Lecture 3
CAPM

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FE5209 Financial Econometrics

Regression analysis and CAPM

Outline

- Capital Market Line (CML), Security Market Line (SML) and Characteristic Line.
- Risk diversification.
- Testing the CAPM, market efficiency and applications.

Reading

SDA chapter 12, 13, and 16

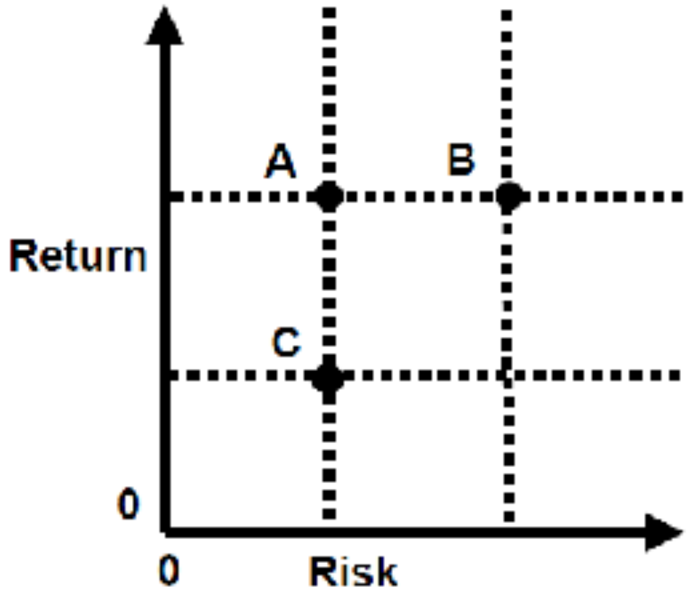
What would be the risk premiums on securities if the following assumptions were true?

1. *The market prices are "in **equilibrium**." In particular, for each asset, supply equals demand.*
2. *Everyone has the **same forecasts** of expected returns and risks.*
3. *All investors choose portfolios optimally according to the principles of **efficient diversification**. This implies that everyone holds a tangency portfolio of risky assets as well as the risk-free asset.*
4. *The market rewards people for assuming unavoidable risk, but there is no reward for needless risks due to inefficient portfolio selection. Therefore, the risk premium on a single security is not due to its "standalone" risk, but rather to its contribution to the risk of the tangency portfolio.*

How do investors regard risk and return?

Two key observations regarding preferences

- **Non-satisfaction**: For a given level of risk, the preferred alternative is one with the highest expected return (A > C)
- **Risk aversion**: For a given level of return, the preferred alternative is one with the lowest level of risk (A > B)



Investors prefer less risk

Consider two investments
– Deposit $10 in a savings account with annual yield of 5%
– Buy stock for $10 with a 50 - 50 chance of selling for $12 or $9 in one year.

Which is more attractive to risk-averse investors?
– Expected return for savings account = 5%
– Expected return for stock = (0.5*(12+ 9) -10)/10*100%= 5%
For same return, investors prefer less risky savings account.

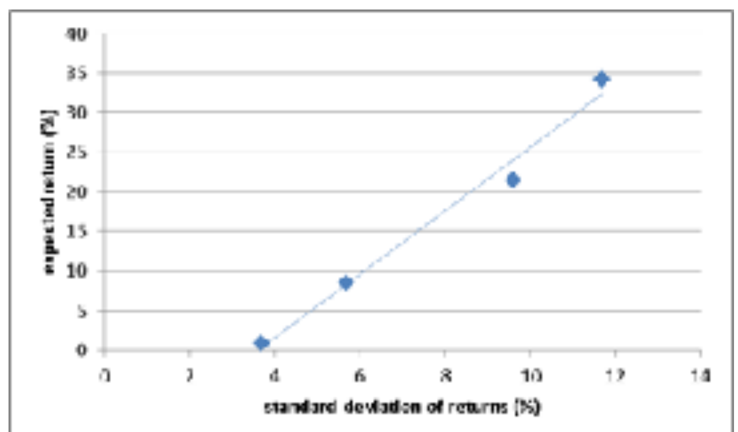
What if stock had a 75% chance of selling for $12?
The riskier investment has higher expected return.

An empirical observation

We observe a clear relationship between risk and return.

Annual	Small Stocks	Large Stocks	Government Bonds	Treasury Bills
Return	11.7%	9.6%	5.7%	3.7%
S.D.	34.1%	21.4%	8.5%	0.9%

Small stocks have the highest annual return but this higher returns are associated with much greater risk.
Variability and expected return are positively correlated with an upward slope.
The riskier investment have historically realized higher returns.
The historical returns of the higher-risk investment classes have higher standard deviations.



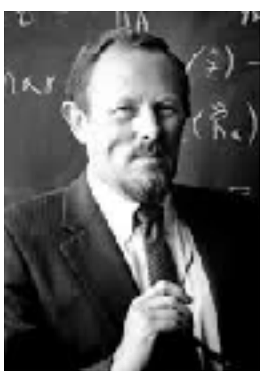
CAPM

How investors' asset demand determines the relation between assets' risk and return in a market equilibrium (when demand equals supply).

The CAPM, initially proposed by Sharpe (1964) and Lintner (1965) has provided a simple and compelling theory of asset market pricing through the association of a portfolio investment to a single risk factor (the Beta factor)

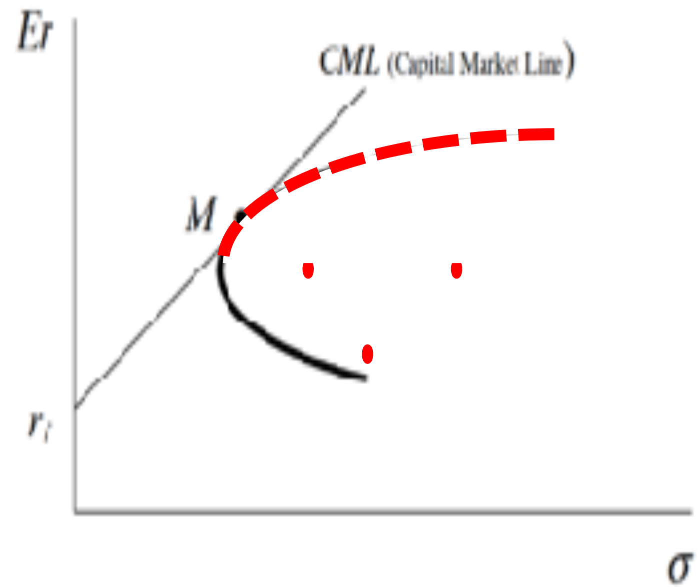
CAPM assumes that investors hold only the risk-free asset and the market portfolio. The market portfolio is the optimally diversified portfolio which includes all risky asset.

According to the CAPM, an asset's systematic risk is measured by its contribution to the risk of the market portfolio.



The capital market line (CML)

Each investor will have a utility maximizing portfolio that is a combination of the risk-free asset and a portfolio of risky assets that is determined by the line drawn from the risk-free rate of return tangent to the investor's efficient frontier of risky assets.



The capital market line: $E\left(R_p\right) = R_f + \sigma_p[(E\left(R_m\right) - R_f)/\sigma_m]$

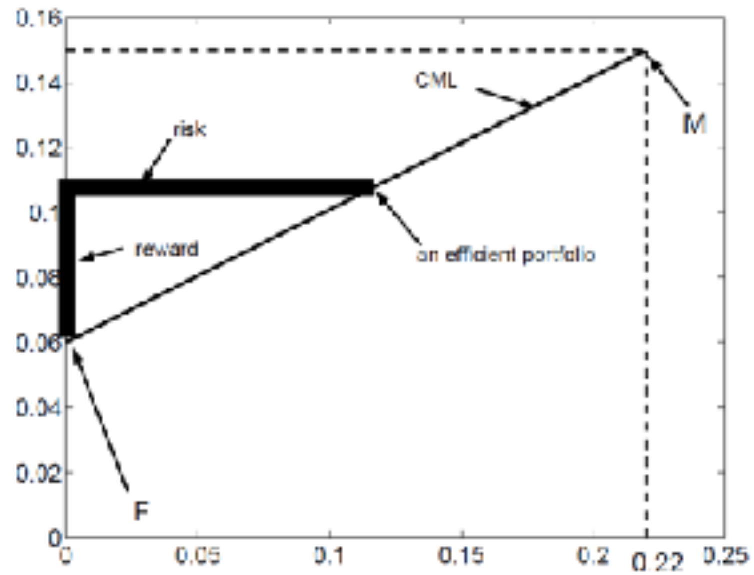
The capital market line (CML) relates the excess expected return on an efficient portfolio to its risk. Excess expected return is the expected return minus the risk-free rate and is also called the risk premium.

Sharpe's "reward-to-risk" ratio

$$E\left(R_p\right) = R_f + \sigma_p[(E\left(R_m\right) - R_f)/\sigma_m]$$

All portfolios on the CML have the same Sharpe ratio as that of the market portfolio: $\dfrac{E\left(R_p\right) - R_f}{\sigma_p} = \dfrac{E(R_m - R_f)}{\sigma_m}$

From the efficient market hypothesis, all portfolios should have a Sharpe ratio less than or equal to the market's.



Derivation of CML

Consider an efficient portfolio that allocates a proportion w of its assets to the market portfolio and (1 - w) to the risk-free asset. Then

$$R_p = wR_M + (1 - w)R_f = R_f + w(R_M - R_f)$$

Taking expectations gives $E(R_p) = R_f + w(E(R_M) - R_f)$.

Also, the risk of the combination equals $\sigma_p = w\sigma_M$

Substituting w in the first equation gives the CML.

The CML says that the optimal way to invest is to

1. decide on the risk σ_p that you can tolerate,
2. calculate $w = \sigma_p/\sigma_M$;
3. invest w proportion of your investment in an index fund, that is, a fund that tracks the market as a whole;
4. invest 1 - w proportion of your investment in risk-free Treasury bills, or a money-market fund.

Security Market Line (SML)

What is the risk-return trade-off for an individual asset?

The security market line (SML) relates the excess return on an asset to the slope of its regression on the market portfolio. *The SML differs from the CML in that the SML applies to all assets while the CML applies only to efficient portfolios.*

Suppose that there are many securities indexed by i.
Define the term β for asset *i*:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{COV(R_i, R_m)}{Var(R_m)}$$

The security market line:

$$E\left(R_i\right) = R_f + \ [E\left(R_m\right) - R_f]\beta_i$$

Derivation of SML

Consider the following portfolio: 100w% of asset *i*, 100(1-w)% of market portfolio.

The expected return is $E\left(R_p\right) = wE\left(R_i\right) + (1-w)E\left(R_m\right)$

The risk is measured as: $\sigma^2\left(R_p\right) = \begin{pmatrix} w & 1-w \end{pmatrix} \begin{pmatrix} \sigma_i^2 & \sigma_{im} \\ \sigma_{im} & \sigma_m^2 \end{pmatrix} \begin{pmatrix} w \\ 1-w \end{pmatrix}$

which gives $\sigma\left(R_p\right) = \sqrt{w^2\sigma_i^2 + (1-w)^2\sigma_m^2 + 2w(1-w)\sigma_{im}}$

Derivation of SML

Consider the effects of a small change in w:

$$\frac{\partial E(R_p)}{\partial w} = E\left(R_i\right) - E\left(R_m\right)$$

$$\frac{\partial \sigma(R_p)}{\partial w} = \frac{2w\sigma_i^2 - 2\sigma_m^2 + 2w\sigma_m^2 + 2\sigma_{im} - 4w\sigma_{im}}{2\sqrt{w^2\sigma_i^2 + (1-w)^2\sigma_m^2 + 2w(1-w)\sigma_{im}}}$$

Evaluate the partial derivatives at w = 0 (equilibrium at M):

$$\frac{\partial E(R_p)}{\partial w} = E\left(R_i\right) - E\left(R_m\right)$$

$$\frac{\partial \sigma(R_p)}{\partial w} = \frac{\sigma_{im} - \sigma_m^2}{\sigma_m}$$

The slope of risk return trade-off evaluated at M:

$$\frac{\partial E(R_p)/\partial w}{\partial \sigma(R_p)/\partial w} = \frac{E\left(R_i\right) - E\left(R_m\right)}{(\sigma_{im} - \sigma_m^2)/\sigma_m}$$

Derivation of SML

This is also the slope of CML, so setting them equal:

$$\frac{E(R_i) - E(R_m)}{(\sigma_{im} - \sigma_m^2)/\sigma_m} = \frac{E(R_m) - R_f}{\sigma_m}$$

Rearranging:

$$E(R_i) = R_f + \frac{\sigma_{im}}{\sigma_m^2}[E(R_m) - R_f]$$

Define the term β for asset *i*.

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{COV(R_i, R_m)}{Var(R_m)}$$

The security market line:

$$E(R_i) = R_f + [E(R_m) - R_f]\beta_i$$

Remark: β_i is a variable in the SML, not the slope. More precisely, $E(R_i)$ is a linear function of β_i with slope $E(R_m) - R_f$.

Beta

$$E(R_i) = R_f + [E(R_m) - R_f]\beta_i$$

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{COV(R_i, R_m)}{Var(R_m)}$$

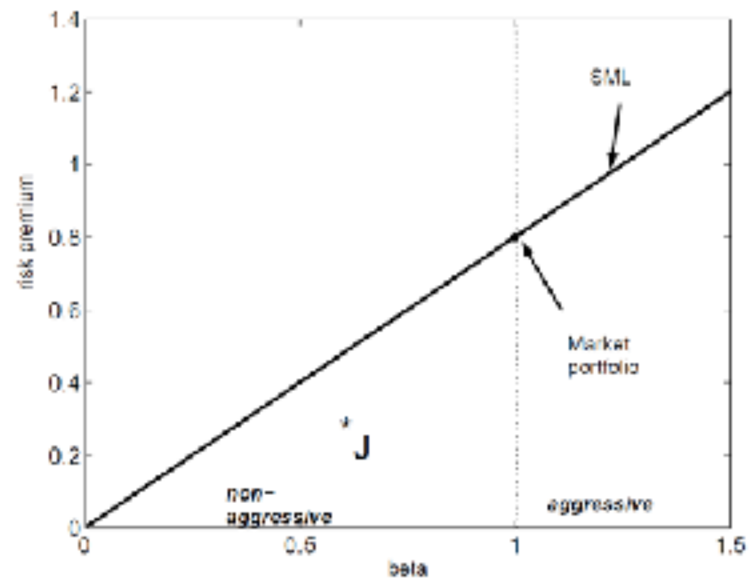
The SML says that the risk premium of the i-th asset is the product of its Beta and the risk premium of the market portfolio $E(R_m) - R_f$. Therefore, Beta measures both the riskiness of the i-th asset and the reward for assuming that riskiness. Consequently, Beta is a measure of how "aggressive" the i-th asset is. By definition, the beta for the market portfolio is 1; i.e., $\beta_M = 1$.

This suggests

$\beta_i > 1$ "aggressive,"

$\beta_i = 1$ "average risk,"

$\beta_i < 1$ "not aggressive."



Beta obtained from online sources

Company	Yahoo Finance (Yahoo.com)	Microsoft Money Central (MSN.com)
Computers and Software		
Apple Inc. (AAPL)	2.90	2.58
Dell Inc. (DELL)	1.81	1.37
Hewlett Packard (HPQ)	1.27	1.47
Utilities		
American Electric Power Co. (AEP)	0.74	0.73
Duke Energy Corp. (DUK)	0.40	0.56
Centerpoint Energy (CNP)	0.82	0.91

Security characteristic line

SML specifies a ***linear relationship*** between the **expected return** of an individual stock and the risk of a well-diversified portfolio:

$$E(R_i) = R_f + \beta_i[E(R_M) - R_f]$$

where $E(R_i)$ = expected return on stock i,

R_f= risk free rate of return

R_M= market portfolio return

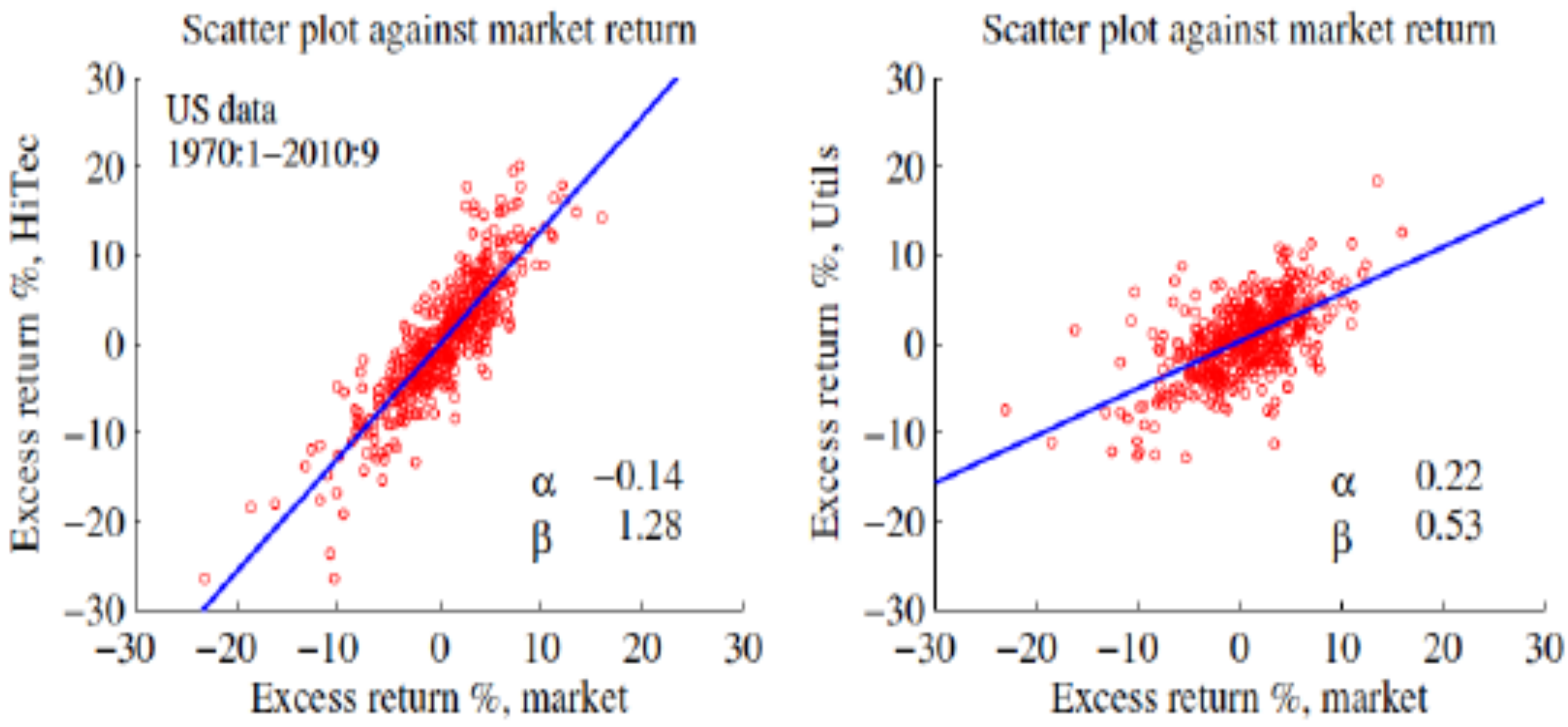
$E(R_M) - R_f$ is called the market risk premium

The security characteristic line is a regression model:

$$R_{it} = R_{ft} + \beta_i\left(R_{Mt} - R_{ft}\right) + \epsilon_{it}$$

where ϵ_{it} is IID noise. The SML gives us information about expected returns, but not about the variance of the returns. The characteristic line is said to be a return-generating process since it gives us a probability model of the returns, not just a model of their expected values.

Estimation of Beta

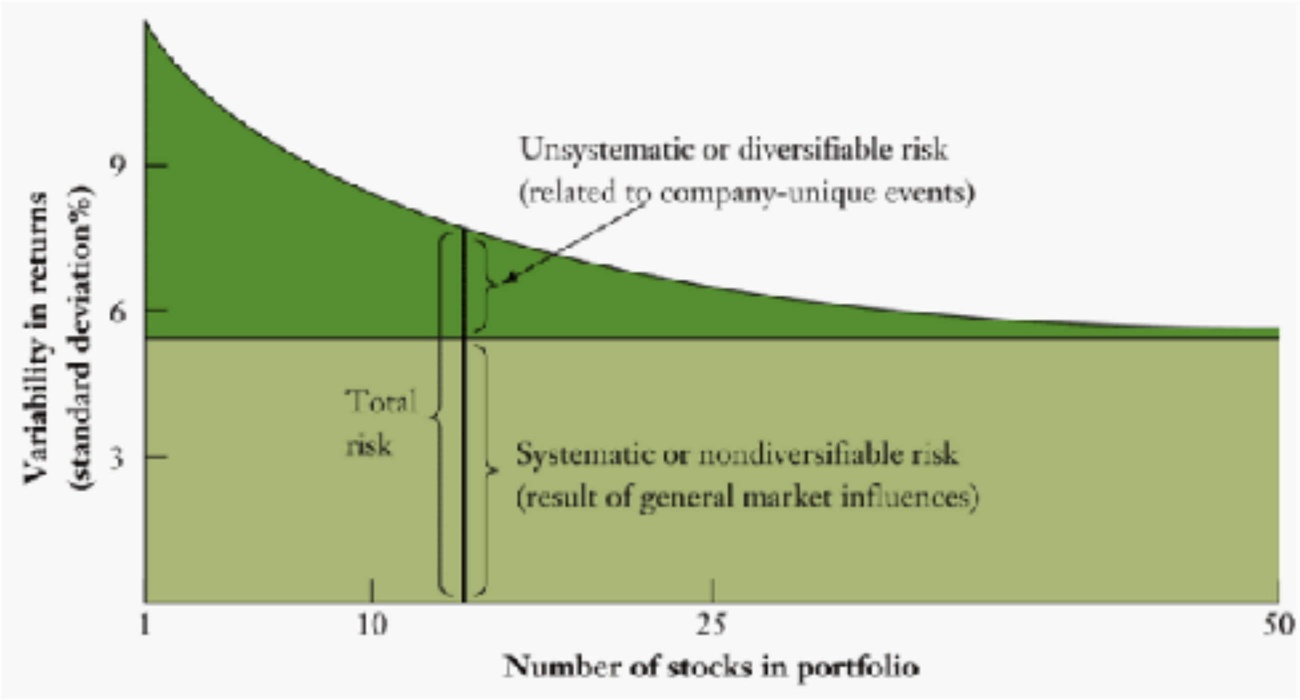


Regression results of CAPM

	HiTec	Utils
constant	-0.14	0.22
	(-0.87)	(1.42)
market return	1.28	0.53
	(32.27)	(12.55)
R2	0.75	0.35
obs	489.00	489.00
Autocorr (t)	-0.76	0.84
White	6.83	19.76
All slopes	364.82	170.92

CAPM and risk diversification

Diversify unique risk by holding a portfolio of many investments.
The effect of lowering risk via appropriate portfolio formulation is called diversification.



By learning how to compute the expected return and risk on a portfolio, we illustrate the effect of diversification.

Correlation and diversification

The market component cannot be reduced by diversification, but the unique component can be reduced or even eliminated by sufficient diversification.

The correlation coefficient ρ that measures linear dependence between two securities ranges from -1 to 1.

- ❑ $\rho = -1$: perfect negative correlation, two assets move in perfectly opposite direction.
- ❑ $\rho = 1$: perfect positive correlation, two assets move exactly together.

As long as the investment returns are not perfectly positively correlated, there exist diversification benefits.

Moreover, the diversification benefits are greater when the correlations are lower or negative.

Diversification

For a portfolio of N stocks, we have

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + \ldots + w_N^2\sigma_N^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 + \ldots + 2w_{N-1}w_N\rho_{N-1,N}\sigma_{N-1}\sigma_N}$$

$$= \sqrt{\sum_i \sum_j w_i w_j \rho_{ij}\sigma_i\sigma_j}$$

Suppose $w_i = w_j = 1/N$ (equal proportions), there are

N weighted variance terms, $i = j \rightarrow \sigma_i^2$
$(N^2 - N)$ weighted cov. terms, $\rightarrow \rho_{ij}\sigma_i\sigma_j$

and $\sigma_p = \sqrt{\left(\frac{1}{N}\right) Average\ Var + \left(1 - \left(\frac{1}{N}\right)\right) Average\ Cov}$.

Variance-Covariance Matrix

	1	2	3	4	5
1	σ^2	COV	COV	COV	COV
2	COV	σ^2	COV	COV	COV
3	COV	COV	σ^2	COV	COV
4	COV	COV	COV	σ^2	COV
5	COV	COV	COV	COV	σ^2

For large N, $1/N \Rightarrow 0$.

Average variance term associated with unique risks becomes irrelevant.
Average covariance term associated with market risk remains.
The diversification gains achieved by adding more investments will depend on the degree of the correlation among the investments. *If the stocks are perfectly moving together, they are essentially the same stock. There is no diversification.*

CAPM and risk diversification

The security characteristic line implies:

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\epsilon_i}^2$$

The squared risk has two components $\beta_i^2 \sigma_M^2$ is called the market or systematic component of risk and $\sigma_{\epsilon_i}^2$ is called the unique, nonmarket, or unsystematic component of risk. CAPM successfully reflects risk diversification. Beta, the reward to holding risk, only applies to market variance, and has nothing to do with idiosyncratic risk.

Market Risk (systematic, non-diversifiable)

—Investments tend to fluctuate with outside markets.

—The systematic risk component measures the contribution of the investment to the risk of the market. For example: war, hike in corporate tax rate.

Unique or Project Risk (idiosyncratic, diversifiable)

—Individual characteristics of investments affect return.

—The idiosyncratic risk is the element of risk that does not contribute to the risk of the market. It can be diversified away when the investment is combined with other investments. For example: product recall, change of management.

Implications of CAPM

- The market portfolio is the tangent portfolio.
- Combining the risk-free asset and the market portfolio gives the portfolio frontier.
- The risk of an individual asset is characterized by its covariation with the market portfolio.
- In equilibrium, only the systematic (market) risk is priced. The non-systematic risk can be diversified away.

CAPM provides a theoretical justification for the widespread practice of passive investing by holding index funds. The validity of the CAPM can only be guaranteed if all of these assumptions are true.

Problems in CAPM estimation

- Training sample: beta estimation problem derived from the fact that beta estimates depend on data interval studied. Difficulty tied to the fact that betas are inherently unstable over a long time interval.
- Data frequency: Daily return is very volatile. Monthly return data may be used to remove noise. However the lower sample size leads to higher variation of estimator.
- What is the appropriate risk-free rate? Short term rates fluctuate, long term rates involve risk premium?
- What is market portfolio? Distortion to beta estimates due to the fact that indexes are imperfect proxies for overall market. No single index consists of all capital assets, including stocks, bonds, real estate, collectibles, etc.

Further criticisms of beta as risk measure

CAPM provides only incomplete description of return volatility - volatility in individual assets can only be described as a function of overall market volatility.

Overall market volatility is very difficult to measure:

- Market index bias: distortion to beta estimates due to the fact that indexes are imperfect proxies for overall market.
- No single index consists of all capital assets, including stocks, bonds, real estate, collectibles, etc.

Model specification bias: CAPM assumes that the variance of return is an adequate measurement of risk. This would be implied by the assumption that returns are normally distributed. Indeed risk in financial investments is not variance in itself, rather it is the probability of losing: it is asymmetric in nature.

Test the CAPM

The CAPM states that, given the assumptions, the expected return on asset is a positive linear function of its index of systematic risk as measured by beta. The higher the βi or beta is, the higher the expected return.

Typically, a methodology referred to as a ***two-pass regression*** is used to test the CAPM.

- The first pass involves the estimation of beta for each security from its characteristic line. The betas from the first-pass regression are then used to form portfolios of securities ranked by portfolio beta.
- Calculate portfolio's beta will give you a measure of its overall market risk.

$$\beta_p = w_1\beta_1 + w_2\beta_2 + \ldots + w_m\beta_m$$

  where w represents the percentage of the stock in your total portfolio.
- The portfolio returns, the return on the risk-free asset, and the portfolio betas are then used to estimate the second-pass regression. Then the following second-pass regression which is the empirical analogue of the CAPM is estimated:

$$R_p - R_F = b_o + b_1\beta_p + \epsilon_p$$

Methodology for Testing the CAPM

$$R_p - R_F = b_o + b_1\beta_p + \epsilon_p$$

According to the CAPM, the following should be found:

1. The intercept b_0 should not be significantly different from zero.
2. The slope estimator b_1 should equal the observed risk premium $(R_M - R_F)$ over which the second-pass regression is estimated.
3. The relationship between beta and return should be linear. That is, if, for example, the following multiple regression is estimated,

$$R_p - R_F = b_0 + b_1\beta_p + b_2\left(\beta_p\right)^2 + \epsilon_p$$

   the parameters b_0 and b_2 should not be significantly different from zero.

4. Beta should be the only factor that is priced by the market.

Representative results of the CAPM test

Fama and French (1993, 1996).

Construct 25 stock portfolios according to two characteristics of the firm:

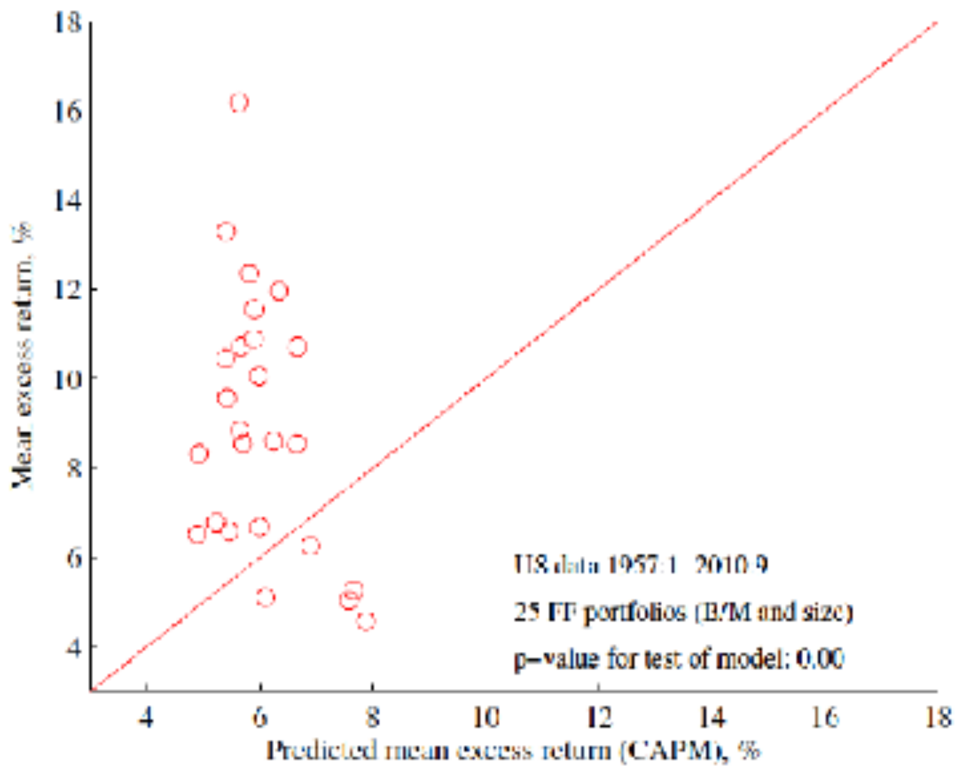
✓ the size

✓ the book-value-to-market-value ratio (B/M)

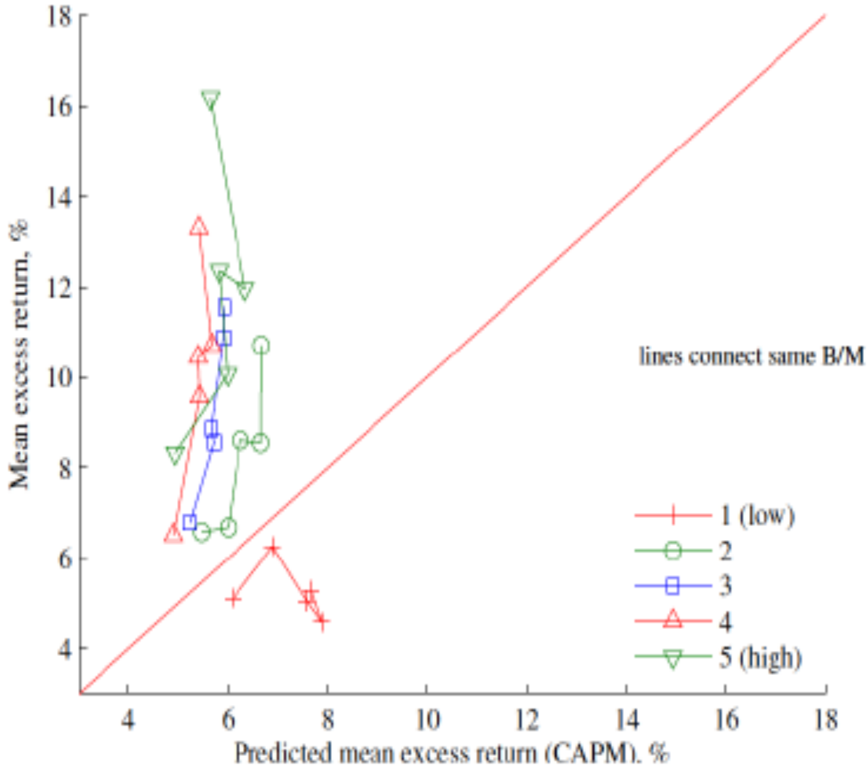
There is almost no relation between $E\left(R_{it}^{e}\right)$ and β_i.

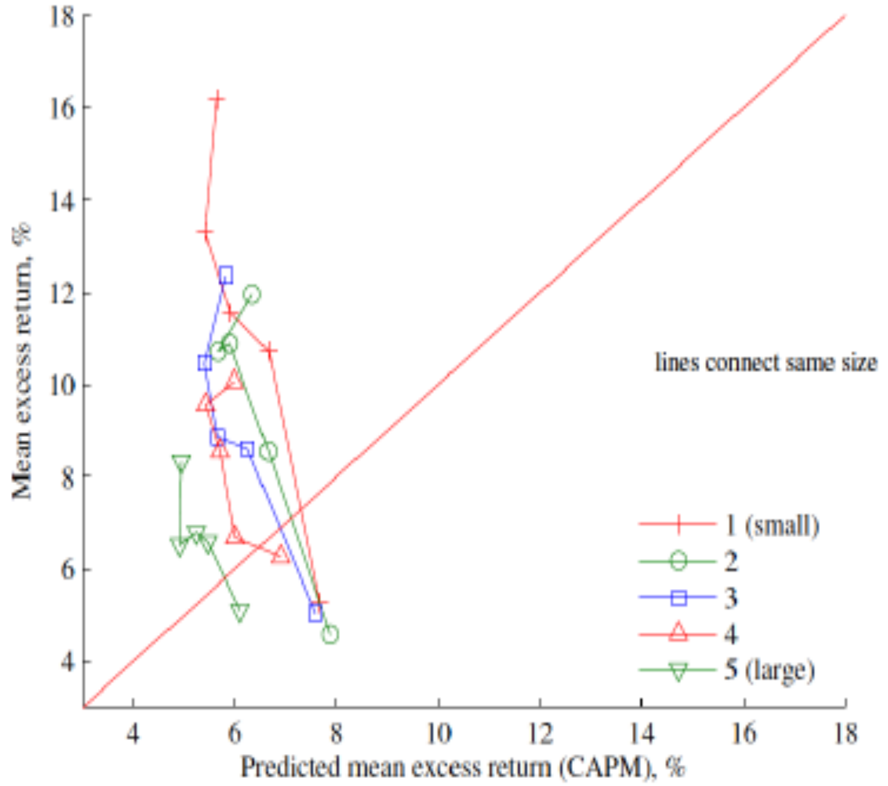
➢ *run a traditional CAPM regression on each of the 25 portfolios*

➢ *then study if the expected excess returns are related to the betas as they should according to CAPM (recall that CAPM implies $E\left(R_{it}^{e}\right) = \beta_i \lambda$ where λ is the risk premium on the market portfolio).*



CAPM on FF portfolios (B/M and size)





Findings of Fama French CAPM test

The general results of the empirical tests of the CAPM are as follows:

1. The estimated intercept term b_0, is significantly different from zero and consequently different from what is hypothesized for this value.
2. The estimated coefficient for beta, b_1, has been found to be less than the observed risk premium $(R_M - R_F)$.
3. The relationship between beta and return appears to be linear.
4. Beta is not the only factor priced by the market. Several studies have discovered other factors that explain stock returns.

Multifactor pricing models

Empirical evidence indicates that CAPM beta does not explain the cross-section of expected returns, such as momentum and anomalies (January, size, book/market effects, for example)

What to do?
Dynamic, equilibrium models or multi-factor models.

Two multi-factor approaches:
1. Ad-hoc theorizing: Fama-French three-factor pricing model; Robert Jones 10-factor model; Chen-Roll-Ross five-factor model.
2. Curve fitting: Arbitrage Pricing Theory (APT).

Robert Jones' multiple factor models

Robert Jones of Goldman Sachs Asset Management regressed monthly stock returns against the following factors: ”value” factors, ”momentum” factors, and risk factors.

4 value factors: book/market ratio, earnings/price ratio, sales/price ratio, and cash flow/price ratio.

3 momentum factors: estimate revisions for earnings, revisions ratio, and price momentum.

3 risk factors: the systematic risk or beta from the CAPM, the residual risk from the CAPM and an uncertainty estimate measure.

The conclusion from the regression results is that there are factors other than the CAPM beta that explain returns.

Chen, Roll, and Ross (1986)

Another example: Chen, Roll, and Ross (1986) use a number of macro variables as factors - along with traditional market indexes. They find that industrial production and inflation surprises are priced factors, while the market index might not be. These factors are considered to influence future cash flows and future discount rates. The CRR model is based on intuitive analysis and empirical investigation, and consists of five factors.

1. yield spread between long and short interest rates
2. expected inflation
3. unexpected inflation (past realizations)
4. industrial production growth
5. yield spread between high- and low-grade corporate bonds.

Diagnostics are performed to determine whether consumption growth or oil prices add any explanatory power.

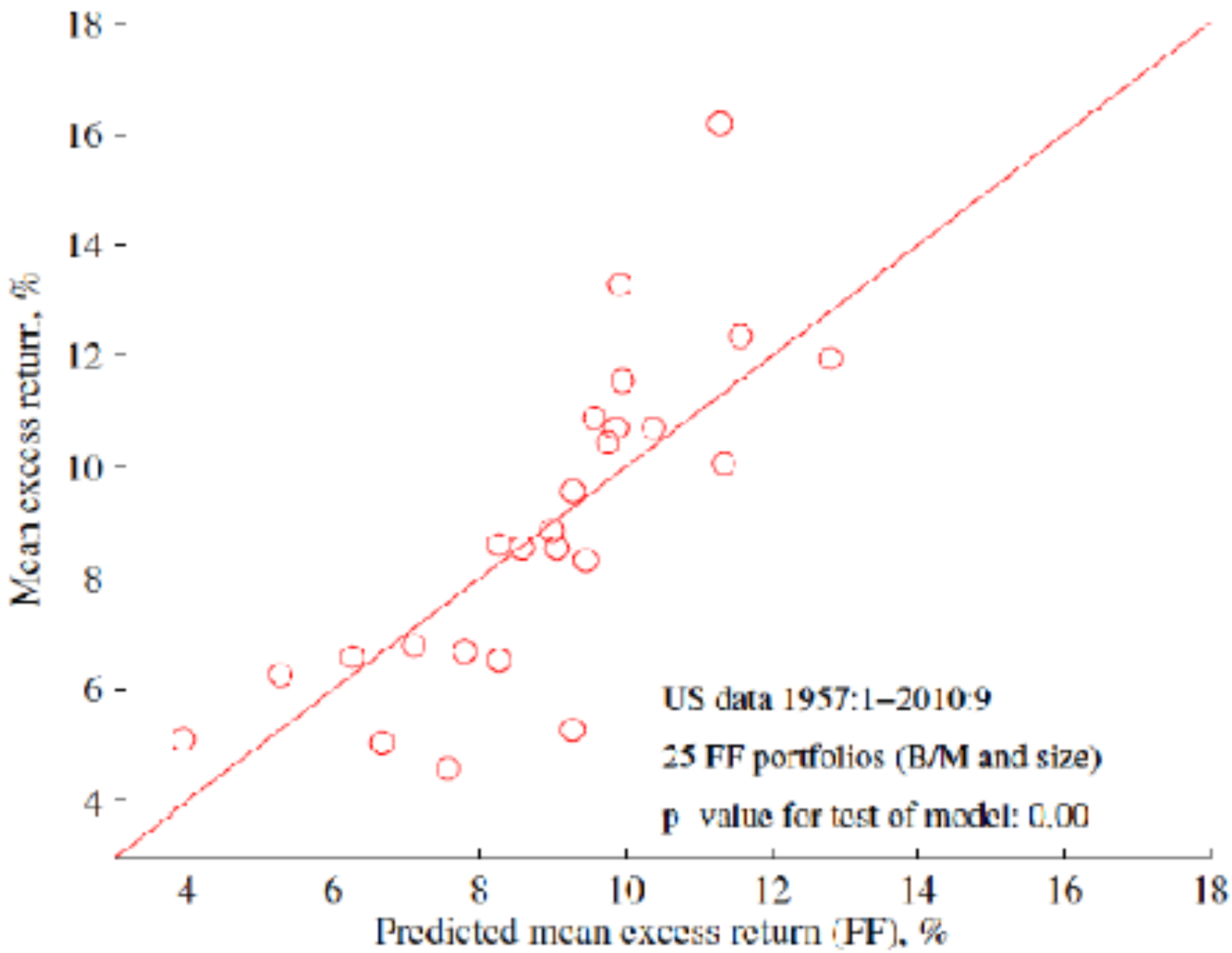
Fama-French three-factor model

Fama and French find that a three-factor model fits the 25 stock portfolios fairly well.

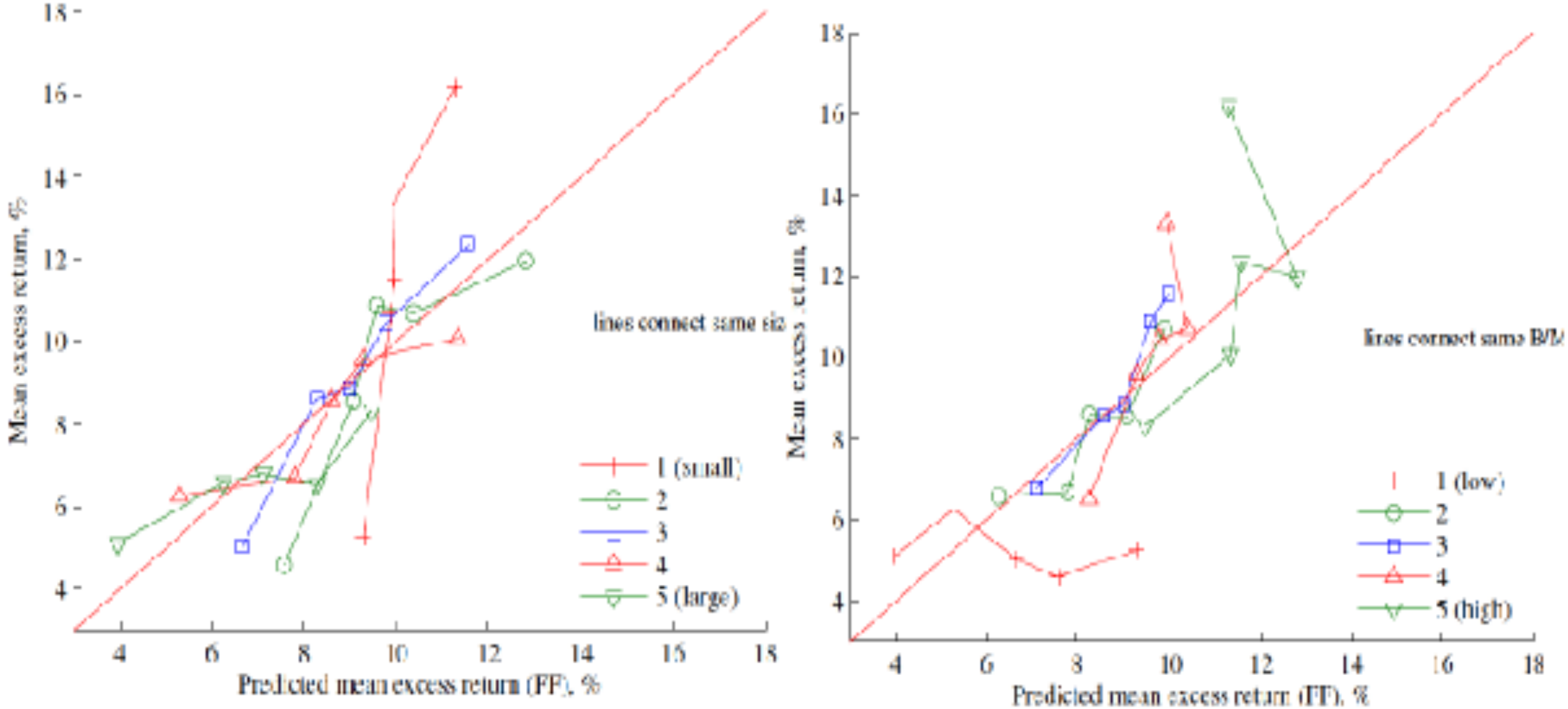
- the market return
- the return on a portfolio of small stocks minus the return on a portfolio of big stocks
- the return on a portfolio with high B/M minus the return on a portfolio with low B/M

The three-factor model is rejected at traditional significance levels, but it can still capture a fair amount of the variation of expected returns.

FF three-factor model



FF three-factor model (B/M and size)



Importance of testing market efficiency

There is a long list of academic papers that have examined the efficiency of capital markets. By far, most studies have focused on the pricing efficiency of the equity markets. Pricing efficiency refers to a market where prices at all times fully reflect all available information that is relevant to the valuation of securities.

Fama classified the pricing efficiency of the stock market into three forms:

1. *weak form: price reflects the past and current prices*
2. *semistrong form: price reflects public information*
3. *strong form: price reflects all information including un-public one*

The distinction among these forms lies in the relevant information that is hypothesized to be impounded in the price of the security.

Select trading strategy/evaluate fund manager's performance:

- An active portfolio strategy uses available information and forecasting techniques to seek a better performance than a portfolio that is simply diversified broadly.
- A passive portfolio strategy involves minimal expected input, and instead relies on diversification to match the performance of some market index.
- There are also hybrid strategies.

The Jensen measure

In evaluating the performance of a money manager, one must adjust for the risks accepted by the manager in generating return.

Subtracting the portfolio's actual return from the expected return gives the ***excess return*** or the ***active return***.

The ***Jensen measure*** (proposed in 1968 by Michael Jensen, also called the ***Jensen index*** or ***Jensen alpha***) is a risk-adjusted performance measure that uses the CAPM to empirically determine whether a portfolio manager outperformed a market index.

The intercept term, α_i, is interpreted as the ***unique return*** realized by the portfolio manager and is the estimated value of the Jensen measure.

Today, the CAPM is not the typical asset pricing model used by professional money manager. Rather, multifactor models are used.

A multifactor model can be used to show where a money manager took risks relative to the benchmark and if those bets paid off.

Test market efficiency

Tests of the pricing efficiency of the market examine whether it is possible to generate abnormal returns.

If a strategy can be shown to consistently outperform the market, then the market is not price-efficient.

However, this test may be misspecified for two reasons:

1. It may fail to consider the appropriate measure of risk.
2. The risk measures may not be estimated properly.

Hence, tests of market efficiency are *joint tests* of both the **efficiency of the market and the validity of the pricing model** employed in the study.

- ➢ Studies of the semi-strong form of pricing efficiency have tested whether investors can outperform the market by selecting securities on the basis of fundamental security analysis
- ➢ One of the main empirical tests of strong-form pricing efficiency has been the study of the performance of professional money managers: mutual fund managers and pension fund managers.

Benchmark selection: Sharpe benchmarks

William Sharpe suggested that a benchmark can be constructed using multiple regression analysis from various specialized market indexes.

A benchmark can be created using regression analysis that adjusts for a manager's index-like tendencies. Such a benchmark is called a ***Sharpe benchmark***.

Sharpe suggested 10 mutually exclusive indexes to provide asset class diversification.

Sharpe benchmarks are determined by regressing periodic returns on various market indexes.

The Sharpe benchmark was reported for one portfolio management firm based on performance:

$$Sharpe\ benchmark$$

$$= 0.43 \times (FRC\ Price/drivenindex) + 0.13 \times \left(FRC\ Earnings - growth\ index\right) + 0.44 \times (FRC\ 2000\ index)$$

where FRC is an index produced by the Frank Russell Company.

Benchmark selection: Sharpe benchmarks

The three indexes were selected because they were the only indexes of the 10 that were statistically significant.

The sum of the three coefficients is equal to one.

The coefficient of determination for this regression was 97.6%.

By subtracting the style benchmark's monthly return from the manager's monthly portfolio return, performance can be measured.

This difference is referred as "added value residuals".

The added value residual for this month would be calculated as follows.

1. Calculate the value of the Sharpe benchmark:

$$\begin{aligned} \textit{Sharpe benchmark} &= 0.43 \times (0.7\%) + 0.13 \times (1.4\%) + 0.44 \times (2.2\%) \\ &= 1.45\% \end{aligned}$$

2. The added value residual is then:

$$\begin{aligned} \textit{Added value residual} &= \textit{Actual return} - \textit{Sharpe benchmark return} \\ &= 1.75\% - 1.45\% = 0.3\% \end{aligned}$$

A test of a strong-form pricing efficiency

As support for the position of strong-form pricing efficiency, many studies have compared the performance of equity mutual fund managers against a suitable stock market index to assess the performance of fund managers in general.

But this is not a fair comparison because it ignores risk.

Robert Jones analyzed the performance of the average large-cap mutual fund adjusted for risk. The variables in his regression model are:

- Y_t = the difference between the returns on a composite mutual fund index and the *S* &*P* 500 in month *t* ;
- $X_{1,t}$ = the difference between the *S*&*P* 500 return and the 90-day Treasury rate for month *t*
- $X_{2,t}$ = the difference between the returns on the Russell 3000 Value Index and the Russell 3000 Growth Index for month *t*
- $X_{3,t}$ = the difference between the returns on the Russell 1000 Index (large-cap stocks) and the Russell 2000 Index (small-cap stocks) for month *t* .

A test of a strong-form pricing efficiency

The results of the regression are reported below with the t –statistic for each parameter shown in parentheses:

$$\hat{Y}_t = \underset{(-0.192)}{-0.007} \quad \underset{(-8.771)}{-0.0083X_{1,t}} \quad \underset{(-3.628)}{-0.071X_{2,t}} \quad \underset{(-17.380)}{-0.244X_{3,t}}$$

Interpretation:

- Relative to the $S\&P$ 500, the average large-cap mutual fund makes statistically significant bets against the market, against value, and against size.
- The adjusted R^2 is 0.63: 63% of the variation in the average large-cap mutual fund’s returns is explained by the regression model.
- Statistically, the intercept term is not significant. So, the average active return is indistinguishable from zero.

Conclusion: the average large-cap mutual funds covers its costs on a risk-adjusted basis.

Representative results on mutual fund performance

Mutual fund evaluations (estimated α_i) typically find:

- ❑ on average neutral performance
- ❑ large funds might be worse
- ❑ perhaps better performed on less liquid market
- ❑ there is very little persistence in performance: α_i for one sample does not predict α_i for subsequent samples (except for bad funds).

R lab

The S&P 500 index will be a proxy for the market portfolio and the 90-day Treasury rate will serve as the risk-free rate.
This lab uses the data set Stock_FX_Bond_2004_to_2006.csv, which is available on the book's website. This data set contains a subset of the data in the data set Stock_FX_Bond.csv used elsewhere.
The R commands will be given in small groups so that they can be explained better. First run the following commands to read the data, extract the prices, and find the number of observations:

```
dat = read.csv("Stock_FX_Bond_2004_to_2006.csv",header=T)
prices = dat[,c(5,7,9,11,13,15,17,24)]
n = dim(prices)[1]
```

Next, run these commands to convert the risk-free rate to a daily rate, compute net returns, extract the Treasury rate, and compute excess returns for the market and for seven stocks. The risk-free rate is given as a percentage so the returns are also computed as percentages.

- Data: Rlab3_Stock_FX_Bond_2004_to_2006.csv
- R: Rlab3.R

R lab

```
dat2 = as.matrix(cbind(dat[(2:n),3]/365, 100*(prices[2:n,]/prices[1:(n-1),] - 1)))
names(dat2)[1] = "treasury"
risk_free = dat2[,1]
ExRet = dat2[,2:9] - risk_free
market = ExRet[,8]
stockExRet = ExRet[,1:7]
```

Now fit model (16.19) to each stock, compute the residuals, look at a scatter-plot matrix of the residuals, and extract the estimated betas.

```
fit_reg = lm(stockExRet~market)
summary(fit_reg)
res = residuals(fit_reg)
pairs(res)
options(digits=3)
betas=fit_reg$coeff[2,]
```

R lab

Problem 1 Would you reject the null hypothesis that alpha is zero for any of the seven stocks? Why or why not?

Problem 2 Use the CAPM to estimate the expected excess return for all seven stocks. Compare these results to using the sample means of the excess returns to estimate these parameters. Assume for the remainder of this lab that all alphas are zero. (Note: Because of this assumption, one might consider reestimating the betas and the residuals with a no-intercept model. However, since the estimated alphas were close to zero, forcing the alphas to be exactly zero will not change the estimates of the betas or the residuals by much. Therefore, for simplicity, do not reestimate.)

Problem 3 Compute the correlation matrix of the residuals. Do any of the residual correlations seem large? Could you suggest a reason why the large correlations might be large? (Information about the companies in this data set is available at Yahoo Finance and other Internet sites.)

R lab

Problem 4 What percentage of the excess return variance for UTX is due to the market?

Problem 5 An analyst predicts that the expected excess return on the market next year will be 4%. Assume that the betas estimated here using data from 2004-2006 are suitable as estimates of next year's betas. Estimate the expected excess returns for the seven stocks for next year.

R lab – Results & Discussions

Problem 1 Would you reject the null hypothesis that alpha is zero for any of the seven stocks? Why or why not?

Selected output is listed below. We can see that the p-values for the intercepts are 0.42, 0.14, 0.39, 0.23, 0.91, 0.41, and 0.31. Since all are large, in particular, well above 0.05, we can accept that all of the alphas are zero.

```
> summary(fit_reg)
Response GM_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.0685     0.0850   -0.81     0.42
market        1.2025     0.1253    9.60   <2e-16 ***

Response F_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.0977     0.0665   -1.47     0.14
market        1.2378     0.0980   12.63   <2e-16 ***
---

Response UTX_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.0296     0.0343    0.86     0.39
market        0.9766     0.0506   19.31   <2e-16 ***
---

Response CAT_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.0568     0.0477    1.19     0.23
market        1.4098     0.0703   20.06   <2e-16 ***
---
```

R lab – Results & Discussions

Problem 2 Use model (16.19) to estimate the expected excess return for all seven stocks. Compare these results to using the sample means of the excess returns to estimate these parameters.

The following commands estimate the expected excess returns in both ways.
>betas=fit_reg$coeff[2,]
>betas * mean(market)
>apply(stockExRet,2,mean)

We see that the estimates from the one-factor model are similar to each other and all are positive. The estimates using sample means are dissimilar to each other and four are negative while three are positive. Thus, the one-factor model produces much less variable estimates of mean returns.

```
> betas * mean(market)
 GM_AC   F_AC UTX_AC CAT_AC MRK_AC PFE_AC IBM_AC
0.0233 0.0240 0.0190 0.0274 0.0146 0.0184 0.0159

> apply(stockExRet,2,mean)
  GM_AC    F_AC  UTX_AC  CAT_AC  MRK_AC  PFE_AC  IBM_AC
-0.0452 -0.0736  0.0485  0.0842  0.0071 -0.0218 -0.0163
```

R lab – Results & Discussions

Problem 3 Compute the correlation matrix of the residuals. Do any of the residual correlations seem large? Could you suggest a reason why the large correlations might be large?

```
> res = residuals(fit_reg)
> options(digits=3)
> cor(res)
          GM_AC     F_AC   UTX_AC CAT_AC  MRK_AC  PFE_AC   IBM_AC
GM_AC   1.00000  0.50911  0.03967 0.0202 -0.0472 -0.0188  0.00785
F_AC    0.50911  1.00000 -0.00714 0.0289  0.0128  0.0114  0.03575
UTX_AC  0.03967 -0.00714  1.00000 0.1498 -0.0154 -0.1110 -0.06949
CAT_AC  0.02023  0.02895  0.14977 1.0000 -0.0757 -0.0650 -0.08342
MRK_AC -0.04715  0.01279 -0.01540 -0.0757 1.0000  0.2833 -0.07817
PFE_AC -0.01877  0.01142 -0.11103 -0.0650 0.2833  1.0000 -0.04606
IBM_AC  0.00785  0.03575 -0.06949 -0.0834 -0.0782 -0.0461  1.00000
```

The residual correlation matrix is above. There is a reasonably high residual correlation of 0.51 between GM and F, which makes sense since they are both in automotive industry. There is also a moderate residual correlation of 0.28 between Pfizer and Merck, which again is sensible since both are pharmaceutical companies.

R lab – Results & Discussions

Problem 4 What percentage of the excess return variance for UTX is due to the market?

We see from the output that R^2 is 0.357, so the percentage of the excess return variance for UTX due to the market is estimated as 35.7%

```
Response UTX_AC :

Call:
lm(formula = UTX_AC ~ market)

Residuals:
    Min      1Q  Median      3Q     Max
-3.8194 -0.5149 -0.0154  0.4938  6.3806

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.0296     0.0343    0.86     0.39
market        0.9766     0.0506   19.31   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.89 on 670 degrees of freedom
Multiple R-squared: 0.357,      Adjusted R-squared: 0.356
F-statistic:  373 on 1 and 670 DF,  p-value: <2e-16
```

R lab – Results & Discussions

Problem 5 An analyst predicts that the expected excess return on the market next year will be 4%. Estimate the expected excess returns for the seven stocks for next year.

```
> 4*betas
GI1_AC    F_AC UTX_AC  CAT_AC   MRK_AC  PFE_AC IBM_AC
4.81      4.95    3.91    5.64     3.00    3.80   3.28
```

Appendix: Standard deviation of a two-asset portfolio

Compute the expected return and standard deviation of the following portfolio consisting of two stocks that have a correlation coefficient $\rho = 0.75$.

Portfolio	Weight	Expected Return	Standard Deviation
Apple	0.5	0.14	0.20
Coca-Cola	0.5	0.14	0.20

Expected return: $E\left(R_p\right) = 0.5 \times 0.14 + 0.5 \times 0.14 = 0.14$.

Standard deviation: Unlike expected return, the standard deviation of a portfolio is not generally equal to the weighted average of the standard deviations of the returns in the portfolio.

For a simple portfolio which has 2 stocks:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2}$$

$$\sigma_p = \sqrt{.5^2 \times .2^2 + .5^2 \times .2^2 + 2 \times .5 \times .5 \times .75 \times .2 \times .2} = \sqrt{0.035} = 0.187 < 0.2$$

Appendix: Diversification and the correlation coefficient

Correlation	E(Return)	Standard Deviation
−1.00	0.14	0%
−0.80	0.14	6%
−0.60	0.14	9%
−0.40	0.14	11%
−0.20	0.14	13%
0.0	0.14	14%
0.20	0.14	15%
0.40	0.14	17%
0.60	0.14	18%
0.80	0.14	19%
1.00	0.14	20%

Appendix: Diversification and the correlation coefficient