EXERCISES FROM *A FRIENDLY INTRODUCTION TO GROUP THEORY* **BY DAVID NASH**

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Contents

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1. Preliminaries

1.1. Sets Exercises.

1. (a)

```
IsUnder30 := n -> n < 30;;


# {n ∈ ℤ | n^2 < 30}
S := [];; n := 0;;
repeat
    Add( S, n^2 );
    n := n + 1;
until not IsUnder30( n^2 );


# {x, y, z ∈ S | x^2 + y^2 + z^2}
T := Set( List( Tuples( S, 3), Sum) );;


# {n ∈ ℤ | n < 30 ∧ ∃ x, y, z ∈ ℤ, x^2 + y^2 + z^2 = n}
Set( Filtered( T, IsUnder30 ) );
```

```
[ 0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18,
  19, 20, 21, 22, 24, 25, 26, 27, 29 ]
```

(b)
```
DigitsInt := function ( n, base )
    local digits;
    digits := [];
    repeat
        Add( digits, RemInt( n, base ), 1 );
        n := QuoInt( n, base );
    until IsZero( n );
    return digits;
end;;


IsAllOddSortedList := function ( digits )
    return ForAll( digits, IsOddInt ) and
           IsSortedList( digits );
end;;


IsAllOddSortedDigitsInt := function ( n )
    return IsAllOddSortedList( DigitsInt( n, 10 ) );
end;;


Set( Filtered( [100 .. 999], IsAllOddSortedDigitsInt ) );
```

```
[ 111, 113, 115, 117, 119, 133, 135, 137, 139, 155, 157, 159,
  177, 179, 199, 333, 335, 337, 339, 355, 357, 359, 377, 379,
  399, 555, 557, 559, 577, 579, 599, 777, 779, 799, 999 ]
```

(c)
```
Cartesian( [ 1, 2, 3 ], [ 1, FLOAT.PI ] );
```

```
[ [ 1, 1 ], [ 1, 3.14159 ], [ 2, 1 ], [ 2, 3.14159 ],
  [ 3, 1 ], [ 3, 3.14159 ] ]
```

2. *Proof.* 2 Let $x \in R \cup (S \cap T)$. By definition $x \in R$ or $x \in S \cap T$.

Case 1: $x \in R$ Since $x \in R$, it follows that $x \in R \cup S$ and $x \in R \cup T$. Thus we have $x \in (R \cup S) \cap (R \cup T)$.

Case 2: $x \in S \cap T$ By definition $x \in S$ and $x \in T$. It follows that $x \in R \cup S$ and $x \in R \cup T$. Thus we have $x \in (R \cup S) \cap (R \cup T)$.

Together, Case 1 and Case 2 demonstrate that if $x \in R \cup (S \cap T)$, then $x \in (R \cup S) \cap (R \cup T)$ as well, which proves that $R \cup (S \cap T) \subseteq (R \cup S) \cap (R \cup T)$.

For the other containment, suppose that $x \in (R \cup S) \cap (R \cup T)$. Then specifically $x \in R \cup S$ and $x \in R \cup T$. These imply respectively that $x \in R$ or $x \in S$ and that $x \in R$ or $x \in T$. Hence, $x \in R \cup (S \cap T)$ and we have $(R \cup S) \cap (R \cup T) \subseteq R \cup (S \cap T)$. By double containment we have shown that $R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$ as desired. $\square$