VE444: Networks

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Review

- Nash Equilibrium Existence Theorem
- If game is zero-sum, we can compute a NE in poly time.
- User optimal v.s. Social optimal

Mechanism Design Basics

Motivating example

- 2012 London Olympics
- Video: 08:00

Motivating example

- 2012 London Olympics
- Phase 1: Round-robin
- 4 teams of 4
- Top 2 teams from each group advance
- Phase 2: Knockout

Motivating example

- Trigger: in group D, Danish PJ upset Chinese team QW
- Next match: Chinese team XY meets Korean team KH to decide who is 1st and 2nd in group A
- Issue: Group A winner would face QW in semis, 2nd-best would only face QW in the final.

- Misalignment between participant and designer's goal.

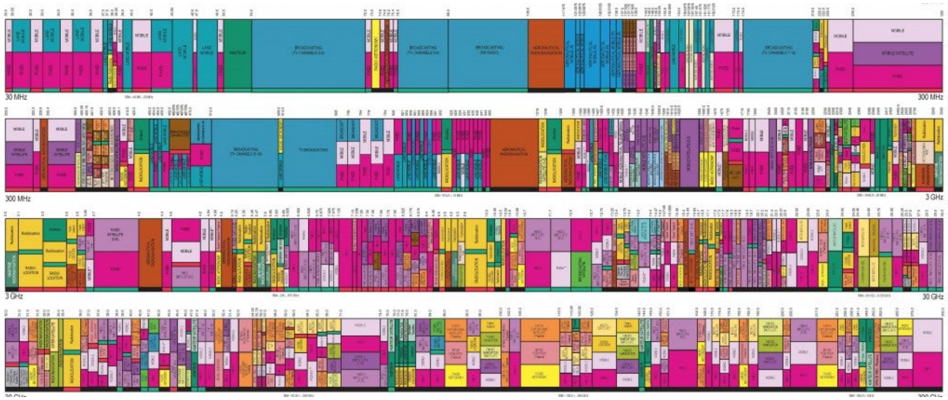
Mechanism design in Practice

- Spectrum auctions since 1994
  - FCC auctions
  - Worldwide innovations in auction design
- Other innovative auctions
  - Electricity
  - Carbon emissions
  - Search auctions
  - Computing resources

FCC completes 3.5 GHz spectrum auction raising $4.5 bn

August 26, 2020

The Federal Communications Commission (FCC) has completed the 3.5 GHz spectrum auction (Auction 105) – raising $4.585 billion.



Mechanism design in Practice

- Kidney exchange
- Resident doctor matching
- Voting

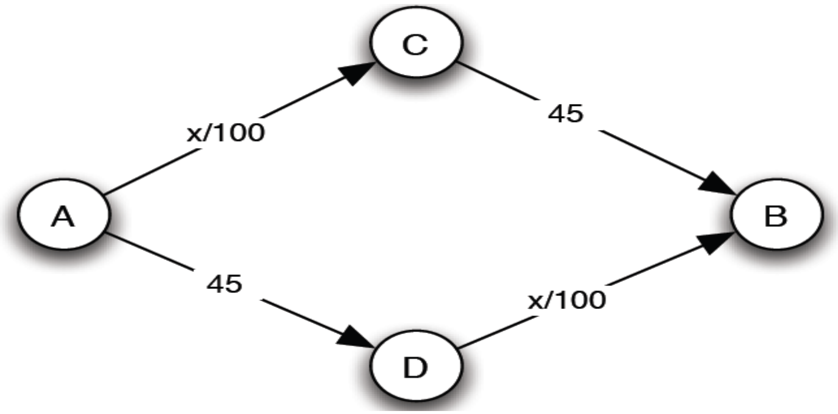
- But first, when is selfish behavior benign?

Traffic as a Game

A game on network structure

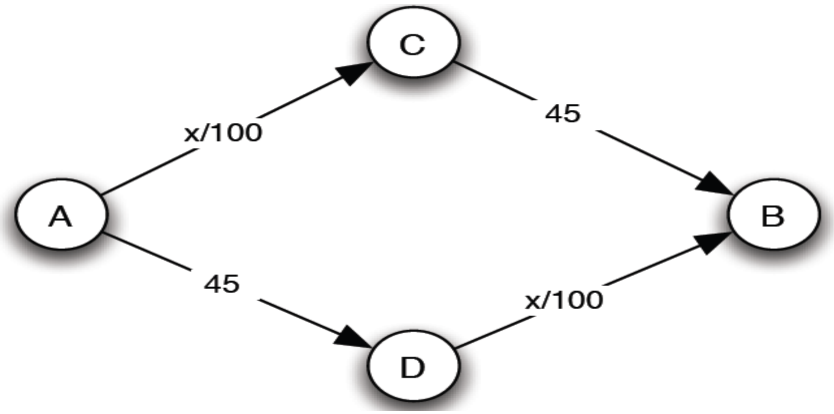
- 4000 vehicles want to travel from A to B

- Players: 4000 drivers
- Strategy set: upper path, lower path
- Payoff: travel time (the less the better, but also depends on other's choices)
- Equilibrium? Payoff matrix?



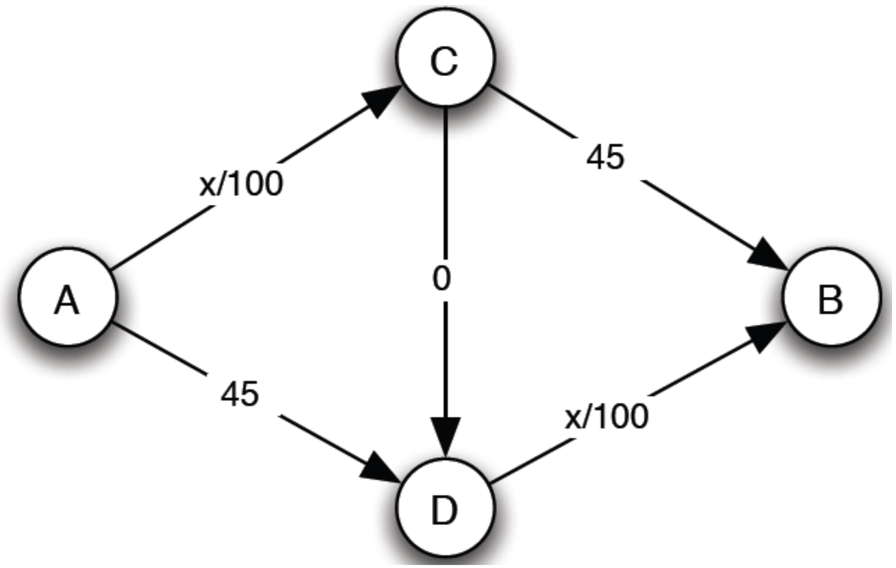
A game on network structure

- Equilibrium: 2000 A-C-B, 2000 A-D-B
- Payoff for each driver: 65
- If anyone deviates, his payoff will be: 2001/100 + 45 > 65



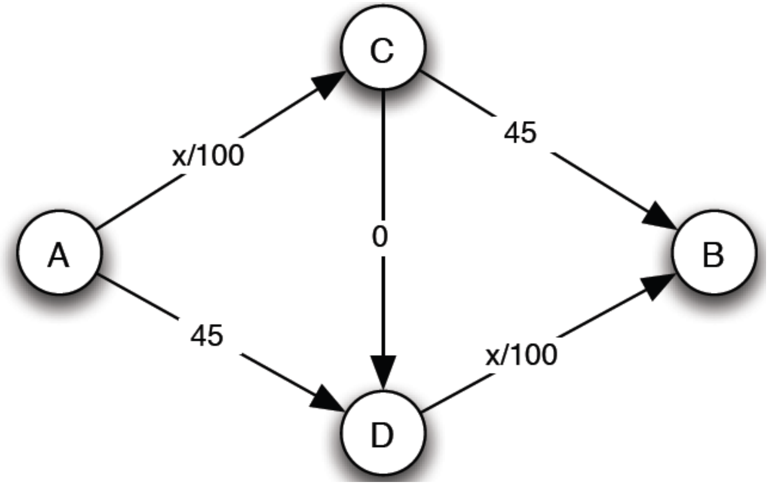
If the government builds a new road

- Assume the government want to do something good: let’s build a new road and it is an express road, e.g., travel time on this road is negligible!
- What will happen?



If the government builds a new road

- Is the previous possible?
  - 2000 A-C-B
  - 2000 A-D-B
- No way, no longer equilibrium
- If you are the one on A-C-B
  - A-C-D-B will be faster, there is incentive to change
- This is called Braess's Paradox



If the government builds a new road

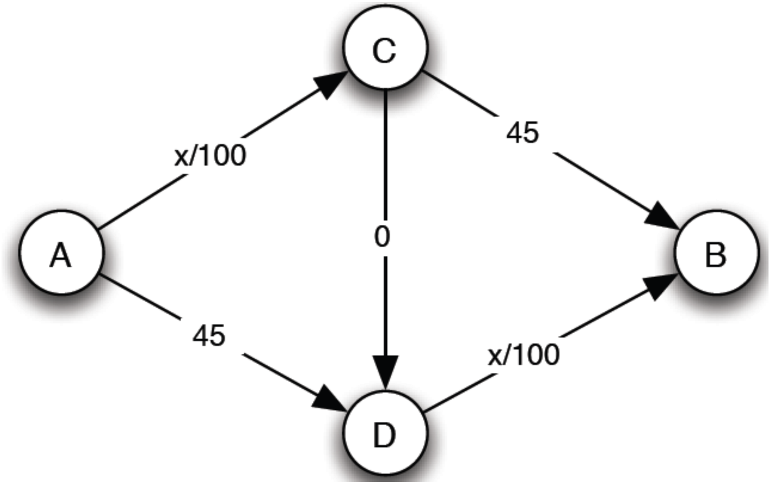
- Equilibrium: every one uses A-C-D-B
  - Travel time is

4000/100+0+4000/100=80

  - If someone tries to deviate,

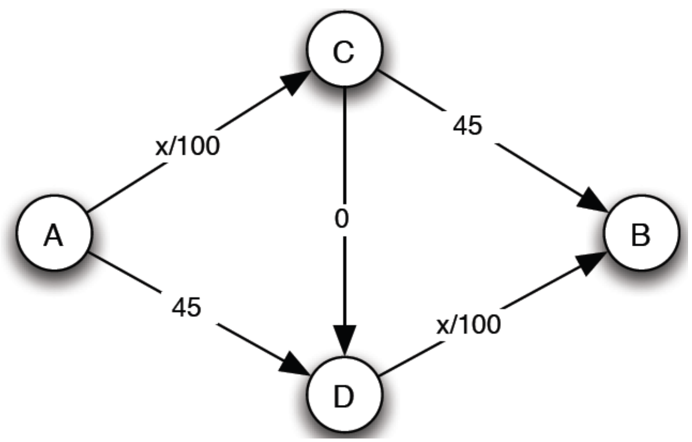
Travel time is: 45+4000/100=85>80

It is worse than 65!



Comparing with the social optimal

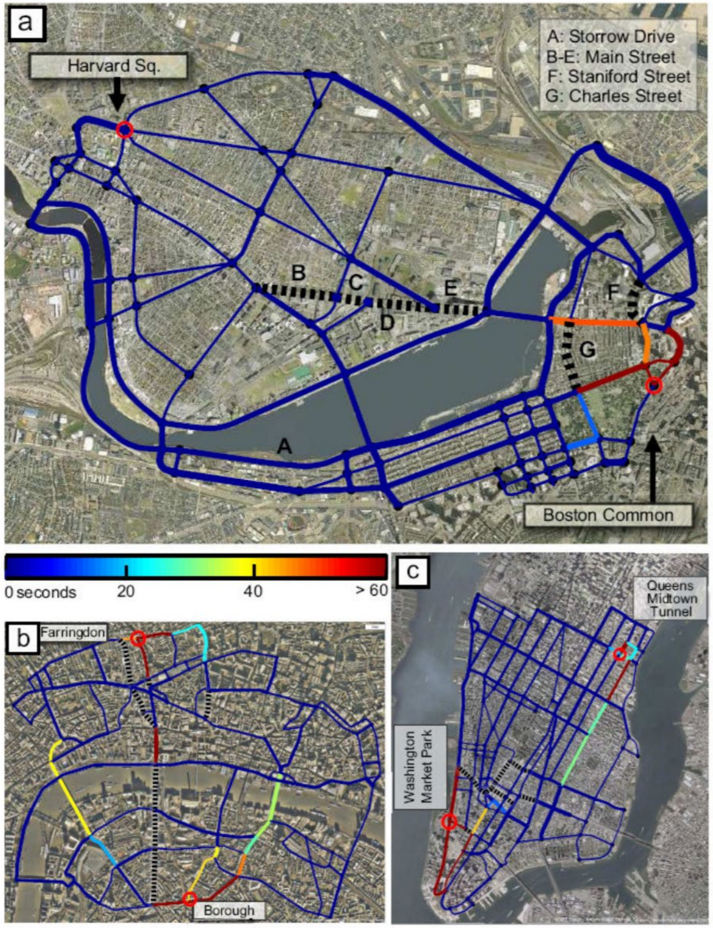
- The outcome of rational behaviors can be inferior to a centrally designed outcome.
- Question is: by how much?
- Price of Anarchy(POA): the ratio between the system performance with strategic players and the best-possible system performance



Real-world practice

- Stuttgart, Germany (1969): The traffic worsened until a newly built road was closed
- New York City (1990): On Earth Day 42nd street was closed and traffic flow improved.
- Seoul, South Korea (2002), replaced a six lane highway with a five mile long park, traffic flow improved.

Real-world practice



Price of Anarchy in Transportation Networks: Efficiency and Optimality Control, Physical Review Letters, 2008

A summary

- A simple example shows a game on network structure
- We see Braess's Paradox
- Invest more resources may not get a good result

- Identify application domains and conditions under which the POA is guaranteed to be close to 1
  - Selfish behavior leads to a near-optimal outcome

Mechanism Design with Money: Auction

An example

- In game and Braess's Paradox, we see the interaction of rational behavior, equilibrium, and network structure
- Can we change the condition and let the players to directly interact (of course, we also need to set up a set of rules for their interaction)?
- Auction can be one of such scenario

Auction is everywhere

- Auctions by Christie and Sotheby’s
- Government auctions the land, license, etc
- Electricity
- Carbon emissions
- Search auctions
- Computing resources

Auction is important

- It is everywhere, with very simple format, but there can be complicated interactions

- It is also a game
  - Participants: sellers and buyers
  - Strategy: bid
  - Payoff: for the buyers: the value of the object, (0 if the auction fails); for the sellers: the paid price, 0 if the auction fails)
  - Equilibrium: best response for each other, i.e., no one has the incentive to deviate

The format of auctions

- The equilibrium depends on the format and regulations of the auctions
- The format also influence the strategy choices of the buyers and sellers

Types of Auction

- Auction: a seller auctioning one item to a set of buyers
- Procurement auctions: a buyer trying to purchase a single time among a set of sellers.

Types of Auction

- Ascending-bid auctions/English auctions: the seller gradually raises the price, bidders drop out until finally only one bidder remains;
  - This is useful for auctions of art works, antiques, etc.

- Descending-bid auctions/Dutch auctions: the seller gradually lowers the price from some high initial value until the first moment when some bidder accepts and pays the current price;
  - This is useful for auctions of flowers, fresh farm products.

Types of Auction

- First-price sealed-bid auctions: bidders submit simultaneous “sealed bids" to the seller, who would then open them all together. The highest bidder wins the object and pays the value of her bid.
  - This is used in call for bid auctions
- Second-price sealed-bid auctions/Vickrey auctions: Bidders submit simultaneous sealed bids to the sellers; the highest bidder wins the object and pays the value of the second-highest bid
  - This is used in advertisement auction in Internet websites
  - In honor of William Vickrey, first game-theoretic analysis of auctions, Nobel Memorial Prize in Economics in 1996.

When are Auctions Appropriate?

- Known value
  - Seller valuation: x, buyer valuation: y
- Surplus: y-x
- Commit to the mechanism
- Unknown value
  - Independent, private values
  - Common value

Relationship between different formats

- Descending-bid
- Ascending-bid
- First-price
- Second-price

A few points in auction formats

- Who get the object
  - Usually the highest or lowest bidder
- What kind of price to pay
  - First price or second price, this influences the strategy of the bidders
- Do the bidders know the price of others
  - Sealed auction, needs to guess about others
  - Unsealed auction, can see other’s bids

A summary

- Auction is common in our life
- Auction has many formats
- Auction different objects may need to choose different auction formats

A few points in auction formats

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Auction: Game perspective

- First-price sealed-bid auctions (FPA)
  - Highest bidder wins and pays the value of her bid
- Second-price sealed-bid auctions (SPA)
  - Highest bidder wins and pays the value of the second-highest bid.
- Formulating as a game
  - Players: bidders
  - Strategy: bid
  - Payoff: true value – payoff, or 0 (if auction fails)
- A game-oriented thinking
  - Equilibrium! Best response to each other, no one changes

Second-price sealed-bid auctions

- An object
  - Different people may have different values for it, v1, v2, …, vk. These are true value/intrinsic value, i.e., player *i* will pay at most *vi*.
  - Players don't know other's true values
  - Every one has a bid, assume b1 > b2 > … > bk
  - By the rule of second-price auction, the payoff of the highest bidder is v1 – b2 and others are 0; here, b1 is the highest bid, v1 is the true value

How do you play the game, i.e., how do you bid?

Second-price sealed-bid auctions

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We assume that people maximize their own profits, i.e., get the object and pay the lowest price possible

What strategy is optimal?

- From game point of view (dominant strategy): cannot get better payoff by changing to other strategies, regardless of the other players' strategy
- Claim: In a sealed-bid second-price auction, it is a dominant strategy for each bidder i to choose a bid bi = vi.

Proof

- Assume in an auction, you consider the object worth \$100 and you bid \$100, now consider whether you can get better payoff using other strategy
- There are two cases:
  - You win the bid: now you have positive payoff (you only need to pay the second high bid, $v1 – b2 > 0$)
    - Increase the bid won't change anything
    - Decrease the bid won't change anything unless less than the second high, then you lose the bid and payoff become 0
  - You lose the bid: now your payoff is 0
    - Decrease the bid won't change anything
    - Increase the bid won't change anything unless becoming the highest; note that this means that this is greater than your value, so payoff becomes negative

A Summary

- In second-price sealed-bid auction, bid the true value is a (weakly) dominant strategy (more in Chapter 15)
- First-price seal-bid auction does not have such property

- It may not be easy to know the "true value"

Mechanism Design without Money: Matching Markets