

EXERCISES FROM
A FRIENDLY INTRODUCTION TO GROUP THEORY
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CONTENTS

1. Preliminaries	2
1.1. Sets	2

1. PRELIMINARIES

1.1. Sets.

1. (a) `IsUnder30 := n -> n < 30;;`

```
# {n ∈ ℤ | n² < 30}
S := [];; n := 0;;
repeat
  Add( S, n^2 );
  n := n + 1;
until not IsUnder30( n^2 );

# {x, y, z ∈ S | x² + y² + z²}
T := Set( List( Tuples( S, 3), Sum) );;

# {n ∈ ℤ | n < 30 ∧ ∃ x, y, z ∈ ℤ, x² + y² + z² = n}
Set( Filtered( T, IsUnder30 ) );
```

```
[ 0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18,
  19, 20, 21, 22, 24, 25, 26, 27, 29 ]
```

```
(b) DigitsInt := function ( n, base )
    local digits;
    digits := [];
    repeat
        Add( digits, RemInt( n, base ), 1 );
        n := QuoInt( n, base );
    until IsZero( n );
    return digits;
end;;

IsAllOddSortedList := function ( digits )
    return ForAll( digits, IsOddInt ) and
        IsSortedList( digits );
end;;

IsAllOddSortedDigitsInt := function ( n )
    return IsAllOddSortedList( DigitsInt( n, 10 ) );
end;;

Set( Filtered( [100 .. 999], IsAllOddSortedDigitsInt ) );
```

```
[ 111, 113, 115, 117, 119, 133, 135, 137, 139, 155, 157, 159,
  177, 179, 199, 333, 335, 337, 339, 355, 357, 359, 377, 379,
  399, 555, 557, 559, 577, 579, 599, 777, 779, 799, 999 ]
```

```
(c) Cartesian( [ 1, 2, 3 ], [ 1, FLOAT.PI ] );
```

```
[ [ 1, 1 ], [ 1, 3.14159 ], [ 2, 1 ], [ 2, 3.14159 ],
  [ 3, 1 ], [ 3, 3.14159 ] ]
```

2. *Proof of* $\textcircled{2}$.

$$\begin{array}{ccc}
\overline{x \in R \cup (S \cap T)} & \overline{x \in R} & \overline{x \in R \cup (S \cap T)} \\
\overline{x \in (R \cup S) \cap (R \cup T)} & \overline{x \in (R \cup S) \cap (R \cup T)} & \overline{x \in R \cup (S \cap T)} \\
\overline{R \cup (S \cap T) \subseteq (R \cup S) \cap (R \cup T)} & & \overline{x \in (R \cup S) \cap (R \cup T)} \\
\overline{R \cup (S \cap T) = (R \cup S) \cap (R \cup T)} & & \overline{R \cup (S \cap T) \subseteq (R \cup S) \cap (R \cup T)}
\end{array}$$

□

Proof of $\textcircled{4}$.

$$\begin{array}{ccccccc}
\overline{x \in R \setminus (S \cap T)} & \overline{x \notin S} & \overline{x \in R \setminus (S \cap T)} & \overline{x \notin T} & \overline{x \in (R \setminus S) \cup (R \setminus T)} & \overline{x \in R \setminus S} & \overline{x \in (R \setminus S) \cup (R \setminus T)} \\
\overline{x \in R \setminus S} & & \overline{x \in R \setminus T} & & \overline{x \in R \wedge x \notin S} & & \overline{x \in R \wedge x \notin T} \\
\overline{x \in (R \setminus S) \cup (R \setminus T)} & & & & \overline{x \in R \setminus (S \cap T)} & & \overline{x \in R \setminus (S \cap T)} \\
\overline{R \setminus (S \cap T) \subseteq (R \setminus S) \cup (R \setminus T)} & & & & \overline{(R \setminus S) \cup (R \setminus T) \subseteq R \setminus (S \cap T)} & & \overline{R \setminus (S \cap T) = (R \setminus S) \cup (R \setminus T)}
\end{array}$$

□