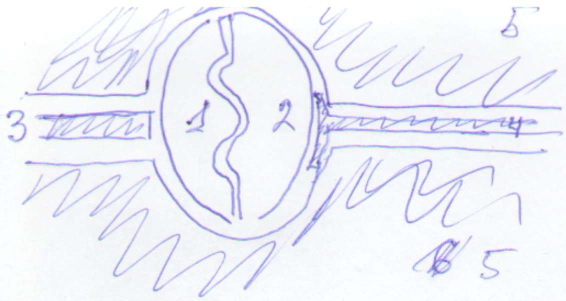
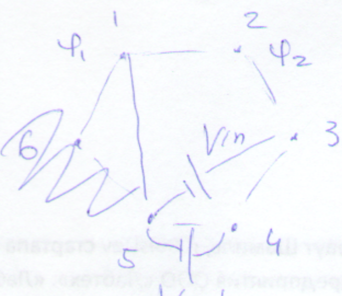
5 [Расчет к двухполюснику.]

Граф K_5

$\boxed{1/3}$





$$\boxed{\varphi_5 = 0}$$

$$\sum J_i^{in} = 0, \forall i \in \{1, \dots, 5\}$$

① : $\cancel{\varphi_1\left(\frac{1}{z_{12}} + \frac{1}{z_{12}} + \right.}$

$$(1)\quad \frac{\varphi_1}{z_1} - \frac{\varphi_2}{z_{12}} - \frac{V_{in}}{z_{13}} - \frac{V_{out}}{z_{14}} = 0$$

$$z_1 = \left(\sum_{i=2}^{5} z_{1i}^{-1}\right)^{-1}$$

$$(2)\quad \frac{\varphi_2}{z_2} - \frac{\varphi_1}{z_{12}} - \frac{V_{in}}{z_{23}} - \frac{V_{out}}{z_{24}} = 0$$

$$(1) \Rightarrow (1')\quad \varphi_1 = \frac{z_1}{z_{12}}\varphi_2 + \frac{z_1}{z_{13}} V_{in} + \frac{z_1}{z_{14}} V_{out}$$

$(1') \to (2)$

$$\frac{\varphi_2}{z_2} - \frac{z_1}{z_{12}^2}\varphi_2 - \frac{z_1}{z_{13} z_{12}} V_{in} - \frac{z_1}{z_{14} z_{12}} V_{out} - \frac{V_{in}}{z_{23}} - \frac{V_{out}}{z_{24}} = 0$$

$$\varphi_2 = \left(\frac{1}{z_2} - \frac{z_1}{z_{12}^2}\right)^{-1}\left[V_{in}\left(\frac{z_1}{z_{13} z_{12}} + \frac{1}{z_{23}}\right) + V_{out}\left(\frac{1}{z_{24}} + \frac{z_1}{z_{14} z_{12}}\right)\right]$$

Где в следствие симметрии достаточно поменять индексы ур-ий (1) и (2) местами $1 \Leftrightarrow 2$

$$\varphi_1 = \left(\frac{1}{z_1} - \frac{z_2}{z_{12}^2}\right)^{-1}\left[V_{in}\left(\frac{z_2}{z_{23} z_{12}} + \frac{1}{z_{13}}\right) + V_{out}\left(\frac{1}{z_{14}} + \frac{z_2}{z_{24} z_{12}}\right)\right]$$

$$\varphi_2 - \varphi_1 = \frac{Z_{12}^2}{Z_{12}^2 - Z_1 Z_2}\Big[V_{in}\Big\{\Big(\frac{Z_1 Z_2}{Z_{13} Z_{12}} + \frac{Z_2}{Z_{23}}\Big) - \Big(\frac{Z_1 Z_2}{Z_{23} Z_{12}} + \frac{Z_1}{Z_{13}}\Big)\Big\} + V_{out}\Big\{\Big(\frac{Z_1 Z_2}{Z_{14} Z_{12}} + \frac{Z_2}{Z_{24}}\Big) - \Big(\frac{Z_1 Z_2}{Z_{24} Z_{12}} + \frac{Z_1}{Z_{14}}\Big)\Big\}\Big]$$

Проверим в Wolfram Mathematica разность $\varphi_2 - \varphi_1$ совпадает с моим ответом.
Файл: Equivalent_scheme_verification.nb раздел

Для импеданса. Ток цепи замкнутой накоротко

$$(1)\quad \frac{\varphi_1}{Z_1} - \frac{\varphi_2}{Z_{12}} - \frac{V_{in}}{Z_{13}} - \frac{V_{out}}{Z_{14}} = J_1^{in}$$

$$(2)\quad \frac{\varphi_2}{Z_2} - \frac{\varphi_1}{Z_{12}} - \frac{V_{in}}{Z_{23}} - \frac{V_{out}}{Z_{24}} = J_2^{in}$$

Решу в предположении известных J_1^{in} и J_2^{in}

$$(1) \Rightarrow (1'):\ \varphi_1 = Z_1 J_1^{in} + \frac{Z_1 \varphi_2}{Z_{12}} + \frac{Z_1 V_{in}}{Z_{13}} + \frac{Z_1 V_{out}}{Z_{14}}$$

$(1') \to (2)$:

$$\varphi_2\Big(\frac{1}{Z_2} - \frac{Z_1}{Z_{12}^2}\Big) = \frac{Z_1}{Z_{12}} J_1^{in} + V_{in}\Big(\frac{Z_2}{Z_{23} Z_{12}} + \frac{1}{Z_{13}}\Big) + \dots + J_2^{in}$$

$$\varphi_2 = \frac{Z_2 Z_{12}^2}{Z_{12}^2 - Z_1 Z_2}\Big(J_1^{in}\frac{Z_1}{Z_{12}} + J_2^{in}\Big) - " - \qquad \varphi_2\big|_{J_i^{in}=0}$$

то же, что и раньше

$$\varphi_2 - \varphi_1 = 0 \qquad J_1^{in} = J \qquad J_2^{in} = -J \qquad \varphi_2 = \frac{Z_1 Z_2}{Z_{12}^2 - Z_1 Z_2} Z_{12} J\Big(\frac{Z_1}{Z_{12}} - 1\Big) + \dots$$

$$\varphi_1 = \frac{Z_1 Z_2}{Z_{12}^2 - Z_1 Z_2} Z_{12} J\Big(-\frac{Z_2}{Z_{12}} + 1\Big)$$

$$\varphi_2 - \varphi_1\big|_{J=0} + \frac{Z_1 Z_2}{Z_{12}^2 - Z_1 Z_2} Z_{12}\Big(\frac{Z_1}{Z_{12}} - 1 - \frac{Z_2}{Z_{12}} + 1\Big) J = 0$$

$$J = \big(\varphi_2 - \varphi_1\big|_{J=0}\big) \times \Big(\frac{Z_{12}^2 - Z_1 Z_2}{Z_1 Z_2} Z_{12}\Big(2 - \frac{Z_1 + Z_2}{Z_{12}}\Big)\Big)$$

$$(1) \Rightarrow (1'): \quad \varphi_1 = Z_1 J_1^{in} + \text{—''—}$$

$$(1') \to (2) \quad \varphi_2 \left(\frac{1}{Z_2} - \frac{Z_1}{Z_{12}^2} \right) = \frac{Z_1}{Z_{12}} J_1^{in} + J_2^{in} + \text{—''—}$$

$$\varphi_2 = \frac{Z_2 Z_{12}}{Z_{12}^2 - Z_1 Z_2} \left(Z_1 J_1^{in} + Z_{12} J_2^{in} \right) + \varphi_2 \Big|_{J_i^{in}=0}$$

$$\varphi_1 = \frac{Z_1 Z_{12}}{Z_{12}^2 - Z_1 Z_2} \left(Z_2 J_2^{in} + Z_{12} J_1^{in} \right) + \varphi_1 \Big|_{J_i^{in}=0}$$

$$\varphi_2 - \varphi_1 = 0 \qquad J_1^{in} = J = -J_2^{in}$$

$$\varphi_2 = \frac{Z_2 Z_{12}}{Z_{12}^2 - Z_1 Z_2} J \left(Z_1 - Z_{12} \right) + \varphi_2 \Big|_{J_i^{in}=0} \qquad \varphi_1 = \frac{Z_1 Z_{12}}{Z_{12}^2 - Z_1 Z_2} J \left(Z_{12} - Z_2 \right) + \varphi_1 \Big|_{J_i^{in}=0}$$

$$\varphi_2 - \varphi_1 = V^{2-1}_{\text{opencircuit}} + J \frac{Z_{12}}{Z_{12}^2 - Z_1 Z_2} \left(Z_2 Z_1 - Z_2 Z_{12} - Z_1 Z_{12} + Z_1 Z_2 \right)$$

$$J = \frac{V^{2-1}_{\text{opencircuit}}}{Z}$$

$$\boxed{Z = \frac{Z_{12}}{Z_{12}^2 - Z_1 Z_2} \left(Z_{12} (Z_1 + Z_2) - 2 Z_1 Z_2 \right) = \frac{Z_{12}}{Z_{12}^2 - Z_1 Z_2} \left[Z_1 (Z_{12} - Z_2) + Z_2 (Z_{12} - Z_1) \right]}$$

$$Z_i \leq Z_{12}$$

Проверим в WM

файл: Equivalent-scheme-verification.nb

раздел: short-circuit-current