Tarea 7.

Cinemática Inversa por Matrices Homogéneas.

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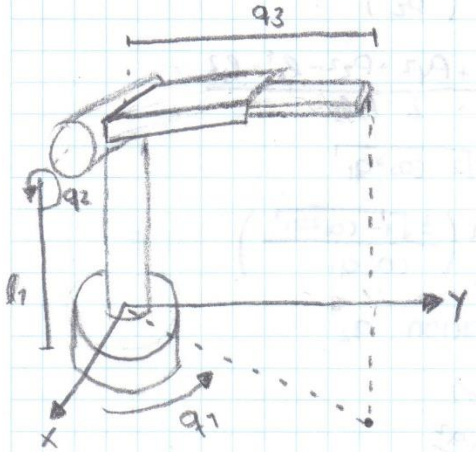
Grado y grupo:
8°A

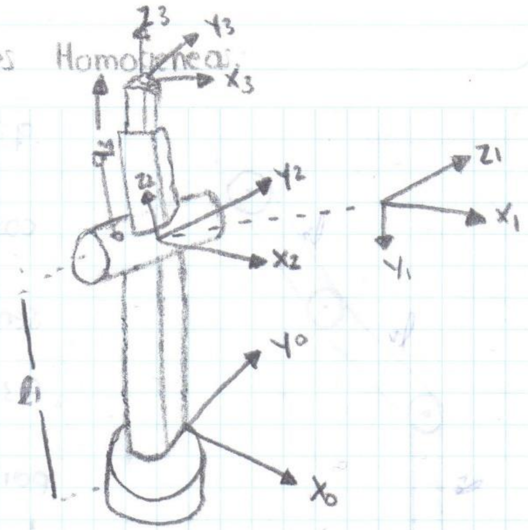
Materia:
Cinemática de Robots.

Carrera:
Ingeniería Mecatrónica.

Cinematica Inversa por Matrices Homogeneas

Robot 3 GDL.





Obtener: T_3^0

A_1^0

A_2^1

A_3^2

tabla D-H

i	θ_i	d_i	a_i	α_i
1	q_1	L_1	Ø	-90°
2	q_2	Ø	Ø	90°
3	Ø	q_3	Ø	Ø

i	θ_i	d_i	a_i	α_i
1	q_1	L_1	0	90°
2	q_2	0	0	-90°
3	q_3	q_3	0	0

$$\begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1^0 \qquad \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2^1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_3^2$$

$$A_2^0 = \begin{bmatrix} C_1C_2 & -S_1 & -C_1S_2 & 0 \\ S_1C_2 & C_1 & -S_1S_2 & 0 \\ S_2 & 0 & C_2 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = \begin{bmatrix} C_1C_2 & -S_1 & -C_1S_2 & -q_3C_1S_2 \\ S_1C_2 & C_1 & -S_1S_2 & -q_3S_1S_2 \\ S_2 & 0 & C_2 & q_3C_2+l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} =$$

$$\begin{bmatrix} n_x & n_y & n_z & -n^T p \\ o_x & o_y & o_z & -o^T p \\ a_x & a_y & a_z & -a^T p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^{0\,-1} = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & 1 & -l_1 \\ S_1 & -C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^{1\,-1} = \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} C_2 & S_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$A_3^{2\,-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Utilizando la primera ecuación:

$$(A_1^0)^{-1} T_3^0 = A_2^1 A_2^3 = \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & 1 & -l_1 \\ S_1 & -C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 & 0 & -S_2 & -S_2 q_3 \\ S_2 & 0 & C_2 & C_2 q_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_1 P_x - C_1 P_y = 0 \rightarrow \tan(q_1) = \left(\frac{P_y}{P_x}\right)$$

$$\boxed{q_1 = \arctan\left(\frac{P_y}{P_x}\right)}$$

Utilizando la segunda ecuación:

$$(A_2^1)^{-1} (A_1^0)^{-1} T = A_3^2 =$$

$$\begin{bmatrix} C_2 & S_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & 1 & -l_1 \\ S_1 & -C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 C_1 & C_2 S_1 & S_2 & -l_1 S_2 \\ -S_1 & C_1 & 0 & 0 \\ -S_2 C_1 & -S_2 S_1 & C_2 & -C_2 l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_2C_1P_x + C_2S_1P_y + S_2P_z - l_1S_2 = 0$$
$$C_2(C_1P_x + S_1P_y) + S_2(p_z - l_1) = 0$$
$$\tan(q_2) = \frac{C_1P_x + S_1P_y}{(p_z - l_1)}$$

$$S_1P_x - C_1P_y = 0$$
$$(S_1P_x - C_1P_y)^2 = S_1^2P_x^2 - C_1^2P_y^2 - 2S_1C_1P_xP_y = 0$$
$$(1 - C_1^2)P_x^2 + (1 - S_1^2)P_y^2 = 2S_1C_1P_xP_y$$
$$C_1^2P_x^2 + S_1^2P_y^2 + 2S_1C_1P_xP_y = P_x^2 + P_y^2$$
$$C_1P_x + S_1P_y = \sqrt{P_x^2 + P_y^2}$$

$$\boxed{q_2 = \arctan\frac{\sqrt{P_x^2+P_y^2}}{l_1 - P_z}}$$

$$-S_2C_1P_x - S_2P_1P_y + C_2P_z - C_1l_1 = q_3$$
$$C_2(P_z - l_1) - S_2(C_1P_x + S_1P_y) = q_3$$

$$\boxed{q_3 = C_2(P_z - l_1) - S_2\sqrt{P_x^2 + P_y^2}}$$

$$q_2 = \arctan\left(-\frac{P_xCq_1 + P_ySq_1}{P_z - L_1}\right)$$

$$q_3 = C_2(P_z - l_1) - S_2(C_1P_z + S_1P_y)$$

eurocolors
Papeles que inspiran