VE444: Networks

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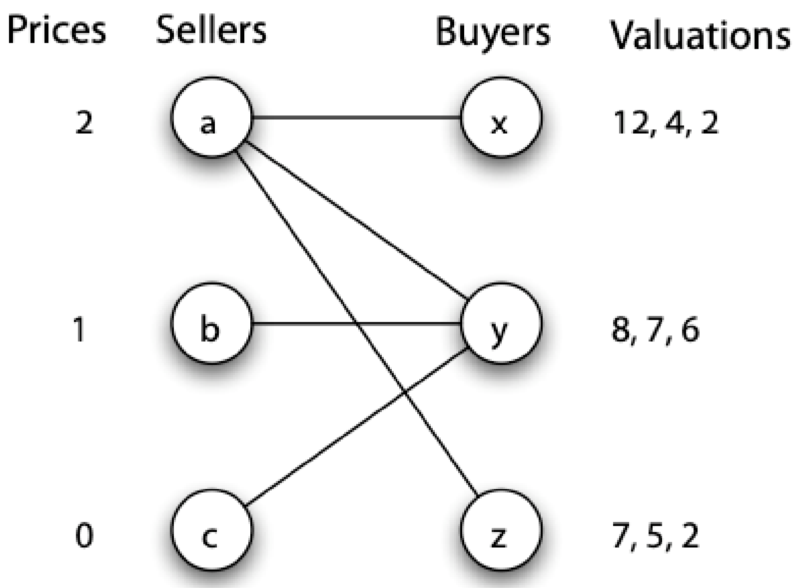
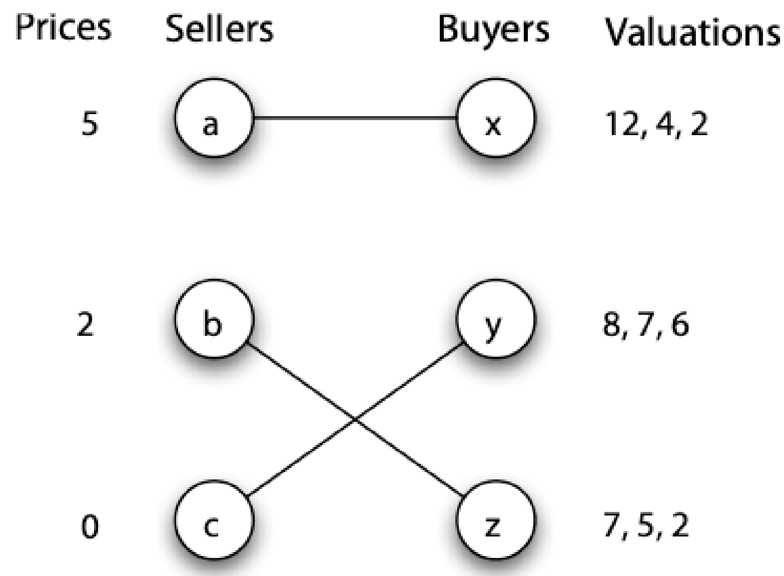
Matching Markets

Market clearing price





A set of price is **market clearing price** if the resulting preferred-seller graph has a perfect matching.

- Coordination over tie-breaking allowed
- Multiple sets of market clearing price could exist

Optimality of the Market-clearing Prices

Optimality of the Market-clearing Prices: For any set of market-clearing prices, a perfect matching in the resulting preferred-seller graph has the maximum total valuation of any assignment of sellers to buyers

Reason:
M: perfect matching
Total Payoff of buyers in M = Total Valuation of buyers in M – Sum of all prices

Optimality of the Market-clearing Prices (v2): For any set of market-clearing prices, a perfect matching in the resulting preferred-seller graph has the maximum sum of payoffs of all sellers to buyers

Existence of Market-clearing prices

Existence of Market-clearing prices: For any set of buyer valuations, there exists a set of market-clearing price.

- Why it always exists?
  - Construct a procedure that stops only when market-clearing prices are found
  - This procedure has limited rounds.

How to find a set of Market-Clearing Prices?

- Two parts: price probing(increase) and price reduction
- Procedure:
  1. At the start of each round, a set of prices, with the smallest one equal to 0
  2. Construct the preferred-seller graph and check whether there is a perfect matching
  3. Stops when perfect matching exists, output current prices
  4. If not, identify a constricted set of buyers, S and their neighbors N(S)
  5. Each seller in N(S) simultaneously raises his price by one unit
  6. Reduce price to guarantee the smallest price equal to 0.
  7. Using the updated price to start a new round

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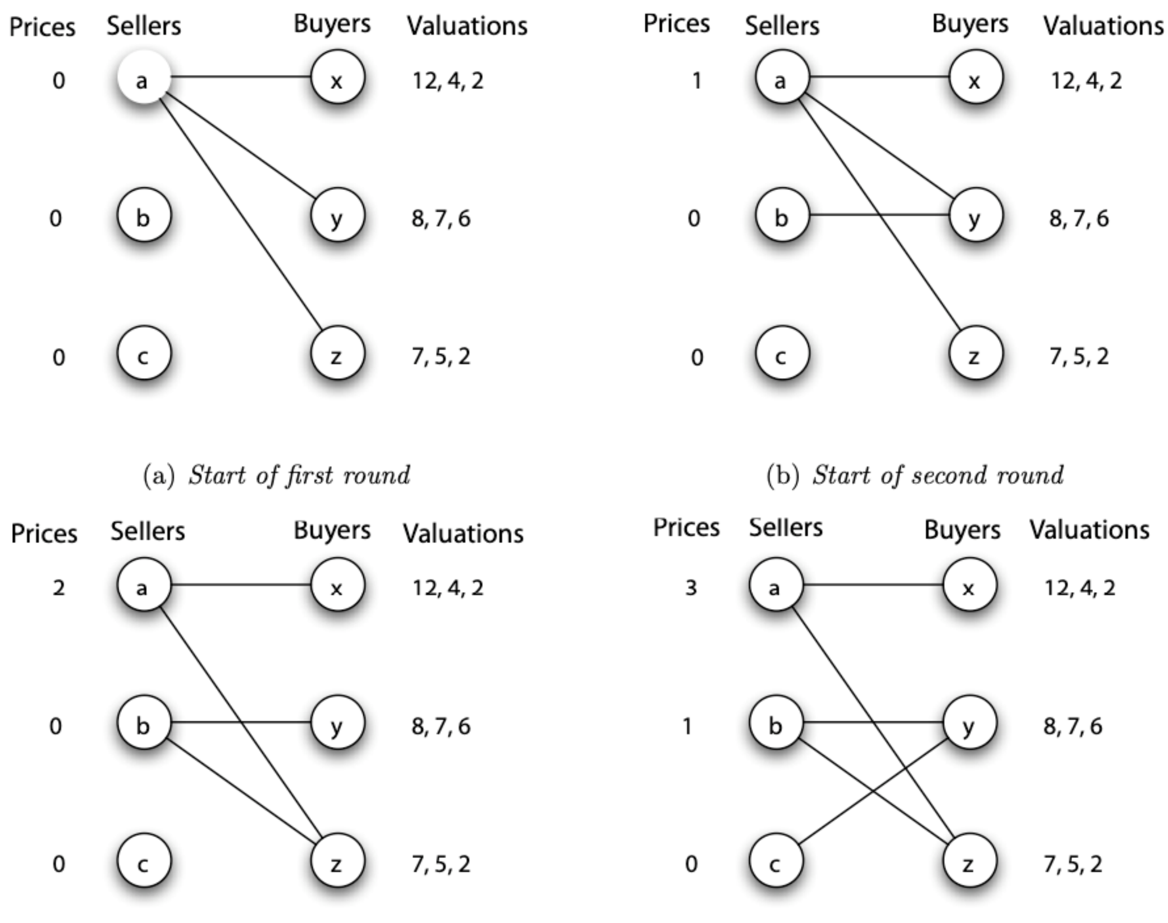


(a) *Start of first round*

(b) *Start of second round*

(c) *Start of third round*

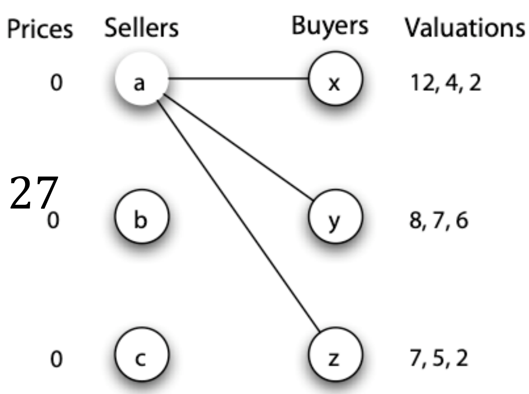
(d) *Start of fourth round*

Existence of the market clearing price

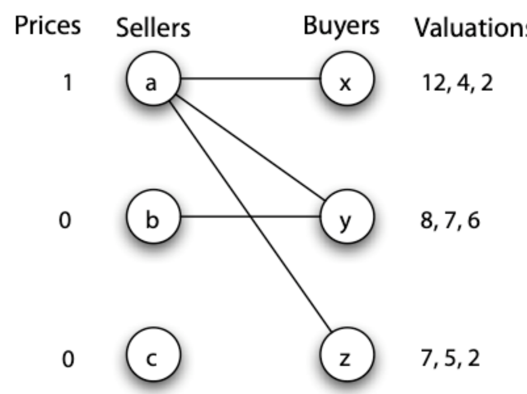
- This procedure stops after finite rounds.
- Potential energy:
  - Potential energy of a buyer: maximum payoff under current prices
  - Potential energy of a seller: current price
  - Potential energy of the auction: sum of potentials of all buyers and sellers
- Potential energy at the begin: P≥ 0
- Potential energy at the start of each round at least zero
- Potential energy only changes when the prices change
  - Price reduction: no change
- Price probing (S: constricted buyer set):
  - Each seller potential goes up by one unit
  - Each buyer potential goes down by one unit
  - But $S \geq N(S)$

Existence of the market clearing price

$P_s: 0$
$P_b: 12 + 8 + 7 = 27$
$P_t: 27$



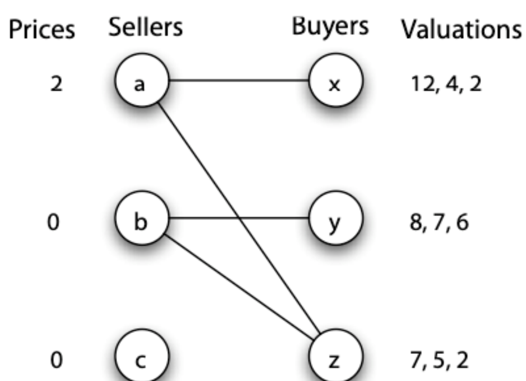
(a) *Start of first round*



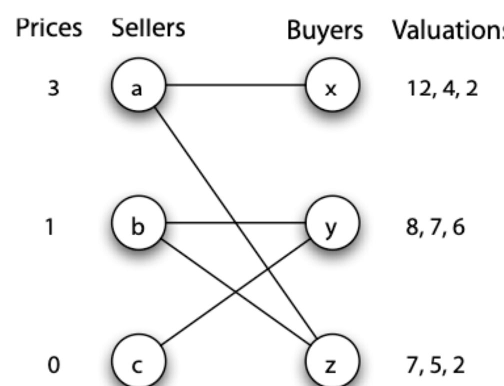
(b) *Start of second round*

$P_s: 1$
P_b:11+7+6=24
P_t:25

$P_s: 2$
P_b:10+7+5=22
P_t:24



(c) *Start of third round*



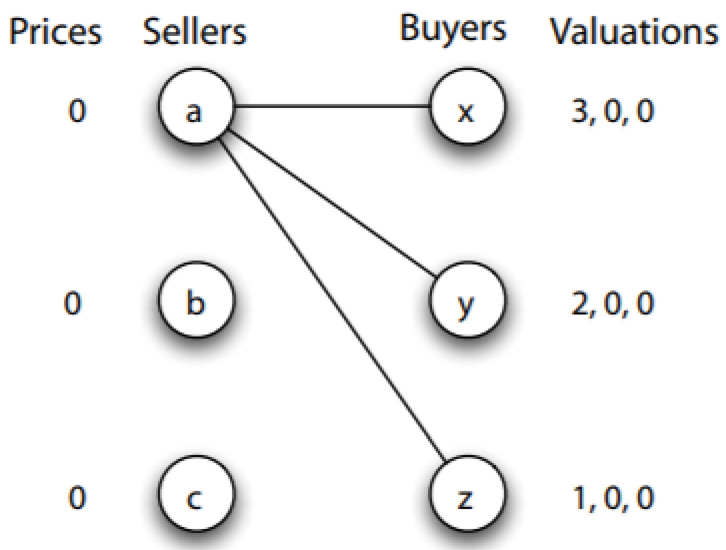
(d) *Start of fourth round*

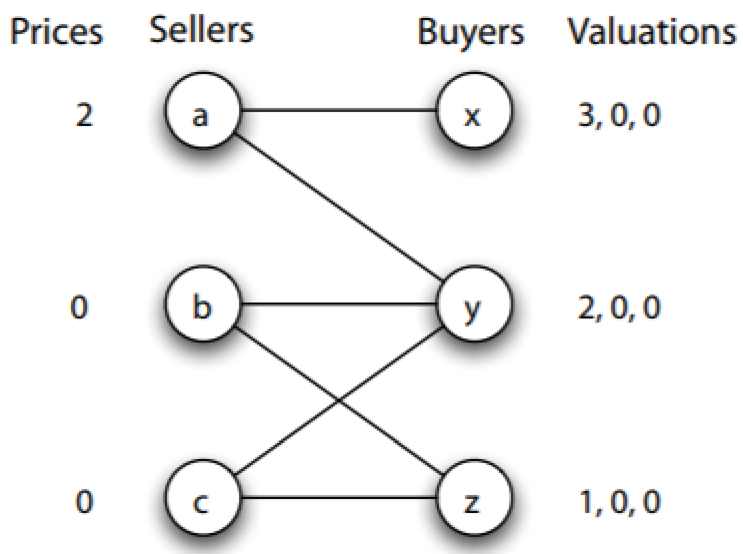
$P_s: 4$
P_b: 9+6+4=19
P_t:23

Relationship with auction?

Relationship with single-item auctions

From one seller, n buyers to perfect matching model





Homework

During the winter break, n students would love to choose a foreign university to do the exchange problem. Each student has a list of universities that they want to go. Naturally, each university has some capacities. For simplicity, let's assume each university can accommodate 3 students at most. *Describe an algorithm that finds if all the students can go to their desired universities for exchange.* (Hint: Embedding the problem into the appropriate graph is the key. The rest is just class content)

Summary

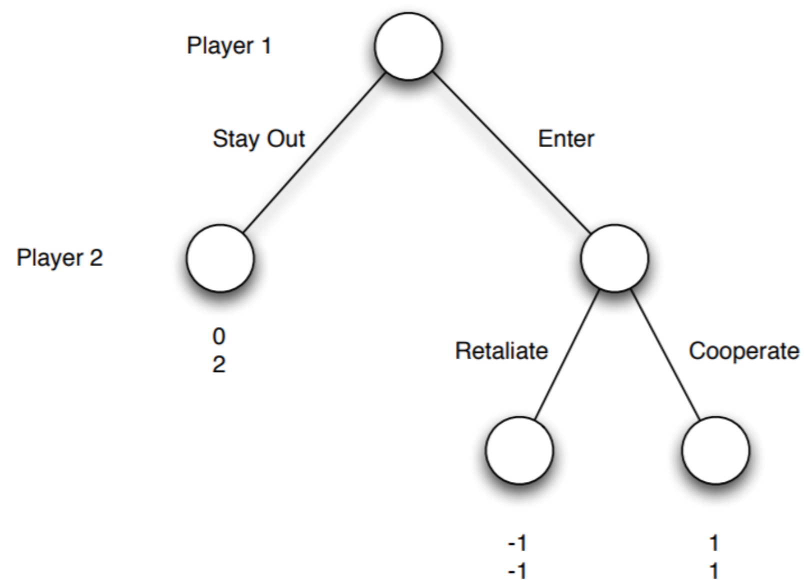
- Perfect matching v.s. constricted set
- Alternating BFS -> perfect matching
  - Augmented path || constricted set
- Existence of the market clearing price
- Optimality of the market clearing price

Dynamic game

- Dynamic game: some player or set of plyaers moves first, others players respond.
  - Example: Board games, negotiations
- Market Entry Game: Consider a region where Firm 2 is currently the only serious participant in a given line of business, and Firm 1 is considering whether to enter the market.
  - The first move in this game is made by Firm 1, who must decide whether to stay out of the market or enter it.
  - If Firm 1 chooses to stay out, then the game ends, with Firm 1 getting a payoff of 0 and Firm 2 keeping the payoff from the entire market
  - If Firm 1 chooses to enter, then the game continues to a second move by Firm 2, who must choose whether to cooperate and divide the market evenly with Firm 1, or retaliate and engage in a price war.
    - If Firm 2 cooperates, then each firm gets a payoff corresponding to half the market
    - If Firm 2 retaliates, then each firm gets a negative payoff.
- Extensive form representation of a game:

Dynamic game

- Extensive form representation of a game:
- Backward induction



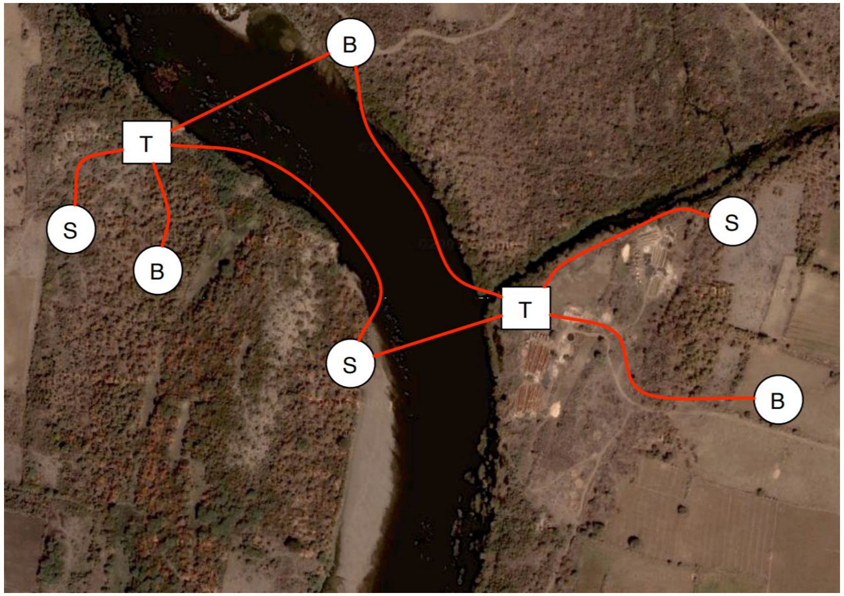
Markets with intermediaries

- Single seller, multiple buyers, private valuations -> auction
- Multiple seller, multiple buyers, direct interaction -> matching
- Trade with Intermediaries: individual buyers and sellers trade through intermediaries
  - Example: Agricultural goods trading, financial assets markets

Network model

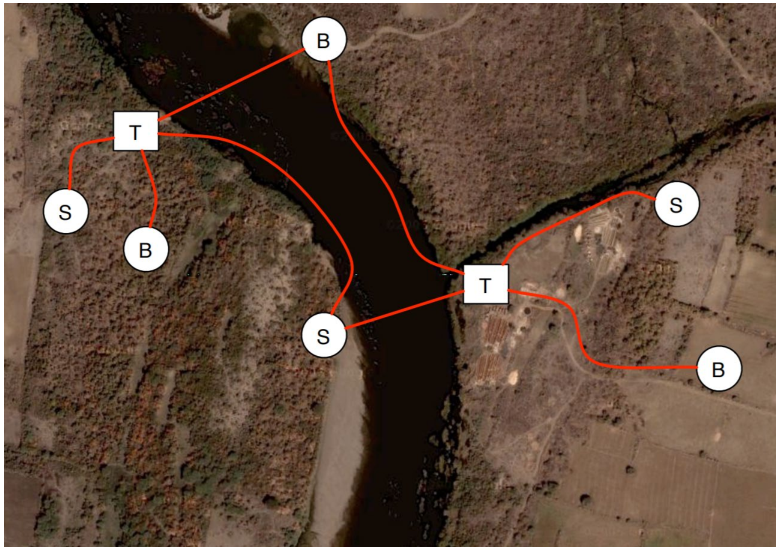
- Individual buyers and sellers often trade through intermediaries
- Not all buyers and sellers have access to the same intermediaries
- Not all buyers and sellers trade at the same price

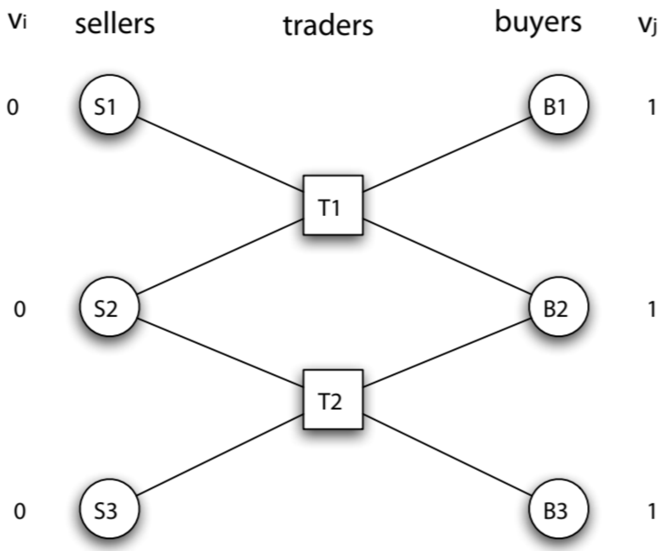
Example: Agricultural trading in a developing country



Poor transportation networks
Perishability of the products

Network model





Single type
Indivisible units
Valuation known to all buyers, sellers, and traders
Difference between matching:

- Intermediaries
- Same valuation
- Network is fixed

Prices in the Network model

Dynamic game: traders first set prices, sellers and buyers react to these prices

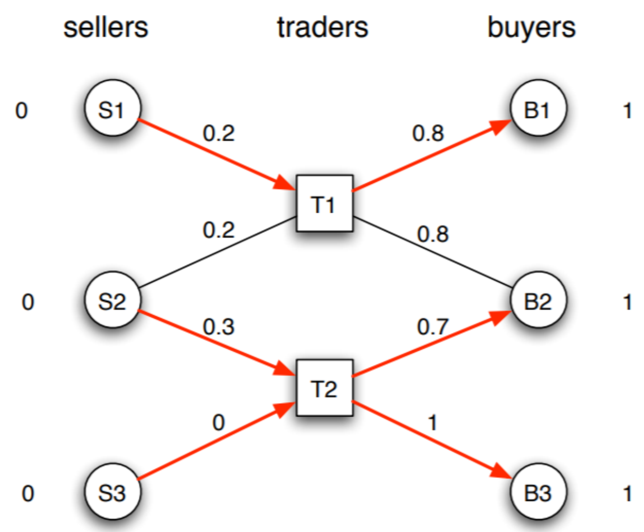
Trader's strategy: choice of bid b_{ti} and ask prices a_{tj}

Buyer/Seller's strategy: which trader to deal with or stay out of the transaction

Indifference exist: tie-breaking

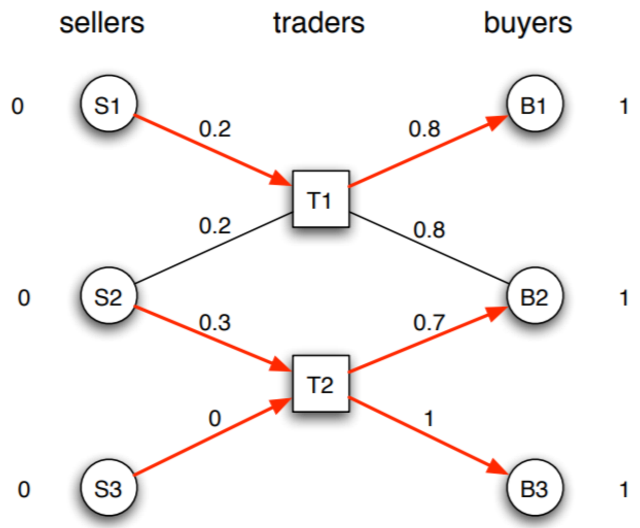
Payoff for players

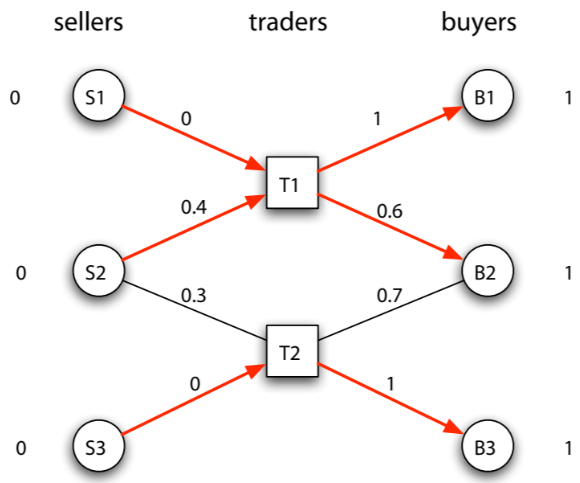
Seller's payoff:Trade b_{ti}, no trade v_i (=0 currently);
Buyer's payoff:Trade: $v_j - a_{tj}$; No Trade: 0
Trader's payoff: sum of askes – sum of bids



Equilibrium?

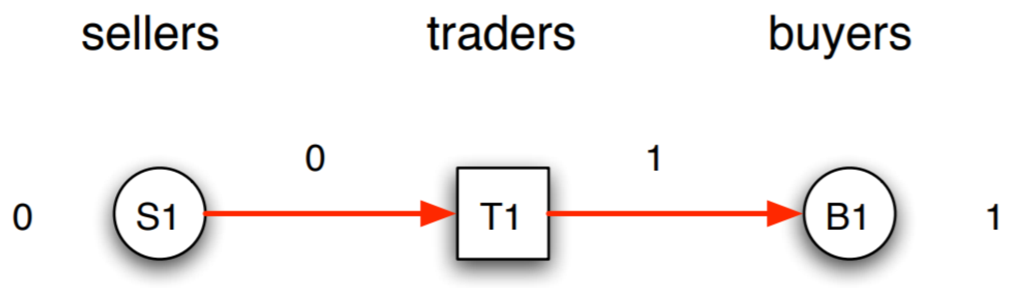
T1 to S2 and B2
T1 to S1 and B1



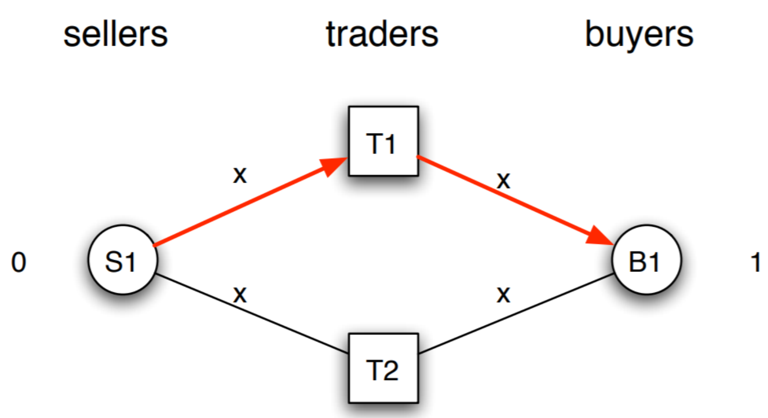


Simple building blocks

Monopoly case:

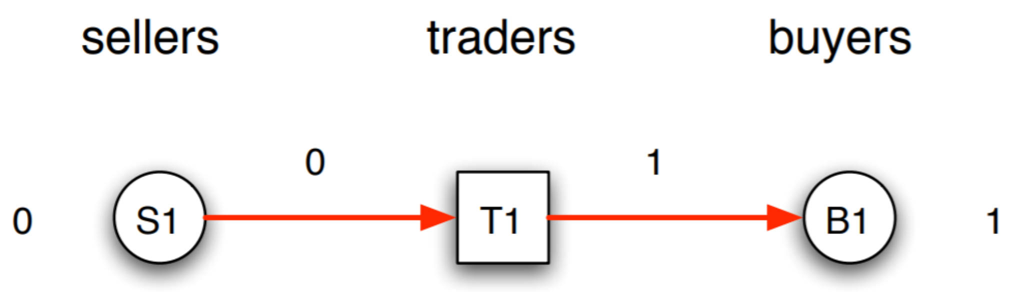


Perfect competition:



Reasoning

Monopoly case:



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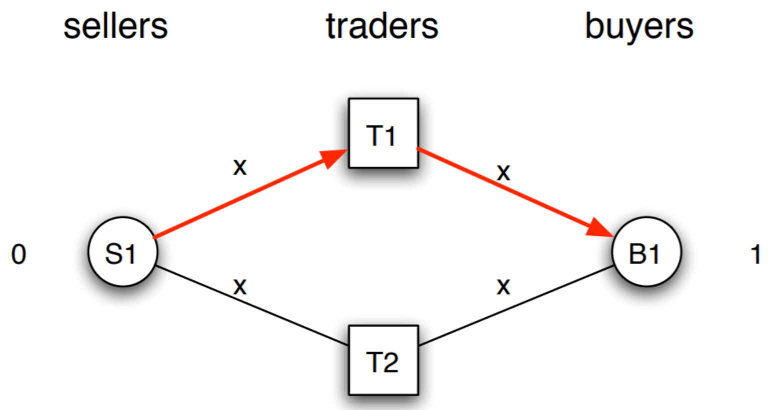

1. Trader performing the trade at the equilibrium must have a payoff of 0
2. Trader not performing the trade must also have bid and ask values of x

The equilibria for the trading network