Class 3. Introduction to OLS

Advanced Econometrics I

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Problem 1

Let the simple linear model on a constant be specified as follows:

$$Y_i = \alpha + \varepsilon_i.$$

Find the OLS estimator of α.

Problem 2*

Let the simple pairwise linear regression model be given:

$$Y_i = \alpha + \beta X_i + \varepsilon_i.$$

Show that:

$$\hat{\beta}_{ols} = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - \overline{X})^2}, \quad \hat{\alpha}_{ols} = \overline{Y} - \hat{\beta} \times \overline{X}.$$

Problem 3

For 16 observations (X, Y) you are given the following:

$$\sum_{i=1}^{n} Y_i^2 = 526, \quad \sum_{i=1}^{n} Y_i = 64, \quad \sum_{i=1}^{n} X_i^2 = 657, \quad \sum_{i=1}^{n} X_i = 96, \quad \sum_{i=1}^{n} X_i Y_i = 492.$$

(a) Estimate a regression: $Y_i = \alpha + \beta X_i + \varepsilon_i$;

(b) Calculate $RSS = \sum_{i=1}^{n} \varepsilon_i^2$;

(c) Calculate the estimate of variance of the error term;

(d) Calculate the estimate of variance of $\hat{\beta}$.

Problem 4

How will the estimates of coefficients change in a simple linear model $Y_i = \alpha + \beta X_i + \varepsilon_i, \quad i = 1, \ldots n$ after the following transformations [1]?

(a) $X^* = X + c$;

(b) $Y^* = Y + c$;

(c) $X^* = X \times c$;

(d) $Y^* = Y \times c$.

[1] $c \neq 0$

Problem 5

Prove that for regression model $Y = X\beta + \varepsilon$ the problem of least square estimation for β is equivalent to the problem of the following linear equation solution:

$$X^T X\beta = X^T Y.$$

Derive the estimator of β in a matrix form.

Problem 6

Let the linear regression model be given: $Y = X\beta + \varepsilon$. Suppose you have the following observations:

$$Y = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad X = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}.$$

(a) Find the OLS estimate of β via the **matrix form**;

(b) Find the OLS estimate of β via the **vector form**.

Problem 7

Prove that the determination coefficient R^2 in the linear model equals the squared correlation coefficient between Y and $\hat{Y}$, i.e., $\rho^2 = corr^2(Y, \hat{Y})$.

Problem 8

Let the lineal regression model be $Y_i = \alpha + \beta_i x_{i1} + \beta_2 x_{i2} + \varepsilon_i, \;\; i = 1, \ldots, n$, with a matrix form $Y = X\beta + \varepsilon$, where $\beta = (\alpha, \beta_1, \beta_2)^T$.

The observations are:

$$Y = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}, \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

To make the calculations easier use the following matrices:

$$X^T X = \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (X^T X)^{-1} = \begin{pmatrix} 1/3 & -1/3 & 0 \\ -1/3 & 4/3 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

(a) Define the number of observations;

(b) Define the number of regressors (including the intercept);

(c) Calculate $TSS = \sum_{i=1}^{n} (Y_i - \overline{Y})^2$

(d) Use the OLS (ordinary least squares) to estimate the vector of coefficients β;

(e) What is the value of $\hat{\varepsilon}_5$ – the OLS residual for the 5th observation?

(f) Calculate $RSS = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$;

(g) Find and give the interpretation for R^2 in terms of the quality of the estimated regression;

(h) Find the value of the unbiased estimator for an unknown parameter σ^2 – the variance of the error term;

(i) Calculate $\hat{\Sigma}_{\hat{\beta}}$ – estimate of the variance-covariance matrix of $\hat{\beta}$;

(j) Find $\hat{\sigma}^2_{\hat{\alpha}}$ – estimate of variance of $\hat{\alpha}$;

(k) Find $\hat{\sigma}^2_{\hat{\beta}_1}$ – estimate of variance of $\hat{\beta}_1$;

(l) Find $\widehat{cov}(\hat{\alpha}, \hat{\beta}_1)$ – estimate of covariance of $\hat{\alpha}$ and $\hat{\beta}_1$;

(m) Calculate $\hat{\sigma}^2_{(\hat{\beta}_1+\hat{\beta}_2)}$, $\hat{\sigma}^2_{(\hat{\beta}_1-\hat{\beta}_2)}$, and $\hat{\sigma}^2_{(\hat{\alpha}+\hat{\beta}_1+2\hat{\beta}_2)}$.