Assignment 08

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In [1]:
```
import PIL.Image as pilimg
import numpy as np
import matplotlib.pyplot as plt
import random
import copy
import math
from numpy.linalg import *
```

In [2]:
```
# Init Function1
def init_func(x) :
    y = x**3 - 16*x
    return y

def init_noise(x_domain) :

    noise_y = np.zeros(len(x_domain), dtype=float)

    for i in range(len(x_domain)) :
        x = x_domain[i]
        noise_y[i] = init_func(x) + random.uniform(-30, 30)

    return noise_y

# Init Function2
def init_func2(x) :
    y = (x**2 - 9) * (x**2 - 36)
    return y

def init_noise2(x_domain) :

    noise_y = np.zeros(len(x_domain), dtype=float)

    for i in range(len(x_domain)) :
        x = x_domain[i]
        noise_y[i] = init_func2(x) + random.uniform(-200, 200)
```

```
        return noise_y


    def plot_points_func(domain_x, domain_y, noise_y, degree) :
        plt.scatter(domain_x, noise_y, c='r', s=5)
        plt.plot(domain_x, domain_y)

        if (degree == 0) :
            plt.title("Original Polynomial")
        else :
            plt.title("Degree : " + str(degree))

        plt.show()
```

In [3]:
```
# Class
class LeastSquare() :
    def __init__(self, degree, x_domain, y_domain) :
        self.degree = degree + 1
        self.noise_x = x_domain
        self.n = len(self.noise_x)

        self.noise_y = np.array(y_domain)
        self.noise_y = self.noise_y.reshape(self.n, 1)


    def calculate_matrix_a(self) :
        self.matrix_a = np.zeros((self.n, self.degree), dtype=float)

        for i in range (self.n) :
            for j in range (self.degree) :
                self.matrix_a[i][j] = self.noise_x[i] ** j


    def pseudo_inverse(self) :

        self.calculate_matrix_a()

        trans_a = self.matrix_a.T
        ata = np.mat(trans_a) * np.mat(self.matrix_a)
        aiat = np.mat(ata.I) * np.mat(trans_a)

        self.coeff = np.mat(aiat) * np.mat(self.noise_y)
        self.reshape_coeff()


    def reshape_coeff(self) :
        prev_coeff = copy.deepcopy(self.coeff)
```

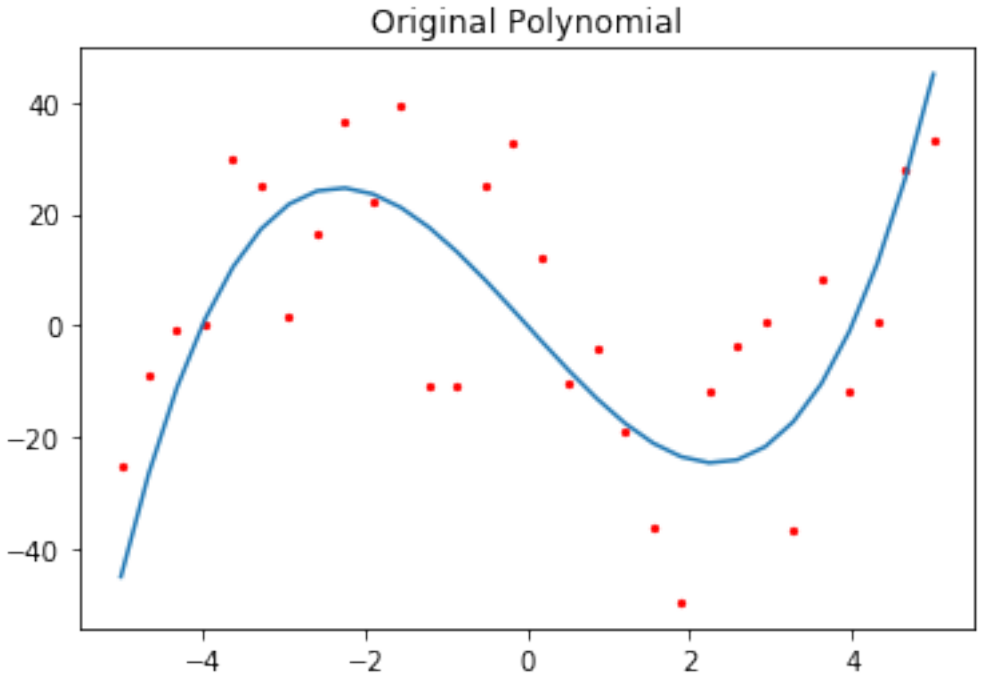
```
            self.coeff = np.zeros(self.degree, dtype=float)

            for i in range (len(prev_coeff)) :
                self.coeff[i] = prev_coeff[i][0]

        #         print(self.coeff)


        def poly_func(self, x) :
            y = 0

            for i in range (self.degree) :
                y += self.coeff[i] * (x ** i)

            return y


        def cal_domain_y(self) :
            self.y_domain = [self.poly_func(i) for i in self.noise_x]

            return self.y_domain

        def calculate_residual(self) :
            residual = 0

            for i in range (self.n) :
                residual += (self.y_domain[i] - self.noise_y[i][0]) ** 2

        #         print(residual)
            return residual
```

In [4]:
```
ls_degree=[1, 2, 3, 5, 9]
ls_y = []
residual_list = []

def do_least_square(degree) :
    least_square = LeastSquare(degree, x_domain, initial_noise)
    least_square.pseudo_inverse()

    ls_y.append(least_square.cal_domain_y())
    residual_list.append(least_square.calculate_residual())
    plot_points_func(x_domain, least_square.cal_domain_y(), initial_noise, degree)
```

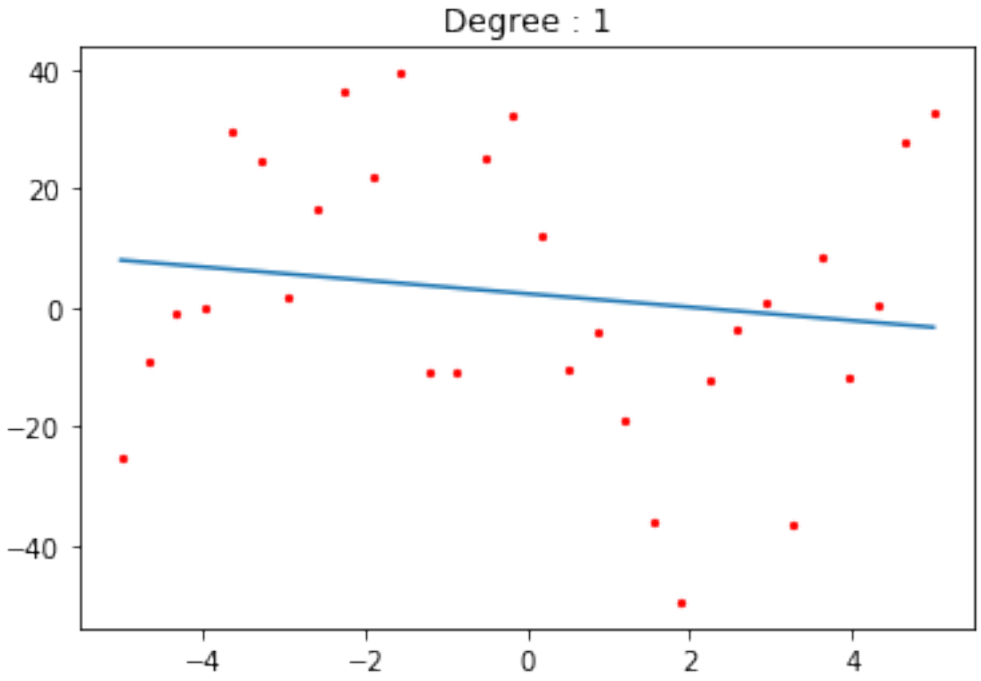
1 $y = x^3 - 16x$

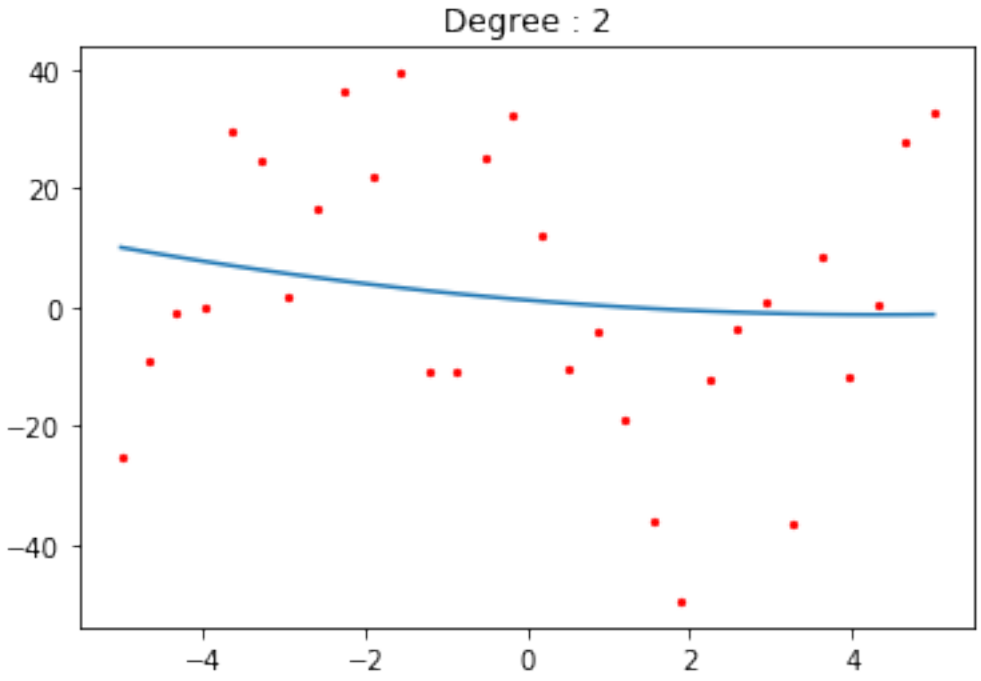
In [5]:
```
# Main 1
x_domain = np.linspace(-5, 5, 30)
```

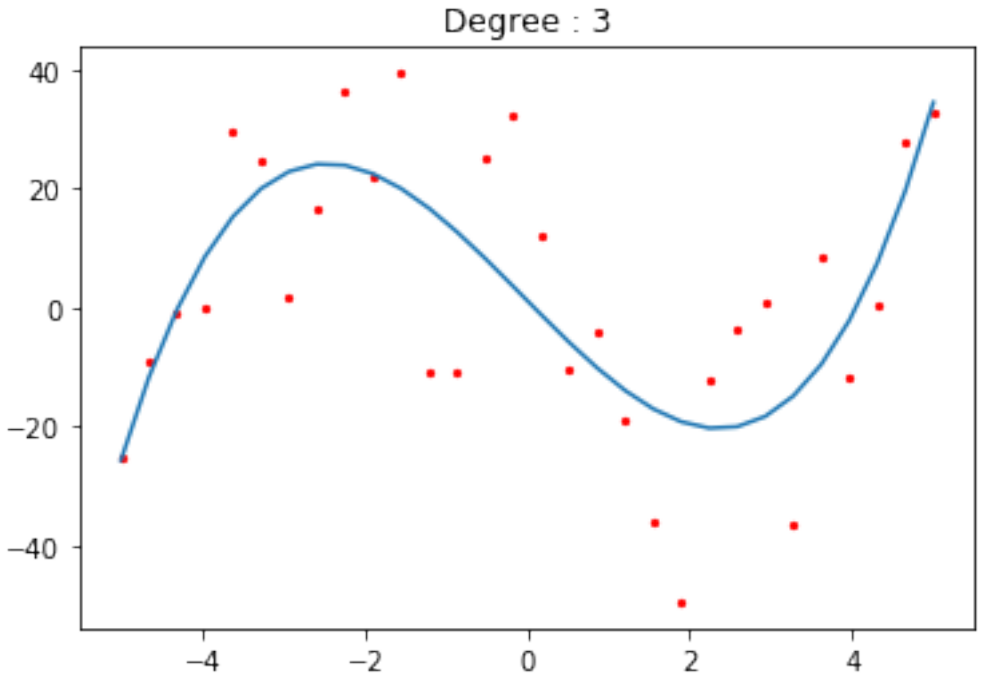
```
y_domain = [init_func(i) for i in x_domain]

initial_noise = init_noise(x_domain)

plot_points_func(x_domain, y_domain, initial_noise, 0)
```

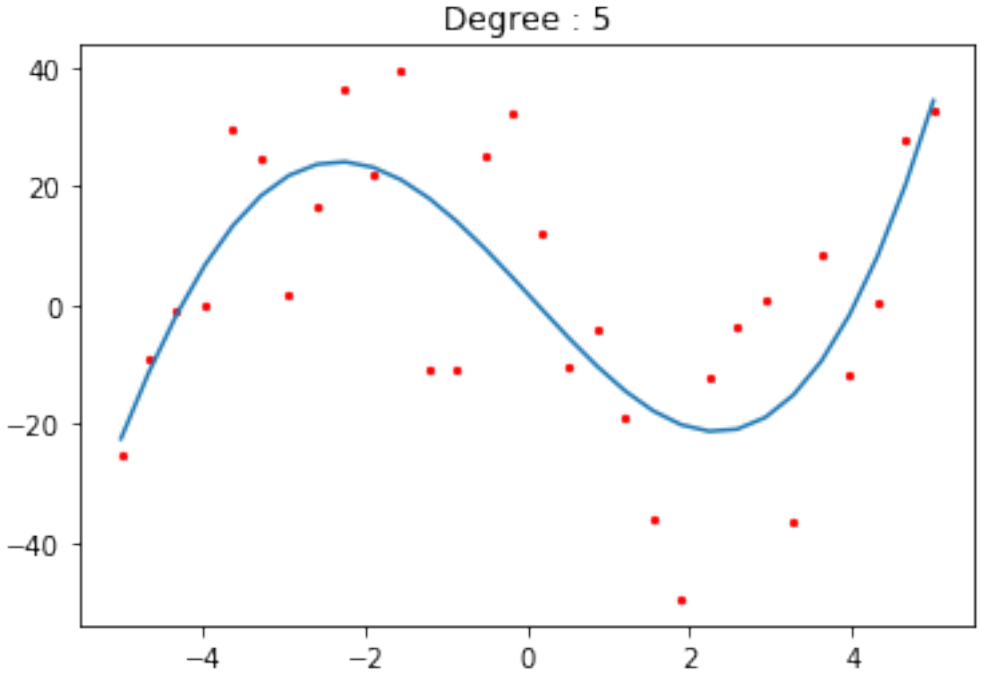


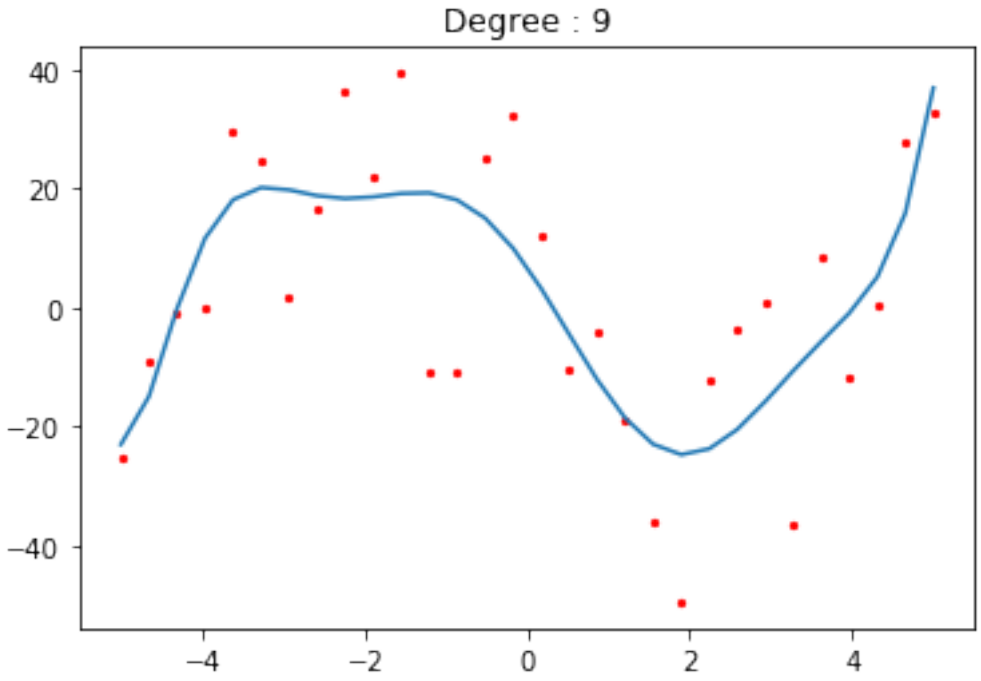

```
In [6]: for i in ls_degree :
            do_least_square(i)
```


Degree : 1
40
20
0
−20
−40
−4
−2
0
2
4

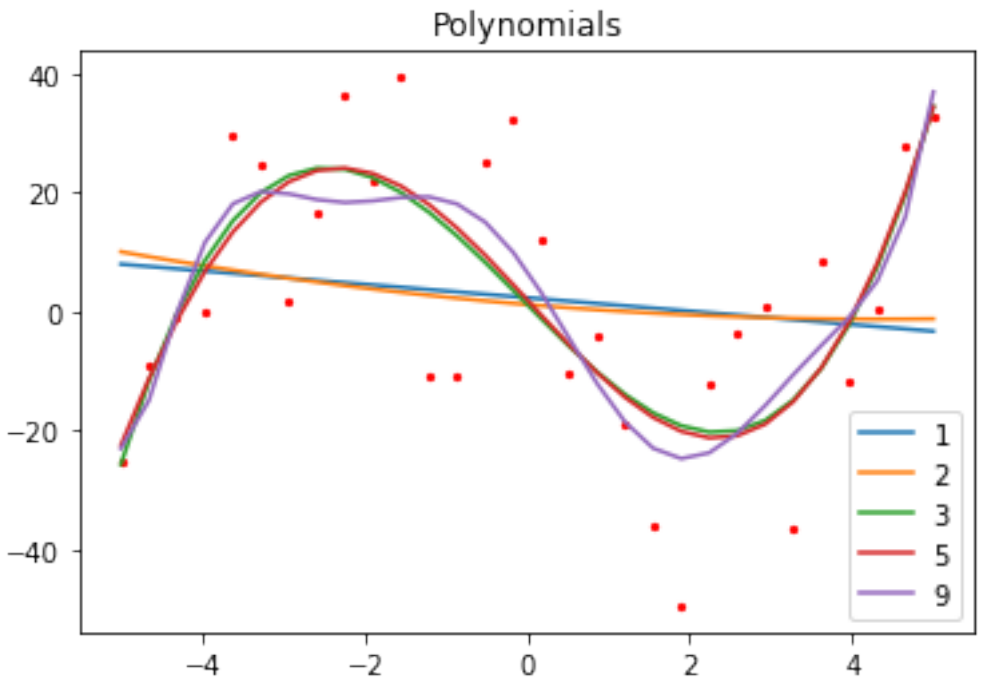

Degree : 2
40
20
0
−20
−40
−4
−2
0
2
4

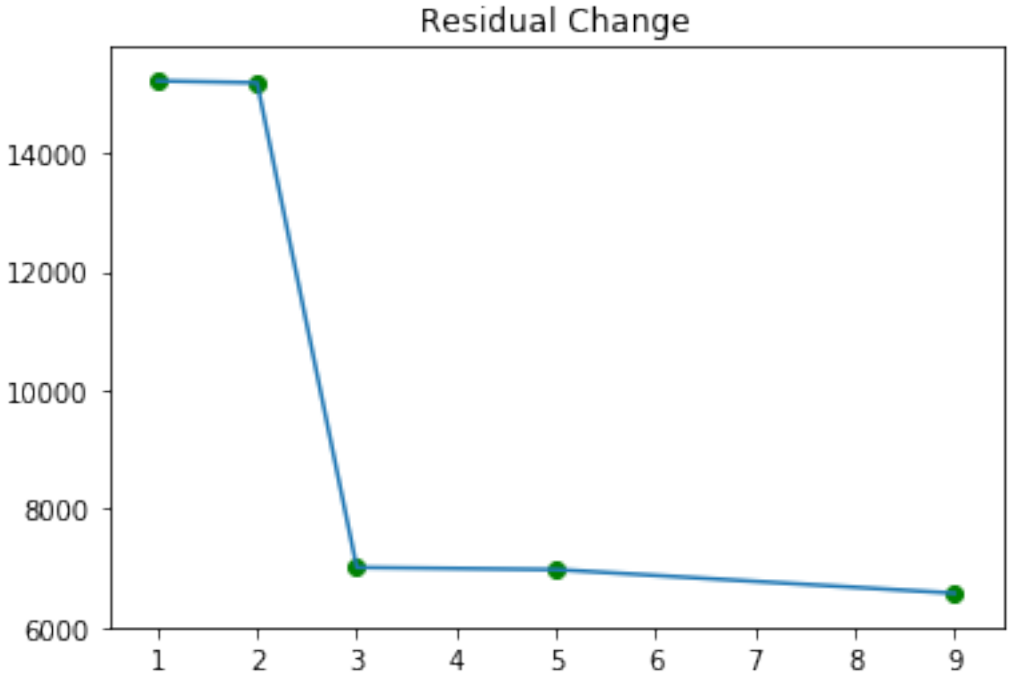

Degree : 3
40
20
0
−20
−40
−4
−2
0
2
4


Degree : 5
40
20
0
−20
−40
−4
−2
0
2
4



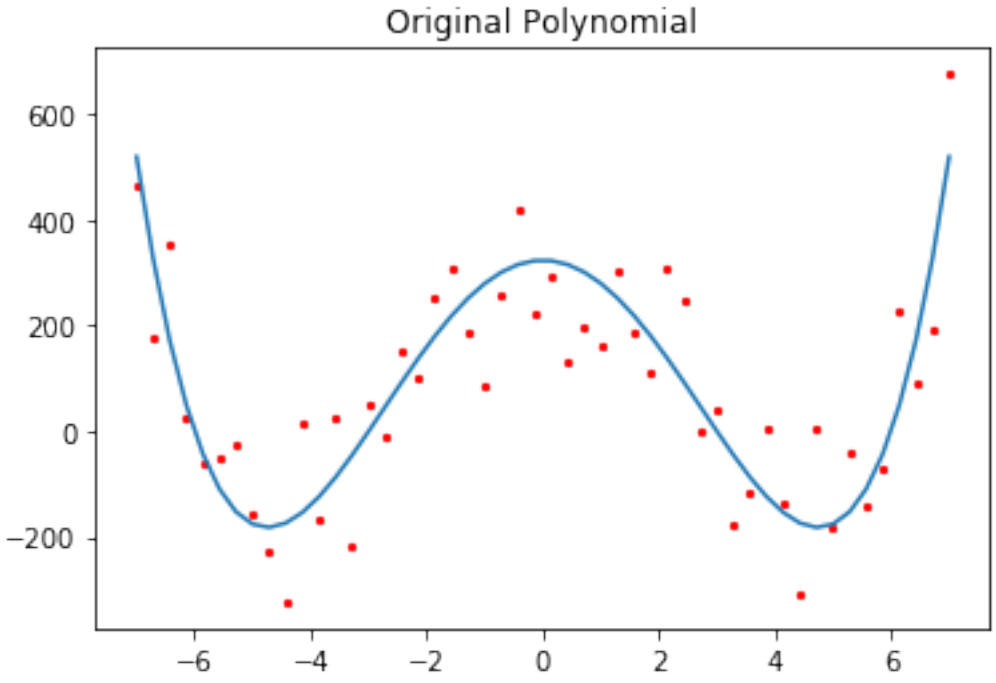

```
In [7]: plt.title("Polynomials")
        plt.scatter(x_domain, initial_noise, c='r', s=5)
        for i in range(len(ls_y)):
            plt.plot(x_domain, ls_y[i])
        plt.legend(ls_degree)
        plt.show()

        plt.title("Residual Change")
        plt.plot(ls_degree, residual_list)
        plt.scatter(ls_degree,residual_list, c='g')
        plt.show()
```


Polynomials
40
20
0
−20
−40
−4
−2
0
2
4
1
2
3
5
9

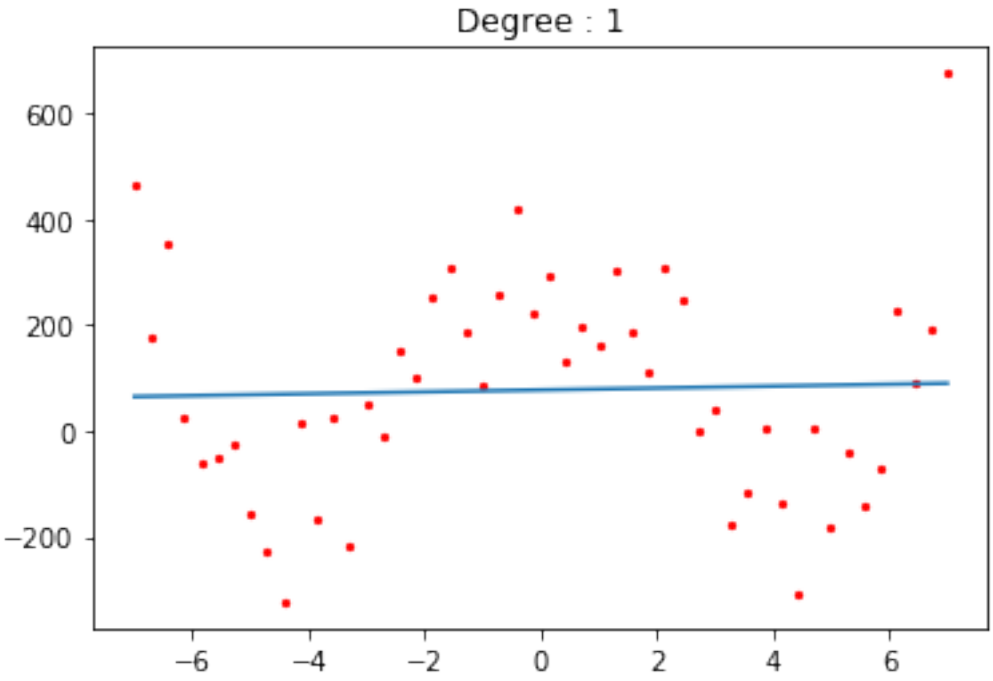

Residual Change
14000
12000
10000
8000
6000
1
2
3
4
5
6
7
8
9

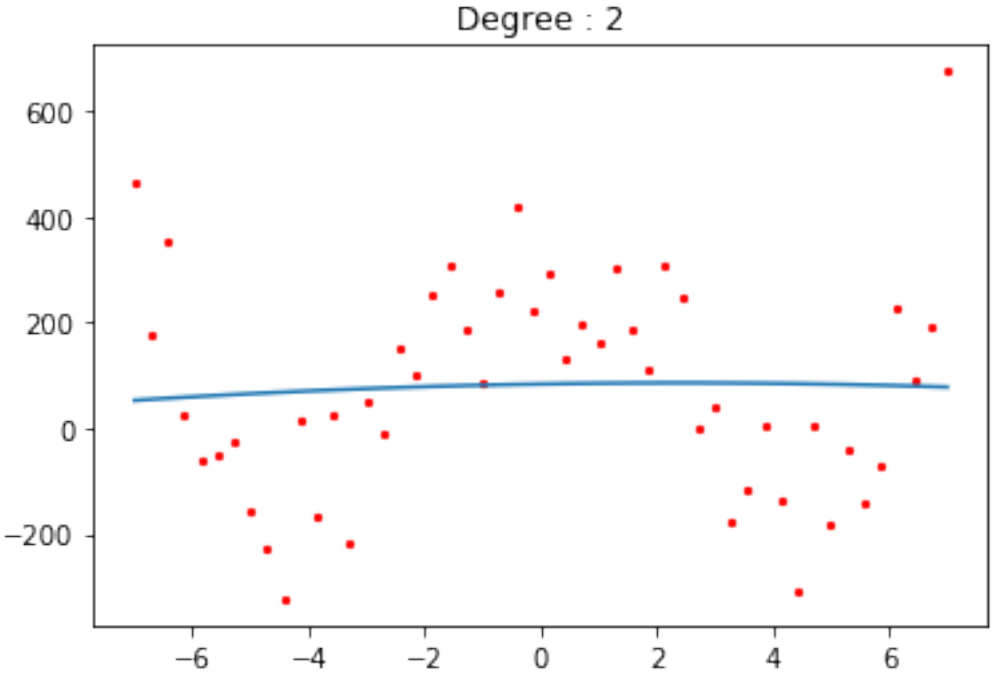
2 $y = (x-3)(x+3)(x-6)(x+6)$

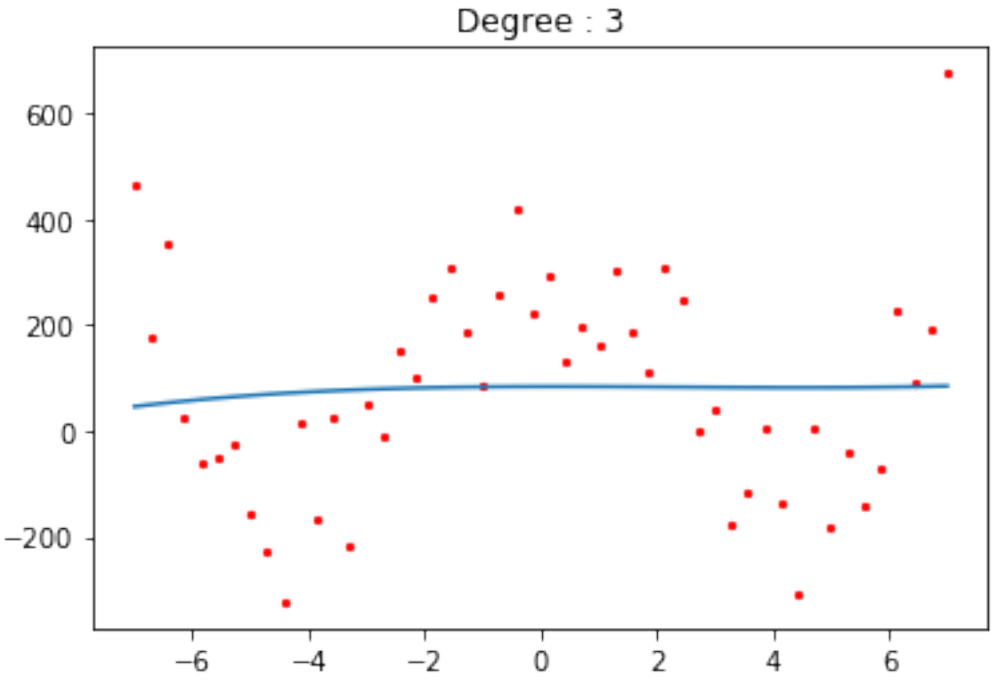
In [8]:
```
ls_y = []
residual_list = []

# Main 2
x_domain = np.linspace(-7, 7, 50)
y_domain = [init_func2(i) for i in x_domain]

initial_noise = init_noise2(x_domain)

plot_points_func(x_domain, y_domain, initial_noise, 0)
```

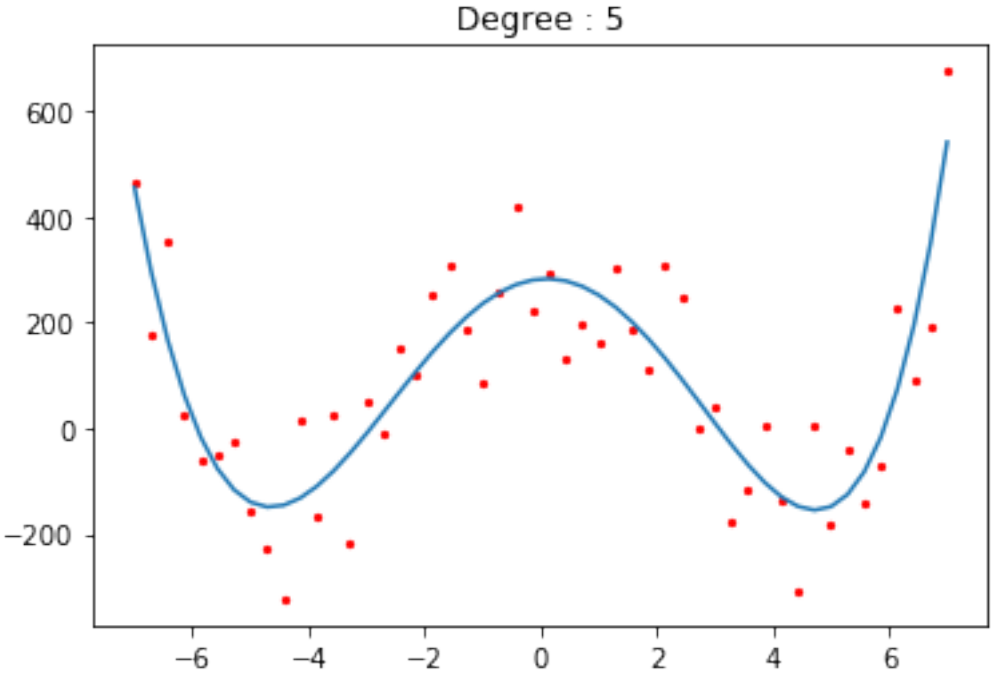


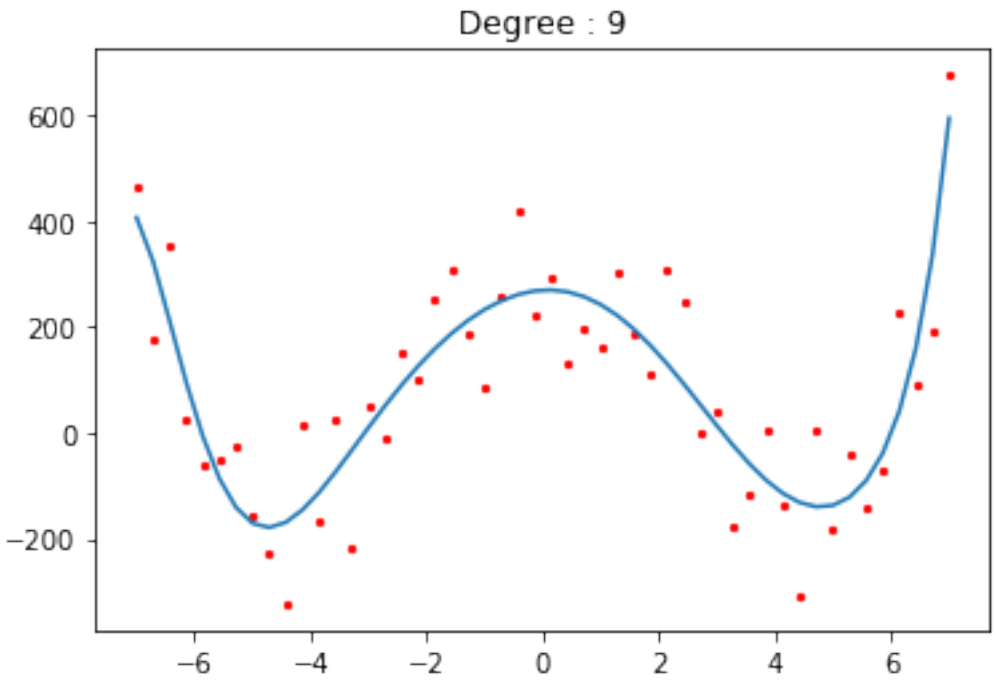
In [9]:
```
for i in ls_degree :
    do_least_square(i)
```


Degree : 1
600
400
200
0
−200
−6
−4
−2
0
2
4
6

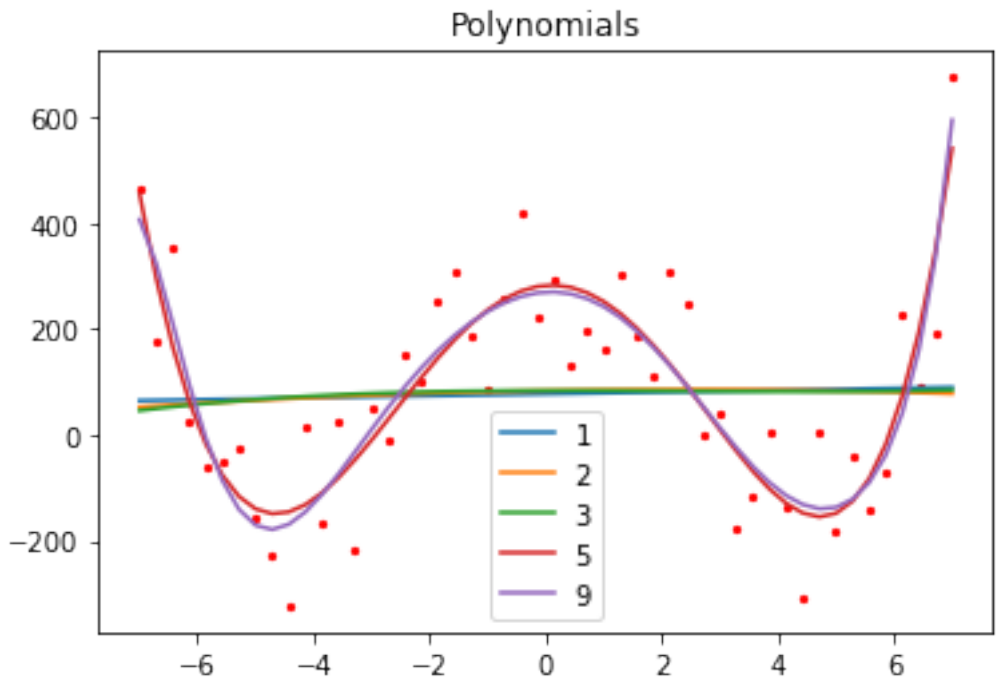

Degree : 2
600
400
200
0
−200
−6
−4
−2
0
2
4
6

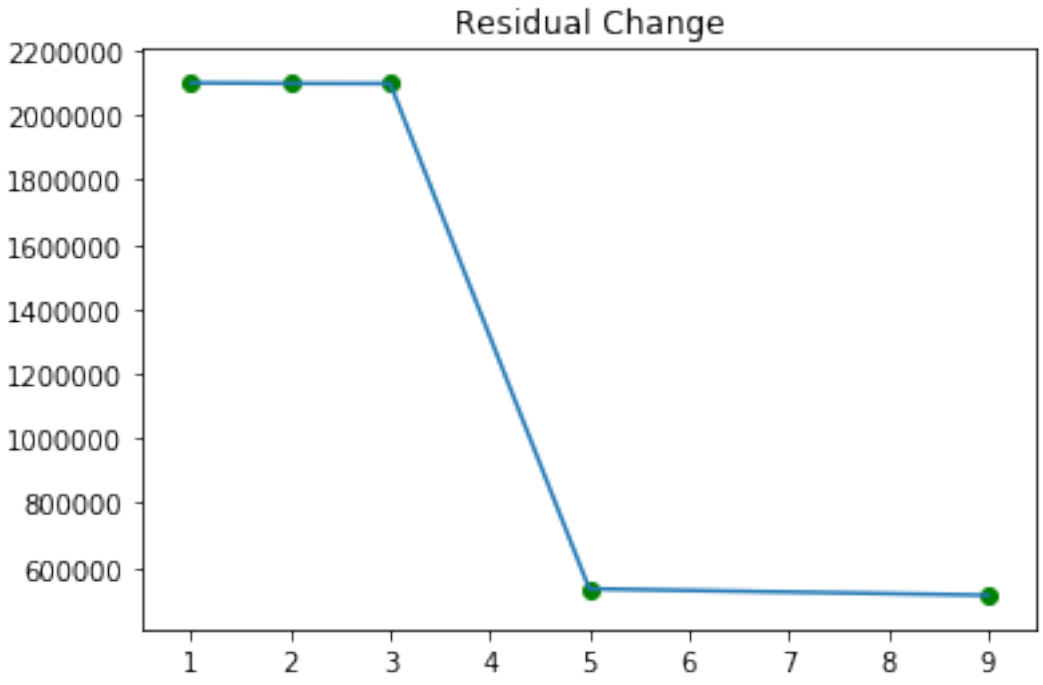

Degree : 3
600
400
200
0
−200
−6
−4
−2
0
2
4
6


Degree : 5
600
400
200
0
−200
−6
−4
−2
0
2
4
6




```
In [10]: plt.title("Polynomials")
         plt.scatter(x_domain, initial_noise, c='r', s=5)
         for i in range(len(ls_y)):
             plt.plot(x_domain, ls_y[i])
         plt.legend(ls_degree)
         plt.show()

         plt.title("Residual Change")
         plt.plot(ls_degree, residual_list)
         plt.scatter(ls_degree,residual_list, c='g')
         plt.show()
```


Polynomials
600
400
200
0
−200
−6
−4
−2
0
2
4
6
1
2
3
5
9


Residual Change
2200000
2000000
1800000
1600000
1400000
1200000
1000000
800000
600000
1
2
3
4
5
6
7
8
9