State University Higher School of Economics

International College of Economics and Finance

Calculus Syllabus

1. Course description

1.1. Course pre-requisites

Students are expected to have a firm grounding in elementary mathematics, algebra, trigonometry, and geometry on the coordinate plane, the properties and graphs of elementary functions at the level of Russian high school.

1.2. Abstract

This course is designed to introduce students to the basic ideas and methods of mathematical analysis and their application to mathematical modeling. Four key concepts of the course, in order of appearance, are Limits, Derivatives, Series and Integrals. For each of the concepts the theoretical foundations are introduced and discussed, but the main focus is on the computational techniques and the applications. The course helps lay the foundation for the entire block of quantitative disciplines that are studied at ICEF, and it also provides some of the analytical tools that are required by advanced courses in economics. The course is taught in English.

2. Learning Objectives & Outcomes

At the conclusion of the course, students should:

- be able to analyze functions of one variable represented in a variety of ways: graphical, numerical, analytical, or verbal, and understand the relationships between these various representations;
- understand the concepts of the limit of an infinite sequence, the limit of a function at a point and the limit of a function as its argument approaches infinity;
- understand the meaning of the derivative in terms of a rate of change and local linear approximation, and be able to use derivatives to solve a variety of problems;
- understand the concept of infinite series and the idea of approximating a function by its Taylor series, use Taylor polynomials to approximate function values;

- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and be able to use integrals to solve a variety of problems;
- understand the relationship between the derivative and the definite integral, as expressed by the Fundamental Theorem of Calculus;
- understand how the concept of definite integral extends to double and triple integrals, and be able to compute multiple integrals by reducing them to iterated integrals;
- be able to communicate mathematics in well-written sentences and to explain the solutions to problems;
- be able to model a written description of a simple economic or physical situation with a function, differential equation, or an integral;
- be able to use mathematical analysis to solve problems, interpret results, and verify conclusions;
- be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.

3. Methods of Instruction

Throughout the academic year there will be 1 lecture and 1 seminar per week for each academic group. In addition, the lector and each of the class teacher will held regular office hours.

Almost each week a homework assignment will be posted following the lecture. The homework will have two parts: a part to be submitted at the seminar in about a week from the posting date and a self-study part. Students are encouraged to do both parts. The self-study part of the homework may contain more complex problems aimed at a deeper understanding of the topic, it can be discussed at the seminars and the office hours.

4. Reading List

4.1. Main textbooks in English

- Stewart J. (editor), Calculus. Early Transcendentals. 6 edition. Thomson Brooks/Cole, 2008 (**S**). Any later edition is good as well.
- Dowling E.T. Introduction to Mathematical Economics. McGraw-Hill, 1980. (**D**)

- Lockshin J., Calculus: theory, examples, exercises. ICEF. This is a problem book compiled specifically for the ICEF Calculus course.

4.2. Recommended textbooks in Russian

- Красс М. С., Высшая математика для экономиста. М., 1998 (**Кр**).
- Кремер Н.Ш., Путко И.М., Фридман М.Н. Высшая математика для экономистов. М, 2000 (**КПФ**)
- Фихтенгольц Г.М. Курс дифференциального и интегрального исчисления (в трех томах). М., 1998 (**Ф**).
- Краснов М. и др., Вся высшая математика, том 1, 2.
- Демидович Б.М. Сборник задач и упражнений по математическому анализу. М., Наука, 1996.

4.3. Supplementary reading

- Simon C.P., Blume L. Mathematics for Economists. W.W.Norton & Company, 1994.
- Chiang A.C. Fundamental Methods of Mathematical Economics. McGraw-Hill, 1984
- Anthony M., Biggs N. Mathematics for Economics and Finance. CUP, 1996.
- Зорич В.П., Математический анализ (в 2-х) томах. Фазис, 1997-1998. An English translation as also availalble.

4.4. Link of the course topics to the textbook chapters

a) Introduction

The application of mathematics to describing phenomena. The role of mathematics and mathematical modeling in economics. Different forms of representation of functions. Elementary concepts: domain and range of a function, even and odd functions, periodic functions. Graphs of elementary functions. Shifts and distortions of graphs. Implicit functions. Examples of functions in economics: utility function, production function, cost function, demand and supply functions.

(S. – pp. 10-72; D. Ch. 7; K. – pp. 23-51; Кр. - pp. 11-14, 46-58, 88-91, 155-161; КПФ – pp. 125-137; Ф. - V.1, pp. 93-114)

b) Sequences. Limit of a sequence

Sequences: bounded and unbounded, infinitely small and infinitely large. Limit of a sequence. Limit theorems for sequences: arithmetic operations, sandwich theorem. Monotone sequences. Convergence of a monotone increasing sequence. The number ***e***.

(S. – pp. 674-686; Кр. - pp. 24-45; КПФ – pp. 141-142; Ф. - V.1, pp. 43-92)

c) Limit of a function

The limit of a function at infinity. Asymptotes of a function at infinity. The limit of a function at a point. Limit theorems for functions. Functions that tend to zero, functions that tend to infinity. First and Second Special Limits. Types of indeterminate forms. Finding limits. Left and right limits.

(S. – pp. 88-118; D. - Ch. 3.1; K. - pp. 71-91; Кр. - pp. 58-73; КПФ – pp. 143-160; Ф. - V.1, pp. 115-145)

d) Continuity

Definition of continuity of a function at point and on an interval. Continuity of elementary functions. Properties of continuous functions. Points of discontinuity. Classification of points of discontinuity. Vertical asymptotes.

(S. – pp. 119-143; D. – Ch. 3.2; K. – pp. 92-95; Кр. - pp. 74-87; КПФ – pp. 161-164; Ф. - V.1, pp. 146-185)

e) The derivative

Definition of the derivative. Tangent lines and normal lines. Geometric, physical and economic interpretations of the derivative. Right and left derivatives. Differentiability at a point. Differentiability and continuity. Differentiation. Rules of differentiation. Derivatives of elementary functions. Differentiation of inverse functions. Logarithmic differentiation. Differentiation of implicit functions. Existence of a differentiable implicit function. Definition and geometric interpretation of differentials. Approximate calculations using differentials. The second derivative. The economic meaning of the second derivative. Higher-order derivatives and differentials. Properties of differentiable functions: Rolle's theorem, the Mean Value theorem, Cauchy’s theorem, and their geometric interpretation.

(S. – pp. 143-166, 172-261; D. - pp. 41-47; K. pp. 109-144, 163-166, 211-214, 216-218; Кр. - pp. 98-123; КПФ – pp. 176-198, 209-211; Ф. V.1, pp. 186-222, 231-245)

f) Applications of the derivative

L’Hospital’s rule. Necessary and sufficient conditions for increasing/decreasing functions. Related rates. Concave and convex functions. Different ways of expressing concavity. Economic interpretation of concave and convex functions. Points of inflection. Local extrema. First-order necessary and sufficient conditions for a local extremum. Second-order necessary and sufficient conditions for a local extremum. Maximum and minimum values of a function on an interval. Geometric and economic applications of optimization. Curve sketching.

(S. – pp. 270-333; D. – Ch. 4; K. – pp. 167-210; Кр. - pp. 124-132, 140-161; КПФ – pp.212-234, 240-241; Ф. – V.1, pp. 268-336)

g) Number series, power series, and Taylor expansions

Necessary condition for convergence of a series. Harmonic series and power series. The ratio test. Comparing series to test for convergence. Alternating series. Sufficient condition for convergence of an alternating series. Absolute convergence. Radius and interval of convergence of a power series. Abel's theorems. Taylor's formula. Taylor and Maclaurin series. Taylor and Maclaurin expansions for elementary functions. Application of Taylor series for analyzing the behavior of a function at a point and for conducting approximate calculations.

(S. – pp. 687-747; D. – Ch. 30.2; Кр. - pp. 133-139; КПФ – pp. 356-372, 379-390; Ф. - V.1, pp. 246-262)

h) Anti-derivatives and the indefinite integral

Anti-derivatives. The indefinite integral and its properties. Table of indefinite integrals. Basic methods of integration: direct integration, substitution and integration by parts. Integration of rational functions.

(S. – pp. 340-345, 452-489; D. –pp. 357-362; K. – pp. 234-257; Кр. - pp. 162-186; КПФ – pp. 251-270; Ф. – V.2, pp. 11-93)

i) The definite integral

Problems that require the definite integral. Definition of the definite integral using Riemann sums. Sufficient condition for the existence of the definite integral. Approximate calculation of definite integrals using rectangles and trapezoids. Simpson's rule. Properties of the definite integral. Differentiation of a definite integral with variable upper bound. The fundamental theorem of calculus. Substitution and integration by parts.

(S. – pp. 354 - 390; D. –pp. 373-375; K. – pp. 273-285; Кр. - pp. 187 - 210, 233-236; КПФ – pp. 283-296, 312-317; Ф. - V.2, pp. 94-168)

j) Applications of the definite integral

Applications of the definite integral in geometry, economics and physics. Area of a flat region, volume of a solid of revolution, volume of a solid with known cross-sections. Use of definite integrals to solve separable differential equations.

(S. – pp. 414-447, 524-561; D. – p. 376; K. – pp. 286-315; Кр. - pp. 211-232; КПФ – pp. 298-306; Ф. - V.2, pp. 169-243)

k) Improper Integrals

Integrals with infinite bounds. Improper integrals of the first kind. Integration of unbounded functions. Improper integrals of the second kind. Principle value. Convergence tests for improper integrals. Absolute and relative convergence of improper integrals.

(S. – pp.508-517; Кр. - pp. 237-248; КПФ – pp. 307-311; Ф. - V.2, pp. 552-653)

l) The double and triple integrals

Definition of double and triple integrals. Reduction of double integrals to iterated integrals. Changing the order of integration in iterated integrals. The geometric interpretation and main properties of double integrals.

(S. – pp. 950- 973; Кр. – с. 390-405; КПФ: pp. 425-427; Ф.: V. 3, pp. 136-141, 154-167)

m) Differential equations and slope fields

Definition of first order differential equations. General and particular solutions. Existence and uniqueness theorem. Isoclines and direction fields. Solution of separable differential equations. Solution of homogeneous differential equations and first-order linear equations. Application of differential equations to physics and economics.

(S. – pp. 566-607; D. –pp. 392, 395-396; K. – pp. 316-323; Кр. - pp. 477-544; КПФ – pp. 325-336; Ф. - V.2, pp. 244-257)

5. Special Equipment and Software Support

The lecture notes, the homework assignments and some other supplementary materials are posted on the ICEF information portal. Thus, the students are required to have some means of accessing that information. The requirements are minimalistic and can be met by any contemporary computer or a smartphone.

No special equipment is required for the Calculus classes or the exams.

While working on their homework the students are free to use contemporary math software and graphing calculators. In many cases using such software helps to develop the deeper understanding of the material. However, the students are required to be able to solve the assignments and present their solutions in the classical way, that is by means of pen and paper only.

6. Grading System and Examination Type

Examination is in writing. Sample materials for knowledge assessment are available in ICEF Information system at https://icef-info.hse.ru.

6.1. Exam Schedule

The academic year 2019 – 2020 at ICEF is divided into 4 modules as follows:

- Fall Semester
  - Module I: from 2nd September 2019 till 26th October 2019;
  - Module II: from 4th November 2019 till 21st December 2019;
- Spring Semester
  - Module III: from 13th January 2020 till 21st March 2020;
  - Module IV: from 30th March 2020 till 30th April 2020.

At the end of each module the students sit a written exam. The exam approximate dates and weights are combined in the table below. The exact dates of the exams are decided during the year and announced in form of the cross-subject exam schedule.

Semester	Fall	Fall	Spring	Spring
Module	I	II	III	IV
Exam time period	28 October – 3 November 2019	23 – 29 December 2018	25 – 31 March 2019	June 2020
Control name	Fall mock	Winter exam	Spring mock	Final exam
Weight in semester grade	0.425	0.575	0.317	0.683
Weight in final grade	17%	23%	19%	41%
Retake		13 – 31 January 2020		September 2020

6.2. Assessment Rules

Each of the four exams is marked out of 100 points. The grades for each of the two semesters are also initially computed out of 100 points using the following formulas:

Fall100 = 0.425 * (Fall mock grade) + 0.575 * (Winter exam grade) + FBC * Fall_Hwk_Quiz

Spring100 = 0.317 * (Spring mock grade) + 0.683 * (Final exam grade) + SBC * Spring_Hwk_Quiz

Here Fall_Hwk_Quiz and Spring_Hwk_Quiz are marks out of 100 points for the homework and in-class activities in the corresponding semesters. It is expected that the scores will be based on at least 5 homework assignments and at least 5 in-class quizzes in each module. The weights FBC and SBC are between 0 and 0.1, the exact values will be determined at the end of each semester.

The grade for the whole course is initially computed as follows:

Course100 = 0.4 * Fall100 + 0.6 * Spring100

The grades Fall100 and Course100 are converted to 10-point scale according to the ICEF guidelines. The resulting grades out of 10 points are the official HSE grades for the first semester and the course. The standard Russian grades out of 5 points are also computed based on the grades out of 10 points and the HSE generic conversion rules.

In case a student misses one of the exams the ICEF generic rules will apply in accordance with the HSE Interim and Ongoing Assessment Regulations (incl. Annex 8 for ICEF). Grade determination after retakes is done in accordance with ICEF Grading Regulations (par. 5). In some cases the student will be allowed to sit a retake exam. For example, if a student missed the Winter exam he/she may be allowed to sit a retake exam in January 2020. There would be no retake exams for the Fall mock or the Spring mock exams, a miss of these exams may be partially compensated according the ICEF rules. The retake exam in September 2020 is reserved for those who missed the final exam or acquired the overall grade Course100 below the level that converts into 4 points out of 10.

7. Course Plan

The following table gives an approximate allocation of time to the course topics. The actual schedule may vary depending on the pace of the lectures. The table also gives the approximate time the students are expected to allocate to their Calculus homework.

No.	Topic titles	TOTAL hours	Lecture hours	Seminar hours	Self-study hours
1	Introduction. Functions of one variable	8	2	2	4
2	Sequences. Limit of a sequence.	16	4	4	8
3	Limit of a function.	16	4	4	8
4	Continuity.	8	2	2	4
5	The derivative.	16	4	4	8
6	Applications of the derivative.	24	6	6	12
7	Infinite series, power series, Taylor series.	24	6	6	12
8	Anti-derivatives and the indefinite integral.	24	6	6	12

9	The definite integral.	24	6	6	12
10	Applications of the definite integral.	40	10	10	20
11	Improper integrals.	16	4	4	8
12	Double integrals, triple integrals.	16	4	4	8
13	Introductions to differential equations.	24	6	6	12
14	Course review.	24	2	2	20
	Total:	280	66	66	148

The detailed course plan per topic is outlined above in the section 4.4.

Author of the original course program: J. Lockshin, 2010.

Updated by A. Akhmetshin, 2012, 2018, 2019.