VE444: Networks

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Game Theory Basics

Interconnectedness at two levels

- **Structural level:**
  - What are the properties of a graph
  - How graph evolve over time
- **Behaviour level:**
  - The outcome for any one in a connected system depends on the combined behaviours of all

Game theory

- Modern game theory started by the works of von Neumann in 1928

Applications of Game Theory

- Theory developed mainly by mathematicians and economists
  - contributions from biologists
- Widely applied in many disciplines
  - from economics to philosophy, including computer science (Systems, Theory and AI)
  - goal is often to understand some phenomena
- “Recently” applied to computer networks
  - Nagle, RFC 970, 1985
    - “datagram networks as a multi-player game”
  - paper in first volume of IEEE/ACM ToN (1993)
  - wider interest starting around 2000

Starting from an example

- Presentation or exam
  - Assume you have two things to do before a deadline tomorrow: study for exam or prepare presentation, and you can do only one
  - Study for exam is predictable, if you study, your grade will be 92, no study, 80
  - Presentation depends on both you and your partner
    - If both prepare presentation, each of you will get 100
    - If only one prepare presentation, each of you will get 92
    - If none of you prepare presentation, each of you will get 84
  - What do you do? (we assume that you and your partner make decisions independently)

Game between Presentation and Exam

- Assume that both of you try to maximize your grades
  - You and your partner all prepare presentation → average grade is (80 + 100) / 2 = 90
  - You and your partner all study the exam → average grade is (92 + 84) / 2 = 88
  - One studies exam and one prepares presentation →
    - The one prepares presentation (80 + 92) / 2 = 86
    - The one studies exam (92 + 92) / 2 = 92

- Question: what would you do?

Basic ingredients of a Game

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

- Player: a set of participants
- Strategies: a set of actions each player could choose
- Utility: assigned values to preferences over *goods*
- Payoff: the utility of an outcome
- $U_{row}, U_{collumn}$

Underlying assumption

- Utility:
  - Given choices, can tell which one is better
  - Consistent, i.e., A > B, B > C, and he must say, A > C
- Rationality: Utility maximization
- Complete information
  - Know the underlying structure: who, strategy set, payoff
  - John Harsanyi, games with incomplete information, 1994 Nobel Prize in Economics
- Independent
  - Make decision independently, no coalition, etc

Reasoning about behavior

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	$90, 90$	$86, 92$
	Exam	$92, 86$	$88, 88$

- Best response: $U_1(S,T) \geq U_1(S',T)$
- Strictly dominant strategy: a strategy is strictly better than all other options, regardless of what the other player does
- Equilibrium point: a set of strategies we expect players to adopt

Why not presentation?

		Your Partner: Presentation	Your Partner: Exam
You	Presentation	90, 90	86, 92
	Exam	92, 86	88, 88

- It is natural if you ask a question, why not (presentation, presentation)
- It cannot stable there, the two players have the incentive to change if it is at (presentation, presentation)
- Our reasoning is rigid, otherwise it violates our assumptions (players are rational)

Prisoner's Dilemma

- Suppose that two suspects have been apprehended by the police and are being interrogated in separate rooms
- The police strongly suspect that these two individuals are responsible for a robbery, but there is not enough evidence to convict either of them of the robbery.
- Each of the suspects is told
  - If you confess, and your partner doesn‘t, then you will be released and your partner will be charged and sent to prison for 10 years.
  - If you both confess, then you will both be convicted and sent to prison for 4 years
  - Finally, if neither of you confesses, then you will be charged for 1 year of resistance.
- Do you want to confess or not?

Prisoner's Dilemma

		Suspect 2: NC	Suspect 2: C
Suspect 1	NC	$-1, -1$	$-10, 0$
Suspect 1	C	$0, -10$	$-4, -4$

- The strictly dominant strategy for both suspect 1 and 2 is confess
- Even though if two NC, they will be sentenced less

Best response vs Dominant strategy

		Suspect 2: NC	Suspect 2: C
Suspect 1	NC	$-1, -1$	$-10, 0$
	C	$0, -10$	$-4, -4$

- **Best response** targets at a single strategy of the opponent (T), and is among all strategies of himself
  - For different T, there may be different best response
- **Dominant strategy** targets at all strategies of the opponent, best response targets at one strategy of the opponent
  - If one has strict dominant strategy, we can assume that he will take it (follow our basic assumptions on game)

Optimization v.s. Game Theory

- Optimization v.s. game theory:
  - Optimization: Suppose you are looking for a particular store in an unfamiliar mall.
  - Game: Suppose you are looking for your lost friend in a mall. Should you stay in a central location so that your friend can find you?

Performance enhancing drugs

		Athlete 2	
		Don't Use Drugs	*Use Drugs*
Athlete 1	*Don't Use Drugs*	3, 3	1, 4
	Use Drugs	4, 1	2, 2

Performance enhancing drugs

		Athlete 2	
		Don't Use Drugs	*Use Drugs*
Athlete 1	*Don't Use Drugs*	3, 3	1, 4
	Use Drugs	4, 1	2, 2

Also called *arms races game*: no good or even harmful for each one internally, but to make sure that each is competitive, have to stay in the competition.

Summary

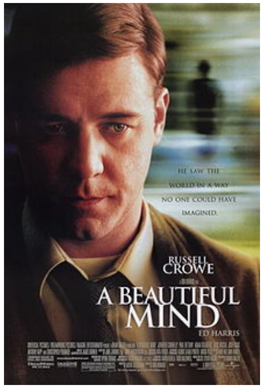
- If both have strictly dominant strategy, they will both adopt their respective strictly dominant strategy
- If one has strictly dominant strategy, he will choose this one and the opponent will choose the best response for this strictly dominant strategy (there must be one!)

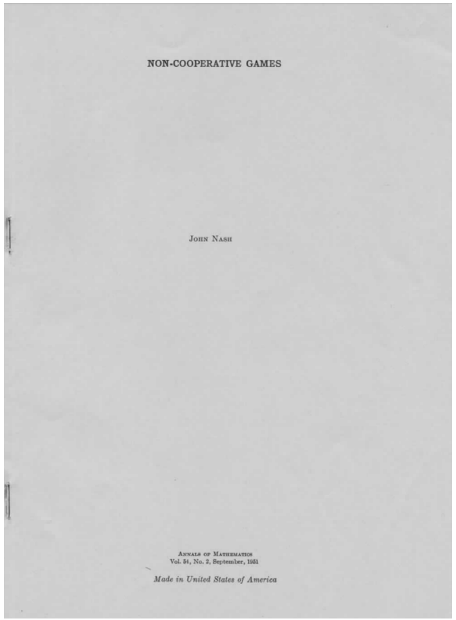
- What if neither has strictly dominant strategy? Where the reasoning starts?

Nash Equilibrium

John Nash

- A legendary life of John Nash
  - Born June 13, 1928
  - B.S and M.S in mathematics at 1948
  - Ph.D, 1950, 28 pages dissertation




NON-COOPERATIVE GAMES

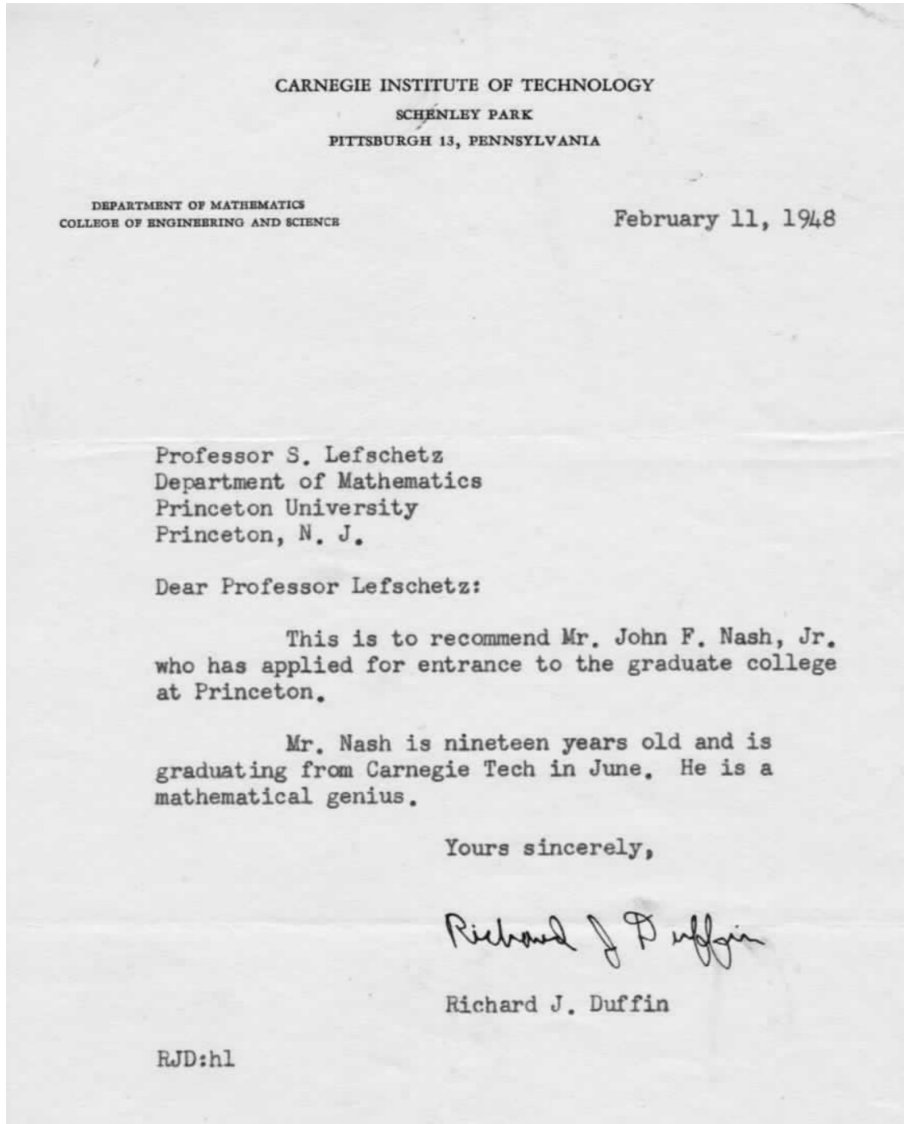
John Nash

Annals of Mathematics
Vol. 54, No. 2, September, 1951

Made in United States of America

https://web.archive.org/web/20170607041209if_/https://webspace.princeton.edu/users/mudd/Digitization/AC105/AC105_Nash_John_Forbes_1950.pdf

John Nash



CARNEGIE INSTITUTE OF TECHNOLOGY
SCHENLEY PARK
PITTSBURGH 13, PENNSYLVANIA

DEPARTMENT OF MATHEMATICS
COLLEGE OF ENGINEERING AND SCIENCE

February 11, 1948

Professor S. Lefschetz
Department of Mathematics
Princeton University
Princeton, N. J.

Dear Professor Lefschetz:

This is to recommend Mr. John F. Nash, Jr. who has applied for entrance to the graduate college at Princeton.

Mr. Nash is nineteen years old and is graduating from Carnegie Tech in June. He is a mathematical genius.

Yours sincerely,

Richard J. Duffin

Richard J. Duffin

RJD:hl

An example with no dominant strategy

- A three client game
  - Two firms that each hope to do business with one of three clients, A, B, C. Each firm has three possible strategies: whether to approach A, B, or C. The results of their two decisions will work out as follows

An example with no dominant strategy

- If the two firms approach the same client, then the client will give half its business to each.
- Firm 1 is too small to attract business on its own, so if it approaches one client while Firm 2 approaches a different one, then Firm 1 gets a payoff of 0.
- If Firm 2 approaches client B or C on its own, it will get their full business. However, A is a larger client, and will only do business with the firms if both approach A.
- Because A is a larger client, doing business with it is worth 8 (and hence 4 to each firm if it's split), while doing business with B or C is worth 2 (and hence 1 to each firm if it's split).

Reasoning

- The payoff matrix

		Firm 2		
		A	B	C
	A	$4, 4$	$0, 2$	$0, 2$
Firm 1	B	$0, 0$	$1, 1$	$0, 2$
	C	$0, 0$	$0, 2$	$1, 1$

- There is no strictly dominant strategy for any firm
- How should we expect the outcome?

Nash Equilibrium

- Assume player 1 selects strategy S, and player 2 selects strategy T. If S is the best response of T, and T is the best response of S, then we say that strategy group (S, T) is a Nash Equilibrium
  - In Nash Equilibrium, no one has the incentive to change their strategy

- Nash Equilibrium: (best responses to each other) No one can become better by unilaterally change his own strategy; though both can become better if both changes.

Reasoning

- The payoff matrix

		Firm 2		
		A	B	C
	A	$4,4$	$0,2$	$0,2$
Firm 1	B	$0,0$	$1,1$	$0,2$
	C	$0,0$	$0,2$	$1,1$

- Which outcome is the Nash Equilibrium point?
- Find Nash Equilibrium
  - Check every strategy-pair, and see if it is the best response for each other
  - Find best response(s), and find the mutual best responses

Reasoning

- The payoff matrix

		Firm 2		
		A	B	C
	A	$4,4$	$0,2$	$0,2$
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	C	$0,0$	$0,2$	$1,1$

- Which outcome is the Nash Equilibrium point?
- Find Nash Equilibrium
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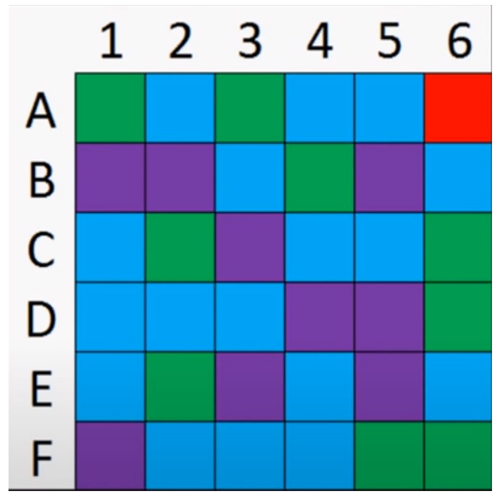
Coordination game

- Pick a square



- If we both select the same square, we win $20

Coordination game

- Pick any day of the year
- If we both select the same day, we win $20

- Your answer?

Coordination game

- Player’s shared goal is to coordinate on the same strategy
  - Example
    - You and your partner are each preparing slides for a joint project presentation (assume you can't reach your partner by phone, do things independently)
    - You have to decide whether to prepare your half of the slides in PowerPoint or in Apple's Keynote software.
    - Either would be fine, but it will be much easier to merge your slides with your partner's if you use the same software

Multiple Equilibria: Coordination game

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	1, 1

- There are two Nash Equilibria (PPT, PPT), (Keynote, Keynote)
- What do players do?
- Schelling focal point model, Social conventions, etc
- Balanced Coordination v.s. unbalanced coordination

Battle of sexes

		Your Partner: *PowerPoint*	Your Partner: *Keynote*
You	*PowerPoint*	1, 2	0, 0
	Keynote	0, 0	2, 1

- Assume you and your partner like different software
- It is difficult to predict what will happen purely based on the structure of game
- Additional information may be needed

Mixed strategy

		Player 2 H	Player 2 T
Player 1	H	$-1, +1$	$+1, -1$
Player 1	T	$+1, -1$	$-1, +1$