VE444: Networks

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Review

- Nash Equilibrium Existence Theorem
- If game is zero-sum, we can compute a NE in poly time.
- User optimal v.s. Social optimal

Mechanism Design Basics

Motivating example

- 2012 London Olympics
- Video: 08:00

Motivating example

- 2012 London Olympics
- Phase 1: Round-robin
- 4 teams of 4
- Top 2 teams from each group advance
- Phase 2: Knockout

Motivating example

- Trigger: in group D, Danish PJ upset Chinese team QW
- Next match: Chinese team XY meets Korean team KH to decide who is 1st and 2nd in group A
- Issue: Group A winner would face QW in semis, 2nd-best would only face QW in the final.

- Misalignment between participant and designer's goal.

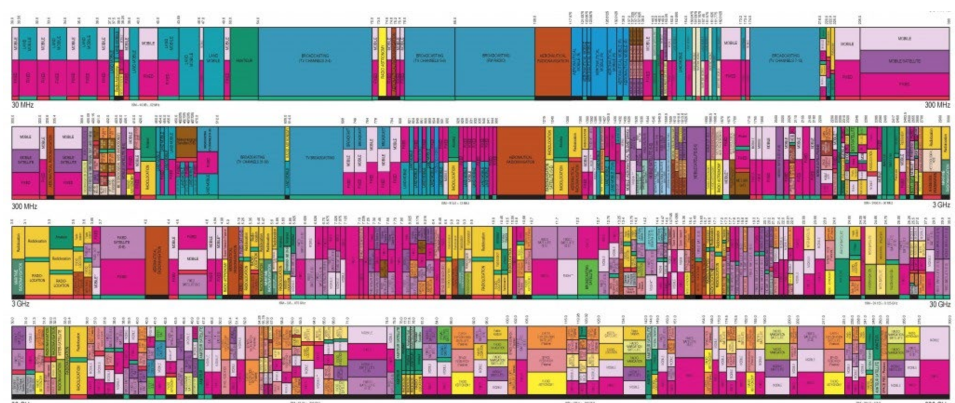
Mechanism design in Practice

- Spectrum auctions since 1994
  - FCC auctions
  - Worldwide innovations in auction design
- Other innovative auctions
  - Electricity
  - Carbon emissions
  - Search auctions
  - Computing resources

FCC completes 3.5 GHz spectrum auction raising $4.5 bn

August 26, 2020

The Federal Communications Commission (FCC) has completed the 3.5 GHz spectrum auction (Auction 105) – raising $4.585 billion.



Mechanism design in Practice

- Kidney exchange
- Resident doctor matching
- Voting

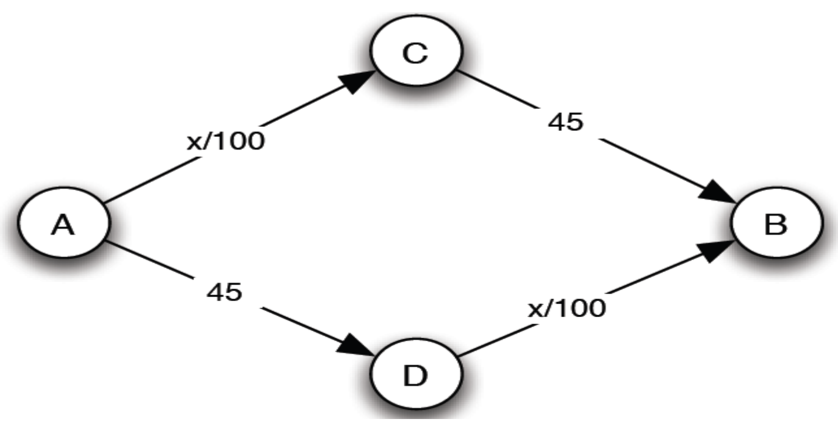
- But first, when is selfish behavior benign?

Traffic as a Game

A game on network structure

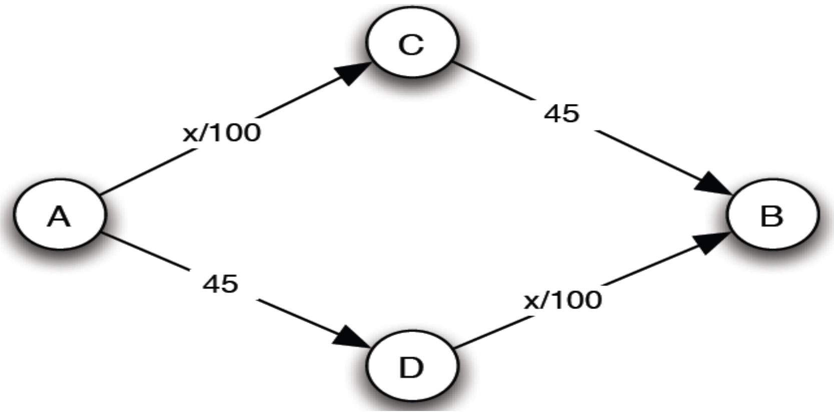
- 4000 vehicles want to travel from A to B

- Players: 4000 drivers
- Strategy set: upper path, lower path
- Payoff: travel time (the less the better, but also depends on other’s choices)
- Equilibrium? Payoff matrix?



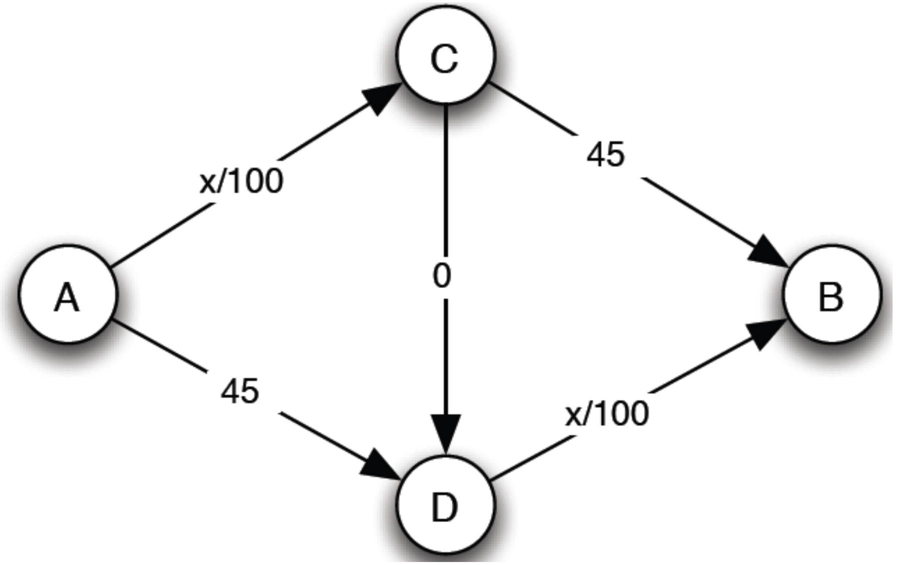
A game on network structure

- Equilibrium: 2000 A-C-B, 2000 A-D-B
- Payoff for each driver: 65
- If anyone deviates, his payoff will be: 2001/100 + 45 > 65



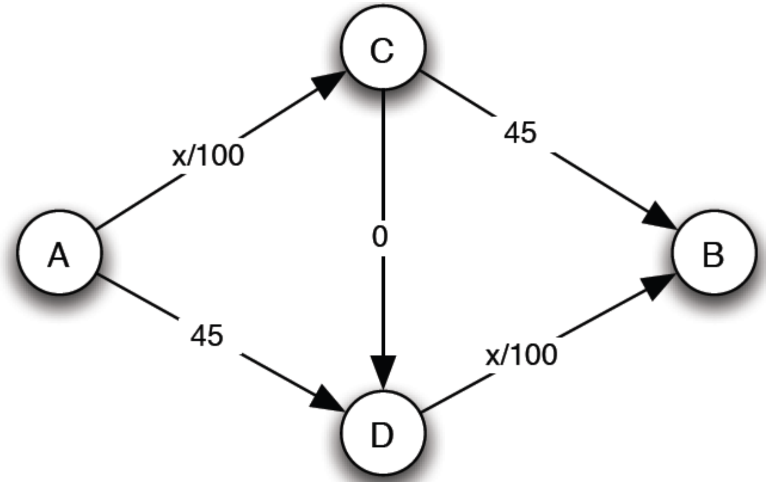
If the government builds a new road

- Assume the government want to do something good: let's build a new road and it is an express road, e.g., travel time on this road is negligible!
- What will happen?



If the government builds a new road

- Is the previous possible?
  - 2000 A-C-B
  - 2000 A-D-B
- No way, no longer equilibrium
- If you are the one on A-C-B
  - A-C-D-B will be faster, there is incentive to change
- This is called Braess's Paradox



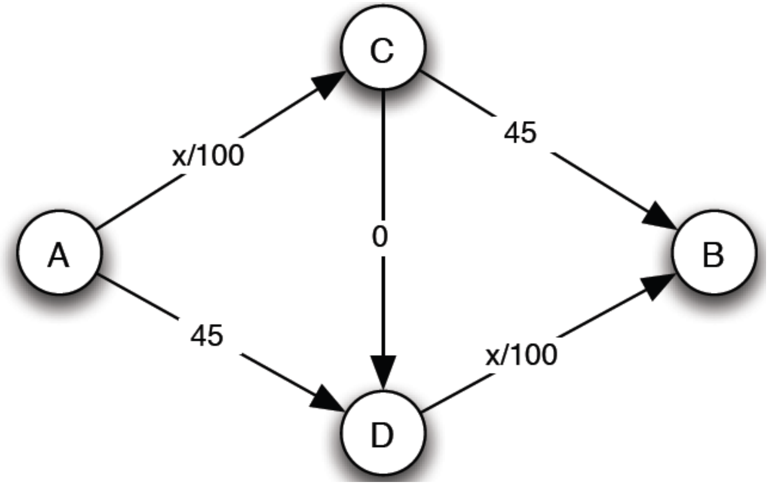
If the government builds a new road

- Equilibrium: every one uses A-C-D-B
  - Travel time is

  4000/100+0+4000/100=80
  - If someone tries to deviate,

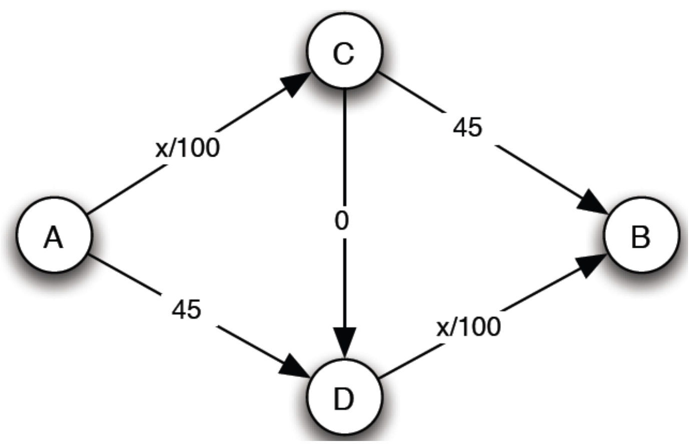
  Travel time is: 45+4000/100=85>80

  It is worse than 65!



Comparing with the social optimal

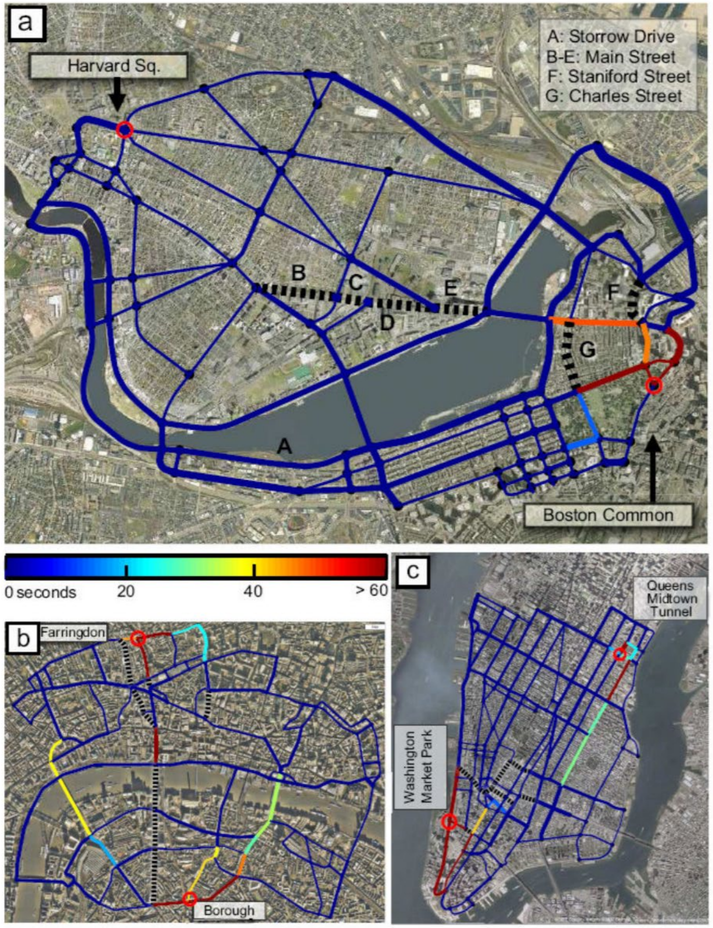
- The outcome of rational behaviors can be inferior to a centrally designed outcome.
- Question is: by how much?
- Price of Anarchy(POA): the ratio between the system performance with strategic players and the best-possible system performance



Real-world practice

- Stuttgart, Germany (1969): The traffic worsened until a newly built road was closed
- New York City (1990): On Earth Day 42nd street was closed and traffic flow improved.
- Seoul, South Korea (2002), replaced a six lane highway with a five mile long park, traffic flow improved.

Real-world practice



Price of Anarchy in Transportation Networks: Efficiency and Optimality Control, Physical Review Letters, 2008

A summary

- A simple example shows a game on network structure
- We see Braess's Paradox
- Invest more resources may not get a good result

- Identify application domains and conditions under which the POA is guaranteed to be close to 1
  - Selfish behavior leads to a near-optimal outcome

Mechanism Design with Money: Auction

An example

- In game and Braess's Paradox, we see the interaction of rational behavior, equilibrium, and network structure
- Can we change the condition and let the players to directly interact (of course, we also need to set up a set of rules for their interaction)?
- Auction can be one of such scenario

Auction is everywhere

- Auctions by Christie and Sotheby’s
- Government auctions the land, license, etc
- Electricity
- Carbon emissions
- Search auctions
- Computing resources

The format of auctions

- The equilibrium depends on the format and regulations of the auctions
- The format also influence the strategy choices of the buyers and sellers

Types of Auction

- (Forward) Auction: a seller auctioning one item to a set of buyers
- Procurement auctions: a buyer trying to purchase a single time among a set of sellers.

Types of Auction

- Ascending-bid auctions/English auctions: the seller gradually raises the price, bidders drop out until finally only one bidder remains;
  - This is useful for auctions of art works, antiques, etc.
- Descending-bid auctions/Dutch auctions: the seller gradually lowers the price from some high initial value until the first moment when some bidder accepts and pays the current price;
  - This is useful for auctions of flowers, fresh farm products.

Types of Auction

- First-price sealed-bid auctions: bidders submit simultaneous “sealed bids" to the seller, who would then open them all together. The highest bidder wins the object and pays the value of her bid.
  - This is used in call for bid auctions
- Second-price sealed-bid auctions/Vickrey auctions: Bidders submit simultaneous sealed bids to the sellers; the highest bidder wins the object and pays the value of the second-highest bid
  - This is used in advertisement auction in Internet websites
  - In honor of William Vickrey, first game-theoretic analysis of auctions, Nobel Memorial Prize in Economics in 1996.

When are Auctions Appropriate?

- Known value
  - Seller valuation: x, buyer valuation: y
- Surplus: y-x
- Commit to the mechanism
- Unknown value
  - Independent, private values
  - Common value

Relationship between different formats

- Descending-bid
- Ascending-bid
- First-price
- Second-price

- Descending-bid is analogous to sealed-bid first price auction
- Ascending-bid is analogous to sealed-bid second price auction

A few points in auction formats

- Who get the object
  - Usually the highest or lowest bidder
- What kind of price to pay
  - First price or second price, this influences the strategy of the bidders
- Do the bidders know the price of others
  - Sealed auction, needs to guess about others
  - Unsealed auction, can see other's bids

Auction: Game perspective

- First-price sealed-bid auctions (FPA)
  - Highest bidder wins and pays the value of her bid
- Second-price sealed-bid auctions (SPA)
  - Highest bidder wins and pays the value of the second-highest bid.
- Formulating as a game
  - Players: bidders
  - Strategy: bid
  - Payoff: true value – payment, or 0 (if auction fails)
- A game-oriented thinking
  - Equilibrium! Best response to each other, no one changes

Second-price sealed-bid auctions

- An object
  - Different people may have different values for it, v1, v2, …, vk. These are true value/intrinsic value, i.e., player *i* will pay at most *vi*.
  - Players don't know other's true values
  - Every one has a bid, assume b1 > b2 > … > bk
  - By the rule of second-price auction, the payoff of the highest bidder is v1 – b2 and others are 0; here, b1 is the highest bid, v1 is the true value

How do you play the game, i.e., how do you bid?

Second-price sealed-bid auctions

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We assume that people maximize their own profits, i.e., get the object and pay the lowest price possible

What strategy is optimal?

- From game point of view (dominant strategy): cannot get better payoff by changing to other strategies, regardless of the other players' strategy

- Claim: In a sealed-bid second-price auction, it is a dominant strategy for each bidder i to choose a bid bi = vi.

Proof

- Assume in an auction, you consider the object worth $100 and you bid $100, now consider whether you can get better payoff using other strategy
- There are two cases:
  - You win the bid: now you have positive payoff (you only need to pay the second high bid, $v1 – b2 > 0$)
    - Increase the bid won't change anything
    - Decrease the bid won't change anything unless less than the second high, then you lose the bid and payoff become 0
  - You lose the bid: now your payoff is 0
    - Decrease the bid won't change anything
    - Increase the bid won't change anything unless becoming the highest; note that this means that this is greater than your value, so payoff becomes negative

First-Price Auction

- In second-price sealed-bid auction, bid the true value is a (weakly) dominant strategy
- First-price seal-bid auction does not have such property
  - Truthful bidding zero payoff
  - Shading requests complex trade-off reasoning

Bidding Strategies in First-Price Auction

- What is the optimal strategy in the equilibrium point?
- Simplified case: two bidders, private value independently and uniformly distributed and uniformly distributed between 0 and 1;
- Strategy: A strategy for a bidder is a function $s(v) = b$ that maps her true value v to a non-negative bid b.
- Assumption:
  - $s(\cdot)$ is a strictly increasing, differentiable function;
  - $s(v) \leq v$ for all v
  - E.G., $s(v)=v$; cv $(c<1)$;
- The assumption helps us narrow down the strategy set.
  - Rules out non-optimal

Bidding Strategies in First-Price Auction

- If bidder i has a value of vi, the probability that this is higher than the value of i's competitor in the interval [0, 1] is exactly vi
- i's expected payoff: g(vi) = vi(vi − s(vi)).
- Equilibrium strategy: for each bidder i, there is no incentive for i to deviate from strategy s(·) if i's competitor is also using strategy s(·).
- Deviations in the bidding strategy function can instead be viewed as deviations in the "true value" that bidder i supplies to her current strategy s(·). -> Revelation Principle
- Condition that I does not want to deviate from strategy s(·):
  - vi(vi − s(vi)) ≥ v(vi − s(v))
- What kind of function satisfies this condition?

Bidding Strategies in First-Price Auction

- What is the optimal strategy in the equilibrium point?
- Equilibrium with many bidders: n bidders, uniform distribution valuation
- For a given bidder i with true value vi, what is the probability that her bid is the highest?

- Expected payoff for bidder i: $G(v_i) = v_i^{n-1}(v_i\text{-s}(v_i))$
- Using Revelation Principle, a deviation from the bidding strategy as supplying a "fake" value v to the function s()
  - $v_i^{n-1}\big(v_i - \mathrm{s}(v_i)\big) \geq v^{n-1}(v_i - \mathrm{s}(v))$ for all v between 0 and 1

- Expected payoff function: $G(v) = v^{n-1}(v_i\text{-s}(v))$
  - $s'(v_i) = (n-1)(1 - \frac{s(v_i)}{v_i})$
  - $\mathrm{s}(v_i) = \left(\frac{n-1}{n}\right) v_i$
- As the number of bidders increases, you generally have to bid more aggressively

Seller Revenue Comparision

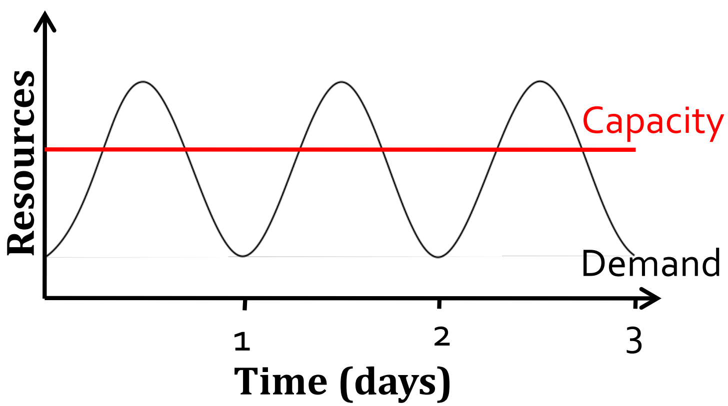
- How to compare the revenue a seller should expect to make in first-price and second-price auctions?
- Assumption: n bidders with values drawn independently from the uniform distribution on the interval [0, 1].
- Background Math Info: Suppose n numbers are drawn independently from the uniform distribution on the interval [0, 1] and then sorted from smallest to largest. The expected value of the number in the kth position on this list is $\frac{k}{n+1}$.
- Revenue equivalence theorem: given certain conditions, any mechanism that results in the same outcomes (i.e. allocates items to the same bidders) also has the same expected revenue.

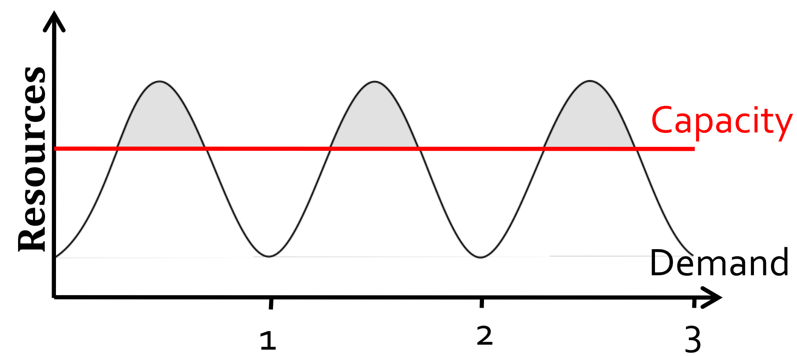
Cloud 101-definition

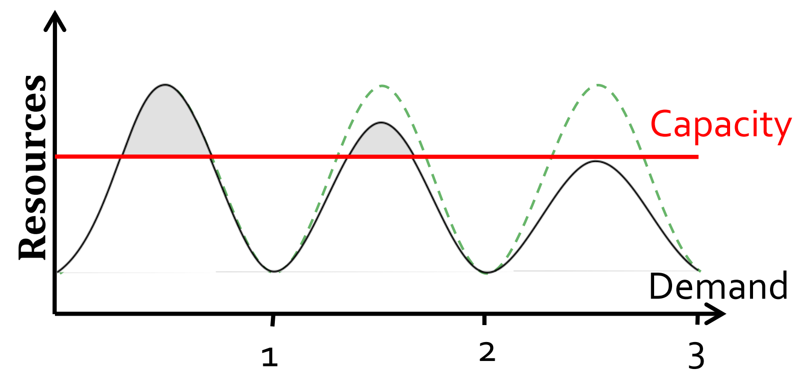
- Definition from ***NIST*** ***(National Institute of Standards and Technology)***
  - Cloud computing is a model for enabling convenient, **on-demand network access** to a **shared pool** of configurable computing resources (e.g., networks, servers, storage, applications, and services) that can be **rapidly provisioned and released** with minimal management effort or service provider interaction.
  - This cloud model promotes **availability** and is composed of five essential characteristics, three service models, and four deployment models.

Cloud 101-dynamic provisioning

- In traditional computing model, two common problems :
- Underestimate system utilization which result in under provision

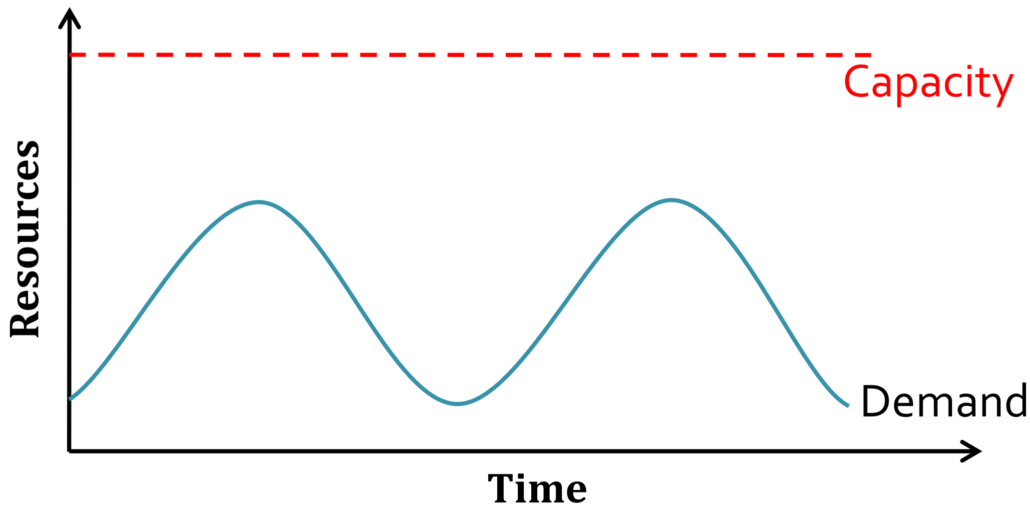






Cloud 101-dynamic provisioning

- Overestimate system utilization which result in low utilization



- How to solve this problem ??
  - Dynamically provision resources

Cloud101-Multi-tenant Design

- What is multi-tenant design ?
  - Multi-tenant refers to a principle in software architecture where a single instance of the software runs on a server, serving multiple client organizations.
  - With a multi-tenant architecture, a software application is designed to virtually partition its data and configuration thus each client organization works with a customized virtual application instance.

IaaS clouds:

- On-demand (pay-as-you-go)
  - Static hourly rate x run hours = Subscription
- Reserved instance
  - One-time subscription fee
  - Free/discounted usage fee during the reservation period
- Spot instance
  - Users bid for computing instances
  - No service guarantee

Mechanism Design without Money: Matching Markets