VE444: Networks

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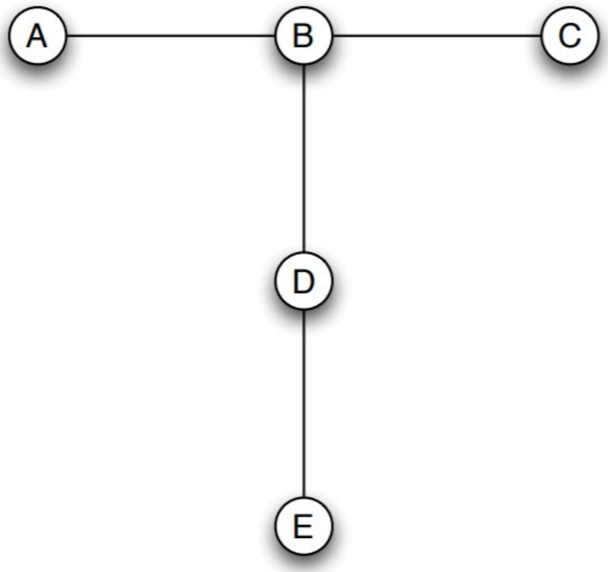
Power

Power

- Is it an individual property?
- Or a result of social relations?
  - Richard Emerson (1962)
  - Social relation between two people produces “values” for them
  - Imbalance of values -> power
  - Division of values: Network exchange theory

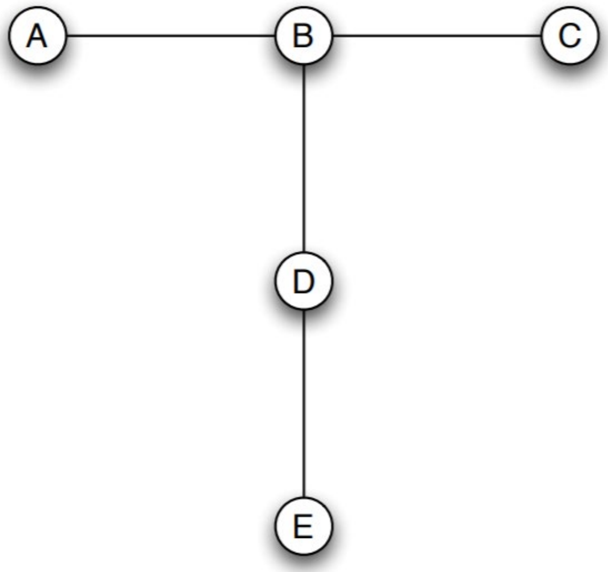
Who is more powerful?

- B
- Why?
  - Dependence: A and C completely depdend on B
  - Exclusion:B can exclude A or C from being his “best friend”
  - Satiation: diminishing rewards for increased amount of something; B will maintain friendship only if he gets a better share
  - Betweenness: unique access point between multiple pairs of nodes



Experimental studies of power

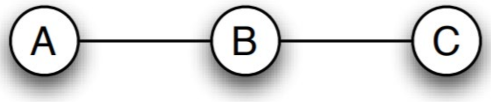
- A small network
- Each individual is a node of the graph
- Each edge contains a fixed amount of $
  - Endpoints negotiate how to split that amount of $
- One-exchange rule: Each node can do transaction with at most one neighbor
  - Results in a matching, which may not be a perfect matching
- This experiment is run for multiple rounds



Experimental results

- 2-Node path
  - Equal deviation

- 3-Node Path
  - $\frac{5}{6} - \frac{1}{6}$
  - Two-exchange: $\frac{1}{2}$-1-$\frac{1}{2}$



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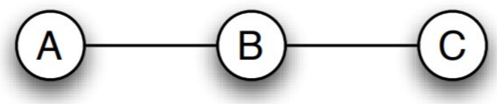
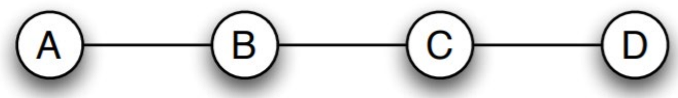


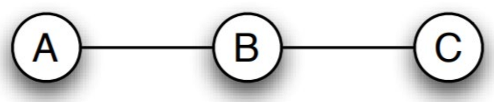
- 4-Node Path
  - A-B exchange: B gets roughly b$etween\ \frac{7}{12}\ and\ \frac{2}{3}$



Experimental results

- 2-Node path
  - Equal deviation

- 3-Node Path
  - $\frac{5}{6} - \frac{1}{6}$ (why not 1-0?)
  - Two-exchange: $\frac{1}{2}$-1-$\frac{1}{2}$



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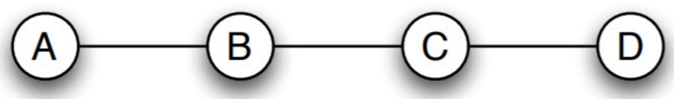
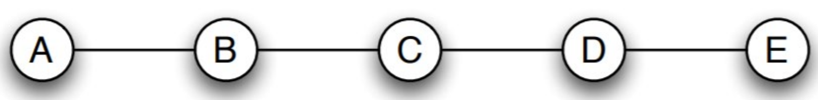
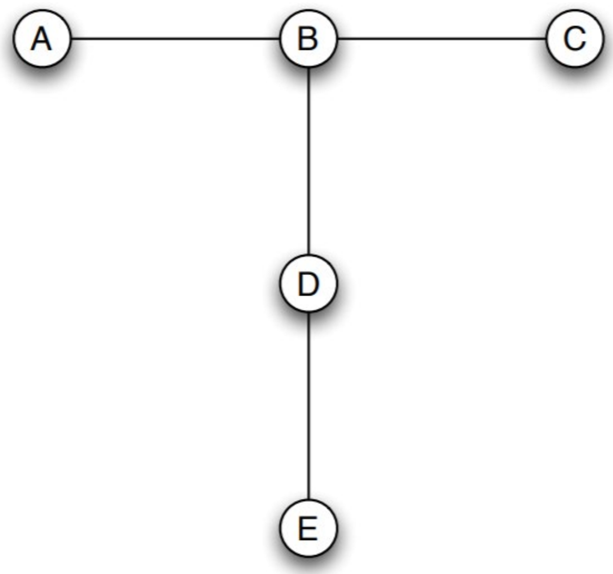


- 5-Node Path
  - Betweenness of C?



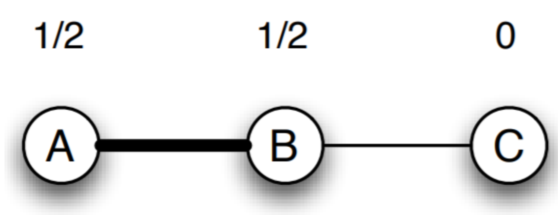
Experimental results

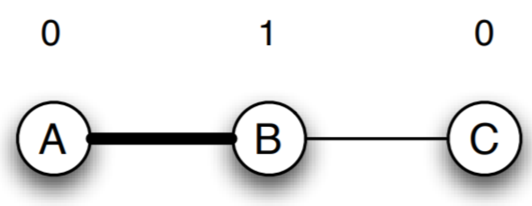


B? D?

Mathematical framework

- Given a exchange graph
- Outcome = (matching, values)
- Stable outcome: no node X can propose an offer to some other node Y that makes both X and Y better off





Instability: Given an outcome consisting of a matching and values for the nodes, an instability in this outcome is an edge not in the matching, joining two nodes X and Y , such that the sum of X's value and Y 's value is less than 1.\

Opportunity + incentive -> unstable

Stable outcomes

- Limitations of stable outcomes
  - Extreme values
    - Ultimatum game

- Ultimatum game
  - Person A is given a dollar and told to propose a division of it to person B. That is, A should propose how much he keeps for himself, and how much he gives to B.
  - Person B is then given the option of approving or rejecting the proposed division.
  - If B approves, each person keeps the proposed amount. If B rejects, then each person gets nothing.

Stable outcomes

- Limitations of stable outcomes
  - Extreme values
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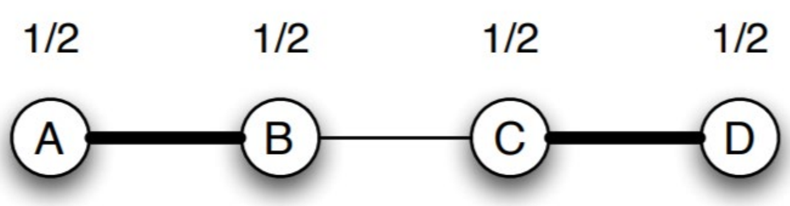
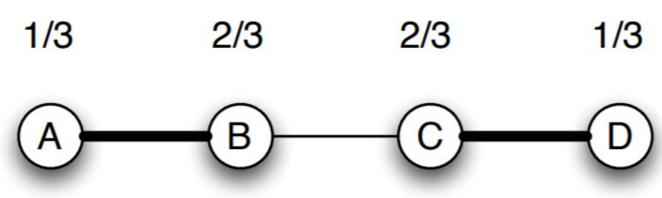
- Ultimatum game
  - If money-maximizer?

- In 1982, Guth, Schmittberger, and Schwarze:
  - An experimental analysis of ultimaturm bargaining, Journal of Economic Behavior and Organization
  - Violate the game-theoretic framework?
  - People play a different game than the one on paper!
  - Stable outcome failed to capture this.
  - More dramatic version: https://www.youtube.com/watch?v=BfE4ZL08twA

Stable outcomes

- Limitations of stable outcomes
  - Extreme values
    - Ultimatum game
  - Ambiguity





Nash bargaining solution

Resolving ambiguity in stable outcome

Rubinstein's Bargaining

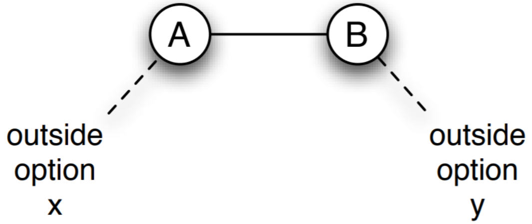
- Bargaining process like this
  - They take turns to make offer
  - A: I'll give you 30% of the dollar
  - B: No, I want 40%
  - A: How about 34%?
  - B: I'll take 36%.
  - A: Agreed.

We want to predict the outcome

Two-person Nash Bargaining

- Nash Bargaining Solution: When A and B negotiate over splitting a dollar, with an outside option of x for A and an outside option of y for B (and x + y ≤ 1), the Nash bargaining outcome is
  - $x+\frac{1}{2}s=\frac{x+1-y}{2}$ to A, and
  - $y+\frac{1}{2}s=\frac{y+1-x}{2}$ to B

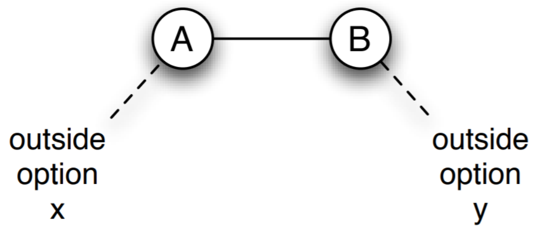
- Surplus: s = 1-x-y



- We will show how Nash Bargaining Solution naturally emerges in the Rubinstein's Bargaining game.

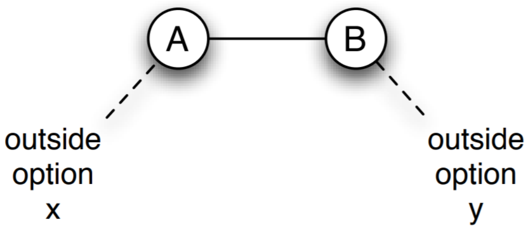
Bargaining as a dynamic game

- Dynamic Game
  - Two people are negotiating over how to split \$1 between them
  - A little extension: They each have outside options
    - $x+y \leq 1$
  - Pressure to reach a deal: probability p that negotiations break down, players forced to accept outside options



- Compared with previous dynamic game
  - Infinite strategy options
  - Infinite-horizon game: sequence of periods can in principle go on forever
    - Backward induction not applicable

Two-person Nash Bargaining: two period

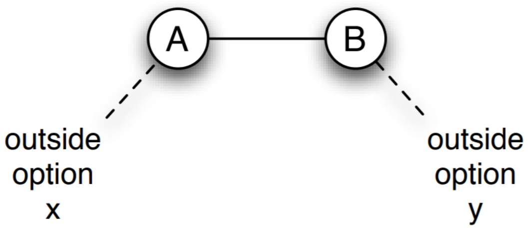


- A sequence of periods
- Two periods
  - (Period 1) A: (a1, b1)? B:
  - (Period 2) B: (a2, b2)? A:
- In period 2:
  - A will accept B's offer if at least x
  - So, B: (x, 1-x)
  - x+y<1 -> B is also ok with this split
  - A : Indifference between outside option x and accepting this offer
- In Period 1:
  - B's expected payoff: py + (1-p)(1-x) if he rejects the offer. Denoted as z
  - A: (1-z, z)
  - Will A has the incentive to offer this?
    - y<1-x
    - z < 1-x
    - 1-z>x

Conclusion :
A will propose (1-z, z) in the first round, and it will be immediately accepted.

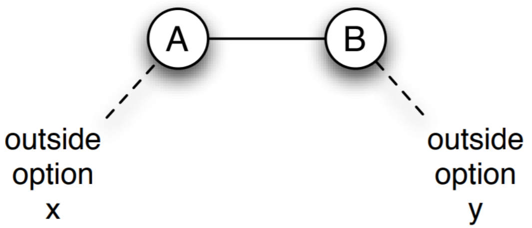
Two-person Nash Bargaining: two period

- Conclusion： A will propose (1-z, z) in the first round, and it will be immediately accepted.
- Discussion:
  - B's expected payoff: py + (1-p)(1-x)
  - When P is close to 1, B's payoff close to his outside option
  - When P is close to 0, A's payoff close to her outside option
  - When P is close to ½, ((x+1-y)/2, (y+1-x)/2)
    - Not general enough



Two-person Nash Bargaining: infinite period

- The structure of this game is stationary over time
- Stationary strategy: the same split in every period in which they are scheduled to propose
  - The split (a1,b1) that A will offer whenever he is scheduled to propose a split;
  - the split (a2,b2) that B will offer whenever she is scheduled to propose a split; and
  - reservation amounts a' and b', constituting the minimum offers that A and B respectively will accept from the other.



- Stationary equilibrium (Stationary strategy)
  - Write a set of equations to describe the stationary strategies in the stationary equilibrium situation
  - Show that when p ->0, Nash Bargaining Solution emerges

Two-person Nash Bargaining: infinite period

- Stationary strategy: the same split in every period in which they are scheduled to propose
  - The split (a1,b1) that A will offer whenever he is scheduled to propose a split;
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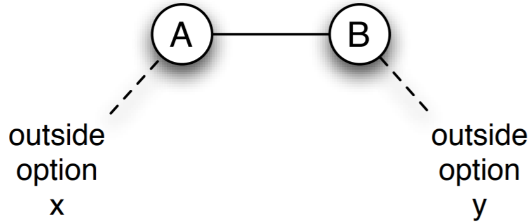
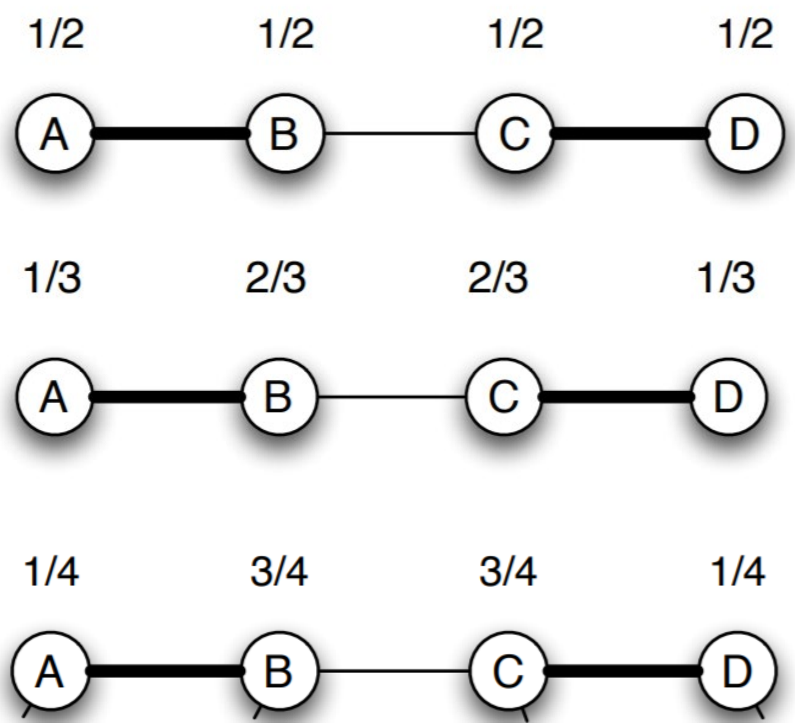
- A will offer B the least he can get B to accept his offer
  - b1=b'
- B will offer A the least he can get A to accept his offer:
  - a2=a'
- B's reservation amount is the indifference amount between accepting A's offer and rejecting A's offer
  - Accept: b1
  - Reject: py+(1-p)b2
  - Indifference: b1=py+(1-p)b2 (1)
- Similarly, a2=px+(1-p)a1 (2)
- Solving (1), (2),
- a1 = $\frac{(1-p)x+1-y}{2-p}$, b1= 1-a1= $\frac{y+(1-p)(1-x)}{2-p}$



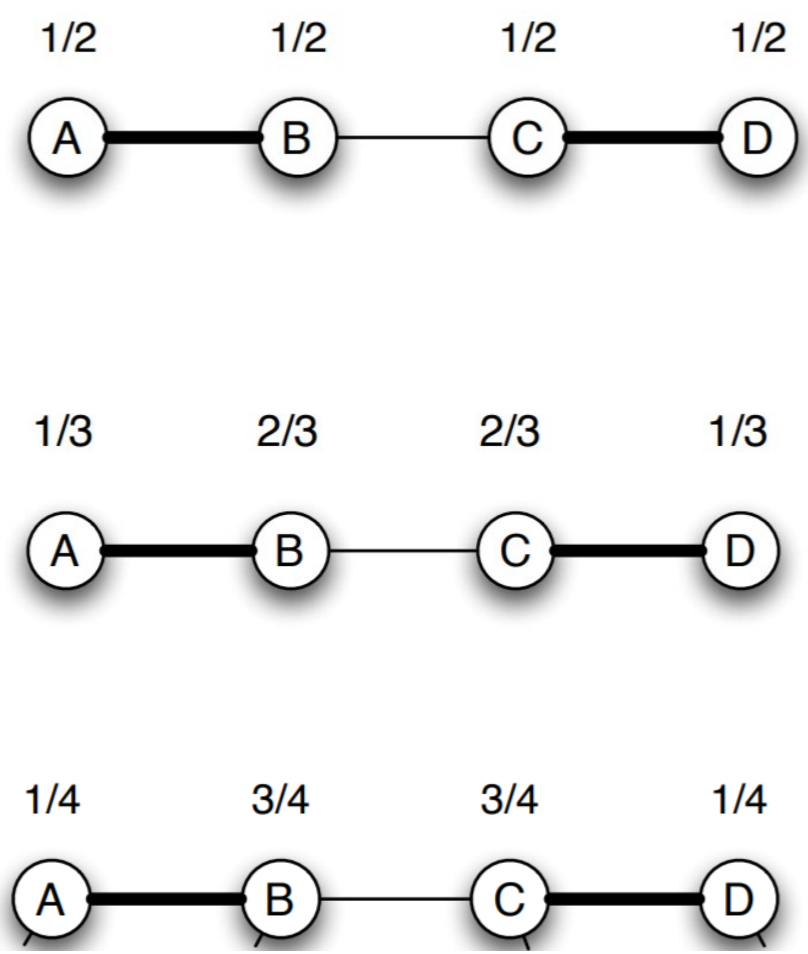
Nash Bargaining solutions?

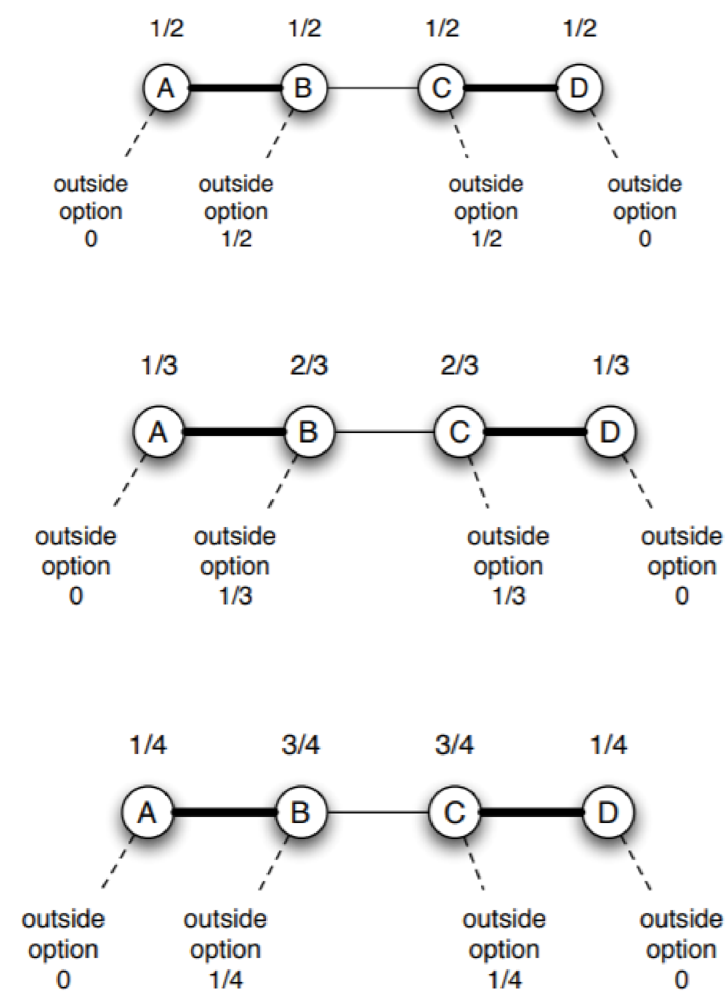
Balanced outcome



Balanced outcome: An outcome (consisting of a matching and node values) is balanced if, for each edge in the matching, the split of the money represents the Nash bargaining outcome for the two nodes involved, given the best outside

Interpretations of Balanced Outcome





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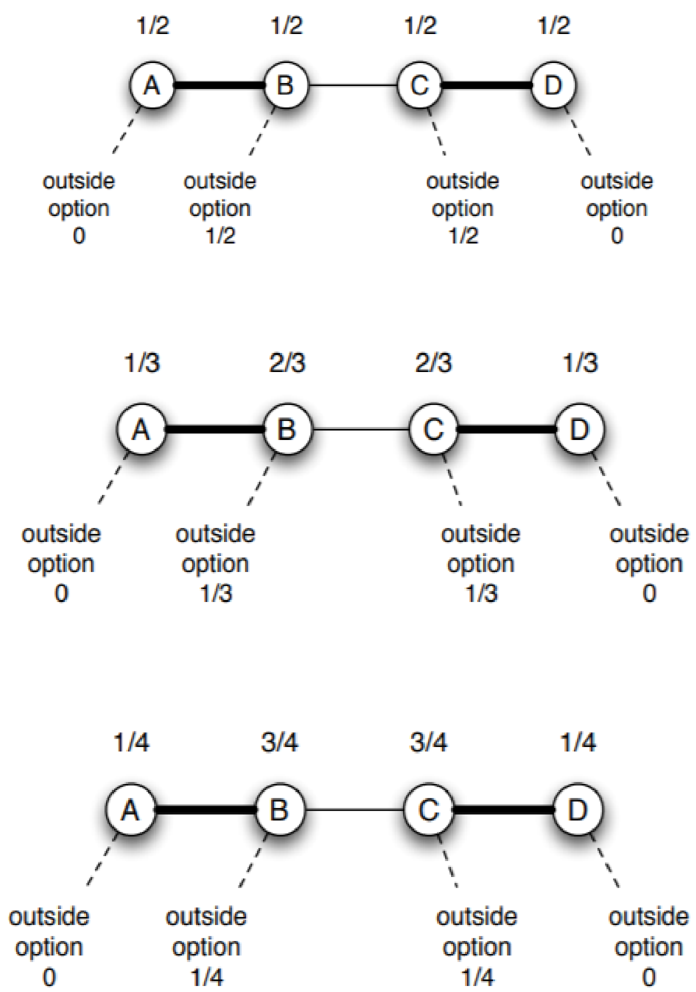


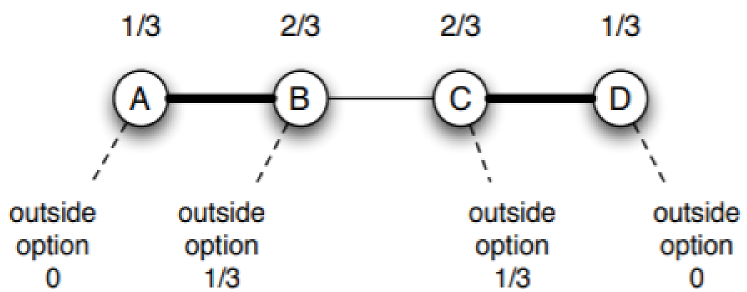
In any network with a stable outcome, there is a balanced outcome

Balance can be used as a way to refine stable outcomes to align with experimental results.

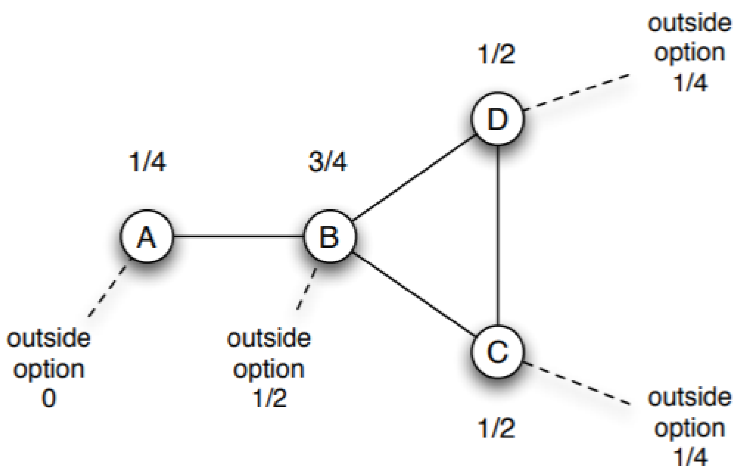
Nash bargaining solution: captures the weak power advantages, and some fairness

Computational issue

Interpretations of Balanced Outcome



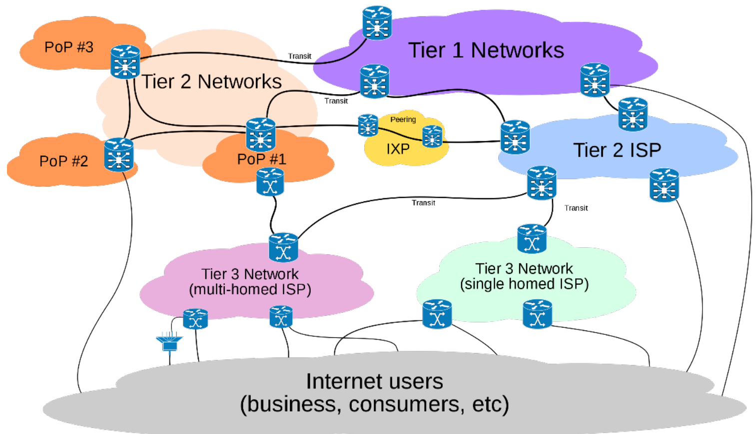
Nash bargaining solution: captures the weak power advantages, and subtle differences because of the network structure



Cooperative game theory: how a collection of players will divide up the value arising from activity

Applications of Nash Bargaining Solution

- Internet Service Providers (ISP): provide connectivity services or "pipe" to transport content
- AT&T, Verizon, Deutsche Telekom



Traffic engineering (TE): adjusting the routing configuration to the traffic to minimize congestion.

Wenjie Jiang, Rui Zhang-Shen, Jennifer Rexford, Mung Chiang. "Cooperative content distribution and traffic engineering in an ISP Network," in ACM SIGMETRICS, 2009.

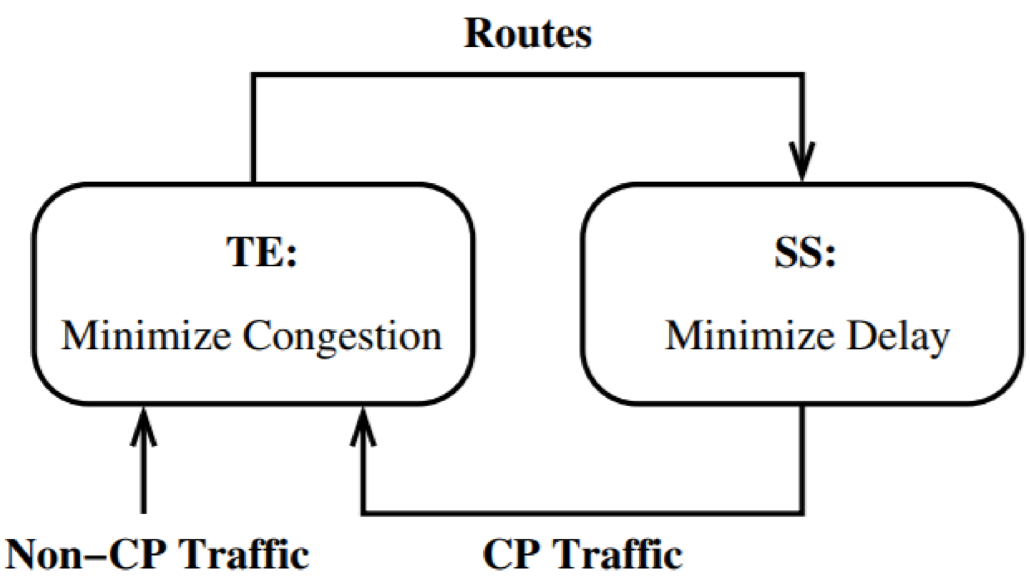
Applications of Nash Bargaining Solution

- Content providers: generate or deliver content to the end users
- Youtube, Akaimai

- Server selection (SS): which servers should deliver content to each end user to minimize network latency, increase throughput

Wenjie Jiang, Rui Zhang-Shen, Jennifer Rexford, Mung Chiang. "Cooperative content distribution and traffic engineering in an ISP Network," in ACM SIGMETRICS, 2009.

The interaction between TE and SS



ISP controls routing matrix, which is constant in SS
CP controls traffic matrix, which is constant in ISP

Wenjie Jiang, Rui Zhang-Shen, Jennifer Rexford, Mung Chiang. "Cooperative content distribution and traffic engineering in an ISP Network," in ACM SIGMETRICS, 2009.

Misalignment between SS and TE

- ISP: minimize congestion
- CP: minimize latency
- But other than CPs, there are background traffic.

Multi-objective misalignment, what should be do?

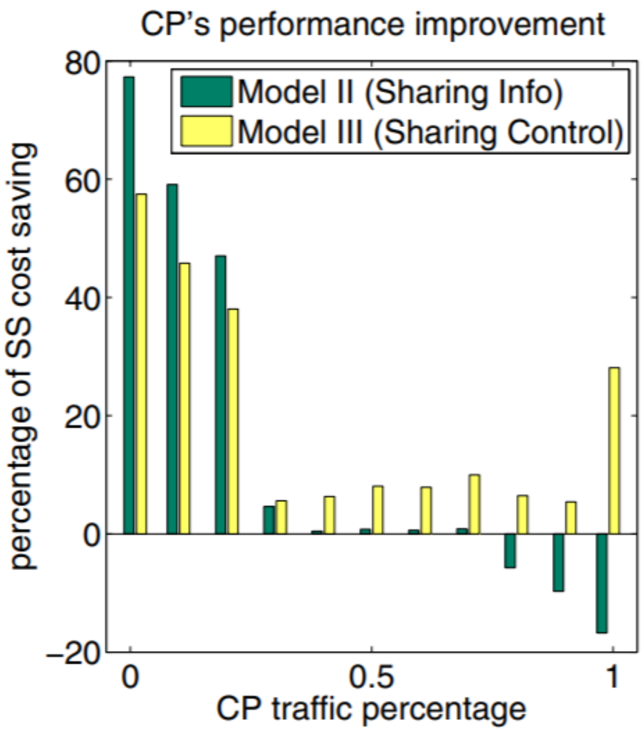
$$\text{minimize} \quad TE + \gamma \cdot SS \text{ ?}$$

- Trial-and-error: impractical and inefficient
- Fair solutions

$$\text{maximize} \quad (TE_0 - TE)(SS_0 - SS)$$

Wenjie Jiang, Rui Zhang-Shen, Jennifer Rexford, Mung Chiang. “Cooperative content distribution and traffic engineering in an ISP Network,” in ACM SIGMETRICS, 2009.

TE and SS performance improvement over selfish non-cooperative



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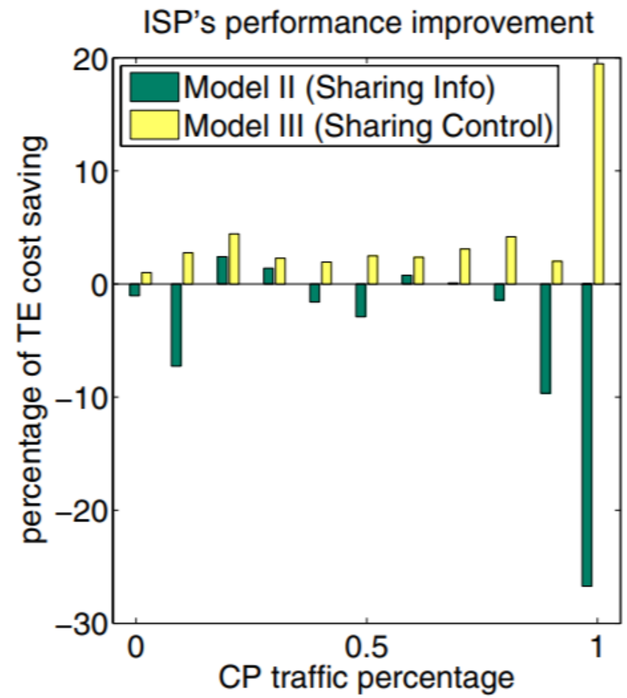

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