Class 4. Hypothesis testing & Goodness-of-fit

Advanced Econometrics I

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Problem 1

The following estimated equation was obtained by ordinary least squares regression using quarterly data for 1960 to 1979 inclusive (N = 80).

$$\hat{y}_t = \underset{(3.4)}{2.20} + \underset{(0.005)}{0.104} x_{1t} + \underset{(2.2)}{3.48} x_{2t} + \underset{(0.15)}{0.34} x_{3t},$$

(standard errors are in parentheses). The explained sum of squares was 112.5, and the residual sum of squares was 19.5.

(a) Which of the slope coefficients are significantly different from zero at the 5% significance level?

(b) Find the 95% confidence interval for β_3;

(c) Give the interpretation to the coefficient estimates.

(d) Calculate the value of R^2 for this regression;

(e) Calculate the value of adjusted R^2;

(f) Are slope coefficients jointly significant?

(g) Check the hypothesis that $\beta_3 = 0.9$;

Problem 2

Given the formula for F-statistic:

$$F = \frac{(RSS_R - RSS_{UR})/q}{RSS_{UR}/(n-K)}$$

derive that

$$F = \frac{(R^2_{UR} - R^2_R)/q}{(1-R^2_{UR})/(n-K)},$$

where q denotes the number of restrictions.

Problem 3

The following regression model is estimated as a production function:

$$\ln Q = 1.37 + \underset{(0.257)}{0.632} \ln K + \underset{(0.219)}{0.452} \ln L, \quad R^2 = 0.98, \quad \widehat{cov}(\hat{\beta}_K, \hat{\beta}_L) = -0.044.$$

The sample size is 40. Test the following hypotheses at the 5% level of significance:

(a) $\beta_K = \beta_L$;

(b) There are constant returns to scale.

Problem 4

If we exclude some regressors from the linear model with constant, it follows that coefficient of adjusted multiple determination R^2_{adj}:

(a) Does not increase;

(b) Does not decrease;

(c) It may increase as well as decrease.

Problem 5

Researcher Frodo investigates the relationship between the power of different armies (in conventional units) and the type of worriers (various Middle-Earth inhabitants). He uses a sample of 10 observations collected by Gandalf.

First, Frodo estimated the following model:

$$Power_i = \alpha + \beta_1 \times Dwarves_i + \beta_2 \times Wizards_i + \beta_3 \times Elves_i + \beta_4 \times Hobbits + \varepsilon_i.$$

For this model he calculated RSS, which is equal to $RSS = 15.29$.

Second, he decided to exclude variables $Wizards$ and $Hobbits$ and estimated the model again:

$$Power_i = \alpha + \beta_1 \times Dwarves_i + \beta_3 \times Elves_i + \varepsilon_i.$$

The values of RSS for this model is equal to $RSS = 41.53$.

Help Mr. Frodo to determine whether Wizards and Hobbits have a jointly significant influence on the power of an army.

Problem 6*

The following models should be estimated:

1. $Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \varepsilon$;
2. $Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \varepsilon$;
3. $Y = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \varepsilon$;
4. $Y = \alpha_0 + \alpha_1 \ln X_1 + \alpha_2 \ln X_2 + \alpha_3 \ln X_3 + \varepsilon$
5. $Y = \alpha_0 \times X_1^{\alpha_1} \times X_2^{\alpha_2} \times X_3^{\alpha_3} \times \varepsilon$

(a) Which of them could be estimated with the use of OLS method?

(b) Which of them could be compared by R^2?