

# VE444: Networks

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# PageRank: The “Flow” Model

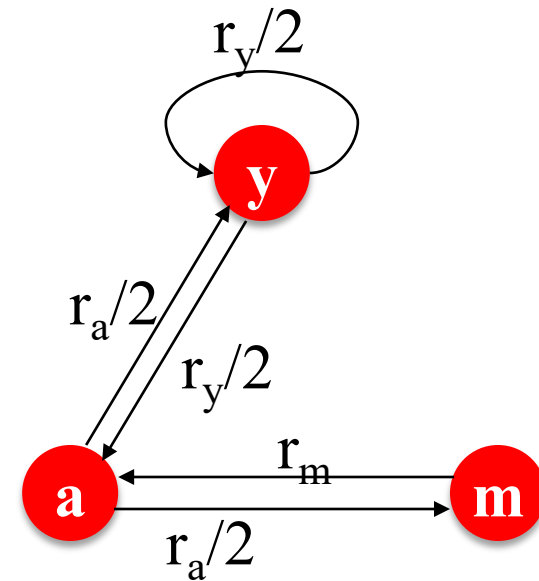
- A page is important if it is pointed to by other important pages
- Define a “rank”  $r_j$  for node  $j$

$$r_j = \sum_{i \rightarrow j} \frac{r_i}{d_i}$$

$d_i$  ... out-degree of node  $i$

You might wonder: Let’s just use Gaussian elimination to solve this system of linear equations. Bad idea!

The web in 1839



“Flow” equations:

$$r_y = r_y/2 + r_a/2$$

$$r_a = r_y/2 + r_m$$

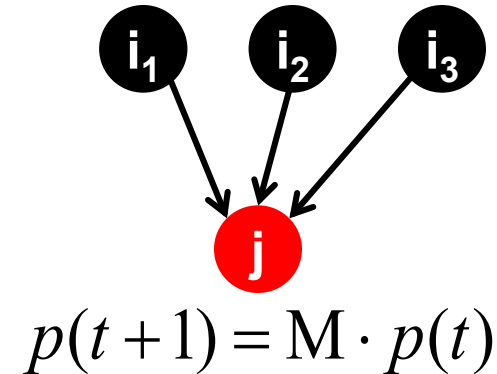
$$r_m = r_a/2$$

# Random walk

- Where is the surfer at time  $t+1$ ?

- Follow a link uniformly at random

$$\mathbf{p}(t+1) = \mathbf{M} \cdot \mathbf{p}(t)$$



- Suppose the random walk reaches a state

$$\mathbf{p}(t+1) = \mathbf{M} \cdot \mathbf{p}(t) = \mathbf{p}(t)$$

then  $\mathbf{p}(t)$  is **stationary distribution** of a random walk

- Our original rank vector  $\mathbf{r}$  satisfies  $\mathbf{r} = \mathbf{M} \cdot \mathbf{r}$

- So,  $\mathbf{r}$  is a stationary distribution for the random walk

# Solution: Random Teleports

- Google's solution that does it all:

At each step, random surfer has two options:

- With probability  $\beta$ , follow a link at random
- With probability  $1-\beta$ , jump to some random page

- **PageRank equation** [Brin-Page, 98]

$$r_j = \sum_{i \rightarrow j} \beta \frac{r_i}{d_i} + (1 - \beta) \frac{1}{N}$$

$d_i$  ... out-degree of node  $i$

This formulation assumes that  $M$  has no dead ends. We can either preprocess matrix  $M$  to remove all dead ends or explicitly follow random teleport links with probability 1.0 from dead-ends.

# Link analysis in Modern Web search

- Combining Links, text, and usage data
  - “I am a student at [SJTI-JI](#)”. (Anchor text)
  - “For more information, [click here](#)”



# Link analysis in Modern Web search

- Current ranking functions are constantly evolving
- Always expect the world to react to what you do (Game theoretic view)
  - Search Engine Optimization/Marketing(SEO, SEM)
  - An example last week

## Links as Votes

- **Idea: Links as votes**
  - Page is more important if it has more links
    - In-coming links? Out-going links?
- **Think of in-links as votes:**
  - [www.stanford.edu](http://www.stanford.edu) has 23,400 in-links
  - [www.nobodyhomepg.com](http://www.nobodyhomepg.com) has 1 in-link
- **Are all in-links equal?**
  - Links from important pages count more
  - Recursive question!

**Ranking Nodes on the Graph**

- All web pages are not equally “important”  
[www. .com](http://www. .com) vs. [www.stanford.edu](http://www.stanford.edu)
- There is large diversity in the web-graph node connectivity.
- So, let's rank the pages using the web graph link structure!

The slide features a network graph diagram on the right side, showing nodes connected by edges. A person is standing next to the slide, and a small video player icon is visible in the bottom right corner.

<https://www.bilibili.com/video/BV1jE41177A4?p=13>

# Sponsored Search Markets

# Google revenue breakdown

- In 2019, Google made \$161.85 billion in revenue
- Out of this amount, \$134.81 billion from advertising!

70.9%



# Sponsored search market

## ■ The search advertising market

The screenshot shows a Google search interface with the query 'keuka lake'. The search bar includes a 'Search' button and links to 'Advanced Search' and 'Preferences'. Below the search bar, a message states 'Customized based on recent search activity. More details'. The results are categorized under 'Web' and 'Books'. The top organic results include 'Welcome to The Keuka Lake Wine Trail', 'A complete guide to the Keuka Lake Wine Country', and 'Keuka Lake - Wikipedia, the free encyclopedia'. A sidebar on the right titled 'Sponsored Links' contains three advertisements: 'Keuka Lake Lodging', 'Keuka Lake Real Estate', and 'Finger Lakes Real Estate'.

Google   [Advanced Search](#) [Preferences](#)

Customized based on recent search activity. [More details](#)

Web [Books](#) Results 1 - 10 of about 381,000 for keuka lake [\[definition\]](#). (0.19 seconds)

**Welcome to The Keuka Lake Wine Trail**  
Information about seven wineries on **Keuka Lake** in the Finger Lakes district. Offers a trail map, event calendar, winery descriptions, tourist services, ...  
[www.keukawinetrail.com/](http://www.keukawinetrail.com/) - 13k - [Cached](#) - [Similar pages](#) - [Note this](#)

**A complete guide to the Keuka Lake Wine Country**  
your own, follow the **Keuka Lake** Wine Trail, or book a wine tour and leave the driving to a pro. From casual to gourmet, hotdogs to haute cuisine, ...  
[www.keukalake.com/](http://www.keukalake.com/) - 24k - [Cached](#) - [Similar pages](#) - [Note this](#)

**Keuka Lake - Wikipedia, the free encyclopedia**  
**Keuka Lake** is an unusual member of the Finger Lakes because it is Y-shaped instead of long and narrow. Because of its shape, it was referred to in the past ...  
[en.wikipedia.org/wiki/Keuka\\_Lake](http://en.wikipedia.org/wiki/Keuka_Lake) - 26k - [Cached](#) - [Similar pages](#) - [Note this](#)

**Seneca Lake (New York) - Wikipedia, the free encyclopedia**  
The two main inlets are Catharine Creek at the southern end and the **Keuka Lake** Outlet.

Sponsored Links

**Keuka Lake Lodging**  
Lakeside vacation rentals on the Finger Lakes in upstate New York.  
[FingerLakesPremierProperties.com](http://FingerLakesPremierProperties.com)

**Keuka Lake Real Estate**  
Looking for Information about **Keuka Lake** Real Estate?  
[www.MarkMalcolm.com](http://www.MarkMalcolm.com)  
New York

**Finger Lakes Real Estate**  
Find your dream home; Lakefront, Lakeview, Cottage, Land or Farm!  
[www.winetrailproperties.com](http://www.winetrailproperties.com)  
New York

# Sponsored search market

## ■ The search advertising market

Google

keuka lake

www.airbnb.com › stays ›  
**Keuka Lake Vacation Rentals & Homes - New York**  
Oct 27, 2020 - Rent from people in Keuka Lake, NY from \$20/night. Find the best of Keuka Lake with local hosts in 191 countries. Belong anywhere with Airbnb.

globalphile.com › keuka-lake-and-wine-trail ›  
**Hammondsport, NY and the Keuka Lake Wine Trail**  
Keuka Lake, almost 20 miles long and shaped like a Y, is among the most beautiful of the Finger Lakes. At the top sits the town of Penn Yan, and at the bottom is Hammondsport.

www.booking.com › Hotels in the United States ›  
**The 10 Best Keuka Lake Wine Trail Hotels — Worldwide**  
Find hotels in Keuka Lake Wine Trail, us. Book online, pay at the hotel or by credit card. Read hotel reviews from real guests.

www.klvineyards.com ›  
**Fine dry wines | Keuka Lake Vineyards | United States**  
Keuka Lake Vineyards is a small winery in the Finger Lakes Region of New York, making fine, premium dry wines.

www.keuka.edu › life-at-kc › keuka-lake ›  
**Keuka Lake | Keuka College**  
Keuka Lake is one of the cleanest and most majestic freshwater lakes in the world. The lake is 19.6 miles long and is, on average, 3/4 mile wide and 101 feet ...

Ad · www.fingerlakeswinecountry.com/ ›  
**Finger Lakes Wine Country - Finger Lakes Distilling**  
Our food scene is definitely something to brag about with many new restaurants & bakeries. Your world-class Wine Country experience awaits, and it's right around the corner. #MyFLX. Plan Your Visit. View Packages & Offers. Subscribe To Updates.

Ad · www.vacationrenter.com/keuka-lake/rentalsbyowner ›  
**Keuka Lake, NY Rentals - (Rentals By Owner)**  
Instant Booking On **Keuka Lake** Vacation Rentals. Book Today & Save Up To 75%! Best Rate Guarantee. Instant Confirmation. 24/7 Customer Service. Top Reviews.  
Rentals By Owner · Best Rentals · Keuka Lake Vacation · Keuka Lake Rentals  
Fall Sale: Up to \$100 off All Bookings

Searches related to keuka lake

keuka lake winery      keuka lake swimming  
keuka lake wineries map      keuka lake state park  
keuka lake weather      keuka lake restaurants  
keuka lake things to do      keuka lake rentals

< Goooooooooooooole >  
Previous 1 2 3 4 5 6 7 8 9 10 Next

www.keuka.edu › life-at-kc › keuka-lake ›  
**Keuka Lake | Keuka College**  
Keuka Lake is one of the cleanest and most majestic freshwater lakes in the world. The lake is 19.6 miles long and is, on average, 3/4 mile wide and 101 feet ...

Ad · www.fingerlakeswinecountry.com/ ›  
**Finger Lakes Wine Country - Finger Lakes Distilling**  
Our food scene is definitely something to brag about with many new restaurants & bakeries. Your world-class Wine Country experience awaits, and it's right around the corner. #MyFLX. Plan Your Visit. View Packages & Offers. Subscribe To Updates.

Ad · www.vacationrenter.com/keuka-lake/rentalsbyowner ›  
**Keuka Lake, NY Rentals - (Rentals By Owner)**  
Instant Booking On **Keuka Lake** Vacation Rentals. Book Today & Save Up To 75%! Best Rate Guarantee. Instant Confirmation. 24/7 Customer Service. Top Reviews.  
Rentals By Owner · Best Rentals · Keuka Lake Vacation · Keuka Lake Rentals  
Fall Sale: Up to \$100 off All Bookings

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keuka lake winery      keuka lake swimming  
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keuka lake things to do      keuka lake rentals

< Goooooooooooooole >  
Previous 1 2 3 4 5 6 7 8 9 10 Next



# Sponsored search market

- We have studied auction
  - We have studied matching
  - We study them in separation!
- 
- Sponsored search market is a combination of matching and auction!

# Three problems

- Question 1: how should advertisers pay for the ads?
  - Question 2: how should websites allocate the ads slots to the advertisers?
  - Question 3: how should websites set the prices on the ad slots
- 
- Sponsored search market is a combination of matching and auction!

# How to sale ads in search engines

# Keyword-base advertisement

- Targeted advertisement, as compared to traditional forced advertisement
- Low cost, especially for less frequently searched keywords
- Easy to evaluate the effectiveness, i.e., the number of users clicked

Question 1: how should advertisers pay for the ads?

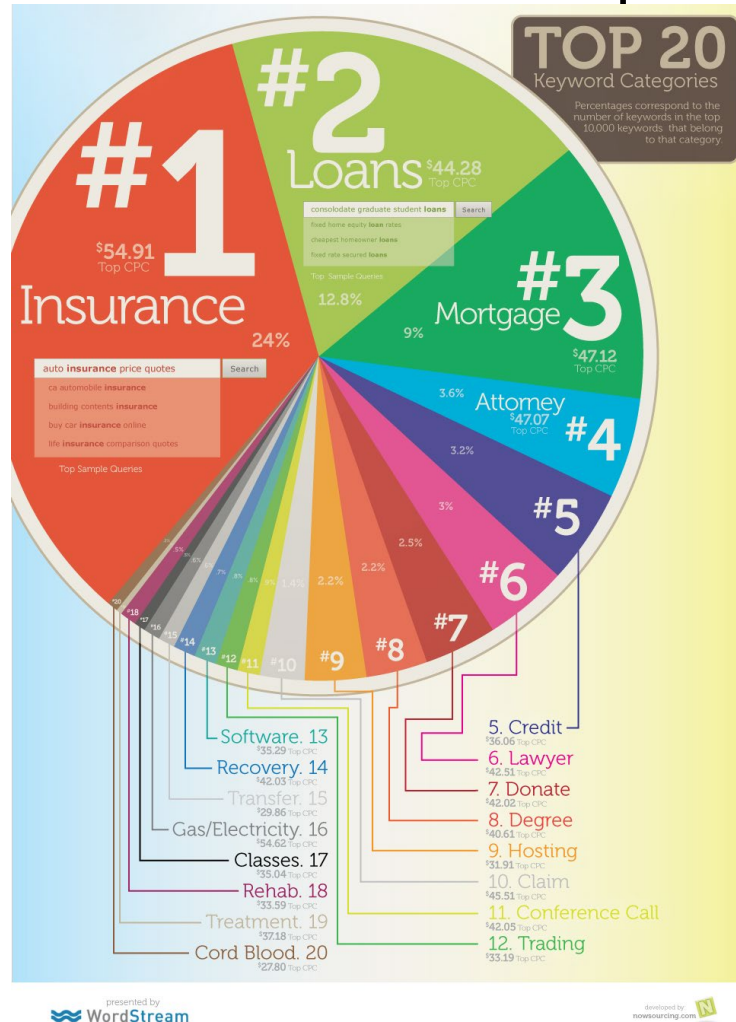
# Payment methods by the websites

- By showing:
  - CPM (cost per mille/cost per thousand impressions)
- By action:
  - CPA (cost per action)
  - CPC (cost per click)
- By sale:
  - CPO (cost per order)



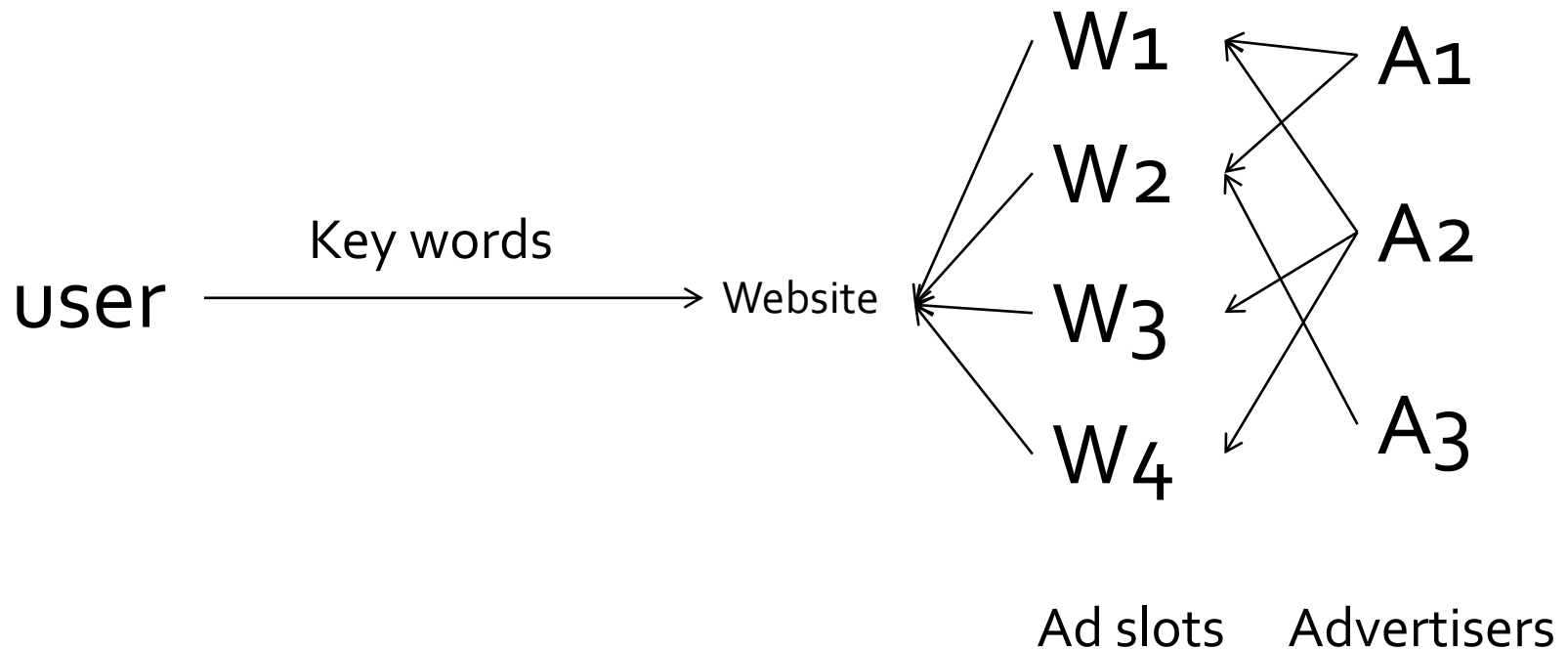
# Payment methods by the websites

- Different key words have different prices



# Keyword-base advertisement

Question 2: how should websites allocate the ads slots to the advertiser?



# Modeling

- Can we model it into the matching problem we have studied before?
- From a real world scenario → problem formulation (recall the matching market that we have learned before)

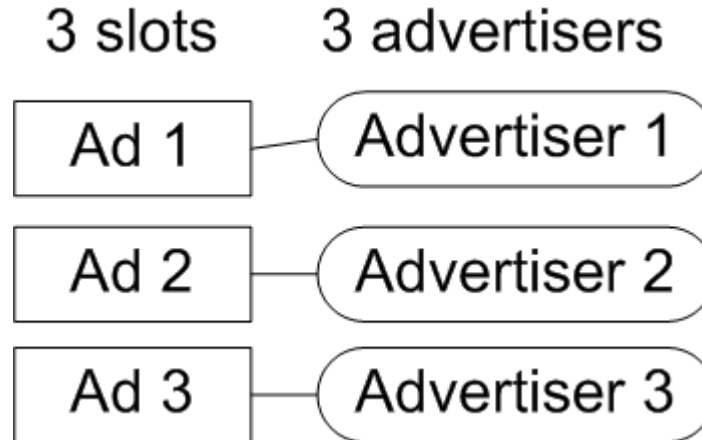
# Some basic concepts

- Advertising slots
- Clickthrough rate: the expected clicks per hour on an advertising slot
- Advertiser's revenue per click: the expected revenue of every click

# Some basic concepts

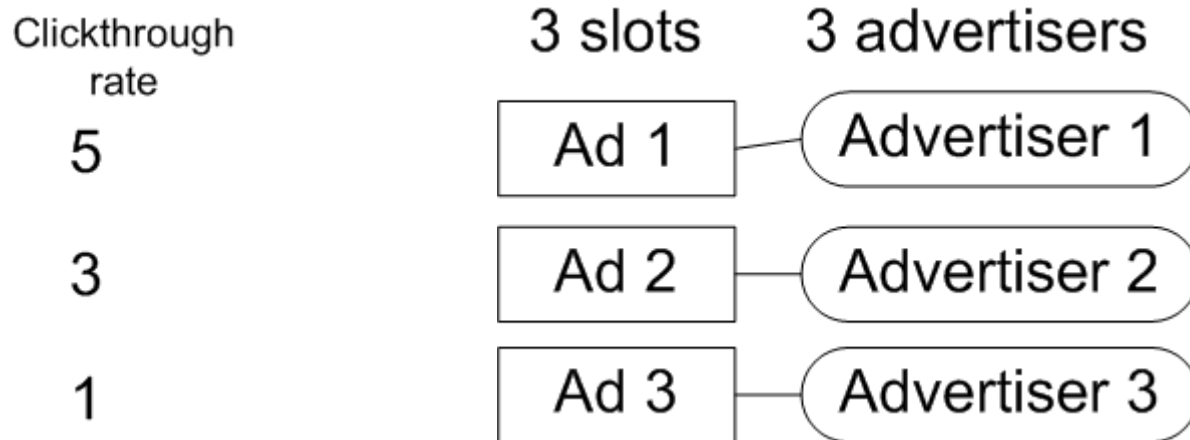
- Advertising slots
- Clickthrough rate: the expected clicks per hour on an advertising slot
- Advertiser's revenue per click: the expected revenue of every click
- Advertiser's valuation (per unit time):  
clickthrough rate \* revenue per click
- Advertiser's payoff: expected revenue - price

# An example



- 3 slots and 3 advertisers

# An example



- 3 slots and 3 advertisers
- Clickthrough rate

# An example

Clickthrough rate	3 slots	3 advertisers	Expected revenue
5	Ad 1	Advertiser 1	Per click 15
3	Ad 2	Advertiser 2	8
1	Ad 3	Advertiser 3	5

- 3 slots and 3 advertisers
- Clickthrough rate, expected revenue per click



# An example

Clickthrough rate	3 slots	3 advertisers	Expected revenue	
			Per click hour	
5	Ad 1	Advertiser 1	15	75
3	Ad 2	Advertiser 2	8	24
1	Ad 3	Advertiser 3	5	5

- 3 slots and 3 advertisers
- Clickthrough rate, expected revenue per click
- Valuation = Expected revenue can be calculated = clickthrough rate \* expected revenue per click

# An example

Clickthrough rate	price	3 slots	3 advertisers	Expected revenue	
5	9	Ad 1	Advertiser 1	15	75
3	4	Ad 2	Advertiser 2	8	24
1	3	Ad 3	Advertiser 3	5	5

- 3 slots and 3 advertisers
- Clickthrough rate, expected revenue per click
- Valuation = Expected revenue can be calculated = clickthrough rate \* expected revenue per click
- Assume we know the price

# An example

Clickthrough rate	price	3 slots	3 advertisers	Expected revenue		Expected payoff
				Per click	hour	Per click
5	9	Ad 1	Advertiser 1	15	75	6
3	4	Ad 2	Advertiser 2	8	24	4
1	3	Ad 3	Advertiser 3	5	5	2

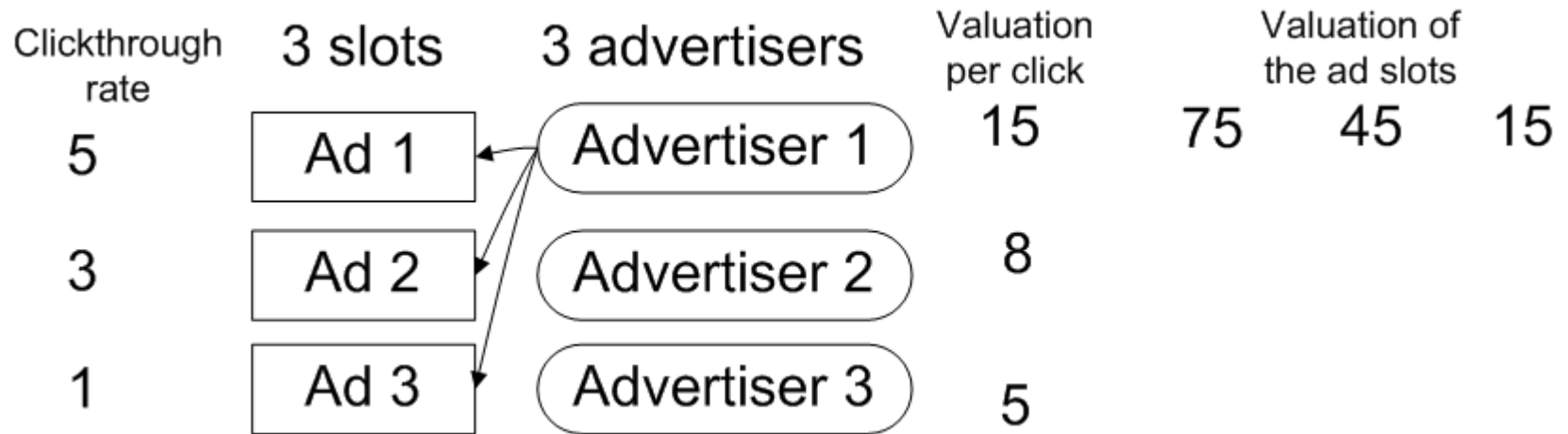
- 3 slots and 3 advertisers
- Clickthrough rate, expected revenue per click
- Valuation = Expected revenue can be calculated = clickthrough rate \* expected revenue per click
- Assume we know the price, expected payoff per click

# An example

Clickthrough rate	price	3 slots	3 advertisers	Expected revenue		Expected payoff	
				Per click	hour	Per click	hour
5	9	Ad 1	Advertiser 1	15	75	6	30
3	4	Ad 2	Advertiser 2	8	24	4	12
1	3	Ad 3	Advertiser 3	5	5	2	2

- Advertiser's valuation (per unit time): clickthrough rate \* expected revenue per click
- Advertiser's payoff: (expected revenue – price) \* clickthrough rate

# An example



- Different slots have different clickthrough rates, so their values are different

# An example

Clickthrough rate	3 slots	3 advertisers	Valuation per click	Valuation of the ad slots		
5	Ad 1	Advertiser 1	15	75	45	15
3	Ad 2	Advertiser 2	8	40	24	8
1	Ad 3	Advertiser 3	5	25	15	5

- Different slots have different clickthrough rates, so their values are different
- Different advertiser's valuation on per click is different, so the valuation of each ad slot is also different

# An example

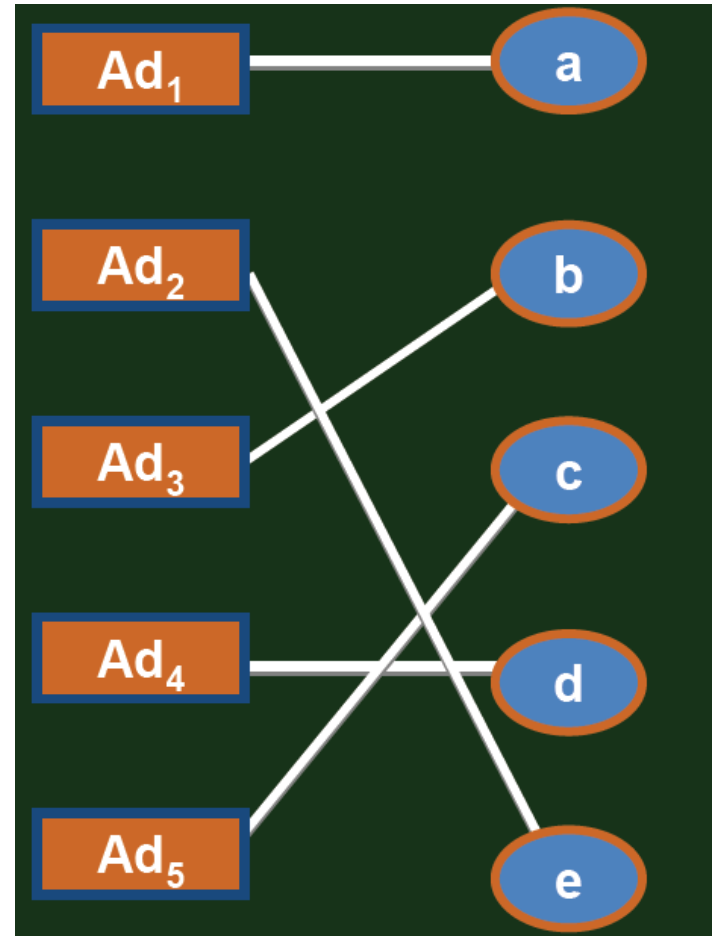
3 slots	3 advertisers	Valuation of the ad slots		
Ad 1	Advertiser 1	75	45	15
Ad 2	Advertiser 2	40	24	8
Ad 3	Advertiser 3	25	15	5

- Seems familiar?
- We have formulated the problem as a matching market

# Pricing the ad slots or the clicks

Question 3: how should websites set the prices on the ad slots

- Matching market: market clearing prices
- Vickrey-Clarke-Groves (VCG) mechanism
- Generalized Second-Price Auction (GSP)





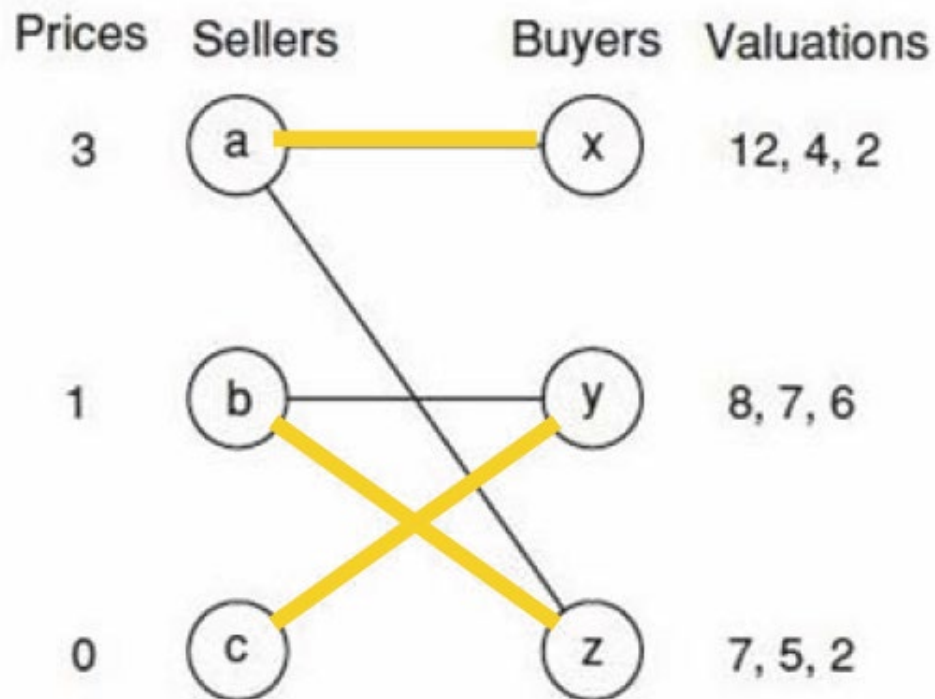
# A summary

- General backgrounds on online ads
- Some general concepts
  - Ad slots, clickthrough rate, advertiser's revenue, advertiser's valuation, advertiser's payoff, etc
- The advertisement market
  - Payment methods
  - Ad slots allocation
  - Pricing the ad slots

# Price setting: Matching of advertiser and ad slots

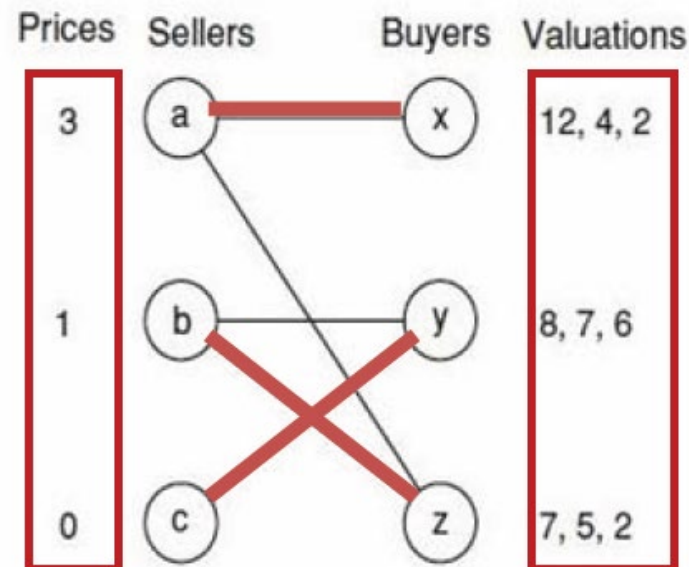
# Matching market

- Matching market: market clearing prices



# Recall some basic elements

- A set of sellers and buyers
- Every buyer has a valuation on every object from the seller
- Every seller has a price for each object
- Every buyer try to maximize his payoff
- If the preferred sellers graph has a perfect match, then this is an **optimal matching**, the price in optimal matching is the **market-clearing price**



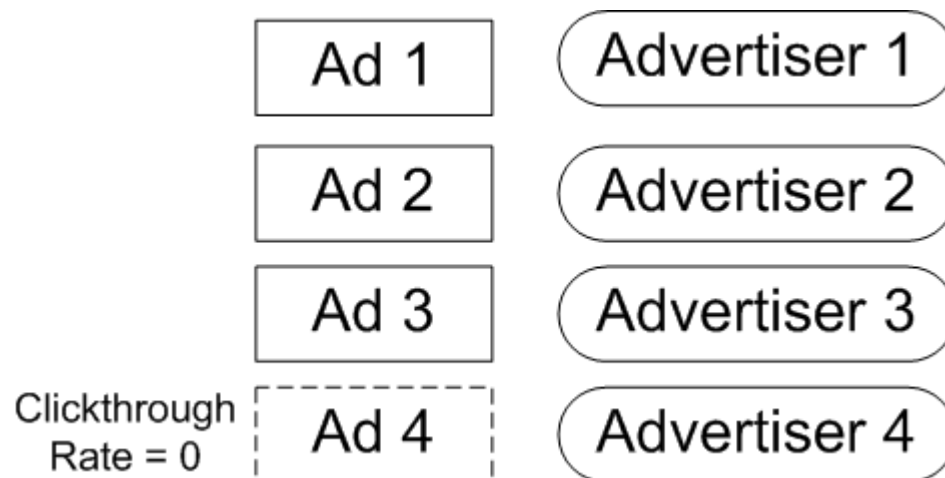
Preferred sellers graph

# The good property of market-clearing prices

- Market-clearing prices satisfy: optimal matching, maximize the total valuations, maximize the total payoff
- If we know the valuations of the advertisers, every seller uses the market-clearing prices for each slot

# The good property of market-clearing prices

- If there are more advertisers than ad slots, we can simply add one slot with clickthrough rate = 0
- 4 advertisers competing for 3 ad slots



# The limitation of market matching

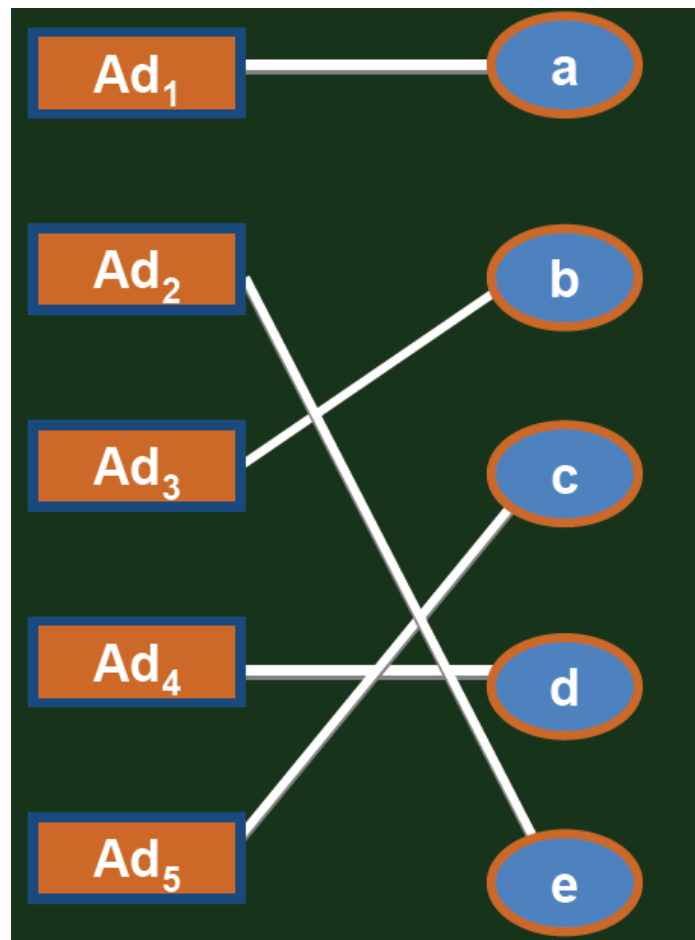
- We have an assumption: every advertiser needs to announce their valuations on the ad slots
- This is not realistic (this can be commercial secrets)

Is there anything else we can do?

# Review: pricing the ad slots or the clicks

Question 3: how should websites set the prices on the ad slots

- Matching market: market clearing prices
- Vickrey-Clarke-Groves (VCG) mechanism
- Generalized Second-Price Auction (GSP)





# Vickrey-Clarke-Groves (VCG) mechanism

# VCG mechanism

- When the advertiser cannot articulate his valuations on the ad slots, e.g., because of secrecy
- Encouraging Truthful Bidding in Matching Markets: The VCG Principle
- This is primarily an auction,
- i.e., if you don't want to say, let's bid











# Which auction format to choose?

- First price auction?
  - There are many disadvantages for first price auction
- Can we do second-price auction?
  - We know that for a single item, second-price auction has very good property: truthful bid
- VCG is an extension of single item second-price auction to multiple items

# Review: Second-price sealed-bid auctions

- An object
  - Different people may have different values for it,  $v_1, v_2, \dots, v_k$ . These are true value/intrinsic value, i.e., player  $i$  will pay at most  $v_i$ .
  - Players don't know other's true values
  - Every one has a bid, assume  $b_1 > b_2 > \dots > b_k$
  - By the rule of second-price auction, the payoff of the highest bidder is  $v_1 - b_2$  and others are 0; here,  $b_1$  is the highest bid, so  $v_1$  is the true value
- Truthful bid is a dominant strategy
- Harm due to the existence of the winner

# An example

slots	advertisers	valuations	prices	slots	advertisers	valuations
		30, 15, 6	13			30, 15, 6
		20, 10, 4	3			20, 10, 4
		10, 5, 2	0			10, 5, 2



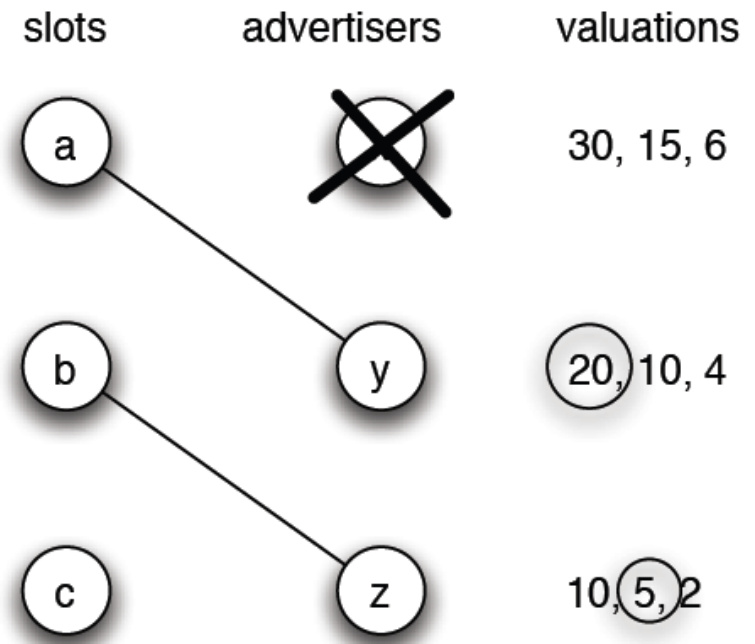
3 slots a, b, c, 3 advertisers x, y z, the valuations are as shown. Using market matching, we can see that the optimal matching is a-x, b-y, c-z, with a total valuation of 42

# An example

If x weren't there, y would do better by  $20 - 10 = 10$ , and z would do better by  $5 - 2 = 3$

The total harm x caused is 13

This is the price x should pay



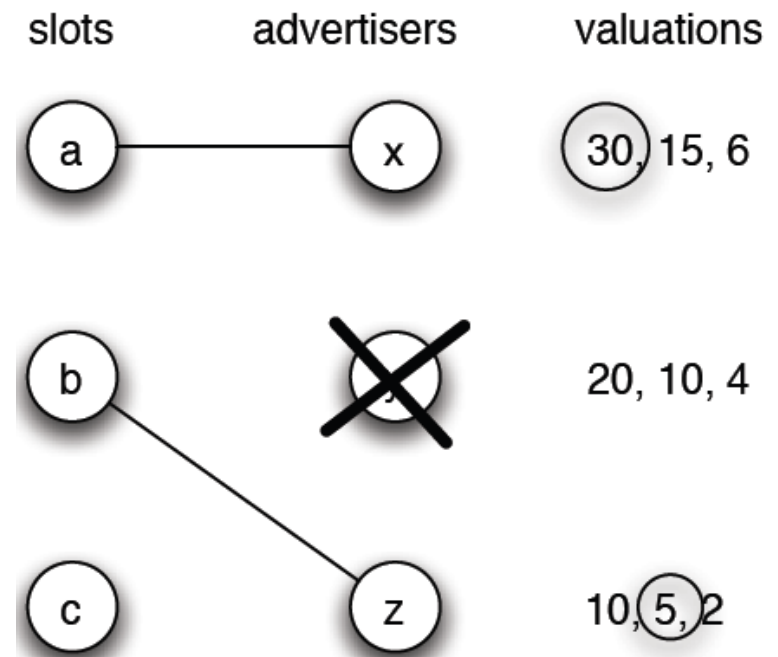
# An example

If y weren't there, x would be unaffected, and z would do better by  $5 - 2 = 3$

The total harm y caused is 3

This is the price y should pay

If z weren't there, z cause no harm to x and y



# Formulating the “harm”

Let  $V_B^S$  be the maximum total valuation over all possible perfect matchings

$V_{B-j}^{S-i}$  be the value of the optimal matching when we take out buyer  $j$  and seller  $i$

$V_{B-j}^S$  be the value of the optimal matching when we take out buyer  $j$

Let  $p_{ij} = V_{B-j}^S - V_{B-j}^{S-i}$

This is how everyone do without  $j$  (where  $i$  is there for sell),  
and how everyone do with  $j$  (where  $i$  is taken by  $j$ )



# The VCG price setting procedure

- Ask buyers to announce valuations for the items (these announcement need not be truthful)
- Choose a socially optimal assignment of items to buyers, i.e., a perfect matching that maximizes the total valuation of each buyer for what they get.
- Charge each buyer the appropriate VCG price: if buyer  $j$  receives item  $i$  under optimal matching, then charge  $j$  a price  $p_{ij}$  (see  $p_{ij}$  in the last slide)

# Analyze the VCG procedure

- Claim: If items are assigned and prices computed according to the VCG procedure, then truthfully announcing valuations is a dominant strategy for each buyer, and the resulting assignment maximizes the total valuation of any perfect matching of slots and advertisers.
- Two good properties:
  - Maximize total valuation
  - Truthful-telling is a dominant strategy

# Proof

- Claim: If items are assigned and prices computed according to the VCG procedure, then truthfully announcing valuations is a dominant strategy for each buyer, and the resulting assignment maximizes the total valuation of any perfect matching of slots and advertisers.
- There are two parts. The second part on maximizing total valuation is easier: as long as they report their valuation truthfully, the total valuation is maximized as the matching is perfect matching

# Proof

- For the first part, truthful-telling, we want to show that no one has the incentive to deviate
- That is:
- If buyer  $j$  bids true valuation and gets slot  $i$ , his payoff is

$$v_{ij} - p_{ij}$$

- Assume he changes his bid and gets slot  $h$ , his payoff is

$$v_{hj} - p_{hj}$$

- We need to show

$$v_{ij} - p_{ij} \geq v_{hj} - p_{hj}$$

# Proof

- We can translate

$$v_{ij} - p_{ij} \geq v_{hj} - p_{hj}$$

- to

$$v_{ij} - [V_{B-j}^S - V_{B-j}^{S-i}] \geq v_{hj} - [V_{B-j}^S - V_{B-j}^{S-h}]$$

- which is

$$v_{ij} + V_{B-j}^{S-i} \geq v_{hj} + V_{B-j}^{S-h}$$

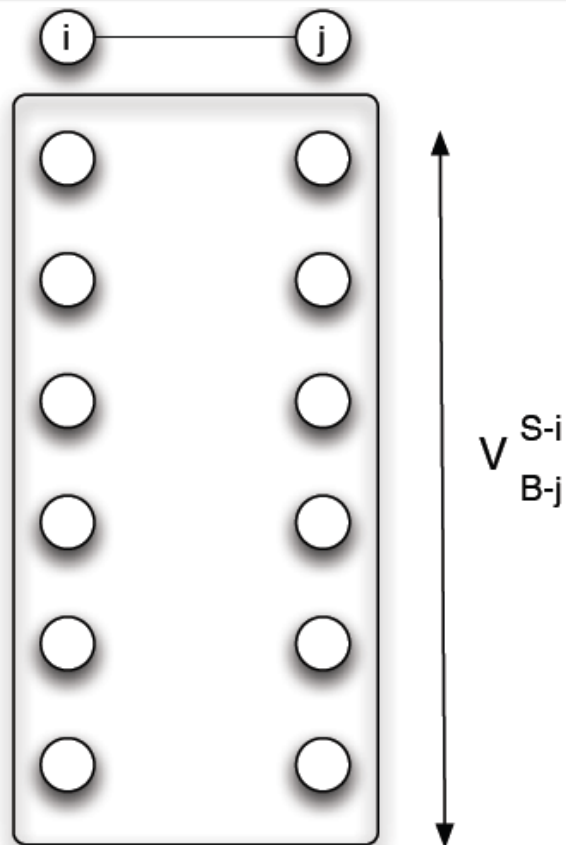
- We know that

$$v_{ij} + V_{B-j}^{S-i} = V_B^S$$

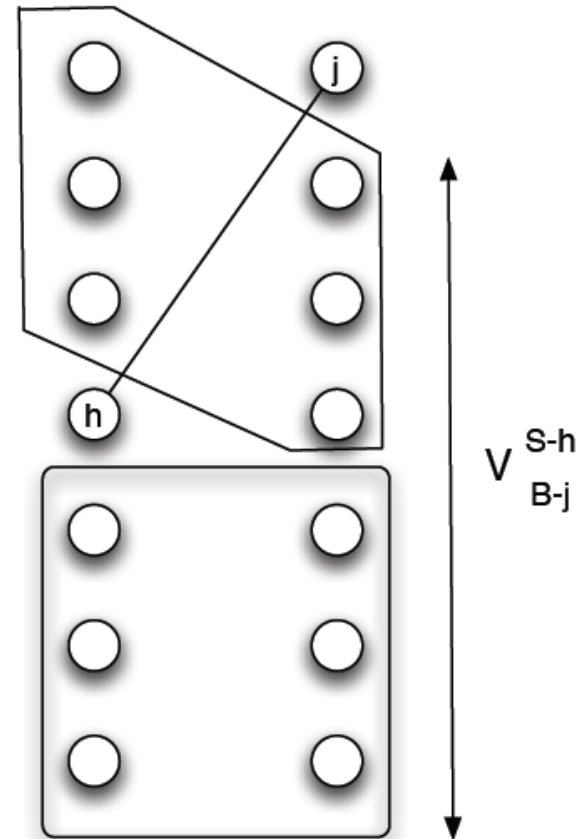
- and

$$v_{hj} + V_{B-j}^{S-h} \leq V_B^S$$

# Proof



(a)  $v_{ij} + V^{S-i}_{B-j}$  is the maximum valuation of any matching.



(b)  $v_{hj} + V^{S-h}_{B-j}$  is the maximum valuation only over matchings constrained to assign  $h$  to  $j$ .

# Price setting: Generalized Second-Price Auction (GSP)

# The GSP price setting procedure

- Each advertiser  $j$  announces a bid consisting of a single number  $b_j$ , i.e., the price he is willing to pay per click
- After each advertiser submits a bid, GSP awards each slot  $i$  to the  $i$ th highest bidder, at a price per click equal to the  $(i+1)$ st highest bid
- In other words, each advertiser who is shown on the results page is paying a price per click equal to the bid of the advertiser just below them



# The GSP price setting procedure

- $n$  slots: click rates  $r_1, r_2, \dots, r_n$ , let's assume in descending order
- $n$  advertisers: bids,  $b_1, b_2, \dots, b_n$ , let's assume in descending order
- Assign  $r_1$  to the 1<sup>st</sup> advertiser, pay by  $b_2$ , assign  $r_2$  to the 2<sup>nd</sup> advertiser, pay by  $b_3$ , ...
- We can create an artificial advertiser to make this balance
- Set the pay as 0 or a “club joining fee”







# Some basic elements of GSP

- GSP's CPC (cost per click):  $b_{i+1}$
- Payoff =  $v_i * r_i - b_{i+1} * r_i$
- $v_i$  is the valuation of advertiser  $i$

# Analysis of GSP

- Truth-telling may not be the dominant strategy for GSP
- In VCG, truth-telling is the dominant strategy

# An example for not truth-telling

clickthrough rates	slots	advertisers	revenues per click
10			7
4			6
0			1

- 3 slots, a, b, c, clickthrough rate 10, 4, 0
- 3 advertisers, x, y, z, every click produces a revenue of 7, 6, 1 for them

# An example for not truth-telling

clickthrough rates	slots	advertisers	revenues per click
10	a	x	7
4	b	y	6
0	c	z	1

- If truth-telling, x will get slot a, pay at a price of 6, his payoff will be  $7 \cdot 10 - 6 \cdot 10 = 10$
- If x bid 5, what will happen?

# An example for not truth-telling

clickthrough rates	slots	advertisers	revenues per click
10	a	x	7
4	b	y	6
0	c	z	1

- If truth-telling, x will get slot a, pay at a price of 6, his payoff will be  $7 \cdot 10 - 6 \cdot 10 = 10$
- If x bid 5, what will happen?
- x will get slot b, pay 1, payoff at  $7 \cdot 4 - 1 \cdot 4 = 24$

# Equilibria

clickthrough rates	slots	advertisers	revenues per click	
10	a	x	7	5
4	b	y	6	4
0	c	z	1	2

- The bids of 5, 4, 2 form a Nash Equilibrium
- Now, x gets a, with payoff  $7 \cdot 10 - 4 \cdot 10 = 30$ . If x bids below 4, he will get b, with payoff  $7 \cdot 4 - 2 \cdot 4 = 20$
- Now, y gets b, with payoff  $6 \cdot 4 - 2 \cdot 4 = 16$ . If y bids above 5, he will get a, with payoff  $6 \cdot 10 - 5 \cdot 10 = 10$ . If y bids below 2, he will get c, with payoff 0

# Equilibria

clickthrough rates	slots	advertisers	revenues per click	
10	a	x	7	5
4	b	y	6	4
0	c	z	1	2

- The bids of 5, 4, 2 form a Nash Equilibrium
- This is also social optimal  $70+24+0=94$



# Equilibria

clickthrough rates	slots	advertisers	revenues per click	
10	a	x	7	3
4	b	y	6	5
0	c	z	1	1

- The bids of 3, 5, 1 also form a Nash Equilibrium
- Now, x gets b, with payoff  $7*4 - 1*4 = 24$ . If x bids above 5, he will get a, with payoff  $7*10 - 5*10 = 20$ . If x bids below 1, he gets c, with payoff 0
- Now, y gets a, with payoff  $6*10 - 3*10 = 30$ . If y bids below 3, he will get b, with payoff  $6*4 - 1*4 = 20$ .

# Equilibria

clickthrough rates	slots	advertisers	revenues per click	
10	a	x	7	3
4	b	y	6	5
0	c	z	1	1

- The bids of 3, 5, 1 also form a Nash Equilibrium
- The social welfare is  $7*4 + 6*10 + 1*0 = 88$
- This is not social optimal

# Some characteristics of GSP

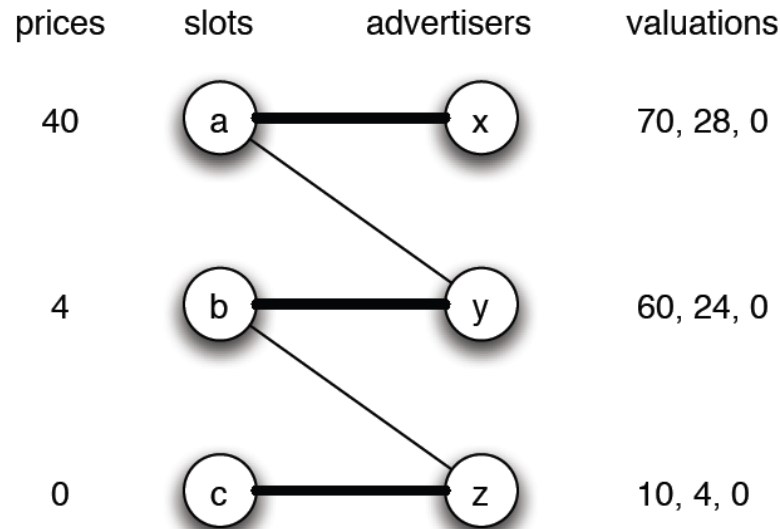
- Truthful-telling is not a dominant strategy
- There can be multiple equilibria, including social optimal and also non-social optimal

# The revenue of the search engine of GSP

clickthrough rates	slots	advertisers	revenues per click		
10	a	x	7	5	3
4	b	y	6	4	5
0	c	z	1	2	1

- For the equilibrium of 5, 4, 2:  $4 * 10 + 2 * 4 = 48$
- For the equilibrium of 3, 5, 1:  $3 * 10 + 1 * 4 = 34$

# The revenue of the search engine of VCG



- The harm is  $(60 - 24) + (4 - 0) = 44$
- VCG may not provide a higher revenue to the search engine (of course, this depends on which equilibrium it ends up to in GSP)

# Why GSP

- Most search engine currently adopts GSP
- VCG targets on social optimal
- VCG is difficult to compute, difficult to understand by many
- Search engine companies care more on revenue, not social optimal
- VCG has assumptions that, no collusion, the price is only related to clickthrough rate, etc

# A summary

- How to auction the advertisement slots
- This is a combination of matching and auction
- VCG has excellent theoretical properties
  - Social optimal
  - Truthful-telling
- GSP (and its variants) has been used widely in practice
  - Easy to understand
  - Search engine may get more