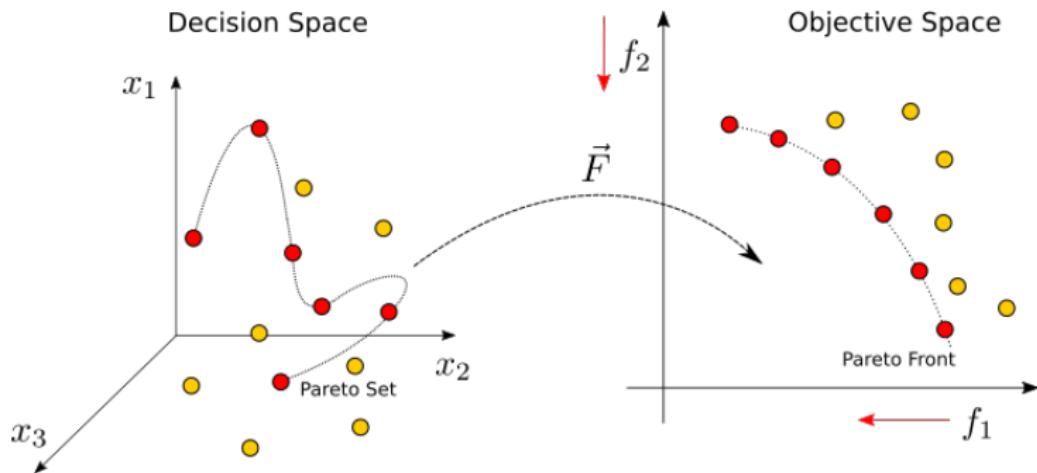
To solve a Multi-objective Optimization Problem (MOP) consists in maximizing or minimizing more than one objective function subject to a set of decision variables.



$$\begin{aligned} & \text{minimize or maximize } \vec{F}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), \dots, f_m(\vec{x})) \\ & \text{subject to } \vec{x} = (x_1, x_2, \dots, x_n) \end{aligned}$$

# Background

To solve a Multi-objective Optimization Problem (MOP) consists in maximizing or minimizing more than one objective function subject to a set of decision variables.



$$\begin{aligned} & \text{minimize or maximize } \vec{F}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), \dots, f_m(\vec{x})) \\ & \text{subject to } \vec{x} = (x_1, x_2, \dots, x_n) \end{aligned}$$

# Background

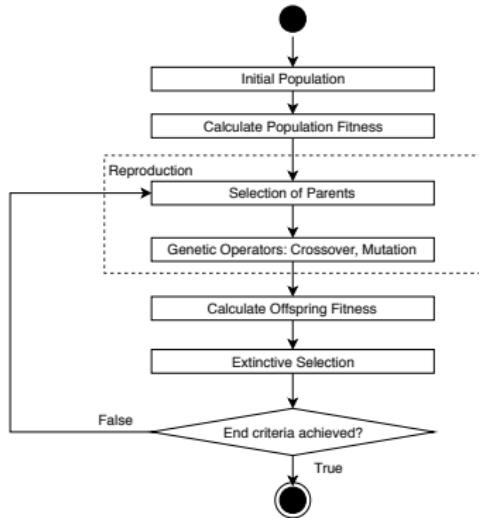
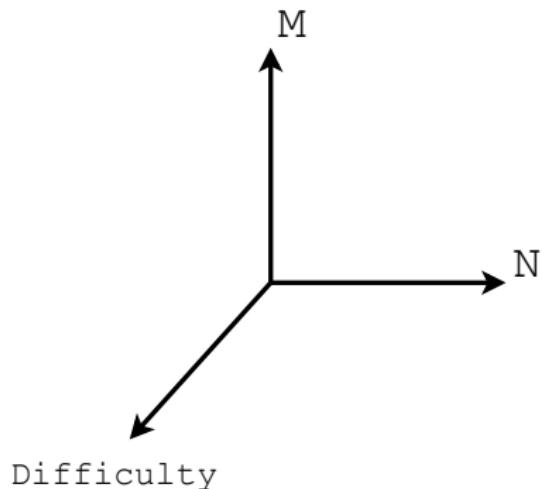


Figure: EA Flowchart.

- Evolutionary algorithms (EA) are heuristic methods based on natural evolution principles that have attracted a lot of attention due to their good performance to deal with MOPs.
- The use of EA to solve MOPs has been motivated mainly because of the population-based nature of EAs which allows the generation of several elements of the Pareto optimal set in a single run.

# Focus of my Research



## Massive Optimization

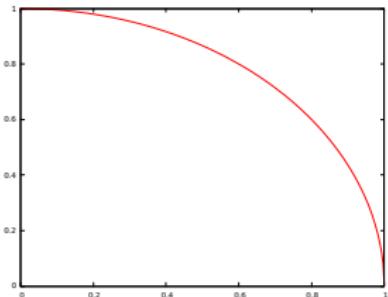
- Large-scale optimization: variable space dimensionality ( $N$ ).
- Any-objective optimization: single-, multi-, and many-objective optimization ( $M$ ).
- Expensive optimization: nature of the problem (Difficulty).
  - Modality;
  - Separability;
  - Topology;
  - etc.

# Focus of my Research

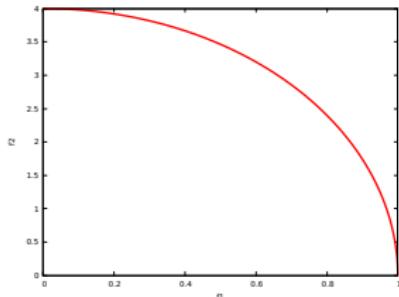
- Difficult topology (geometries) of decision space and variable linkages are commonly seen in real world optimization problems, and they can affect MOEAs' performance (Li and Qingfu, 2009).
- Okabe et al. observed that the Pareto set (PS) topology of most artificial test problems, such as DTLZ, have an oversimplified geometry, arguing that we should not expect such simplification on real problems (Okabe, 2004).
- In this work, we study how MOPs with difficult PS topologies can affect MOEAs, taking a closer look on its interference to decision space scalability.
- To do so, we applied popular MOEAs to solve these kind of problems, and evaluated the results.

# Artificial Test Problems

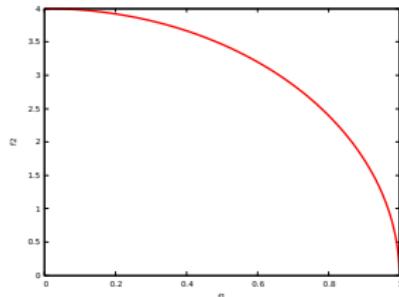
# Artificial Test Problems



(a) DTLZ2

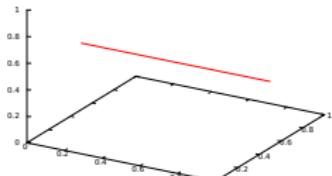


(b) ZCAT3

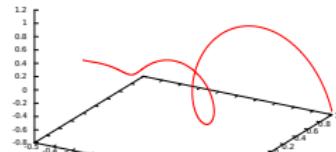


(c) ZCAT9

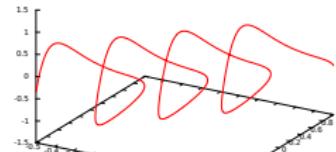
Figure: Objective space for 2 objectives problem.



(a) DTLZ2



(b) ZCAT3



(c) ZCAT9

Figure: Decision space ( $x_1, x_2, x_3$ ).

## Algorithms and Experimental Setup

# Algorithms and Experimental Setup

Table: Algorithms in comparison.

Algorithm	Selection Method	Reproduction Operator
NSGA-II	Pareto dominance	Differential Evolution
A $\varepsilon$ S $\varepsilon$ H	Pareto $\varepsilon$ -dominance	Polynomial Mutation
MOEA/D*	Decomposition	

## Experimental Setup

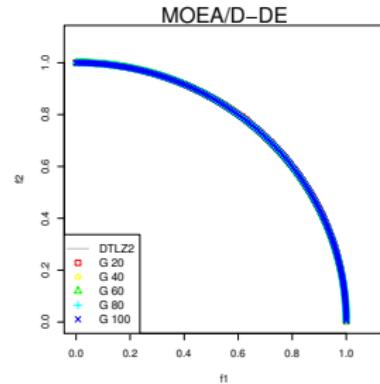
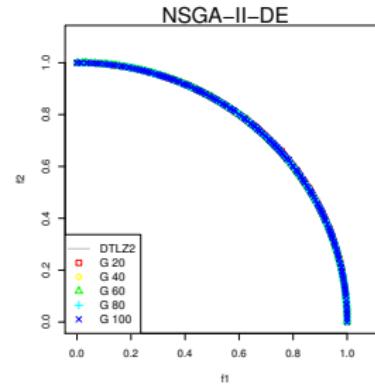
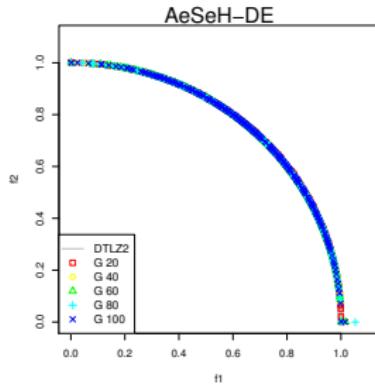
- Problems: DTLZ2, ZCAT3, ZCAT9
- DE Parameter F: 0.5
- Runs: 30
- Number of Generations: 100
- Population Size: 200
- M=2 and N=4, 6, 8, 10, 15, 20, 30, 40, 50
- Number of variables to mutate: 1

## Experimental Results for DTLZ2

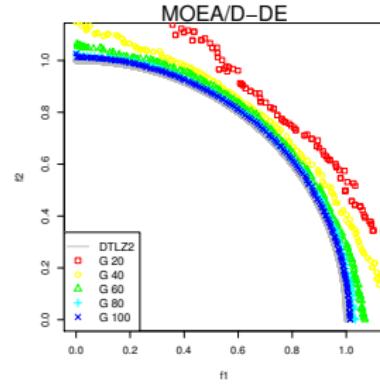
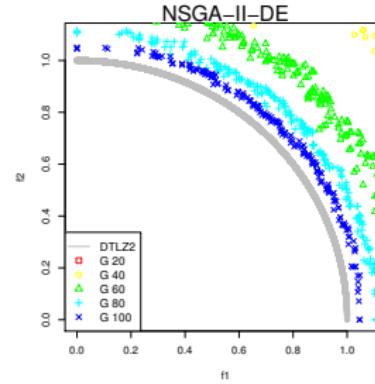
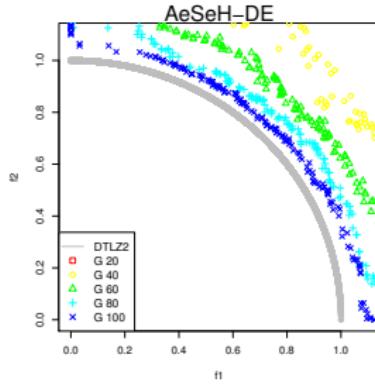
# Experimental Results

## DTLZ2 - 4 and 50 Variables

4 Variables



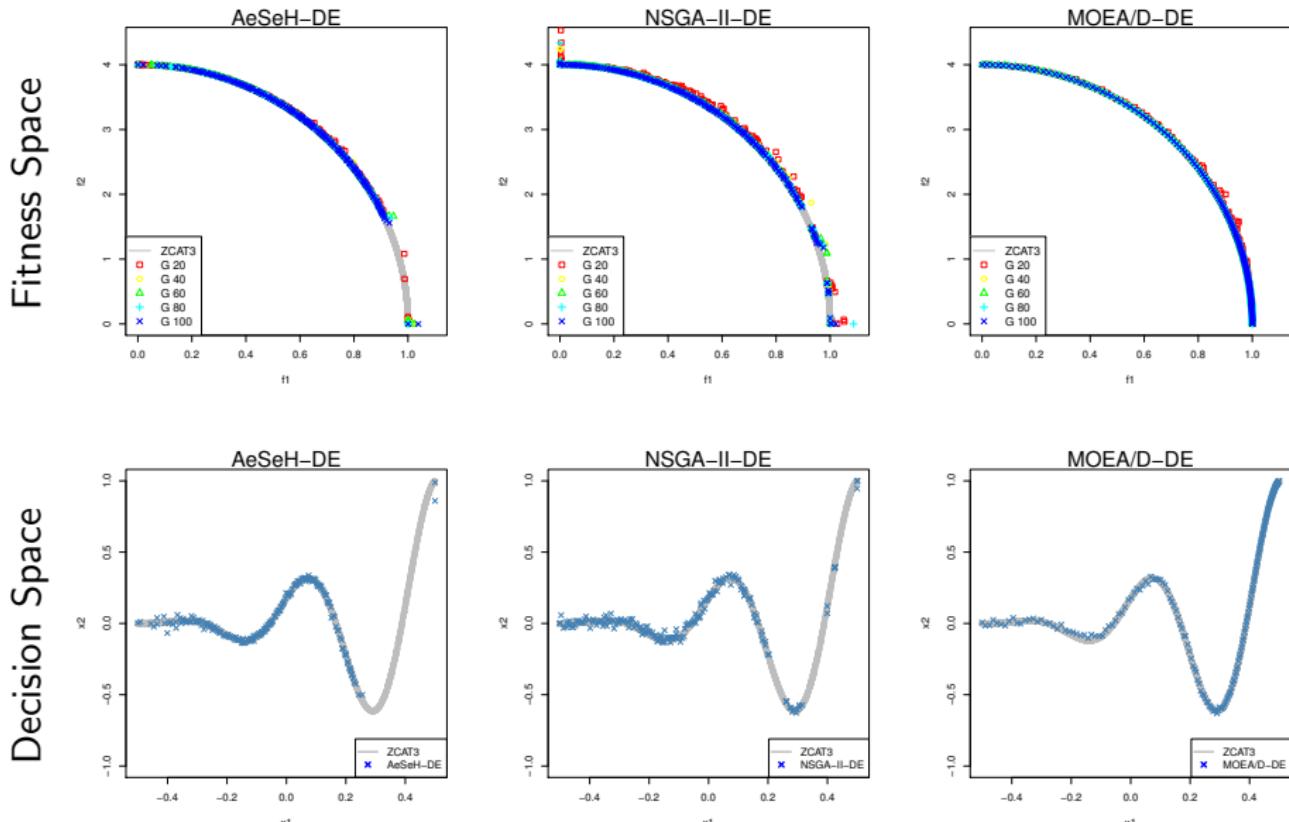
50 Variables



## Experimental Results for ZCAT3 and ZCAT9

# Experimental Results

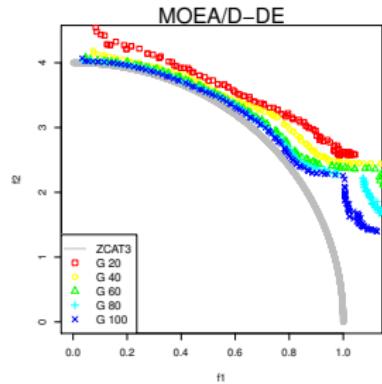
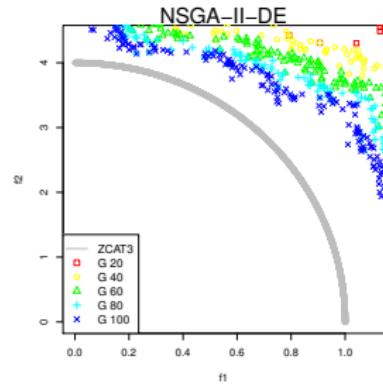
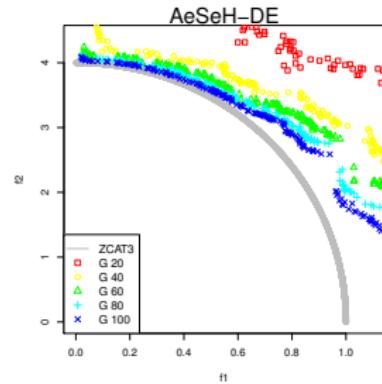
## ZCAT3 - Unimodal/Separable - 4 Variables



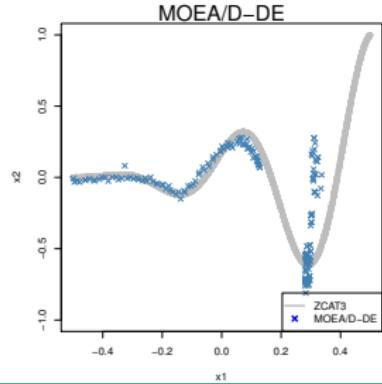
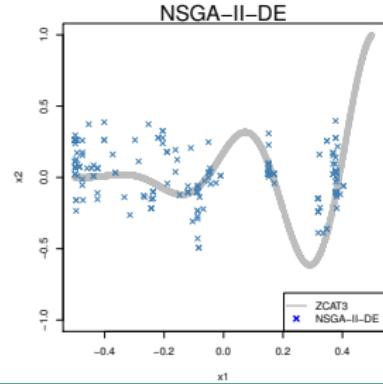
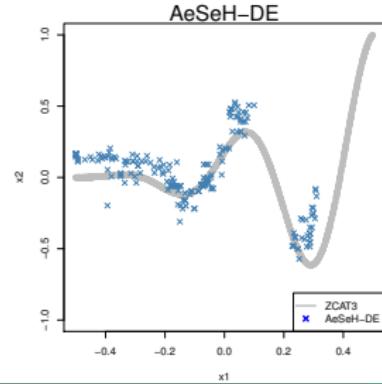
# Experimental Results

## ZCAT3 - Unimodal/Separable - 50 Variables

Fitness Space



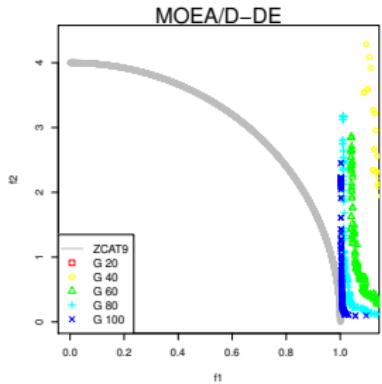
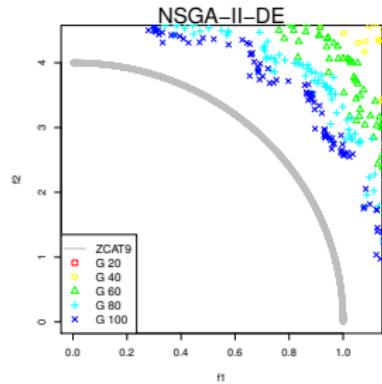
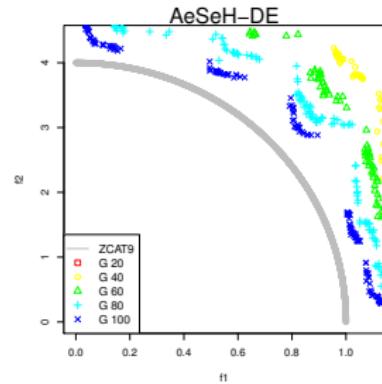
Decision Space



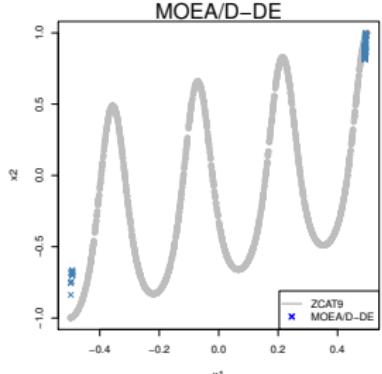
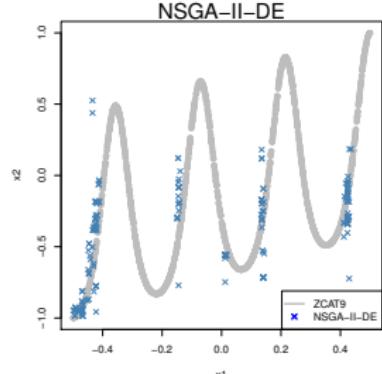
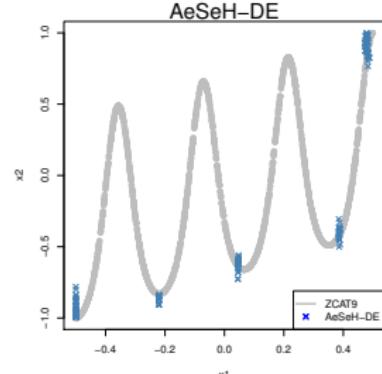
# Experimental Results

## ZCAT9 - Unimodal/Separable - 50 Variables

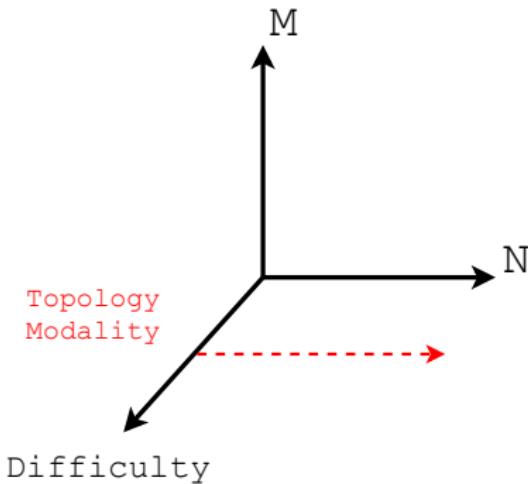
Fitness Space



Decision Space



# Conclusion and Future Work



- Decision space scalability were very impacted by the PS topologies of the problem.
- Most of MOEAs try to focus on objective space (Pareto dominance, crowding distance, decomposition), relying only on genetic operators to find new solutions.
- For future work, we are developing models to improve algorithms' performance on problems with difficult topologies.

Thank you for your attention!