VE444: Networks

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Game Theory Basics

Review

- Basic ingredients
- Equilibrium
- Dominant strategy
- Nash Equilibrium

Multiple Equilibria: Coordination game

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	$1, 1$	$0, 0$
	Keynote	$0, 0$	$1, 1$

- There are two Nash Equilibria (PPT, PPT), (Keynote, Keynote)
- What do players do?
- Schelling focal point model, Social conventions, etc
- Balanced Coordination v.s. unbalanced coordination

Battle of sexes

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	$1, 2$	$0, 0$
	Keynote	$0, 0$	$2, 1$

- Assume you and your partner like different software
- Battle of sexes:
  - A husband and wife want to see a movie together
  - They need to choose between action movie and romantic comedy movie
- It is difficult to predict what will happen purely based on the structure of game
- Additional information may be needed

Stag Hunt game

		Hunter 2	
		Hunt Stag	*Hunt Hare*
Hunter 1	*Hunt Stag*	4, 4	0, 3
	Hunt Hare	3, 0	3, 3

- Two people are out hunting;
  - if they work together, they can catch a stag (which would be the highest-payoff outcome), payoff, say, 4
  - On their own, each can catch a hare, payoff, say, 3
  - If one hunter tries to catch a stag on his own, he will get nothing, while the other one can still catch a hare
- Which equilibrium?

Need to choose between high payoff and risk of non-cooperation

Hawk-Dove Game

		Animal 2	
		D	H
Animal 1	D	$3, 3$	$1, 5$
	H	$5, 1$	$0, 0$

- Two animals are contesting a piece of food.
- Each animal can choose to behave aggressively (the Hawk strategy) or conservatively (the Dove strategy).
  - If the two animals both behave conservatively, they divide the food evenly, and each get a payoff of 3.
  - If one behaves aggressively while the other conservatively, then the aggressor gets most of the food, obtaining a payoff of 5, while the conservative one only gets a payoff of 1.
  - If both animals behave aggressively, they destroy the food (and possibly injure each other), each getting a payoff of 0.

Hawk-Dove Game

		Animal 2 D	Animal 2 H
Animal 1	D	3, 3	1, 5
	H	5, 1	0, 0

- The equilibria are (H, D) or (D, H)
- However, it is not easy to predict which choice
- Nash equilibrium may help to narrow down the choices, but may not predict the sole outcome
  - We don't need to consider non-Nash equilibirum

Mixed strategy

		Player 2 H	Player 2 T
Player 1	H	$-1, +1$	$+1, -1$
Player 1	T	$+1, -1$	$-1, +1$

- Two people each hold a penny and simultaneously choose whether to show heads or tails on their penny
- Player 1 loses his penny to Player 2 if they match
- Player 1 wins Player 2's penny if they don't match

Mixed strategy

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

- zero-sum game: payoff of the players sum to zero in every outcome
- No pair of strategies are best response to each other.
- We need to extend the concept of strategy

Mixed strategy

- Introduce probability, i.e., randomness
- A player will adopt a strategy with certain probabilities; we call it a *distribution*;
- This strategy is called as mixed strategy

- For example
  - Player 1 will choose H with probability p, and T with 1 – p
  - Player 2 will choose H with probability q, and T with 1 – q

- How many strategies one can have? Infinite (differs from pure strategy)
- Pure strategy: when p=1, pure strategy H, etc.

Payoffs from the mixed strategies

		Player 2 H	Player 2 T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

- Player, strategies, what about payoff?
- How to rank different outcomes?

The payoff for mixed strategy is an expectation of the payoffs of the two pure strategies

- Infinite number of strategies
- Payoff at strategies under (p,q)

Calculation of the payoff

- P1(p, q) is the expected payoff where, with probability p, player 1 choose U and, with probability (1-p), player 1 choose D
- What is the payoff when player 1 chooses U? →
- P1(U, q) is the expected payoff where, with probability q, player 2 chooses L, and with probability (1-q), player 2 chooses R

p \ q	0.1	0.2	0.3	0.4	...
0.1	2.74, ?	?, ?	?, ?	?, ?	
0.2	?, ?	?, ?	?, ?	?, ?	
0.3	?, ?	?, ?	?, ?	?, ?	
0.4	?, ?	?, ?	?, ?	?, ?	

	L(q)	R(1-q)
U(p)	4, 4	0, 3
D(1-p)	3, 0	3, 3

Calculation of the payoff

- P1(p,q) = p P1(U,q) + (1-p) P1(D,q)
- P1(U,q) = q P1(U,L) + (1-q) P1(U,R) ➔
  - = 0.1 * 4 + 0.9 * 0 = 0.4
- P1(D,q) = q P1(D,L) + (1-q) P1(D,R) ➔
  - = 0.1*3 + 0.9*3 = 3
- P1(p,q) = 0.1*0.4+0.9*3 = 2.74

p \ q	0.1	0.2	0.3	0.4	...
0.1	2.74, ?	?, ?	?, ?	?, ?	
0.2	?, ?	?, ?	?, ?	?, ?	
0.3	?, ?	?, ?	?, ?	?, ?	
0.4	?, ?	?, ?	?, ?	?, ?	

	L(q)	R(1-q)
U(p)	4, 4	0, 3
D(1-p)	3, 0	3, 3

Calculation of the payoff

- However! When working on a mixed strategy, we usually don't care too much on the exact payoff according to each strategy
- We care more on
  - Whether an equilibrium will be derived
  - When and in what strategy combination such equilibrium will be derived
  - Which probability pair will be the best responses for each other

p \ q	0.1	0.2	0.3	0.4	…
0.1	2.74, ?	?, ?	?, ?	?, ?	
0.2	?, ?	?, ?	?, ?	?, ?	
0.3	?, ?	?, ?	?, ?	?, ?	
0.4	?, ?	?, ?	?, ?	?, ?	

	L(q)	R(1-q)
U(p)	4, 4	0, 3
D(1-p)	3, 0	3, 3

Calculation of the payoff

- We care more on
  - Whether an equilibrium will be derived
  - When and in what strategy combination such equilibrium will be derived
  - Which probability pair will be the best responses for each other

	Stag	**Hare**	**q-mix**
Stag	4,4	0,3	4q, 4q+3(1-q)
Hare	3,0	3,3	3q+3(1-q), 3(1-q)
p-mix	4p+3(1-p), 4p	3(1-p), 3p+3(1-p)	

	L(q)	R(1-q)
U(p)	4, 4	0, 3
D(1-p)	3, 0	3, 3

Calculation of the payoff

	L(q)	R(1-q)
U(p)	4, 4	0, 3
D(1-p)	3, 0	3, 3

	Stag	Hare	q-mix
Stag	4,4	0,3	4q, 4q+3(1-q)
Hare	3,0	3,3	3q+3(1-q), 3(1-q)
p-mix	4p+3(1-p), 4p	3(1-p), 3p+3(1-p)	

- Best response curve:
  - Best q-response to p-mix
  - Best p-response to q-mix
  - Expected payoff for general p varies linearly with p
- Nash equilibrium?

Calculation of the payoff

	Stag	Hare	q-mix
Stag	4,4	0,3	4q, 4q+3(1-q)
Hare	3,0	3,3	3q+3(1-q), 3(1-q)
p-mix	4p+3(1-p), 4p	3(1-p), 3p+3(1-p)	

- Best p-response to q-mix
  - Player 1's best response p as function of player 2's q
  - Pure strategy Stag (p=1) better than pure strategy Hare(p=0) if 4q>3q+3(1-q). -> q>3/4
- Player 1's expected payoff for general p
  - p(4q)+(1-p)(3q+3(1-q)) = (4q-3)p+3 varies linearly with p
  - In same cases (q>3/4)
    - P=1 is also better than any other p in the range from 0 to 1
    - P=1(pure Stag) is Player 1's best response if q>3/4

Calculation of the payoff

	Stag	Hare	q-mix
Stag	4,4	0,3	4q, 4q+3(1-q)
Hare	3,0	3,3	3q+3(1-q), 3(1-q)
p-mix	4p+3(1-p), 4p	3(1-p), 3p+3(1-p)	

- Best p-response to q-mix
  - Player 1's expected payoff for general p

$p(4q)+(1-p)(3q+3(1-q)) = (4q-3)p+3$ varies linearly with p

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    - P=1 is also better than any other p in the range from 0 to 1
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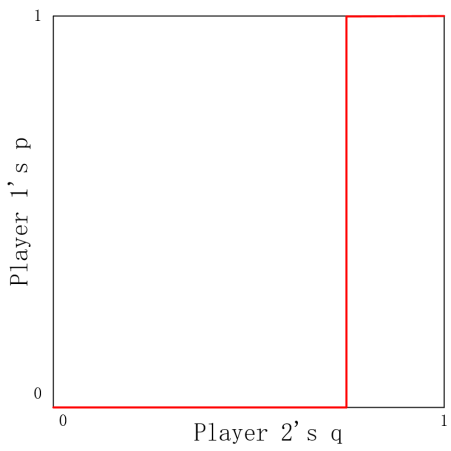

Calculation of the payoff

	Stag	Hare	q-mix
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Hare	3,0	3,3	3q+3(1-q), 3(1-q)
p-mix	4p+3(1-p), 4p	3(1-p), 3p+3(1-p)	

- Best q-response to p-mix
  - Player 2's best response q as function of player 1's p
  - Pure strategy Stag (q=1) better than pure strategy Hare(q=0) if 4p>3p+3(1-p). -> p>3/4
- Player 2's expected payoff for general q
  - q(4p)+(1-q)(3p+3(1-p)) = (4p-3)q+3 varies linearly with q
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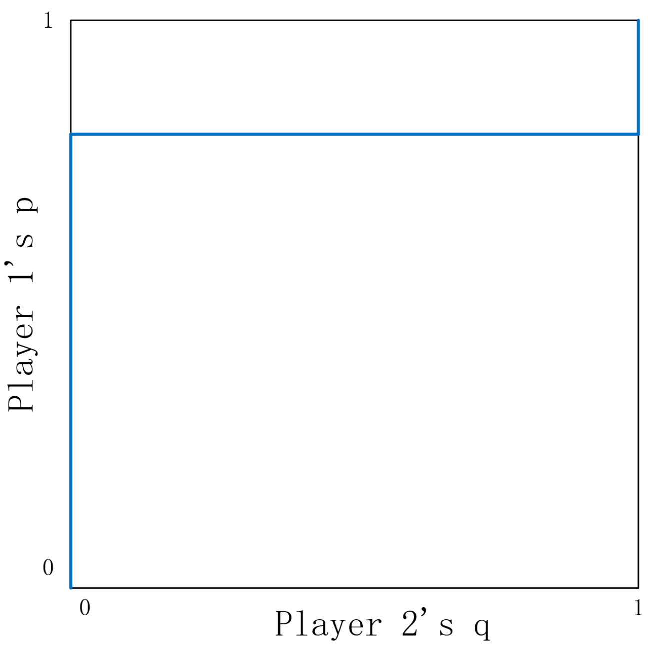
Calculation of the payoff

	Stag	Hare	q-mix
Stag	4,4	0,3	4q, 4q+3(1-q)
Hare	3,0	3,3	3q+3(1-q), 3(1-q)
p-mix	4p+3(1-p), 4p	3(1-p), 3p+3(1-p)	

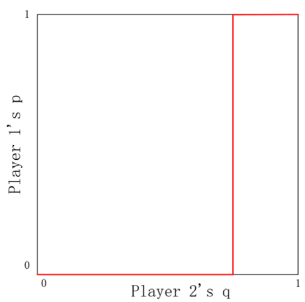
- Best q-response to p-mix
  - Player 2's expected payoff for general q

  q(4p)+(1-q)(3p+3(1-p)) = (4p-3)q+3 varies linearly with q
  - In same cases (p>3/4)
    - q=1 is also better than any other q in the range from 0 to 1
    - q=1(pure Stag) is Player 2's best response if p>3/4

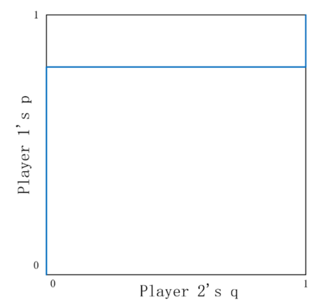


Mixed Strategy Nash Equilibrium

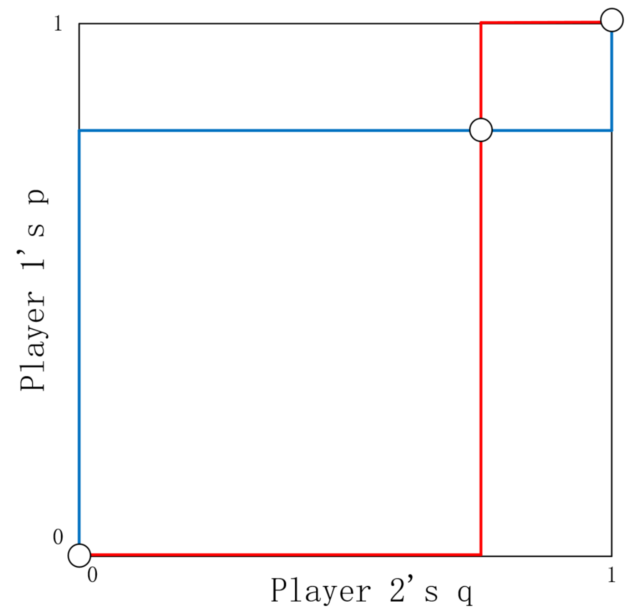
- Player 1’s best response



- Player 2’s best response



- Mutual best response



- Three Nash Equilibrium:
- (p=1, q=1), pure strategy (Stag,Stag)
- (p=0,q=0), pure strategy(Hare, Hare)
- (p=3/4,q=3/4), mixed strategy with expected payoff
  - P*q*(4,4)+(1-p)*(1-q)*(3,3)+p*(1-q)*(0,3)+(1-p)*q*(3,0) = (3,3)

Coming back to the Head-Tail Game

- If player 2 chooses 0.5, it doesn't matter what player 1's strategy is; this is also called indifference.
- In other words, every strategy for player 1 is a best response
- The other way round is also true, i.e., if player 1 chooses 0.5

- What is the best response of player 1 as a function of player 2' q:

	H	T	q-mix
H	-1,+1	+1, -1	-q+1(1-q)
T	+1, -1	-1,+1	q – (1-q)

Indifference point is (p=0.5, q=0.5)

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

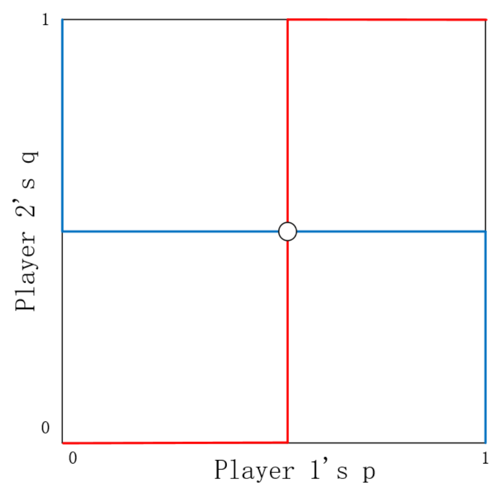
Coming back to the Head-Tail Game

- Best response curve of two players

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

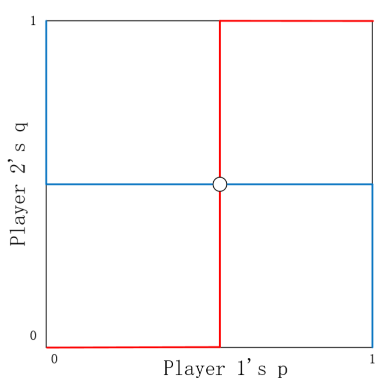
	H	T	q-mix
H	-1,+1	+1, -1	-q+1(1-q)
T	+1, -1	-1,+1	q – (1-q)

Indifference point is (p=0.5, q=0.5)



Coming back to the Head-Tail Game

- For two players, two strategies, no Nash equilibrium for pure strategies
- Recall the matching pennies game
  - If you know that the other player will play H with probability 0.7, what do you do? What if the probability is 0.2?



		Player 2: H	Player 2: T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

Coming back to the Head-Tail Game

- For two players, two strategies, no Nash equilibrium for pure strategies
- Recall the matching pennies game
  - If you (player 1) know that the other player(player 2) will play H with probability 0.7, what do you do? What if the probability is 0.2?
  - You can always play T if you know he plays 0.7 H
  - You can always play H if you know he plays 0.2 H

		Player 2 H	Player 2 T
Player 1	H	$-1, +1$	$+1, -1$
Player 1	T	$+1, -1$	$-1, +1$

Coming back to the Head-Tail Game

- For two players, two strategies, no Nash equilibrium for pure strategies
- Recall the matching pennies game
  - If you know that the other player will play H with probability 0.7, what do you do? What if the probability is 0.2?
  - You can always play T if you know he plays 0.7 H
  - You can always play H if you know he plays 0.2 H
  - Why you choose a fixed strategy now?

  i.e., with probability 1, you play T if you know he plays 0.7 H
  - What if with probability 0.7, you play T? ➔
  - The other 0.3 you play H …

  seems you will suffer a loss

		Player 2: H	Player 2: T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

Finding Nash Equilibrium

- Recall the matching pennies game
  - If you know that the other player will play H with probability 0.7, what do you do? What if the probability is 0.2?
  - You can always play T if you know he plays 0.7 H
  - You can always play H if you know he plays 0.2 H

  - Why you choose a fixed strategy now?

i.e., with probability 1, you play T if you know he plays 0.7 H

  - What if with probability 0.8, you play T? ➔
  - The other 0.2 you play H …

seems you will suffer a loss

		Player 2: H	Player 2: T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

  - If you know that the other player will play H with probability 0.5, what do you do?

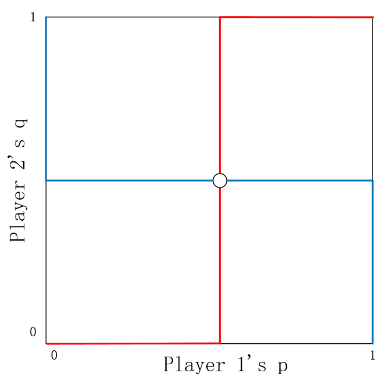
What is special for 0.5?

- If player 2 chooses 0.5, it doesn't matter what player 1's strategy is; this is also called indifference.
- In other words, every strategy for player 1 is a best response
- The other way round is also true, i.e., if player 1 chooses 0.5

- What is the best response of player 1 as a function of player 2' q:

	H	T	q-mix
H	-1,+1	+1, -1	-q+1(1-q)
T	+1, -1	-1,+1	q – (1-q)

Indifference point is (p=0.5, q=0.5)



The existence of the NE

- Every finite game has a mixed strategy Nash equilibrium
  - Finite game: games with finite strategy sets, finite number of players

- Why is this important?
  - Without knowing the existence of an equilibrium, it is difficult (perhaps meaningless) to try to understand its properties.
  - Armed with this theorem, we also know that every finite game has an equilibrium, and thus we can simply try to locate the equilibria.

Mixed strategy game: summary

- For a game of mixed strategy, the three basic ingredients are
  - *Players*, the same as a pure game
  - *Strategy*, a probability distribution on the strategy set of pure game
    - There can be infinite number of strategies
  - *Payoff* at strategies under (p, q), i.e., the *expected* payoff when the probability is p, q for certain strategies
    - Different p, q will have different payoff

Mixed strategy game: summary

- The necessary condition for a pair of mixed strategies to be best responses of each other is that it makes the other player *indifferent* in payoffs under all his pure strategies
- This is how we compute the mixed strategy: indifference
- Method: assume the mixed strategy for player 1 is p, write the expected payoff for both pure strategies for player 2. The Nash equilibrium is the strategy that make the two functions equal

The best strategy is the one that making the other player doesn't know which of his pure strategy is better

Empirical analysis

- The Penaltv-Kick Game

		Goalie	
		L	R
Kicker	L	$0.58, -0.58$	$0.95, -0.95$
	R	$0.93, -0.93$	$0.70, -0.70$

- Ignacio Palacios-Huerta in 2002
- Ball moves to the goal fast enough -> simultaneous move
- Empirical probability of scoring for each of the four basic outcomes
- Differs from the idea matching pennies game:
  - A kicker still has good chance of winning even if dives the correct direction
  - Right-footed, not symmetric

Empirical analysis

- The Penaltv-Kick Game

		Goalie L	Goalie R
Kicker	L	$0.58, -0.58$	$0.95, -0.95$
	R	$0.93, -0.93$	$0.70, -0.70$

- Indifference point (Theoretically):
  - q is the probability that a goalie chooses L
  - To let kicker indifferent between his two options:
    - (.58)(q)+0.95(1-q)=(.93)(q)+(.70)(1-q)
    - q=0.42, similarly, p = 0.39
- In reality :
  - Real q is 0.42
  - Real p is 0.40

Pareto Optimality

Pareto Optimality

- Previous examples

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

		Suspect 2: NC	Suspect 2: C
Suspect 1	NC	$-1, -1$	$-10, 0$
	C	$0, -10$	$-4, -4$

- The eventual outcome is not in any sense good
- "Good for the society"

Pareto Optimality

- Pareto Optimality:
  - Italian economist Vilfredo Pareto
  - many economic solutions helped some people while hurting others. He was interested in finding solutions that helped some people without hurting anyone else.
  - A choice of strategies – one by each player – is Pareto-optimal if there is no other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.

Pareto Optimality

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

- The only outcome that is not Pareto-optimal is the one corresponding to the unique Nash equilibrium.

Social Optimality

- A stronger condition
- Social optimality: A choice of strategies – one by each player – is a social welfare maximizer (or socially optimal) if it maximizes the sum of the player's payoffs

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	$90, 90$	$86, 92$
	Exam	$92, 86$	$88, 88$

- Which outcome is social optimal?
- Social optimal v.s. pareto optimal?

Social Optimality and Nash Equilibrium

- The two can be the same
  - (Presentation, presentation) is both social optimal and Nash equilibrium

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	98, 98	94, 96
	Exam	96, 94	92, 92

- It is an ideal system if social optimal is also Nash equilibrium

Application of Pareto Optimality

- Originated from the concept of efficiency in economy
- Widely applied in multi-criteria optimization

  - Example:
    - A small business advertises through traditional media and personal appearances. Each ad campaign in traditional media costs about \$2000, generating 2 new customers and 1 positive rating per month. Each personal appearance costs \$500, generating 2 new customers and 5 positive ratings. The company wants at least 16 new customers and 28 positive ratings per month. Minimize the advertising costs for this company

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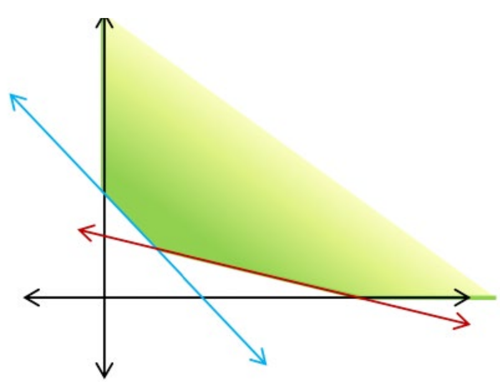
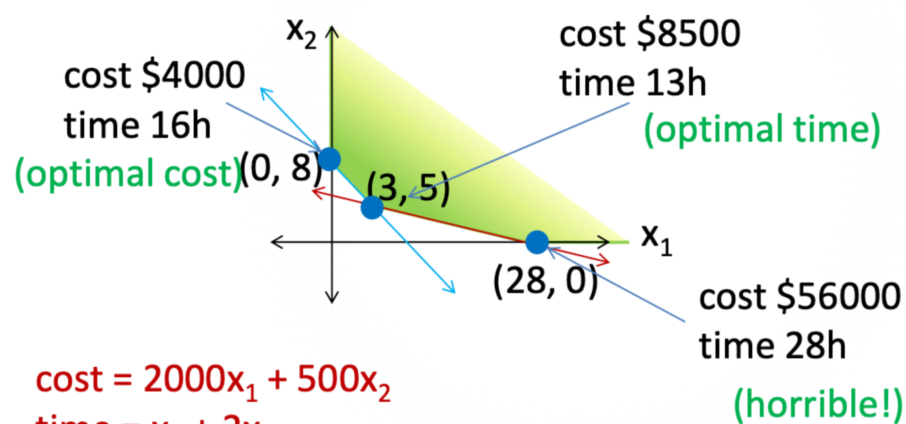
  - Hidden cost: person appearances cost too much time, they need to hire another executive

- Multi-objective problem with constraints
  - Constraints: Minimum number of customers and positive ratings

Application of Pareto Optimality

- Design space
- Criterial space
- Pareto front
- Suppose each personal appearance takes 2 hours and each ad campaign takes 1 hour

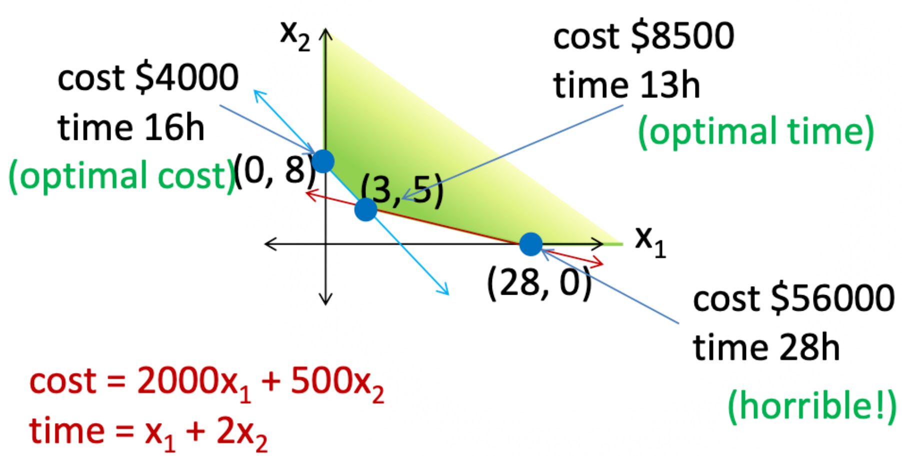


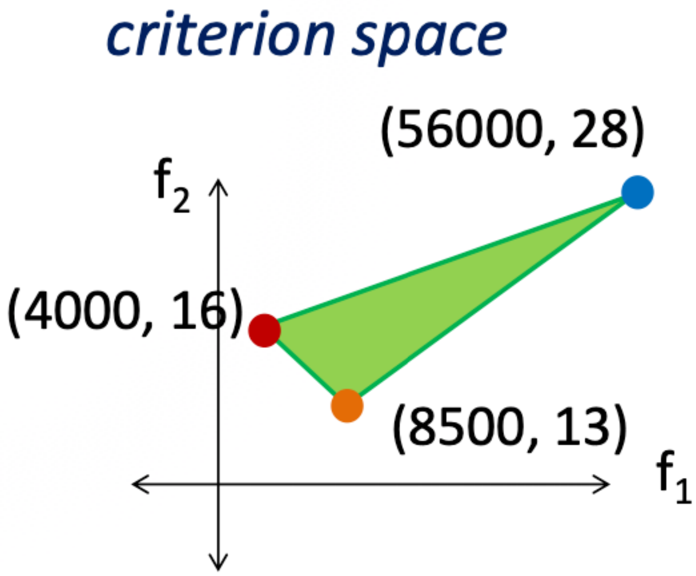


cost = $2000x_1 + 500x_2$
time = $x_1 + 2x_2$

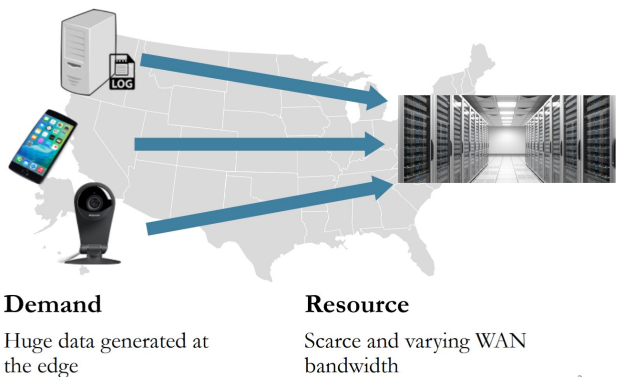
Application of Pareto Optimality

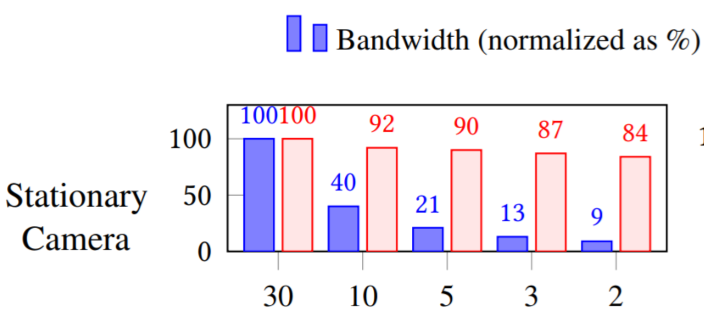
- Design space
- Criterial space
- Pareto front: a set of nondominated solutions if no objective can be improved without sacrificing at least one other objective



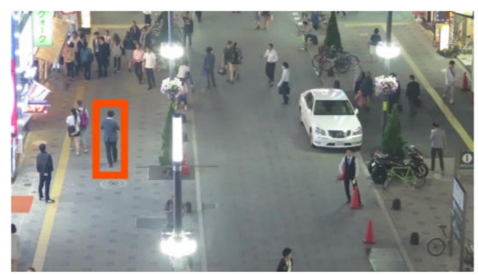


Application of Pareto Optimality: AWStream





(a) Frame Rate
(resolution = 1080p)



(c) t = 0s
small targets in far-field views

(d) t = 1s
small difference compared to t=0s

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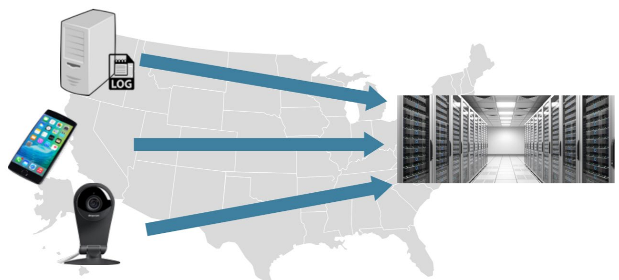
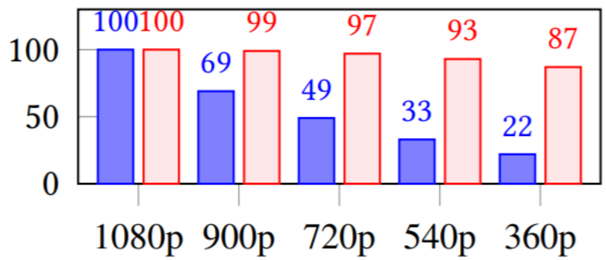
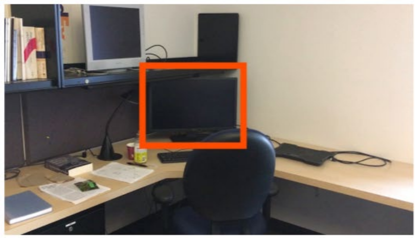


Demand
Huge data generated at the edge

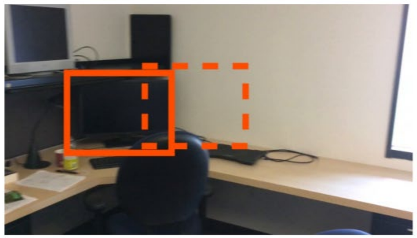
Resource
Scarce and varying WAN bandwidth



(f) Resolution
(frame rate = 30)



(g) t = 0s
nearby and large targets



(h) t = 1s
large difference compared to t=0s

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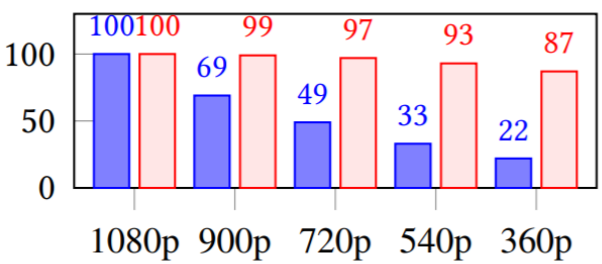
Demand

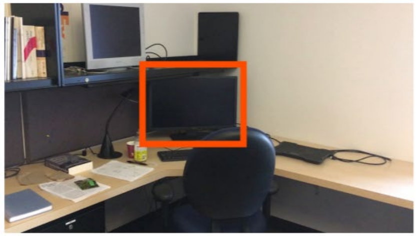
Huge data generated at the edge

Resource

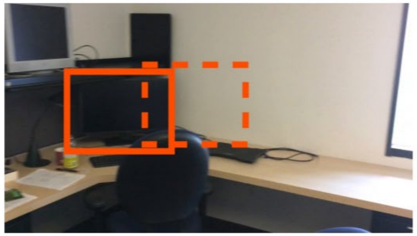
Scarce and varying WAN bandwidth



(f) Resolution
(frame rate = 30)



(g) t = 0s
nearby and large targets



(h) t = 1s
large difference compared to t=0s

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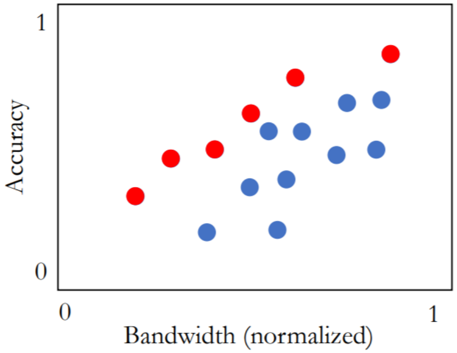
Application	Knobs	Accuracy	Dataset
Augmented Reality	resolution frame rate quantization	F1 score [70]	iPhone video clips training: office (24 s) testing: home (246 s)
Pedestrian Detection	resolution frame rate quantization	F1 score	MOT16 [53] training: MOT16-04 testing: MOT16-03

configuration	bandwidth	accuracy
c1	10.7	1.0
c2	8.3	0.88
c3	6.3	0.87
c4	9.3	0.90
…	….	…



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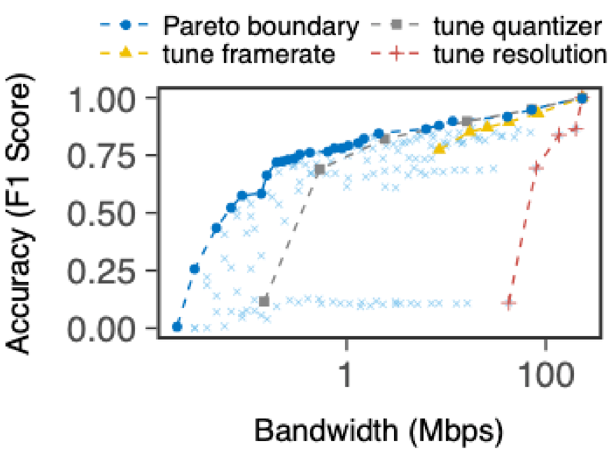

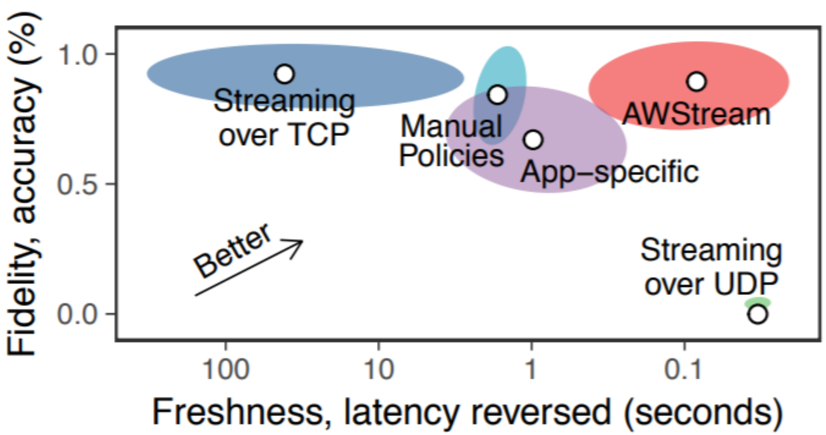

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Logic flow of the Game part

- Game assumptions
- Three basic ingredients of a game
- Strict dominant strategy
- Best response
- Nash equilibrium: best responses to each other
- Multiple equilibria
- No equilibrium
- Mixed strategy
- Solving mixed strategy
- Group optimality: Pareto optimal and social optimal