Wireless networked control over lossy uplinks abstracted by finite-state Markov channels ⋆

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Abstract: Networked control systems using wireless links to convey information among sensors, controllers, and actuators greatly benefit from having an accurate estimate of the communication channel condition. To this end, the finite-state Markov channel abstraction allows for reliable channel state estimation. This paper develops a Markov jump linear system representation for wireless networked control with intermittent channel state observation, message losses, and generalized hold-input dropout compensation. Furthermore, it exploits the emerging structural properties of the system to solve the finite-horizon linear quadratic regulation problem efficiently.

Keywords: Telecommunication-based automation systems

1. INTRODUCTION

Wireless networked control receives considerable attention from industry and academia thanks to its mission-critical applications in industrial automation, intelligent transportation, telesurgery, and smart grids. See, e.g., Park et al. (2018), Eisen et al. (2019), Pezzutto et al. (2020), and Liu et al. (2021) as an overview of significant recent advances in the wireless networked control systems (WNCSs) research. One of the central topics in this research area is estimation and control over fading channels, explored, e.g., in Schenato et al. (2007), Gupta et al. (2009), Heemels et al. (2010), Gonçalves et al. (2010), Ding (2011), Pajic et al. (2011), Minero et al. (2013), Quevedo et al. (2014), Yu and Fu (2015), Zacchia Lun and D'Innocenzo (2019), and Impicciatore et al. (2021, 2022).

Dealing with control systems that exploit wireless links often highlights the communication channels' stochastic behavior. Indeed, wireless links are subject to path loss, shadowing, and fading when mobility is involved, which translates into time-varying message dropouts, message delays, and jitter (Goldsmith, 2005). Thus, having an estimate of the channel condition at the control application level is highly desired for correctly modeling the stochastic properties of a WNCS.

The finite-state Markov channel (FSMC, see, e.g., Sadeghi et al. (2008)) represents a simple yet powerful analytic model capturing the main features of the wireless link. Despite the availability of the FSMC model, when dealing with the application level, packet dropouts dynamics are often modeled as realizations of a Bernoulli process (Schenato et al., 2007; Hu et al., 2021), which may result in an oversimplification of the complex communication subsystem dynamics and incorrect evaluation of the control subsystem behavior, for instance in terms of stability as proven in Zacchia Lun and D'Innocenzo (2019).

This paper exploits an FSMC wireless links abstraction to model packets dropouts, as in Zacchia Lun et al. (2020). Specifically, a uniform partitioning of possible values of signal-to-interference-plus-noise ratio (SINR) provides the channel state space. The probability of the instant value of the SINR being between two adjacent thresholds gives us steady-state probabilities. Finally, integrating the joint probability density function of the SINR between two consecutive packet transmissions determines the transition probabilities between states. Moreover, the SINR is analytically computed based on a real scenario where the characteristics of the communication protocol used in the wireless link are taken into account together with channel impairments.

Similarly to Zacchia Lun and D'Innocenzo (2019), this paper assumes that a remote controller receives the FSMC state estimates with acknowledgments (ACKs) of the successful transmissions. However, this paper addresses for the first time a different scenario where a negative ACK (NACK) mechanism does not provide for delivering the FSMC state information. Such a scenario happens when to save communication resources, the network does not allow NACK transmissions. Furthermore, the simplest versions of link layer protocols only include ACKs: they do not need extensive numbering of frames and only require limited buffering efforts on both transmitter and receiver sides. Then, the controller has no access to the wireless channel state information for the whole duration of a packet error burst. Thus, we introduce a novel wireless networked control design where the feedback gain depends on the available Markov channel state information and packet-loss length. Furthermore, we account for a generalized hold-input dropout compensation strategy at the actuation end. The considered setting has remarkably high complexity

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and requires a deep understanding of the structural properties of the system to obtain a computationally treatable accurate solution, as discussed in Sections 3 and 4.

Our main contribution is deriving a rigorous Markov jump linear system (MJLS) model for wireless networked control over lossy links with intermittent channel state observation and generalized hold-input dropout compensation. We also provide a finite-horizon linear quadratic regulation (LQR) solution to the optimal control problem that exploits the structural properties and reduces the computational complexity compared to a standard MJLS solution.

The rest of the paper has the following structure. Section 2 presents the WNCS model for the considered scenario, Section 3 provides the equivalent MJLS form derivation, and Section 4 introduces an efficient solution to the LQR problem in the finite horizon setting. Section 5 gives the concluding remarks.

2. MODEL FORMULATION

Consider a linear stochastic system with intermittent control packets due to the lossy communication channel and generalized hold-input dropout compensation strategy:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k, \\ u_k = \delta_k u_k^c + (1-\delta_k)\phi u_{k-1}, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is a system state, $u_k \in \mathbb{R}^{n_u}$ is the control input to the actuator, A and B are state and input matrices of appropriate size, $u_k^c \in \mathbb{R}^{n_u}$ is the desired control input computed by the remote controller, and $w_k \in \mathbb{R}^{n_x}$ is a Gaussian white process noise with zero mean and covariance matrix Σ_w. The process noise w_k is assumed to be independent of the initial state x_0 and of the binary stochastic variable δ_k, which models the packet loss between the controller and the actuator: if the packet is correctly delivered, $u_k = u_k^c$; otherwise, if lost, the actuator applies the last available control input multiplied by a scalar ϕ. This is the generalized hold-input dropout compensation strategy (Moayedi et al., 2013), that covers both the zero-input dropout compensation strategy (with $\phi = 0$) and hold-input strategy (with $\phi = 1$) as special cases. See Moayedi et al. (2013) and Yu and Fu (2015) for a more general discussion of dropout compensation.

The number of consecutive control packet dropouts (a.k.a. packet-loss length (Lu et al., 2018)) observed by the controller at a given time step k is modeled as a random variable $\mathfrak{z}_k$. The value of $\mathfrak{z}_k$ is incremented by 1 when the reception of the control packet is not acknowledged, i.e., when the remote controller believes that the last control packet sent to the actuator was corrupted and discarded. The ACK message is considerably shorter than the control one and much less likely to be corrupted. For this reason here we make an idealistic assumption that all the ACKs are always successfully delivered [1]. Thus, the absence of an ACK means that $\delta_{k-1} = 0$. Since the counter $\mathfrak{z}_k$ refers to a number of consecutive dropouts, it is reset to 0 when the transmission of the control packet is successful, i.e. when the ACK is received. This is the case of $\delta_{k-1} = 1$. Formally,

$$\mathfrak{z}_k = (1-\delta_{k-1})(\mathfrak{z}_{k-1}+1). \tag{2}$$

[1] This assumption allows focusing on the problem at hand without burdening readers with tedious technical details. For the sake of completeness, we underline that the assumption of an error-free ACK link can be readily relaxed by introducing an additional binary stochastic variable modeling the successful delivery of the ACK that would also evolve according to an adequately derived FSMC describing the corresponding wireless link.

Notice that, by iterating (2) over multiple time steps,

$$\mathfrak{z}_k = \ell \Leftrightarrow \delta_{k-1-\ell} = 1 \wedge \delta_{k-t} = 0 \ \forall t \in \mathbb{Z} \text{ s.t. } 1 \le t \le \ell. \tag{3}$$

We remark that if $\ell = 0$ then $\{t\}_{t=1}^{0} = \varnothing$, which means that (3) becomes simply $\mathfrak{z}_k = 0 \Leftrightarrow \delta_{k-1} = 1$.

This paper concerns a wireless networked control problem, where an FSMC abstracts the communication link between a controller and an actuator. Zacchia Lun et al. (2020) showed how to obtain a consistent and accurate FSMC model suitable for a wireless industrial automation scenario. Such a model can capture the average and extreme behavior of the radio link through the link quality metrics, such as the long-run mean packet error probability and the maximal number of consecutive dropouts (from now on, denoted by L). Clearly,

$$0 \le \mathfrak{z}_k \le L. \tag{4}$$

A state of an FSMC is the output of a discrete-time Markov chain that takes values in a finite set $\mathbb{S} \triangleq \{s_i\}_{i=1}^{N}$. The probability of successful packet delivery and packet loss are conditioned to the state of the Markov channel:

$$\mathbb{P}(\delta_k = 1 \mid \theta_k = s_i) = \hat{\delta}_i, \quad \mathbb{P}(\delta_k = 0 \mid \theta_k = s_i) = 1 - \hat{\delta}_i. \tag{5}$$

In other words, each state s_i is associated with a binary symmetric channel (BSC) with error probability $1-\hat{\delta}_i$. We remark that the definition of L presented in Zacchia Lun et al. (2020) without loss of generality requires $\hat{\delta}_N = 1$.

The evolution of the FSMC is determined by the transitions between its states, which occur with probabilities

$$p_{ij} \triangleq \mathbb{P}(\theta_k = s_j \mid \theta_{k-1} = s_i) \ge 0, \quad \textstyle\sum_{j=1}^{N} p_{ij} = 1. \tag{6}$$

These probabilities are gathered in a transition probability matrix (TPM) of the Markov channel denoted by $\boldsymbol{P}_c$:

$$\boldsymbol{P}_c \triangleq [p_{ij}]_{i,j=1}^{N}. \tag{7}$$

In many practical wireless communication scenarios, for instance, when the networks are based on IEEE 802.15.4 compatible hardware [2], the state of the FSMC is available to the receiver (Zacchia Lun et al., 2020). So, similarly to Zacchia Lun and D'Innocenzo (2019), we assume that the controller observes Markov channel states via ACKs, which become available only after the most recent decision on the control gain to apply has been made and sent through the channel since the actual success of the transmission is not known in advance. However, the ACKs are sent only when the control packets are received (i.e., when $\delta_{k-1} = 1$).

Without a negative acknowledgement mechanism the controller cannot know the state of the FSMC estimated by the receiver. Thus, unlike Zacchia Lun and D'Innocenzo (2019), we consider that the state of the FSMC is not available to the controller for the whole duration of the packet error burst, i.e., the information set available to the state-feedback controller is formally defined as

$$\mathcal{I}_k = \left\{ (x_t)_{t=0}^{k}, (\mathfrak{z}_t)_{t=1}^{k}, \left(\theta_{t-1-\mathfrak{z}_t}\right)_{t=1}^{k} \right\}. \tag{8}$$

We remark that here we make an idealistic assumption that all the system state variables are measured and sent to a controller over an error-free link. This assumption is mainly made to streamline the presentation. In our future work, we plan to extend the developed results to the output-feedback control affected by the packet losses over all three wireless links present in a general networked control system architecture, namely those between sensors and remote controller (sensing link, or downlink), between controller and actuators (actuation link, or uplink), and between actuators and controller (acknowledgement link).

[2] The standard networking protocols for wireless industrial automation, such as WirelessHART, ISA100.11a, and Zigbee PRO 2015, are all using IEEE 802.15.4 standard radios.

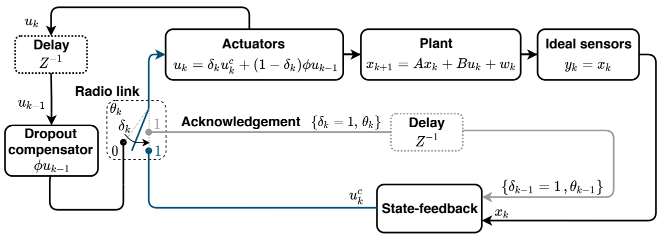

Fig. 1. The architecture of a closed-loop system with control inputs delivered to actuators over a radio link. The link state θ_k is measured for each received packet and fed back to the controller with an acknowledgement issued only after successful transmission. The transmission outcome is indicated by binary random variable δ_k. The generalized hold-input dropout compensation strategy is applied when a control packet is dropped out.

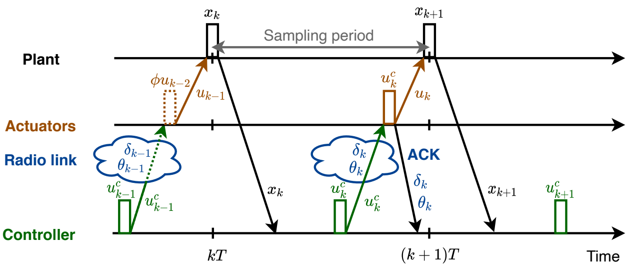

Fig. 2. A timing diagram for a closed-loop system with packet losses on the radio link between the controller and actuators. In this example, at time step k–1, the control packet containing u^c_{k-1} is corrupted during the transmission, which we indicate by a dotted line. The receiver detects the error and discards the message, while actuators apply a dropout compensation strategy by sending a scaled version of the previous input signal, i.e., $u_{k-1} = \phi u_{k-2}$ depicted by a dotted rectangular. At time step k, the control signal u^c_k is received correctly, as shown by a solid line, so the ACK is sent to the controller ($\delta_k = 1$), together with the current estimation θ_k of the state of the link. In this case, actuators apply the received control signal, $u_k = u^c_k$, so the related rectangular is shown as solid.

To this end, we have already shown in Impicciatore et al. (2021) how to design the optimal output-feedback controller that can be obtained by solving the optimal state-feedback control problem and the optimal filtering problem separately (for the case of a wireless networked control scenario where the packet losses occur in both sensing and actuation links, but with an additional negative acknowledgement mechanism for the uplink as in Zacchia Lun and D'Innocenzo (2019), and the zero-hold compensation strategy for all the considered radio links).

Fig. 1 shows the architecture of the closed-loop system, while the related timing diagram is reported in Fig. 2.

2.1 Problem statement

We are interested in a controller that fully exploits its information set. Thus, we study a state-feedback controller which gain depends on both Markov channel state and packet-loss length. Formally, we search for a controller of the following form.

$$u^c_k = K_{(\mathfrak{z}_k, \theta_{k-1-\mathfrak{z}_k})} x_k. \tag{9}$$

The controllers that are independent from the packet-loss length and/or FSMC state are just more conservative special cases of the considered control scheme because they impose an additional constraint of having the same gain for all packet-loss lengths and/or for all observed states of the Markov channel.

3. MARKOV JUMP LINEAR SYSTEM MODEL DERIVATION

In this section, we derive an MJLS for a generalized hold-input dropout compensation strategy over a lossy channel modeled as an FSMC by assuming only ACK messages to communicate the channel status to the controller:

Proposition 1. Given a system defined by (1), an FSMC model defined by (5) and (6), an information set defined by (8), and a control law defined by (9), the state-feedback closed-loop system dynamics are given by an MJLS

$$\begin{cases} \mathbb{x}_{k+1} = \mathbb{A}_{\omega_k} \mathbb{x}_k + \mathbb{B}_{\omega_k} u_k + \mathbb{w}_k, \\ u_k = \mathbb{K}_{\omega_{k-1}} \mathbb{y}_k, \\ \mathbb{y}_k = \mathbb{C}_{\omega_k} \mathbb{x}_k, \end{cases} \tag{10}$$

where $\mathbb{A}_{\omega_k} \in \mathbb{R}^{(L+1)n_x \times (L+1)n_x}$, $\mathbb{B}_{\omega_k} \in \mathbb{R}^{(L+1)n_x \times n_u}$, $\mathbb{K}_{\omega_k} \in \mathbb{R}^{n_u \times n_x}$, $\mathbb{C}_{\omega_k} \in \mathbb{R}^{n_x \times (L+1)n_x}$, and TPM $[\mathbb{p}_{\mathbb{i}\mathbb{j}}]_{\mathbb{i},\mathbb{j}=1}^{\mathbb{L}}$, with $\mathbb{L} = (L+1)^2 N^2$, characterizing the Markov chain ω_k, are defined as in the proof.

Proof. From (1) and (9), for any packet-loss length $\mathfrak{z}_k < L$,

$$x_{k+1} = \left(A + \delta_k B K_{(\mathfrak{z}_k, \theta_{k-1-\mathfrak{z}_k})}\right) x_k + \\ + \sum_{i=1}^{\mathfrak{z}_k+1} \prod_{j=0}^{i-1} (1-\delta_{k-j}) \delta_{k-i} \phi^i B K_{(\mathfrak{z}_{k-i}, \theta_{k-1-i-\mathfrak{z}_{k-i}})} x_{k-i} + w_k. \tag{11}$$

From (3), $\mathfrak{z}_k = \ell$ completely characterizes the sequence of ℓ+1 observed values of δ_{k-1-t}, for $t \in \{i\}_{i=0}^{\ell}$. Thus, for $\mathfrak{z}_k = \ell$,

$$x_{k+1} = \left(A + \delta_k B K_{(\ell, \theta_{k-1-\ell})}\right) x_k + \\ + (1-\delta_k) \phi^{\ell+1} B K_{(\mathfrak{z}_{k-1-\ell}, \theta_{k-2-\ell-\mathfrak{z}_{k-1-\ell}})} x_{k-1-\ell} + w_k. \tag{12}$$

It is evident from (12) that the reception of the control packet or the lack thereof determines two distinct states of the system. This fact allows us to partition the state space of the system based on the last two states of the Markov channel observed by the controller after the last transmission and the related sequences of acknowledgement messages for two consecutive packet-loss intervals. Specifically, the value of δ_k is encoded by the value of $\mathfrak{z}_{k+1}$ which is available in the information set $\mathcal{I}_{k+1}$. Then, from (2), (3), (4), and (11), we have that, for all $0 \le n \le L$,

$$x_{k+1} = \Big(A^{n+1} + \Big(\sum_{j=1}^{n} \phi^{n-j} A^j\Big) B K_{(\mathfrak{z}_{k-n}, \theta_{k-1-n-\mathfrak{z}_{k-n}})} \\ + \phi^n B K_{(\mathfrak{z}_{k-n}, \theta_{k-1-n-\mathfrak{z}_{k-n}})}\Big) x_{k-n} + w_k, \quad \text{for } \mathfrak{z}_{k+1} = n. \tag{13}$$

Since A, B, ϕ, and n are known parameters in (13), it is convenient to define the following parameter-dependent matrix.

$$\Phi_{(n)} = \sum_{j=1}^{n} \phi^{n-j} A^j B + \phi^n B. \tag{14}$$

Thanks to the distributive property of the matrix multiplication and to (14), we can rewrite (13) as

$$x_{k+1} = \left(A^{n+1} + \Phi_{(n)} K_{(\mathfrak{z}_{k-n}, \theta_{k-1-n-\mathfrak{z}_{k-n}})}\right) x_{k-n} + w_k, \ \ \mathfrak{z}_{k+1} = n. \tag{15}$$

It is evident from (15) that x_{k+1} depends on x_{k-n}, where $n \le L$. So, from now on, we consider the current system state x_k and the previous L states in a single augmented state $\mathbb{x}_k$ defined as

$$\mathbb{x}_k = \left(\bigoplus_{t=0}^{L} x_{k-t}^{\top}\right)^{\top} = \left[x_k^{\top} \ x_{k-1}^{\top} \ \cdots \ x_{k-L}^{\top}\right]^{\top}, \tag{16}$$

which recalls that the transpose of the horizontal concatenation (denoted by $\oplus$) of the transposed matrices amounts to the vertical concatenation of the same matrices.

By convention, for any time index $0 < t \le L$, let $x_{-t} = x_0$, so the initial state of the augmented system is given by

$$\mathbb{x}_0 = \left(\bigoplus_{t=0}^{L} x_{-t}^{\top}\right)^{\top} = \left(\bigoplus_{t=0}^{L} x_0^{\top}\right)^{\top} = \left[x_0^{\top} \ x_0^{\top} \ \cdots \ x_0^{\top}\right]^{\top}. \tag{17}$$

Clearly, the size of $\mathbb{x}_k$ is $(L+1)n_x$. The augmented state for the process noise has the same size and is defined by

$$\mathbb{w}_k = \left(w_k^{\top} \oplus \left(\bigoplus_{t=1}^{L} \mathbf{0}^{\top}\right)\right)^{\top} = \left[w_k^{\top} \ \mathbf{0}^{\top} \ \cdots \ \mathbf{0}^{\top}\right]^{\top}. \tag{18}$$

Even if the value of $\theta_{k-\mathfrak{z}_{k+1}}$ does not appear in (15), it is available in the information set $\mathcal{I}_{k+1}$, and together with $\mathfrak{z}_{k+1}$, it defines the next control input to be transmitted to actuators. So, we need to include $\theta_{k-\mathfrak{z}_{k+1}}$ in the state space partitioning. At this point, we can define the operational mode of the system after the last attempted transmission of the control packet as

$$\omega_k \triangleq \left(\mathfrak{z}_{k+1}, \theta_{k-\mathfrak{z}_{k+1}}, \mathfrak{z}_{k-\mathfrak{z}_{k+1}}, \theta_{k-1-\mathfrak{z}_{k+1}-\mathfrak{z}_{k-\mathfrak{z}_{k+1}}}\right). \tag{19}$$

This definition of ω_k allows us to couch the systems in the Markov framework, as presented next.

By convention, $\forall t > 0$, let $\theta_{-t} = \theta_0$ and $\delta_{-t} = 1$, so from (2) and (19), we have $\mathfrak{z}_{-t} = 0$ and $\omega_{-t} = (0, \theta_0, 0, \theta_0)$.

Consider arbitrary indices $1 \le i_{\mathbf{f}}, i_{\mathbf{t}}, j_{\mathbf{f}}, j_{\mathbf{t}} \le N$ of the states of the Markov channel, and arbitrary control packet-loss lengths $0 \le h, \ell, m, n \le L$. Let $\mathfrak{z}_{k+1} = n$, $\theta_{k-n} = s_{j_{\mathbf{t}}}$, $\mathfrak{z}_{k-n} = m$, and $\theta_{k-1-n-m} = s_{j_{\mathbf{f}}}$, so that $\omega_k = \mathring{\jmath}$, and $\mathfrak{z}_k = \ell$, $\theta_{k-1-\ell} = s_{i_{\mathbf{t}}}$, $\mathfrak{z}_{k-1-\ell} = h$, $\theta_{k-2-\ell-h} = s_{i_{\mathbf{f}}}$, so that $\omega_{k-1} = \mathring{\imath}$, where $\mathring{\imath}$, $\mathring{\jmath}$ are the indices of the operational modes associated to the given Markov channel states and packet-loss lengths. Clearly, $1 \le \mathring{\imath}, \mathring{\jmath} \le (L+1)^2 N^2 \triangleq \mathbb{L}$. In the rest of this paper, we will use interchangeably the notation $\omega_k = \mathring{\jmath}$ and $\omega_k = (n, s_{j_{\mathbf{t}}}, m, s_{j_{\mathbf{f}}})$ depending on whether we prefer to emphasize the general properties of the MJLSs or to put attention to the structural properties of our specific problem. The assignment of the values to h, ℓ, m, and n uniquely identifies the related sequences of ACK messages for each two consecutive packet-loss intervals, namely $(\delta_t)_{t=k-2-\ell-h}^{k-1}$, and $(\delta_t)_{t=k-1-n-m}^{k}$.

Let $\mathcal{E}_i$ denote a matrix of the standard basis for $n_x \times (L+1)n_x$ block matrices with blocks of size $n_x \times n_x$. Its i-th block is the identity matrix, while all the other L blocks are zero matrices. Let $\mathcal{E}_0$ denote the nil matrix of the same size. These standard basis block matrices help define the filling block, common to all augmented system state matrices associated with the operational modes. We denote this $Ln_x \times (L+1)n_x$ block matrix by I and define it as $I \triangleq \left(\mathbb{O}_{i=1}^{L} \mathcal{E}_i^\top\right)^\top$. For notational convenience, we denote the nil matrix of the same size as I by O. Formally, $O \triangleq 0\,I$. Then, for each $\omega_k = \mathring{\jmath} = (n, s_{j_{\mathbf{t}}}, m, s_{j_{\mathbf{f}}})$, we define the augmented system state, input, and output matrices.

$$\mathbb{A}_{(n, s_{j_{\mathbf{t}}}, m, s_{j_{\mathbf{f}}})} = \left(\left(A^{n+1}\mathcal{E}_{n+1}\right)^\top \mathbb{O}\, I^\top\right)^\top = \mathbb{A}_{(n)}. \tag{20}$$

Thus, the augmented system state matrix $\mathbb{A}_{\omega_k}$ depends only on the value of the packet-loss length $\mathfrak{z}_{k+1}$, which is observed after the last attempted transmission of the control packet.

$$\mathbb{B}_{\left(n, s_{j_{\mathbf{t}}}, m, s_{j_{\mathbf{f}}}\right)} = \mathcal{E}_1^\top \Phi_{(n)} = \mathbb{B}_{(n)}. \tag{21}$$

It is clear from (21) that, in addition to the obvious dependence on the parameter ϕ characterizing the adopted generalized hold-input dropout compensation strategy, also the augmented system input matrix $\mathbb{B}_{\omega_k}$ depends only on the value of $\mathfrak{z}_{k+1}$.

From (15), it is evident that the control input available to the actuator depends on $x_{k-\mathfrak{z}_{k+1}}$, which is one of the components of the augmented system state $\mathbb{x}_k$. Thus, the desired control input is filtered through the augmented system output matrix $\mathbb{C}_{\omega_k}$:

$$\mathbb{C}_{(n, s_{j_{\mathbf{t}}}, m, s_{j_{\mathbf{f}}})} = \mathcal{E}_{n+1} = \mathbb{C}_{(n)}. \tag{22}$$

As before, the augmented system output matrix depends only on the value of the packet-loss length $\mathfrak{z}_{k+1}$.

Then, the system (1) with a linear state-feedback control law as in (9) is expressed in the augmented form as (10). It is immediate to verify that the augmented state system (10) is equivalent to (15) when

$$\begin{aligned}\mathbb{K}_{\omega_{k-1}} &= K_{(\mathfrak{z}_{k-\mathfrak{z}_{k+1}}, \theta_{k-1-\mathfrak{z}_{k+1}-\mathfrak{z}_{k-\mathfrak{z}_{k+1}}})} = \\ &= \begin{cases} K_{(\mathfrak{z}_k, \theta_{k-1-\mathfrak{z}_k})} & \text{if } \mathfrak{z}_{k+1} = 0; \\ K_{(\mathfrak{z}_{k-1-\mathfrak{z}_k}, \theta_{k-2-\mathfrak{z}_k-\mathfrak{z}_{k-1-\mathfrak{z}_k}})} & \text{otherwise;} \end{cases}\end{aligned} \tag{23}$$

due to the constraints imposed by (2). Specifically, (23) underlines that the last two components of ω_k in (19) correspond to the first two components of ω_{k-1} when the last control input is received correctly, i.e., when $\delta_k = 1$ so that $\mathfrak{z}_{k+1} = 0$ and $u_k = u_k^c$, while they are equal to the last two components of ω_{k-1} when the control input is lost and the dropout compensation strategy is applied. Notice that $\mathbb{K}_{\omega_{k-1}}$ describes the control gain as it is seen from the perspective of actuators, and $\mathfrak{z}_k$ appears in the expressions of both gains at the right hand side of (23).

From (10) it is evident that the augmented system state $\mathbb{x}_k$ evolves according to the operational mode ω_k, which is observed only after the last transmission of the control input, and to the feedback control gain $\mathbb{K}_{\omega_{k-1}}$, that depends on one time-step delayed operational mode observation. For characterizing the stochastic behavior of the system, we need to compute the transition probabilities between its operational modes, that we denote by $\mathbb{p}_{\mathring{\imath}\mathring{\jmath}}$. Formally,

$$\mathbb{p}_{\mathring{\imath}\mathring{\jmath}} \triangleq \mathbb{P}\left(\omega_k = \mathring{\jmath} \mid \omega_{k-1} = \mathring{\imath}\right). \tag{24}$$

To write the explicit expressions of these probabilities in a compact way for different values of the states of the Markov channel, and control packet-loss lengths, we need to define two additional matrices related to the TPM of the Markov channel. The first matrix, denoted by $\boldsymbol{P_1}$, stores the probabilities of successful packet delivery in a state of FSMC starting from a certain previous state. Specifically,

$$\boldsymbol{P_1} = \left[p_{ij}\hat{\delta}_j\right]_{i,j=1}^{N}. \tag{25}$$

The second one is denoted by $\boldsymbol{P_0}$ and describes the probability of the control packet dropout in a state of Markov channel starting from a specific previous state. Formally,

$$\boldsymbol{P_0} = \left[p_{ij}(1-\hat{\delta}_j)\right]_{i,j=1}^{N}. \tag{26}$$

Obviously, we have that

$$\boldsymbol{P_0} + \boldsymbol{P_1} = \boldsymbol{P_c}. \tag{27}$$

Depending on values of indices $1 \le i_{\mathbf{f}}, i_{\mathbf{t}}, j_{\mathbf{f}}, j_{\mathbf{t}} \le N$ of the FSMC states and on values of control packet-loss lengths $0 \le h, \ell, m, n \le L$ used in assignments $\mathfrak{z}_{k+1} = n$, $\theta_{k-n} = s_{j_{\mathbf{t}}}$, $\mathfrak{z}_{k-n} = m$, $\theta_{k-1-n-m} = s_{j_{\mathbf{f}}}$, and $\mathfrak{z}_k = \ell$, $\theta_{k-1-\ell} = s_{i_{\mathbf{t}}}$, $\mathfrak{z}_{k-1-\ell} = h$, $\theta_{k-2-\ell-h} = s_{i_{\mathbf{f}}}$, there are three different expressions for the probabilities of transition between the operational modes. Due to the augmented system structure, most transitions are impossible and thus have zero probability.

Specifically, when the current control packet is received, then the previous control input is not applied anymore. This means that after the successful transmission $\delta_k = 1$ and $\mathfrak{z}_{k+1} = 0$, so that $n = 0$, and $\mathfrak{z}_{k-n} = \mathfrak{z}_k$. In other words, when $n = 0$, only the transitions between the operational modes with $m = \ell$ have non-zero probability of occurrence:

$$\begin{aligned}&\mathbb{P}(\mathfrak{z}_{k+1} = 0, \theta_k = s_{j_{\mathbf{t}}}, \mathfrak{z}_k = m \ne \ell, \theta_{k-1-m} = s_{j_{\mathbf{f}}} \mid \\ &\quad \mathfrak{z}_k = \ell, \theta_{k-1-\ell} = s_{i_{\mathbf{t}}}, \mathfrak{z}_{k-1-\ell} = h, \theta_{k-2-\ell-h} = s_{i_{\mathbf{f}}}) = 0\end{aligned} \tag{28}$$

since the two events are disjoint and thus cannot both occur at the same time. When $n = 0$ and $m = \ell$, the observed states of the Markov channel in two operational modes should be consistent, since any $j_{\mathbf{f}} \ne i_{\mathbf{t}}$ makes the transition impossible:

$$\begin{aligned}&\mathbb{P}(\mathfrak{z}_{k+1} = 0, \theta_k = s_{j_{\mathbf{t}}}, \mathfrak{z}_k = \ell, \theta_{k-1-\ell} = s_{j_{\mathbf{f}}} \ne s_{i_{\mathbf{t}}} \mid \\ &\quad \mathfrak{z}_k = \ell, \theta_{k-1-\ell} = s_{i_{\mathbf{t}}}, \mathfrak{z}_{k-1-\ell} = h, \theta_{k-2-\ell-h} = s_{i_{\mathbf{f}}}) = 0.\end{aligned} \tag{29}$$

At this point, let us denote by $\boldsymbol{e}_i$ the vector of the standard basis of $\mathbb{R}^N$: it has the i-th component equal to 1, while all its other components equal 0. When $n = 0$, $m = \ell$, and $j_{\mathbf{f}} = i_{\mathbf{t}}$, we have that

$$\begin{aligned}&\mathbb{P}(\mathfrak{z}_{k+1} = 0, \theta_k = s_{j_{\mathbf{t}}}, \mathfrak{z}_k = \ell, \theta_{k-1-\ell} = s_{i_{\mathbf{t}}} \mid \mathfrak{z}_k = \ell, \theta_{k-1-\ell} = s_{i_{\mathbf{t}}}, \\ &\quad \mathfrak{z}_{k-1-\ell} = h, \theta_{k-2-\ell-h} = s_{i_{\mathbf{f}}}) = \boldsymbol{e}_{i_{\mathbf{t}}}^\top \boldsymbol{P_c}^\ell \boldsymbol{P_1} \boldsymbol{e}_{j_{\mathbf{t}}}.\end{aligned} \tag{30}$$

When the control packet sent at time step k is not received correctly, the actuators apply the properly scaled last available control input. In this case, $\delta_k=0$, and the counter of consecutive control packet dropouts is incremented. Specifically, $\mathfrak{z}_{k+1}\neq 0$, and from (2), we have that $\mathfrak{z}_{k+1}$ must necessarily be equal to $\mathfrak{z}_k+1$, meaning that $\forall\, n\in\mathbb{Z},\, 1\leq n\leq L$,

$$\mathbb{P}(\mathfrak{z}_{k+1}=n\mid \mathfrak{z}_k\neq n-1)=0. \tag{31}$$

A consequence of (31) is that

$$\begin{aligned}&\mathbb{P}(\mathfrak{z}_{k+1}=n,\,\theta_{k-n}=s_{j_\mathbf{t}},\,\mathfrak{z}_{k-n}=m,\,\theta_{k-1-n-m}=s_{j_\mathbf{f}}\mid\\ &\quad\mathfrak{z}_k=\ell\neq n-1,\,\theta_{k-1-\ell}=s_{i_\mathbf{t}},\,\mathfrak{z}_{k-1-\ell}=h,\,\theta_{k-2-\ell-h}=s_{i_\mathbf{f}})=0.\end{aligned} \tag{32}$$

So, consider $\mathfrak{z}_{k+1}=n>0$ and $\mathfrak{z}_k=\ell=n-1$. It is obvious that in this case $k-1-\ell=k-n$, so that a transition between two operational modes has a non-zero probability of occurrence only if $j_\mathbf{t}=i_\mathbf{t}$, $m=h$ and $j_\mathbf{f}=i_\mathbf{f}$. Formally, we have

$$\begin{aligned}&\mathbb{P}(\mathfrak{z}_{k+1}=n,\,\theta_{k-n}=s_{j_\mathbf{t}}\neq s_{i_\mathbf{t}},\,\mathfrak{z}_{k-n}=m,\,\theta_{k-1-n-m}=s_{j_\mathbf{f}}\mid\\ &\quad\mathfrak{z}_k=n-1,\,\theta_{k-n}=s_{i_\mathbf{t}},\,\mathfrak{z}_{k-n}=h,\,\theta_{k-1-n-h}=s_{i_\mathbf{f}})=0;\end{aligned} \tag{33}$$

$$\begin{aligned}&\mathbb{P}(\mathfrak{z}_{k+1}=n,\,\theta_{k-n}=s_{i_\mathbf{t}},\,\mathfrak{z}_{k-n}=m\neq h,\,\theta_{k-1-n-m}=s_{j_\mathbf{f}}\mid\\ &\quad\mathfrak{z}_k=n-1,\,\theta_{k-n}=s_{i_\mathbf{t}},\,\mathfrak{z}_{k-n}=h,\,\theta_{k-1-n-h}=s_{i_\mathbf{f}})=0;\end{aligned} \tag{34}$$

$$\begin{aligned}&\mathbb{P}(\mathfrak{z}_{k+1}=n,\,\theta_{k-n}=s_{i_\mathbf{t}},\,\mathfrak{z}_{k-n}=h,\,\theta_{k-1-n-h}=s_{j_\mathbf{f}}\neq s_{i_\mathbf{f}}\mid\\ &\quad\mathfrak{z}_k=n-1,\,\theta_{k-n}=s_{i_\mathbf{t}},\,\mathfrak{z}_{k-n}=h,\,\theta_{k-1-n-h}=s_{i_\mathbf{f}})=0\end{aligned} \tag{35}$$

since the events in (33), (34), (35) are pairwise disjoint. The only remaining case is $n=\ell+1$, $j_\mathbf{t}=i_\mathbf{t}$, $m=h$ and $j_\mathbf{f}=i_\mathbf{f}$.

$$\begin{aligned}&\mathbb{P}(\mathfrak{z}_{k+1}=\ell+1,\,\theta_{k-1-\ell}=s_{i_\mathbf{t}},\,\mathfrak{z}_{k-1-\ell}=h,\,\theta_{k-2-\ell-h}=s_{i_\mathbf{f}}\mid\mathfrak{z}_k=\ell,\\ &\quad\theta_{k-1-\ell}=s_{i_\mathbf{t}},\,\mathfrak{z}_{k-1-\ell}=h,\,\theta_{k-2-\ell-h}=s_{i_\mathbf{f}})=\boldsymbol{e}_{i_\mathbf{t}}^\top\boldsymbol{P}_c^\ell\boldsymbol{P}_0\mathbf{1}\end{aligned} \tag{36}$$

by the chain rule of probability, (3), the δ_k independence of δ_{k-t} and θ_{k-t} (that holds for all $t\geq 1$), the law of total probability, (5), the Markov property, (6), (7), (26), and the definitions of the matrix product, the vectors of the standard basis, and the vector of all ones.

From (28)–(30) and (32)–(36), we obtain the general expression for the augmented system transition probabilities between operational modes for all $1\leq i_\mathbf{f},i_\mathbf{t},j_\mathbf{f},j_\mathbf{t}\leq N$, $0\leq h,\ell,m,n\leq L$, with $h,\ell,m,n,i_\mathbf{f},i_\mathbf{t},j_\mathbf{f},j_\mathbf{t}\in\mathbb{Z}$:

$$\begin{aligned}&\mathbb{P}\big(\omega_k=(n,s_{j_\mathbf{t}},m,s_{j_\mathbf{f}})=\mathfrak{j}\mid\omega_{k-1}=(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}})=\mathfrak{i}\big)=\mathbb{p}_{\mathfrak{i}\mathfrak{j}}=\\ &=\begin{cases}\boldsymbol{e}_{i_\mathbf{t}}^\top\boldsymbol{P}_c^\ell\boldsymbol{P}_1\boldsymbol{e}_{j_\mathbf{t}} & \text{if } n=0,\ m=\ell,\ \text{and } j_\mathbf{f}=i_\mathbf{t};\\ \boldsymbol{e}_{i_\mathbf{t}}^\top\boldsymbol{P}_c^\ell\boldsymbol{P}_0\mathbf{1} & \text{if } n=\ell+1,\ m=h,\ j_\mathbf{f}=i_\mathbf{f},\ \text{and } j_\mathbf{t}=i_\mathbf{t};\\ 0 & \text{otherwise.}\end{cases}\end{aligned} \tag{37}$$

For any number N of Markov channel states and arbitrary packet-loss length L, (37) defines an $(L+1)^2N^2\times(L+1)^2N^2$ stochastic matrix having $(L+1)^2N^3+L^2N^2\ll(L+1)^4N^4$ non-zero entries with up to $N^2(L+1)+NL$ different values. For example, with $N=3$ and $L=3$, there will be at most 45 different values within 513 possible non-zero entries among the total of 20736. For notational convenience, we refer to this TPM as $[\mathbb{p}_{\mathfrak{i}\mathfrak{j}}]_{\mathfrak{i},\mathfrak{j}=1}^{\mathbb{L}}$, where $\mathbb{L}=(L+1)^2N^2$ is the operational modes index set cardinality.

Remark 1. The sheer size of the augmented system may make it intractable in general, so the MJLSs framework is used only as a tool for deriving the structural properties of the system (1) in a systematic way, by exploiting the sparsity of the above TPM. For notational convenience, we will indicate the non-zero entries of $[\mathbb{p}_{\mathfrak{i}\mathfrak{j}}]_{\mathfrak{i},\mathfrak{j}=1}^{\mathbb{L}}$ as follows.

$$\mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}}\triangleq\boldsymbol{e}_{i_\mathbf{t}}^\top\boldsymbol{P}_c^\ell\boldsymbol{P}_1\boldsymbol{e}_{j_\mathbf{t}},\qquad\forall\ell\leq L,\,\ell\geq 0; \tag{38a}$$

$$\mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}}\triangleq\boldsymbol{e}_{i_\mathbf{t}}^\top\boldsymbol{P}_c^\ell\boldsymbol{P}_0\mathbf{1},\qquad\forall\ell\leq L-1,\,\ell\geq 0. \tag{38b}$$

It is also opportune defining $\mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}}$ for $\ell=L$:

$$\mathbf{p}_{i_\mathbf{t}L\mathbf{1}}\triangleq 0, \tag{38c}$$

allowing us to write exact expressions concisely.

4. OPTIMAL FINITE-HORIZON LQR

In this section, we derive a control law as in (9) that efficiently solves the finite-horizon optimal LQR problem. Our considerations on the computational complexity are provided at the end of the section. The controller optimizes the average cost function defined as follows. Let $Q=Q^\top\geq 0$ and $R=R^\top>0$ be the state weighting and control weighting matrices of size $n_x\times n_x$ and $n_u\times n_u$. Let $\mathcal{U}$ be the set of all admissible control laws satisfying (9) according to the informational set (8), observed from the actuators perspective. Let $u\in\mathcal{U}$ be an admissible control law seen on the actuation end of the communication link. Formally, $u=(u_t)_{t=0}^k$. For any time horizon $T\geq L$ and any admissible control law, consider the following quadratic cost function

$$\mathcal{J}_T(x_0,\theta_0,u)=\mathbb{E}\Big(\textstyle\sum_{k=0}^{T-1}\left(x_k^\top Qx_k+u_k^\top Ru_k\right)+x_T^\top Qx_T\mid\mathcal{I}_0\Big). \tag{39}$$

In view of (11)–(13), and (15), it is convenient to represent the system in the augmented form. Thus, we define

$$\mathbb{Q}=\mathcal{E}_1^\top Q\mathcal{E}_1. \tag{40}$$

Clearly, $Q=Q^\top\geq 0$ implies that $\mathbb{Q}=\mathbb{Q}^\top\geq 0$. Then, for any time horizon $T\geq L$ and any $u\in\mathcal{U}$, the quadratic cost function $\mathcal{J}_T$ can be applied to the augmented system, i.e.,

$$\mathcal{J}_T(\mathbb{x}_0,\omega_{-1},u)=\mathbb{E}\Big(\textstyle\sum_{k=0}^{T-1}\left(\mathbb{x}_k^\top\mathbb{Q}\mathbb{x}_k+u_k^\top Ru_k\right)+\mathbb{x}_T^\top\mathbb{Q}\mathbb{x}_T\mid\mathcal{I}_0\Big). \tag{41}$$

The optimal finite-horizon cost-to-go function is defined as

$$\begin{aligned}&\mathcal{J}_T^\star(\mathbb{x}_k,\omega_{k-1})\triangleq\mathcal{J}_T\Big(\mathbb{x}_k,\omega_{k-1},(u_t^\star)_{t=k}^{T-1}\Big)=\\ &=\min_{(u_t)_{t=k}^{T-1}\in\mathcal{U}}\mathbb{E}\Big(\textstyle\sum_{t=k}^{T-1}\left(\mathbb{x}_t^\top\mathbb{Q}\mathbb{x}_t+u_t^\top Ru_t\right)+\mathbb{x}_T^\top\mathbb{Q}\mathbb{x}_T\mid\mathcal{I}_k\Big).\end{aligned} \tag{42}$$

We first show that it can be expressed as follows.

$$\mathcal{J}_T^\star(\mathbb{x}_k,\omega_{k-1})=\mathbb{x}_k^\top\mathbb{X}_{(k,\omega_{k-1})}\mathbb{x}_k+g_{(k,\omega_{k-1})}, \tag{43}$$

where, for all values of ω_{k-1}, $\mathbb{X}_{(k,\omega_{k-1})}$ are symmetric positive semi-definite matrices with $\mathbb{X}_{(T,\omega_{T-1})}=\mathbb{Q}$, which are the solution to the coupled Riccati difference equations (CRDEs) defined next, and $g_{(k,\omega_{k-1})}$ is a nonnegative additive term of the cost-to-go that depends on the process noise characteristics, with $g_{(T,\omega_{T-1})}=0$. Since the system (1) is affected by the process noise, we recall that w_k is independent from the initial condition (x_0,θ_0) and from the Markov process (δ_t,θ_t), that describes the FSMC characterizing the communication link between the controller and actuators, for all values k and t of discrete time. From (2), (3), (8), and (16)–(19), this implies the $\mathbb{w}_k$ independence of $\mathbb{x}_0$, ω_t, and $\mathcal{I}_t$, for all $k\geq 0$, $t\geq 0$. Since $\mathbb{E}(w_k)=\mathbf{0}$, we have from (18) that

$$\mathbb{E}(\mathbb{w}_k)=\Big(\textstyle\bigoplus_{t=0}^L\mathbf{0}^\top\Big)^\top=\left[\mathbf{0}^\top\ \cdots\ \mathbf{0}^\top\right]^\top=0\,\mathbb{w}_k=\mathbf{0}. \tag{44}$$

Let $\Sigma_\mathbb{w}$ denote the covariance matrix of the process noise affecting the augmented system (10).

$$\Sigma_\mathbb{w}\triangleq\mathcal{E}_1^\top\Sigma_w\mathcal{E}_1. \tag{45}$$

From (18), (45), (44), and the basic properties of the covariance matrix, we have that

$$\mathbb{E}\big(\mathbb{w}_k\mathbb{w}_k^\top\big)=\Sigma_\mathbb{w}+\mathbb{E}(\mathbb{w}_k)\left(\mathbb{E}(\mathbb{w}_k)\right)^\top=\Sigma_\mathbb{w}. \tag{46}$$

We will see how $\Sigma_\mathbb{w}$ affects the value of the additive term $g_{(k,\omega_{k-1})}$ later in this subsection.

Meanwhile, let us denote the main components of the solution to the CRDEs at time step k by $X_{(i,k,\omega_{k-1})}$, i.e.,

$$\mathbb{X}_{(k,\omega_{k-1})}\triangleq\textstyle\bigoplus_{i=0}^L X_{(i,k,\omega_{k-1})}\triangleq\operatorname{diag}\big(X_{(0,k,\omega_{k-1})},\ldots,X_{(L,k,\omega_{k-1})}\big). \tag{47}$$

The derivation of the explicit expression for $\mathbb{X}_{(k,\omega_{k-1})}$ and $g_{(k,\omega_{k-1})}$ is a part of the proof of (43) via a backward induction, which is only outlined here due to space limits. The base case at the final time step $k=T$ results in (43) with $\mathbb{X}_{(T,\omega_{T-1})}=\mathbb{Q}$ and $g_{(T,\omega_{T-1})}=0$ since from (8) and (16) we know that $\mathbb{x}_T$ is $\mathcal{I}_T$-measurable, and, by its definition (40), $\mathbb{Q}=\mathbb{Q}^\top\geq 0$ is a known augmented state weighting matrix. By the inductive hypothesis, for any $0\leq k\leq T-1$ there exist $\mathbb{X}_{(k+1,\omega_k)}=\mathbb{X}_{(k+1,\omega_k)}^\top\geq 0$ and $g_{(k+1,\omega_k)}\geq 0$ such that (43) is verified for $\mathbb{x}_{k+1}$ and ω_k, so we can use the definition (42) of the optimal cost-to-go at time steps $k+1$ and k, the tower property of the conditional expectation, and the Bellman's principle of optimality to state that

$$\mathcal{J}_T^\star(\mathbb{x}_k,\omega_{k-1}) = \min_{(u_t)_{t=k}^{T-1}\in\mathcal{U}} \mathbb{E}\big(\mathbb{x}_k^\top \mathbb{Q}\,\mathbb{x}_k + u_k^\top R u_k + \mathcal{J}_T^\star(\mathbb{x}_{k+1},\omega_k)\,|\,\mathcal{I}_k\big).$$

After substituting $\mathcal{J}_T^\star(\mathbb{x}_{k+1},\omega_k)$ in the previous equation with its expression from (43), we also substitute $\mathbb{x}_{k+1}$ and u_k with their expressions from (10), obtaining (48), reported on the following page. By the inductive hypothesis, $\mathbb{X}_{(k+1,\,\mathbb{j})} = \mathbb{X}^\top_{(k+1,\,\mathbb{j})} \geq 0$, which, combined with (47), implies that all main components of the solution to CRDEs are symmetric positive semi-definite, i.e., $\forall 0\leq i\leq L$,

$$X_{(i,\,k+1,\,\mathbb{j})} = X^\top_{(i,\,k+1,\,\mathbb{j})} \geq 0. \tag{49}$$

Thus, the result of $\mathbb{w}_k^\top \mathbb{X}_{(k+1,\,\omega_k)}\mathbb{w}_k$ is a real nonnegative scalar, that may be always seen as a 1×1 matrix. Then,

$$\mathbb{w}_k^\top \mathbb{X}_{(k+1,\,\omega_k)}\mathbb{w}_k = \operatorname{tr}\left(\mathbb{w}_k^\top \mathbb{X}_{(k+1,\,\omega_k)}\mathbb{w}_k\right) = \operatorname{tr}\left(\mathbb{X}_{(k+1,\,\omega_k)}\mathbb{w}_k\mathbb{w}_k^\top\right). \tag{50}$$

From the definition of the conditional expectation, considering that $\mathbb{x}_k$ and ω_{k-1} are $\mathcal{I}_k$-measurable, as can be seen from (8), (16), and (19), by the linearity of the expected value, conditional expectation, and trace, by the distributive property of the matrix product, the $\mathbb{w}_k$ independence of $\mathbb{x}_0$, ω_t, and $\mathcal{I}_t$, for all $k\geq 0$, $t\geq 0$ (which by induction implies the independence between $\mathbb{w}_k$ and $\mathbb{x}_k$), (44), (46), and (50), we obtain (51) shown on the next page.

From (37), (38), and (20)–(23), we rewrite (51) explicitly for any value of $\omega_{k-1}=(\ell, s_{i_\mathbf{t}}, h, s_{i_\mathbf{f}})$. The last two addends in (51) are independent of $\mathbb{x}_k$ and define the term $g_{(k,\omega_{k-1})}$. In particular, since the trace of a block diagonal matrix equals the sum of traces of its diagonal blocks, we obtain from (45) and (47) the explicit definition of $g_{(k,\omega_{k-1})}$ for any $\omega_{k-1}=(\ell, s_{i_\mathbf{t}}, h, s_{i_\mathbf{f}})$ in (52) on the following page.

Since, by the inductive hypothesis, $g_{(k+1,\omega_k)}\geq 0$ for any value of ω_k, from (49) and $Q=Q^\top\geq 0$, we have $g_{(k,\omega_{k-1})}\geq 0$. We remark that for $\ell = L$, both addends of the factor multiplying $\mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}}$ in (52) are formally undefined, since, by its definition, $0\leq\ell\leq L$, but thanks to (38c), they disappear in the case above. We will apply the same consideration to all the following expressions involving $\mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}}$. Then, to find the optimal values of gains $K_{(k,\,\ell,\,s_{i_\mathbf{t}})}$ and $K_{(k,\,h,\,s_{i_\mathbf{f}})}$ imposed by (23), we need to search the stationary point of (51) with respect to the gains mentioned above. So, we can perform the matrix differentiation. The stationary points of (51) for $K_{(k,\,\ell,\,s_{i_\mathbf{t}})}$ are defined for all $\ell\leq L$ by (53), shown on the next page. From (16), (20)–(22), (14), and (47), we have that (53) is equivalent to

$$\textstyle\sum_{j_\mathbf{t}=1}^N \mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}}\Big(\big(R + B^\top X_{(0,\,k+1,\,(0,\,s_{j_\mathbf{t}},\,\ell,\,s_{i_\mathbf{t}}))}B\big)K_{(k,\,\ell,\,s_{i_\mathbf{t}})} + B^\top X_{(0,\,k+1,\,(0,\,s_{j_\mathbf{t}},\,\ell,\,s_{i_\mathbf{t}}))}A\Big)x_k x_k^\top = \mathbf{0}. \tag{54}$$

The previous expression should hold for all possible values of $x_k x_k^\top$. Thus $x_k x_k^\top$ should multiply the matrix of all zeros. From (6), (7), (24)–(27), (37), and (38),

$$\textstyle\sum_{j_\mathbf{t}=1}^N \mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}} + \mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}} = 1. \tag{55}$$

For all $i_\mathbf{x}$ such that $0\leq i_\mathbf{x}\leq L$, we denote by $\Psi_{(i_\mathbf{x},k,\ell,i_\mathbf{t})}$ the sum in $j_\mathbf{t}$ of $X_{(i_\mathbf{x},k,(0,s_{j_\mathbf{t}},\ell,s_{i_\mathbf{t}}))}$ weighted by $\mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}}$, i.e.,

$$\Psi_{(i_\mathbf{x},k,\ell,i_\mathbf{t})} = \textstyle\sum_{j_\mathbf{t}=1}^N \mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}} X_{(i_\mathbf{x},k,(0,s_{j_\mathbf{t}},\ell,s_{i_\mathbf{t}}))}. \tag{56}$$

Then, from (56), (55), (49), and $R=R^\top>0$, the matrix multiplying $K_{(k,\ell,s_{i_\mathbf{t}})}$ in (54) is positive-definite. Thus, we can express the optimal control gain $K_{(k,\ell,s_{i_\mathbf{t}})}$ as follows.

$$K^\star_{(k,\ell,s_{i_\mathbf{t}})} = -\big((1-\mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}})R + B^\top\Psi_{(0,k+1,\ell,i_\mathbf{t})}B\big)^{-1}B^\top\Psi_{(0,k+1,\ell,i_\mathbf{t})}A. \tag{57}$$

We recall that $K_{(k,\ell,s_{i_\mathbf{t}})}$ is used when the control input is received correctly, while $K_{(k,h,s_{i_\mathbf{f}})}$ is adopted by actuators when the control input is lost. Specifically, the gain $K_{(k,h,s_{i_\mathbf{f}})}$ is an integral part of the actuation strategy described by (23) implicitly stating via (19) that $\mathfrak{z}_{k-1-\mathfrak{z}_k}=h$ and $\theta_{k-2-\mathfrak{z}_k-h}=s_{i_\mathbf{f}}$ for $\mathfrak{z}_k=\ell$. Thus, the stationary points of (51) for $K_{(k,h,s_{i_\mathbf{f}})}$ are defined for the corresponding value of ℓ (being $\ell\leq L-1$, since from (2) combined with (4) having $\mathfrak{z}_k=L$ would imply $\mathfrak{z}_{k+1}=0$ and thus the application of the control gain $K_{(k,\ell,s_{i_\mathbf{t}})}$ from (23)) by

$$2\,\mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}}\Big(\big(R + \mathbb{B}^\top_{(\ell+1)}\mathbb{X}_{(k+1,\,(\ell+1,\,s_{i_\mathbf{t}},\,h,\,s_{i_\mathbf{f}}))}\mathbb{B}_{(\ell+1)}\big)K_{(k,\,h,\,s_{i_\mathbf{f}})}\cdot \cdot\,\mathbb{C}_{(\ell+1)} + \mathbb{B}^\top_{(\ell+1)}\mathbb{X}_{(k+1,\,(\ell+1,\,s_{i_\mathbf{t}},\,h,\,s_{i_\mathbf{f}}))}\mathbb{A}_{(\ell+1)}\Big)\mathbb{x}_k\mathbb{x}_k^\top\mathbb{C}^\top_{(\ell+1)} = \mathbf{0}.$$

By following the same line of reasoning used to derive (57), we obtain the following expression for optimal $K_{(k,h,s_{i_\mathbf{f}})}$:

$$K^\star_{(k,h,s_{i_\mathbf{f}})} = -\big(R + \Phi^\top_{(\ell+1)}X_{(0,\,k+1,\,(\ell+1,\,s_{i_\mathbf{t}},\,h,\,s_{i_\mathbf{f}}))}\Phi_{(\ell+1)}\big)^{-1}\cdot \cdot\,\Phi^\top_{(\ell+1)}X_{(0,\,k+1,\,(\ell+1,\,s_{i_\mathbf{t}},\,h,\,s_{i_\mathbf{f}}))}A^{\ell+2}. \tag{58}$$

We remark that $\mathcal{J}_T^\star(\mathbb{x}_k,\omega_{k-1})$ is a positive quadratic function, its stationary point corresponds to a local minimum. Since for each gain we have only one stationary point, this point constitutes a global minimum. Based on (14), (20)–(22), (47), (57), and (58), we obtain (43), with $g_{(k,\omega_{k-1})}$ given by (52), and the following expression for $\mathbb{X}_{(k,\omega_{k-1})}$:

$$\begin{aligned}\mathbb{X}_{(k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} &= \mathbb{Q} + \textstyle\sum_{j_\mathbf{t}=1}^N \mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}}\mathbb{A}^\top_{(0)}\mathbb{X}_{(k+1,\,(0,\,s_{j_\mathbf{t}},\,\ell,\,s_{i_\mathbf{t}}))}\mathbb{A}_{(0)} + \\ &+ \textstyle\sum_{j_\mathbf{t}=1}^N \mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}}\mathbb{A}^\top_{(0)}\mathbb{X}_{(k+1,\,(0,\,s_{j_\mathbf{t}},\,\ell,\,s_{i_\mathbf{t}}))}\mathbb{B}_{(0)}K^\star_{(k,\,\ell,\,s_{i_\mathbf{t}})}\mathbb{C}_{(0)} + \\ &+ \mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}}\mathbb{A}^\top_{(\ell+1)}\mathbb{X}_{(k+1,\,(\ell+1,\,s_{i_\mathbf{t}},\,h,\,s_{i_\mathbf{f}}))}\mathbb{B}_{(\ell+1)}K^\star_{(k,\,h,\,s_{i_\mathbf{f}})}\mathbb{C}_{(\ell+1)} + \\ &+ \mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}}\mathbb{A}^\top_{(\ell+1)}\mathbb{X}_{(k+1,\,(\ell+1,\,s_{i_\mathbf{t}},\,h,\,s_{i_\mathbf{f}}))}\mathbb{A}_{(\ell+1)}.\end{aligned} \tag{59}$$

By writing (59) explicitly in a matrix form for each ℓ in $\omega_{k-1}=(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}})$, it is easy to see that $\mathbb{X}_{(k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))}$ is a block diagonal matrix, so we can write the expressions of the elements $X_{(i,k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))}$ on its main diagonal as follows. The first block of $\mathbb{X}_{(k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))}$ is

$$\begin{aligned}X_{(0,k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} &= Q + A^\top\Psi_{(0,k+1,\ell,i_\mathbf{t})}A + \Psi_{(1,k+1,\ell,i_\mathbf{t})} + \\ &+ \mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}}X_{(1,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} - A^\top\Psi_{(0,k+1,\ell,i_\mathbf{t})}B\big((1-\mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}})R + \\ &+ B^\top\Psi_{(0,k+1,\ell,i_\mathbf{t})}B\big)^{-1}B^\top\Psi_{(0,k+1,\ell,i_\mathbf{t})}A.\end{aligned} \tag{60}$$

The last block of $\mathbb{X}_{(k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))}$ is expressed by (61) on the following page. All the remaining components of $\mathbb{X}_{(k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))}$ satisfy yet another equation: for any integer $1\leq i_\mathbf{x}\leq L-1$, we have (62) shown on the next page.

In conclusion, (43) is verified through (47), (52), (56), and (60)–(62). The optimal control gain is given by (57).

We remark that (57) takes into account the generalized dropout compensation strategy, since $K^\star_{(k,\,\ell,\,s_{i_\mathbf{t}})}$ depends on the first component of $\mathbb{X}_{(k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))}$, which, through (60), takes into account possible relevant values of the second block of $\mathbb{X}_{(k+1,\omega_k)}$, that itself depends on $\Phi_{(\ell+1)}$ defined in (14) and on the values of the first and third block of $\mathbb{X}_{(k+2,\omega_{k+1})}$, for each admissible value of k and ℓ. We also underline that each of the $L+1$ blocks defining a solution to the CRDEs has the size of $n_x\times n_x$. By considering (56) and (60)–(62) instead of (59), we solve $(L+1)^3N^2$ equations of size $n_x\times n_x$ instead of $(L+1)^2N^2$ equations of size $(L+1)n_x\times(L+1)n_x$. Since the lower bound for the matrix multiplication (and inversion) is $\Omega(m^2\log m)$ for any square matrix of the size m (Raz, 2002), dealing with considerably smaller matrices translates into much faster and less memory-consuming operations.

5. CONCLUSIONS

This paper introduced a wireless networked control design for the systems without access to channel state information during packet error bursts. It also presented an optimal finite-horizon linear quadratic regulator exploiting the structural properties of the system. The next step is

$$
\begin{aligned}
\mathcal{J}_T^\star(\mathbb{x}_k, \omega_{k-1}) = \min_{\mathbb{K}_{(k,\,\omega_{k-1})}} \mathbb{E}\Big(& \mathbb{x}_k^\top \mathbb{Q}\,\mathbb{x}_k + g_{(k+1,\,\omega_k)} + \mathbb{x}_k^\top \mathbb{C}_{\omega_k}^\top \mathbb{K}_{(k,\,\omega_{k-1})}^\top R\,\mathbb{K}_{(k,\,\omega_{k-1})} \mathbb{C}_{\omega_k} \mathbb{x}_k + \\
&+ \left(\mathbb{w}_k^\top + \mathbb{x}_k^\top \left(\mathbb{A}_{\omega_k}^\top + \mathbb{C}_{\omega_k}^\top \mathbb{K}_{(k,\,\omega_{k-1})}^\top \mathbb{B}_{\omega_k}^\top\right)\right) \mathbb{X}_{(k+1,\,\omega_k)} \left(\left(\mathbb{A}_{\omega_k} + \mathbb{B}_{\omega_k} \mathbb{K}_{(k,\,\omega_{k-1})} \mathbb{C}_{\omega_k}\right) \mathbb{x}_k + \mathbb{w}_k\right) \mid \mathcal{I}_k \Big).
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
\mathcal{J}_T^\star(\mathbb{x}_k, \omega_{k-1}) = \min_{\mathbb{K}_{(k,\,\omega_{k-1})}} \mathbb{x}_k^\top \Big(& \mathbb{Q} + \textstyle\sum_{\mathbb{j}=1}^{\mathbb{L}} \mathbb{p}_{\omega_{k-1}\mathbb{j}}\, \mathbb{A}_{\mathbb{j}}^\top \mathbb{X}_{(k+1,\,\mathbb{j})} \mathbb{B}_{\mathbb{j}} \mathbb{K}_{(k,\,\omega_{k-1})} \mathbb{C}_{\mathbb{j}} + \sum_{\mathbb{j}=1}^{\mathbb{L}} \mathbb{p}_{\omega_{k-1}\mathbb{j}}\, \mathbb{C}_{\mathbb{j}}^\top \mathbb{K}_{(k,\,\omega_{k-1})}^\top \mathbb{B}_{\mathbb{j}}^\top \mathbb{X}_{(k+1,\,\mathbb{j})} \mathbb{A}_{\mathbb{j}} + \sum_{\mathbb{j}=1}^{\mathbb{L}} \mathbb{p}_{\omega_{k-1}\mathbb{j}}\, \mathbb{A}_{\mathbb{j}}^\top \mathbb{X}_{(k+1,\,\mathbb{j})} \mathbb{A}_{\mathbb{j}} + \\
&+ \textstyle\sum_{\mathbb{j}=1}^{\mathbb{L}} \mathbb{p}_{\omega_{k-1}\mathbb{j}}\, \mathbb{C}_{\mathbb{j}}^\top \mathbb{K}_{(k,\,\omega_{k-1})}^\top \left(R + \mathbb{B}_{\mathbb{j}}^\top \mathbb{X}_{(k+1,\,\mathbb{j})} \mathbb{B}_{\mathbb{j}}\right) \mathbb{K}_{(k,\,\omega_{k-1})} \mathbb{C}_{\mathbb{j}} \Big) \mathbb{x}_k + \sum_{\mathbb{j}=1}^{\mathbb{L}} \mathbb{p}_{\omega_{k-1}\mathbb{j}} \operatorname{tr}\left(\mathbb{X}_{(k+1,\,\mathbb{j})} \Sigma_{\mathbb{w}}\right) + \sum_{\mathbb{j}=1}^{\mathbb{L}} \mathbb{p}_{\omega_{k-1}\mathbb{j}}\, g_{(k+1,\,\mathbb{j})}.
\end{aligned} \tag{51}
$$

$$
g_{(k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} = \sum_{j_\mathbf{t}=1}^{N} \mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}} \Big(\operatorname{tr}\Big(X_{(0,k+1,(0,s_{j_\mathbf{t}},\ell,s_{i_\mathbf{t}}))} \Sigma_w\Big) + g_{(k+1,(0,s_{j_\mathbf{t}},\ell,s_{i_\mathbf{t}}))}\Big) + \mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}} \Big(\operatorname{tr}\Big(X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} \Sigma_w\Big) + g_{(k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))}\Big) \tag{52}
$$

$$
2 \sum_{j_\mathbf{t}=1}^{N} \mathbf{p}_{i_\mathbf{t}\ell j_\mathbf{t}} \left(\left(R + \mathbb{B}_{(0)}^\top \mathbb{X}_{(k+1,\,(0,\,s_{j_\mathbf{t}},\,\ell,\,s_{i_\mathbf{t}}))} \mathbb{B}_{(0)}\right) K_{(k,\,\ell,\,s_{i_\mathbf{t}})} \mathbb{C}_{(0)} \mathbb{x}_k \mathbb{x}_k^\top \mathbb{C}_{(0)}^\top + \mathbb{B}_{(0)}^\top \mathbb{X}_{(k+1,\,(0,\,s_{j_\mathbf{t}},\,\ell,\,s_{i_\mathbf{t}}))} \mathbb{A}_{(0)} \mathbb{x}_k \mathbb{x}_k^\top \mathbb{C}_{(0)}^\top\right) = \mathbf{0}. \tag{53}
$$

$$
X_{(L,k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} = \begin{cases} \mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}} \Big(\left(A^{\ell+2}\right)^\top X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} A^{\ell+2} - \left(A^{\ell+2}\right)^\top X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} \Phi_{(\ell+1)} \cdot & \\ \qquad \cdot \left(R + \Phi_{(\ell+1)}^\top X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} \Phi_{(\ell+1)}\right)^{-1} \Phi_{(\ell+1)}^\top X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} A^{\ell+2} \Big) & \text{if } \ell = L-1; \\ \mathbf{0} & \text{otherwise.} \end{cases} \tag{61}
$$

$$
\begin{aligned}
X_{(i_\mathbf{x},k,(\ell,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} &= \Psi_{(i_\mathbf{x}+1,k+1,\ell,i_\mathbf{t})} + \mathbf{p}_{i_\mathbf{t}\ell\mathbf{1}} \Big(X_{(i_\mathbf{x}+1,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} + \left(A^{\ell+2}\right)^\top X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} A^{\ell+2} - \\
&- \left(A^{\ell+2}\right)^\top X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} \Phi_{(\ell+1)} \left(R + \Phi_{(\ell+1)}^\top X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} \Phi_{(\ell+1)}\right)^{-1} \Phi_{(\ell+1)}^\top X_{(0,k+1,(\ell+1,s_{i_\mathbf{t}},h,s_{i_\mathbf{f}}))} A^{\ell+2} \Big).
\end{aligned} \tag{62}
$$

validating the results on extensive case studies, expanding the results to the infinite time horizon case, and investigating the impact of the dropout compensation strategy on the performance of the networked system.

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