# EXERCISES FROM *A FRIENDLY INTRODUCTION TO GROUP THEORY* BY DAVID NASH

ERIC BAILEY

Contents



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# 1. Preliminaries

## 1.1. **Sets.**

**1.** (a)
```
IsUnder30 := n -> n < 30;;


# {n ∈ ℤ | n² < 30}
S := [];; n := 0;;
repeat
    Add( S, n^2 );
    n := n + 1;
until not IsUnder30( n^2 );


# {x, y, z ∈ S | x² + y² + z²}
T := Set( List( Tuples( S, 3), Sum) );;


# {n ∈ ℤ | n < 30 ∧ ∃ x, y, z ∈ ℤ, x² + y² + z² = n}
Set( Filtered( T, IsUnder30 ) );
```

```
[ 0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18,
  19, 20, 21, 22, 24, 25, 26, 27, 29 ]
```

(b)
```
DigitsInt := function ( n, base )
    local digits;
    digits := [];
    repeat
        Add( digits, RemInt( n, base ), 1 );
        n := QuoInt( n, base );
    until IsZero( n );
    return digits;
end;;


IsAllOddSortedList := function ( digits )
    return ForAll( digits, IsOddInt ) and
           IsSortedList( digits );
end;;


IsAllOddSortedDigitsInt := function ( n )
    return IsAllOddSortedList( DigitsInt( n, 10 ) );
end;;


Set( Filtered( [100 .. 999], IsAllOddSortedDigitsInt ) );
```

```
[ 111, 113, 115, 117, 119, 133, 135, 137, 139, 155, 157, 159,
  177, 179, 199, 333, 335, 337, 339, 355, 357, 359, 377, 379,
  399, 555, 557, 559, 577, 579, 599, 777, 779, 799, 999 ]
```

(c)
```
Cartesian( [ 1, 2, 3 ], [ 1, FLOAT.PI ] );
```

```
[ [ 1, 1 ], [ 1, 3.14159 ], [ 2, 1 ], [ 2, 3.14159 ],
  [ 3, 1 ], [ 3, 3.14159 ] ]
```

**2.** *Proof of* 2.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{x \in R \cup (S \cap T)} \quad x \in R}{x \in (R \cup S) \cap (R \cup T)} \qquad \dfrac{\dfrac{}{x \in R \cup (S \cap T)} \quad x \in S \cap T}{x \in (R \cup S) \cap (R \cup T)}}{R \cup (S \cap T) \subseteq (R \cup S) \cap (R \cup T)} \qquad \dfrac{\dfrac{\dfrac{}{x \in R \cup (S \cap T)}}{x \in (R \cup S) \cap (R \cup T)}}{R \cup (S \cap T) \subseteq (R \cup S) \cap (R \cup T)}}{R \cup (S \cap T) = (R \cup S) \cap (R \cup T)}$$

□

*Proof of* 4.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{x \in R \setminus (S \cap T)} \quad x \notin S}{x \in R \setminus S} \qquad \dfrac{\dfrac{}{x \in R \setminus (S \cap T)} \quad x \notin T}{x \in R \setminus T}}{x \in (R \setminus S) \cup (R \setminus T)}}{R \setminus (S \cap T) \subseteq (R \setminus S) \cup (R \setminus T)} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{x \in (R \setminus S) \cup (R \setminus T)} \quad x \in R \setminus S}{x \in R \ \wedge \ x \notin S} \qquad \dfrac{\dfrac{}{x \in (R \setminus S) \cup (R \setminus T)} \quad x \in R \setminus T}{x \in R \ \wedge \ x \notin T}}{x \in R \setminus (S \cap T)}}{(R \setminus S) \cup (R \setminus T) \subseteq R \setminus (S \cap T)}}{R \setminus (S \cap T) = (R \setminus S) \cup (R \setminus T)}$$

□

**3.** (a) *Proof.*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x \in (A \cup B) \cup C}{x \in A \cup B \vee x \in C}}{x \in A \vee x \in B \vee x \in C}}{x \in A \vee x \in B \cup C}}{x \in A \cup (B \cup C)}}{(A \cup B) \cup C \subseteq A \cup (B \cup C)} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x \in A \cup (B \cup C)}{x \in A \vee x \in (B \cup C)}}{x \in A \vee x \in B \vee x \in C}}{x \in A \cup B \vee x \in C}}{x \in (A \cup B) \cup C}}{A \cup (B \cup C) \subseteq (A \cup B) \cup C}}{(A \cup B) \cup C = A \cup (B \cup C)}$$

□

(b) *Proof.*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x \in (A \cap B) \cap C}{x \in A \cap B \wedge x \in C}}{x \in A \wedge x \in B \wedge x \in C}}{x \in A \wedge x \in B \cap C}}{x \in A \cap (B \cap C)}}{(A \cap B) \cap C \subseteq A \cap (B \cap C)} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x \in A \cap (B \cap C)}{x \in A \wedge x \in (B \cap C)}}{x \in A \wedge x \in B \wedge x \in C}}{x \in A \cap B \wedge x \in C}}{x \in (A \cap B) \cap C}}{A \cap (B \cap C) \subseteq (A \cap B) \cap C}}{(A \cap B) \cap C = A \cap (B \cap C)}$$

□

(c) *Proof.*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x \in A \setminus (A \setminus B)}{x \in A \wedge x \notin (A \setminus B)}}{x \in A \wedge (x \notin A \vee x \in B)}}{x \in A \wedge x \in B}}{x \in A \cap B}}{A \setminus (A \setminus B) \subseteq A \cap B} \quad \dfrac{\dfrac{\dfrac{\dfrac{x \in A \cap B}{x \in A \wedge x \in B}}{x \in A \wedge x \notin A \setminus B}}{x \in A \setminus (A \setminus B)}}{A \cap B \subseteq A \setminus (A \setminus B)}}{A \setminus (A \setminus B) = A \cap B}$$

□