Hw4

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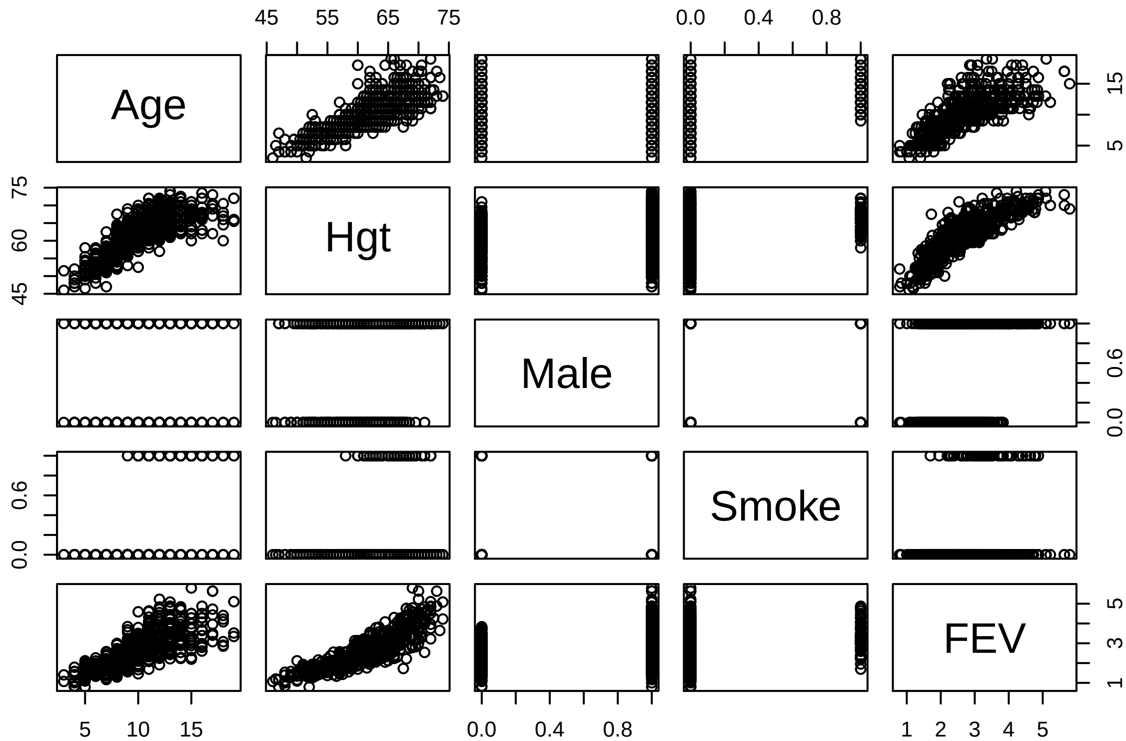
Problem 1

```
FEV.data = read.table(file="FullFEVdataExercise9-21.txt",header=T,sep="")
attach(FEV.data)
head(FEV.data)
```

```
##     Id Age   FEV  Hgt Male Smoke
## 1  301   9 1.708 57.0    0     0
## 2  451   8 1.724 67.5    0     0
## 3  501   7 1.720 54.5    0     0
## 4  642   9 1.558 53.0    1     0
## 5  901   9 1.895 57.0    1     0
## 6 1701   8 2.336 61.0    0     0
```

```
pairs(~Age+Hgt+Male+Smoke+FEV,labels=c("Age","Hgt","Male","Smoke","FEV"),data =FEV.data)
```



(a)

Compare Age and FEV, Height and FEV we can see positive relationship tendency as age increases, FEV will also increases; as height increases FEV will increases.

Compare Male and FEV, we can see that Male has larger FEV value interval than women.

Compare Smoke and FEV, we can see that Nosmoker has lareger FEV value interval than women.

(b)

(prior construction)

Choose BCJ method of constructing an informative prior β. BIDA (p234)

$$\beta \sim N_r(\beta_0, C_0) \perp \tau \sim Gamma(a, b)$$

$$E[Y|\tilde{X}] = \tilde{m} = \tilde{X}\beta \sim N_r(\tilde{Y}, D(\tilde{w}))$$

$$\beta \sim N_r(\tilde{X}^{-1}\tilde{Y}, \tilde{X}^{-1}D(\tilde{w})\tilde{X}^{-1'})$$

Note we can choose mean(beta) = 0 if we wnat high degree predictor.

$$\sqrt{w} = \frac{99th\ percentile - \tilde{m}}{2.33}$$

Since no expert expetation, we choose

$$a = 0.001, b = 0.001$$

Note that we will use the method in week 8 disscusion to modify beta and incov to make it hanlde higher dimension features.

```r
expert.info = (rbind(c(18,70,1,0),c(16,70,0,1),c(13,66,1,1),c(12,60,0,1)))
print(expert.info)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   18   70    1    0
## [2,]   16   70    0    1
## [3,]   13   66    1    1
## [4,]   12   60    0    1
```

```r
expert.mean = c(4.0,4.2,3.4,2.7)
expert.upper = c(4.8,5.0,4.0,3.5)
weight.mat = diag(((expert.upper-expert.mean)/2.33)^2)
```

```r
library(robustHD)
```

```
## Warning: package 'robustHD' was built under R version 3.6.3

## Loading required package: perry

## Warning: package 'perry' was built under R version 3.6.3

## Loading required package: parallel

## Loading required package: robustbase

## Warning: package 'robustbase' was built under R version 3.6.3

##
## Attaching package: 'robustbase'

## The following object is masked from 'package:survival':
##
##     heart
```

```r
Age.st = Age
Hgt.st = Hgt
FEV.st = FEV
Age_Male.st = Age*Male
Age_Hgt.st = Age*Hgt
Age_Smoke.st = Age*Smoke
Age2.st = Age^2
Hgt2.st = Hgt^2
```

(predictor selection)

1. linear
2. linear + age*male
3. linear + age*height
4. linear + age*smoke
5. linear + age^2
6. linear + height^2
7. linear + age^2 + height^2

```r
X.mat <- list()
X.mat[[1]] = model.matrix(~ Age.st+Hgt.st+as.factor(Smoke)+as.factor(Male))
X.mat[[2]] = model.matrix(~ Age.st+Hgt.st+as.factor(Smoke)+as.factor(Male)+Age_Male.st)
X.mat[[3]] = model.matrix(~ Age.st+Hgt.st+as.factor(Smoke)+as.factor(Male)+Age_Hgt.st)
X.mat[[4]] = model.matrix(~ Age.st+Hgt.st+as.factor(Smoke)+as.factor(Male)+Age_Smoke.st)
```

```
X.mat[[5]] = model.matrix(~ Age.st+Hgt.st+as.factor(Smoke)+as.factor(Male)+Age2.st)
X.mat[[6]] = model.matrix(~ Age.st+Hgt.st+as.factor(Smoke)+as.factor(Male)+Hgt2.st)
```

```
X.mat_cal <- list()
X.mat_cal[[1]] = cbind(c(1,1,1,1),expert.info)
X.mat_cal[[2]] = cbind(X.mat_cal[[1]],c(expert.info[,1]*expert.info[,3]))
X.mat_cal[[3]] = cbind(X.mat_cal[[1]],c(expert.info[,1]*expert.info[,2]))
X.mat_cal[[4]] = cbind(X.mat_cal[[1]],c(expert.info[,1]*expert.info[,4]))
X.mat_cal[[5]] = cbind(X.mat_cal[[1]],c(expert.info[,1]*expert.info[,1]))
X.mat_cal[[6]] = cbind(X.mat_cal[[1]],c(expert.info[,2]*expert.info[,2]))
X.mat_cal[[7]] = cbind(X.mat_cal[[6]],c(expert.info[,1]*expert.info[,1]))

mu.lst <- list()
C0inv <- list()
XnegY = solve(X.mat_cal[[1]][,1:4],expert.mean)
XinvDXinvt = solve(X.mat_cal[[1]][,1:4])*weight.mat*t(solve(X.mat_cal[[1]][,1:4]))
for(i in 1:7)
{
  n = dim(X.mat_cal[[i]])[1]
  m = dim(X.mat_cal[[i]])[2]
  mu.lst[[i]] = matrix(0L, nrow =m, ncol = 1)
  mu.lst[[i]][1:4] = XnegY
  C0inv[[i]] = diag(0.001, m, m)
  C0inv[[i]][1:4,1:4] = XinvDXinvt
}
```

```
j = 1
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.001, b=0.001## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[4]*Xmat[i,5]
  like[i] <- dnorm(Y[i],mu[i],tau)
  invlike[i] <- 1/like[i]
  pw_logf[i] <- log(like[i])
}

beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)
}"
```

```
jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0)))
jags.param <- c("beta","tau","like", "invlike", "pw_logf")
```

```
FEV.fit1 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

```
## module glm loaded
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 6368
##
## Initializing model
```

```
DIC.lst <- c()
BIC.lst <- c()
LPML.lst <- c()
r=dim(X.mat[[j]])[2]
n=dim(X.mat[[j]])[1]
pm_tau=FEV.fit1$BUGSoutput$summary["tau", "mean"]
pm_coeff=FEV.fit1$BUGSoutput$summary[c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]"), "mean"]
BIC1 <- -n*log(pm_tau)+n*log(2*pi) + pm_tau*sum((FEV-(pm_coeff[1]+pm_coeff[2]*Age.st+pm_coeff[3]*Hgt.st
CPO1 <- 1/FEV.fit1$BUGSoutput$mean$invlike ## invlike is a vector of length n
LPML1 <- sum(log(CPO1))
DIC.lst = c(DIC.lst,FEV.fit1$BUGSoutput$DIC)
BIC.lst = c(BIC.lst,BIC1)
LPML.lst= c(LPML.lst,LPML1)
```

```
j = 2
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.001, b=0.001## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[5]*Xmat[i,5]+beta[6
  like[i] <- dnorm(Y[i],mu[i],tau)
  invlike[i] <- 1/like[i]
  pw_logf[i] <- log(like[i])
}

beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)
}"

jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0,0)))
jags.param <- c("beta","tau","like", "invlike", "pw_logf")
```

```r
FEV.fit1 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 7056
##
## Initializing model
```

```r
r=dim(X.mat[[j]])[2]
n=dim(X.mat[[j]])[1]
pm_tau=FEV.fit1$BUGSoutput$summary["tau", "mean"]
pm_coeff=FEV.fit1$BUGSoutput$summary[c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]","beta[6]"), "me
BIC1 <- -n*log(pm_tau)+n*log(2*pi) + pm_tau*sum((FEV-(pm_coeff[1]+pm_coeff[2]*Age.st+pm_coeff[3]*Hgt.st
CPO1 <- 1/FEV.fit1$BUGSoutput$mean$invlike ## invlike is a vector of length n
LPML1 <- sum(log(CPO1))
DIC.lst = c(DIC.lst,FEV.fit1$BUGSoutput$DIC)
BIC.lst = c(BIC.lst,BIC1)
LPML.lst= c(LPML.lst,LPML1)
```

```r
j = 3
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.001, b=0.001## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[5]*Xmat[i,5]+beta[6
  like[i] <- dnorm(Y[i],mu[i],tau)
  invlike[i] <- 1/like[i]
  pw_logf[i] <- log(like[i])
}

beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)
}"

jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0,0)))
jags.param <- c("beta","tau","like", "invlike", "pw_logf")

FEV.fit1 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
```

```
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 7280
##
## Initializing model
```

```
r=dim(X.mat[[j]])[2]
n=dim(X.mat[[j]])[1]
pm_tau=FEV.fit1$BUGSoutput$summary["tau", "mean"]
pm_coeff=FEV.fit1$BUGSoutput$summary[c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]","beta[6]"), "me
BIC1 <- -n*log(pm_tau)+n*log(2*pi) + pm_tau*sum((FEV-(pm_coeff[1]+pm_coeff[2]*Age.st+pm_coeff[3]*Hgt.st
CPO1 <- 1/FEV.fit1$BUGSoutput$mean$invlike ## invlike is a vector of length n
LPML1 <- sum(log(CPO1))
DIC.lst = c(DIC.lst,FEV.fit1$BUGSoutput$DIC)
BIC.lst = c(BIC.lst,BIC1)
LPML.lst= c(LPML.lst,LPML1)
```

```
j = 4
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.001, b=0.001## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[5]*Xmat[i,5]+beta[6
  like[i] <- dnorm(Y[i],mu[i],tau)
  invlike[i] <- 1/like[i]
  pw_logf[i] <- log(like[i])
}

beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)
}"

jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0,0)))
jags.param <- c("beta","tau","like", "invlike", "pw_logf")

FEV.fit1 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 7050
##
```

```
## Initializing model
```

```r
r=dim(X.mat[[j]])[2]
n=dim(X.mat[[j]])[1]
pm_tau=FEV.fit1$BUGSoutput$summary["tau", "mean"]
pm_coeff=FEV.fit1$BUGSoutput$summary[c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]","beta[6]"), "me
BIC1 <- -n*log(pm_tau)+n*log(2*pi) + pm_tau*sum((FEV-(pm_coeff[1]+pm_coeff[2]*Age.st+pm_coeff[3]*Hgt.st
CPO1 <- 1/FEV.fit1$BUGSoutput$mean$invlike ## invlike is a vector of length n
LPML1 <- sum(log(CPO1))
DIC.lst = c(DIC.lst,FEV.fit1$BUGSoutput$DIC)
BIC.lst = c(BIC.lst,BIC1)
LPML.lst= c(LPML.lst,LPML1)
```

```r
j = 5
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.001, b=0.001## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[5]*Xmat[i,5]+beta[6
  like[i] <- dnorm(Y[i],mu[i],tau)
  invlike[i] <- 1/like[i]
  pw_logf[i] <- log(like[i])
}

beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)
}"

jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0,0)))
jags.param <- c("beta","tau","like", "invlike", "pw_logf")

FEV.fit1 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 7055
##
## Initializing model
```

```r
r=dim(X.mat[[j]])[2]
n=dim(X.mat[[j]])[1]
pm_tau=FEV.fit1$BUGSoutput$summary["tau", "mean"]
```

```
pm_coeff=FEV.fit1$BUGSoutput$summary[c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]","beta[6]"), "me
BIC1 <- -n*log(pm_tau)+n*log(2*pi) + pm_tau*sum((FEV-(pm_coeff[1]+pm_coeff[2]*Age.st+pm_coeff[3]*Hgt.st+
CPO1 <- 1/FEV.fit1$BUGSoutput$mean$invlike ## invlike is a vector of length n
LPML1 <- sum(log(CPO1))
DIC.lst = c(DIC.lst,FEV.fit1$BUGSoutput$DIC)
BIC.lst = c(BIC.lst,BIC1)
LPML.lst= c(LPML.lst,LPML1)
```

```
j = 6
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.001, b=0.001## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[5]*Xmat[i,5]+beta[6]
  like[i] <- dnorm(Y[i],mu[i],tau)
  invlike[i] <- 1/like[i]
  pw_logf[i] <- log(like[i])
}

beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)
}"

jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0,0)))
jags.param <- c("beta","tau","like", "invlike", "pw_logf")

FEV.fit1 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 7094
##
## Initializing model
```

```
r=dim(X.mat[[j]])[2]
n=dim(X.mat[[j]])[1]
pm_tau=FEV.fit1$BUGSoutput$summary["tau", "mean"]
pm_coeff=FEV.fit1$BUGSoutput$summary[c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]","beta[6]"), "me
BIC1 <- -n*log(pm_tau)+n*log(2*pi) + pm_tau*sum((FEV-(pm_coeff[1]+pm_coeff[2]*Age.st+pm_coeff[3]*Hgt.st+
CPO1 <- 1/FEV.fit1$BUGSoutput$mean$invlike ## invlike is a vector of length n
LPML1 <- sum(log(CPO1))
```

```
DIC.lst = c(DIC.lst,FEV.fit1$BUGSoutput$DIC)
BIC.lst = c(BIC.lst,BIC1)
LPML.lst= c(LPML.lst,LPML1)
```

```
print(DIC.lst)
```

```
## [1] 716.7616 680.7741 667.1273 706.1877 681.7507 713.7077
```

```
print(BIC.lst)
```

```
## [1] 889.5185 710.9452 684.9721 736.8619 712.5955 726.5361
```

```
print(LPML.lst)
```

```
## [1] -358.6577 -340.5238 -327.5528 -353.6191 -341.5075 -347.8008
```

(predictor selection)

```
df <- cbind(c("model 1","model 2","model 3","model 4","model 5","model 6"),DIC.lst,BIC.lst,LPML.lst)
pander(df)
```

	DIC.lst	BIC.lst	LPML.lst
model 1	716.761573148418	889.518474096389	-358.657748946456
model 2	680.774139673271	710.945199411815	-340.523778251755
model 3	667.127266892029	684.972129629967	-327.552820006869
model 4	706.187651522222	736.86191796947	-353.619088476789
model 5	681.750712717644	712.595547935484	-341.507488657307
model 6	713.707720241945	726.536052356127	-347.800815701966

I will choose model 3 for it has least DIC, BIC and has highest LPML

(convergence and model diagostics)

```
FEV.fit1$BUGSoutput$DIC
```

```
## [1] 713.7077
```

X.mat[[3]] = model.matrix(~ Age.st+Hgt.st+as.factor(Smoke)+as.factor(Male)+Age_Hgt.st)

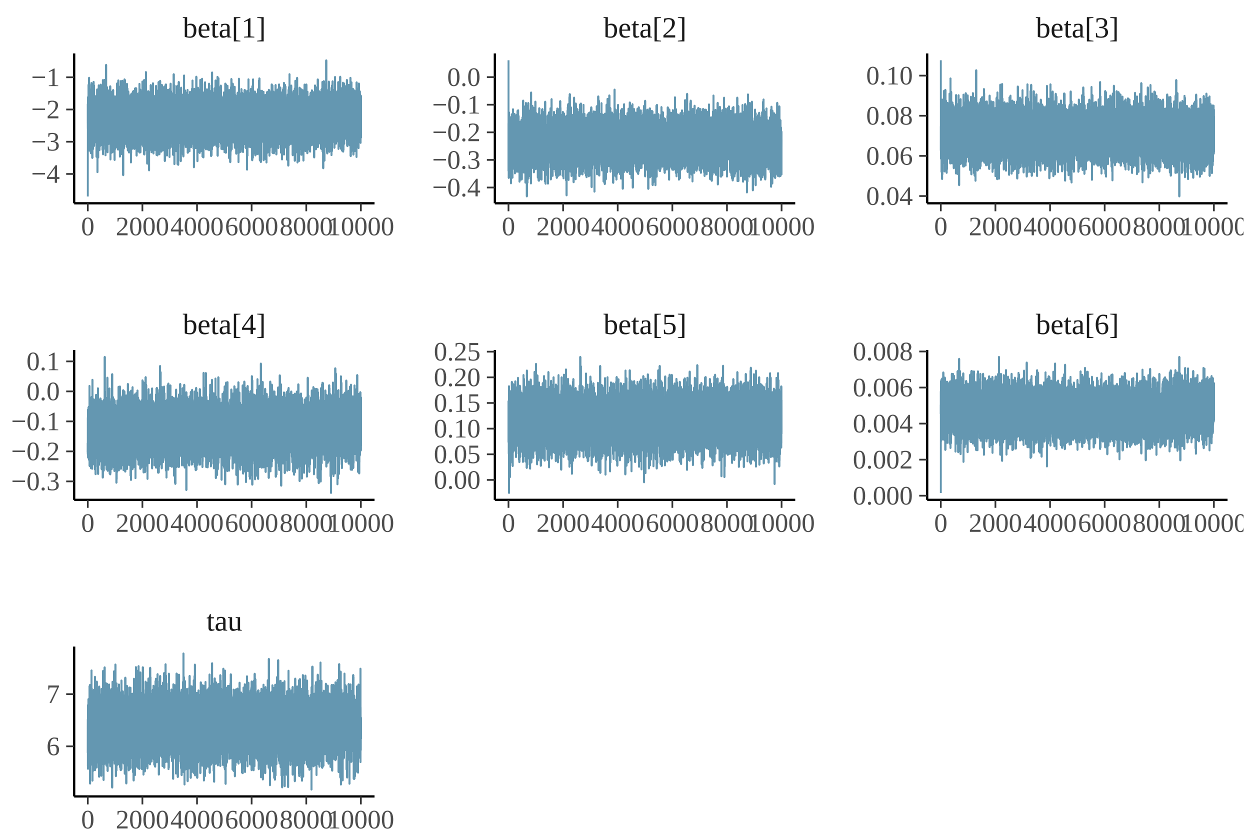
```
j = 3
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.001, b=0.001## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[5]*Xmat[i,5]+beta[6
}
```

```
beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)

}"

jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0,0)))
jags.param <- c("beta","tau")

FEV.fit1 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 5326
##
## Initializing model
```

(Convergence and Model Diagnosis)

```
jags.mcmc = as.mcmc(FEV.fit1)
mcmc_trace(jags.mcmc,pars=c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]","beta[6]","tau"))
```



From the above plots, we can see that our model parameters converges and mix well.

(c)

```
FEV.fit1
```

(posterior reference)

```
## Inference for Bugs model at "4", fit using jags,
##  1 chains, each with 12000 iterations (first 2000 discarded)
##  n.sims = 10000 iterations saved
##          mu.vect sd.vect    2.5%     25%     50%     75%   97.5%
## beta[1]   -2.300   0.442  -3.159  -2.603  -2.299  -2.000  -1.445
## beta[2]   -0.237   0.052  -0.339  -0.271  -0.237  -0.201  -0.135
## beta[3]    0.071   0.008   0.056   0.066   0.071   0.076   0.085
## beta[4]   -0.128   0.057  -0.241  -0.167  -0.128  -0.090  -0.018
## beta[5]    0.114   0.033   0.050   0.092   0.114   0.135   0.180
## beta[6]    0.005   0.001   0.003   0.004   0.005   0.005   0.006
## tau        6.372   0.359   5.688   6.128   6.361   6.612   7.082
## deviance 646.328   6.416 635.915 641.695 645.579 650.115 660.881
##
## DIC info (using the rule, pD = var(deviance)/2)
## pD = 20.6 and DIC = 666.9
## DIC is an estimate of expected predictive error (lower deviance is better).
```

Note that beta[2] is almost negative, this is not without reasons, since feature six is age*height which will incease our FEV simulation a lot.

```
j = 3
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.001, b=0.001## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[5]*Xmat[i,5]+beta[6]
}

beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)

mean15hgtm66ns = beta[1] + beta[2]*15 + beta[3]*66 + beta[4]*0+beta[5]*1+beta[6]*15*66
mean15hgtm66s = beta[1] + beta[2]*15 + beta[3]*66 + beta[4]*1+beta[5]*1+beta[6]*15*66
mean16hgtfm66ns = beta[1] + beta[2]*16 + beta[3]*66 + beta[4]*0+beta[5]*0+beta[6]*16*66
mean16hgtfm66s = beta[1] + beta[2]*16 + beta[3]*66 + beta[4]*0+beta[5]*1+beta[6]*16*66
mean17hgtm70s = beta[1] + beta[2]*17 + beta[3]*70 + beta[4]*1+beta[5]*1+beta[6]*17*70
mean17hgtm70ns = beta[1] + beta[2]*17 + beta[3]*70 + beta[4]*0+beta[5]*1+beta[6]*17*70
mean17hgtfm70s = beta[1] + beta[2]*17 + beta[3]*70 + beta[4]*1+beta[5]*0+beta[6]*17*70
mean17hgtfm70ns = beta[1] + beta[2]*17 + beta[3]*70 + beta[4]*0+beta[5]*0+beta[6]*17*70
```

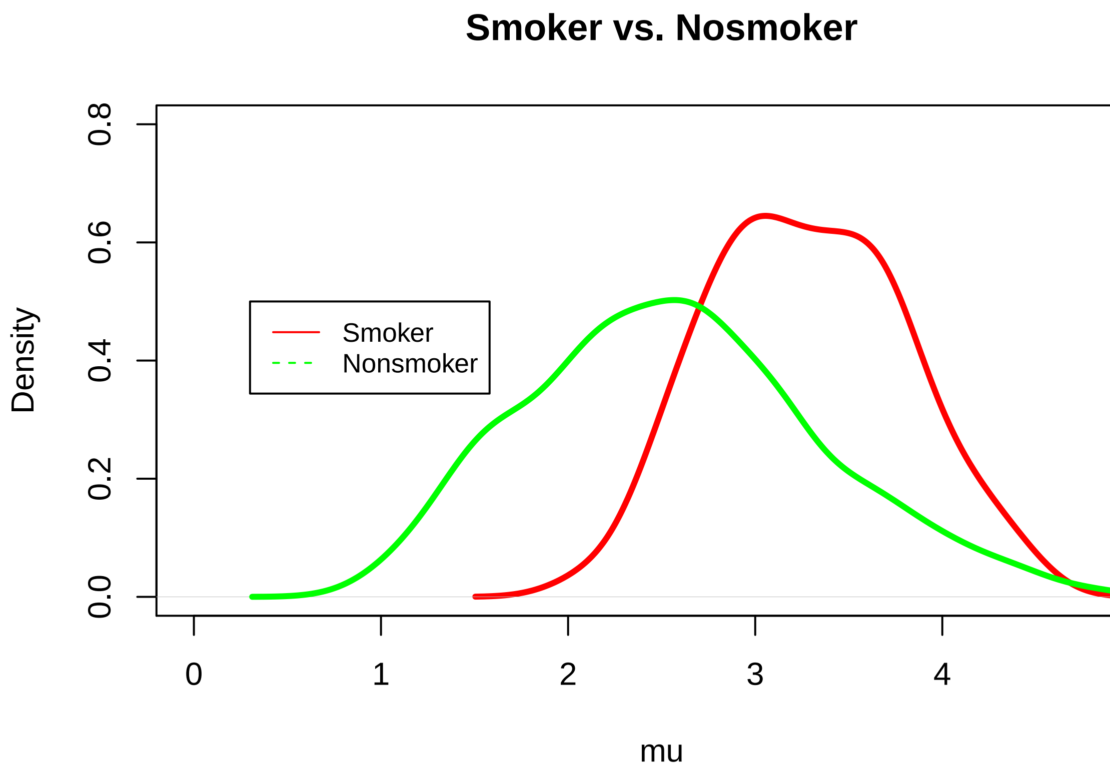
```
}"

jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0,0)))
jags.param <- c("mean15hgtm66ns","mean15hgtm66s","mean16hgtfm66ns","mean16hgtfm66s","mean17hgtm70s","mea

FEV.fit1 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 5344
##
## Initializing model
```

```
which(Smoke ==1)
```

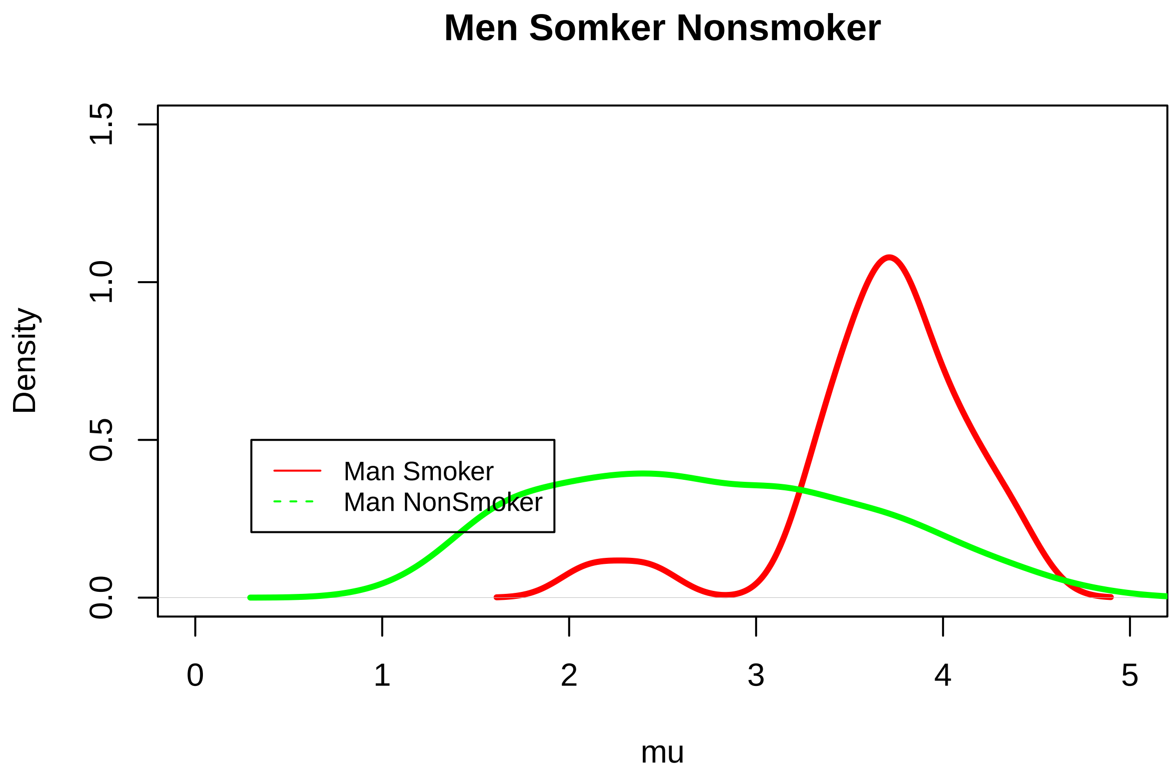
```
##  [1] 191 332 358 366 369 370 372 381 384 388 403 414 422 435 439 441 446 456 461
## [20] 472 479 483 484 488 494 496 498 506 518 522 523 541 556 559 574 590 594 595
## [39] 600 601 602 604 607 610 611 616 617 618 621 622 623 629 633 634 635 637 638
## [58] 640 644 645 646 649 650 651 653
```

```
# FEV.fit1$BUGSoutput$sims.matrix[,which(Smoke ==1)]
plot(main="Smoker vs. Nosmoker",density(FEV.fit1$BUGSoutput$mean$mu[which(Smoke ==1)]),col='red',type='l
lines(density(FEV.fit1$BUGSoutput$mean$mu[which(Smoke ==0)]),col='green',type='l',xlab ='mu',lwd=3,xlim=
legend(0.3,0.5,legend=c("Smoker","Nonsmoker"),col=c("red","green"),lty=1:2, cex=0.8)
```

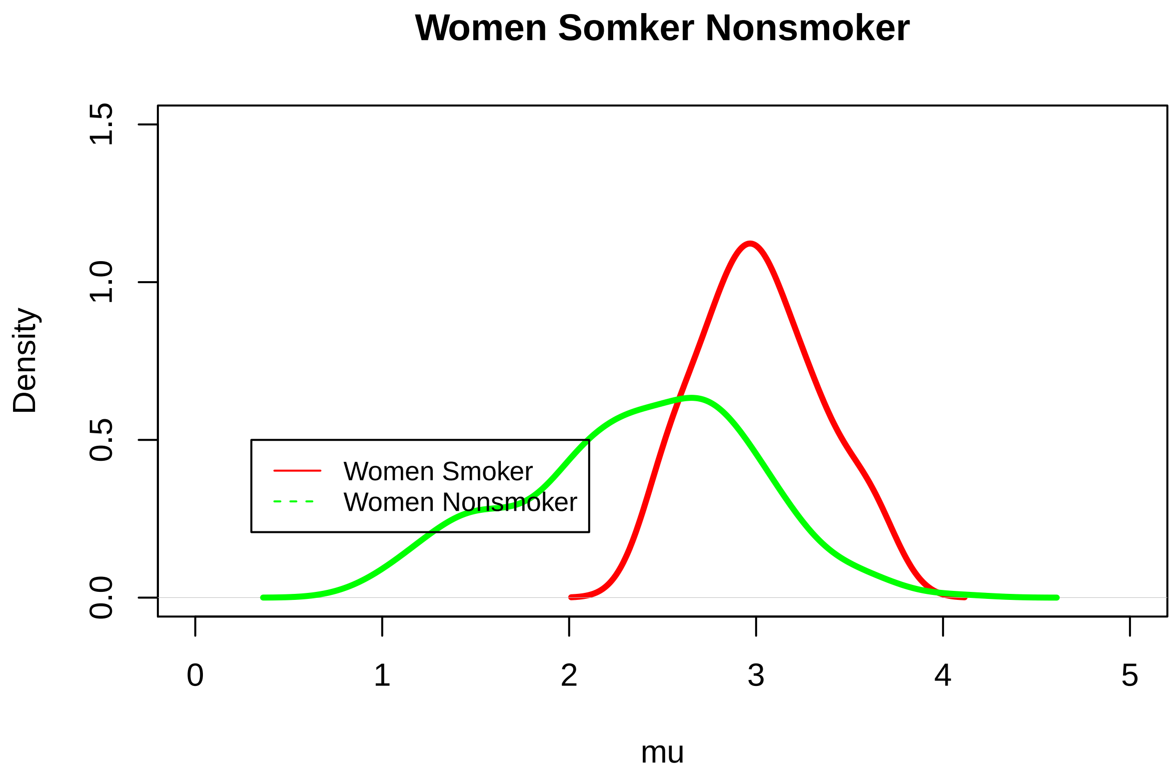


(subpopulation means)

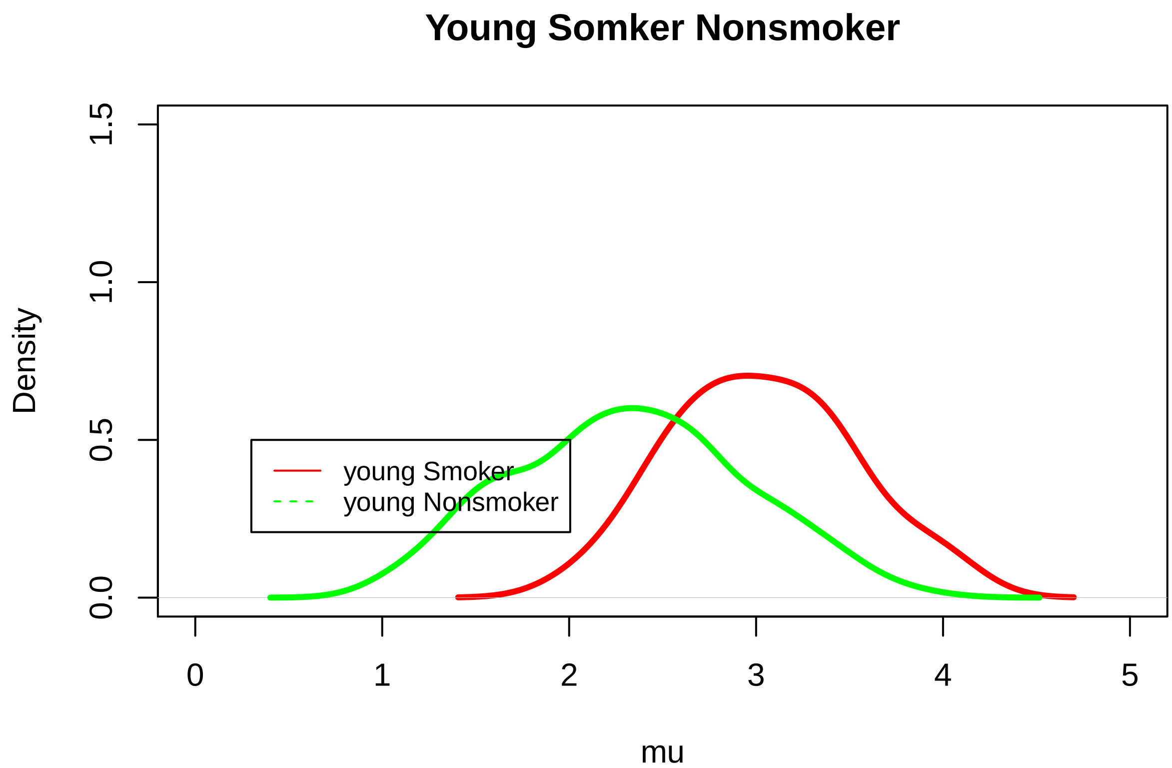
```
plot(main="Men Somker Nonsmoker",density(FEV.fit1$BUGSoutput$mean$mu[which(Male ==1 & Smoke ==1)]),col=
lines(density(FEV.fit1$BUGSoutput$mean$mu[which(Male ==1& Smoke ==0)]),col='green',type='l',xlab ='mu',l
legend(0.3,0.5,legend=c("Man Smoker","Man NonSmoker"),col=c("red","green"),lty=1:2, cex=0.8)
```



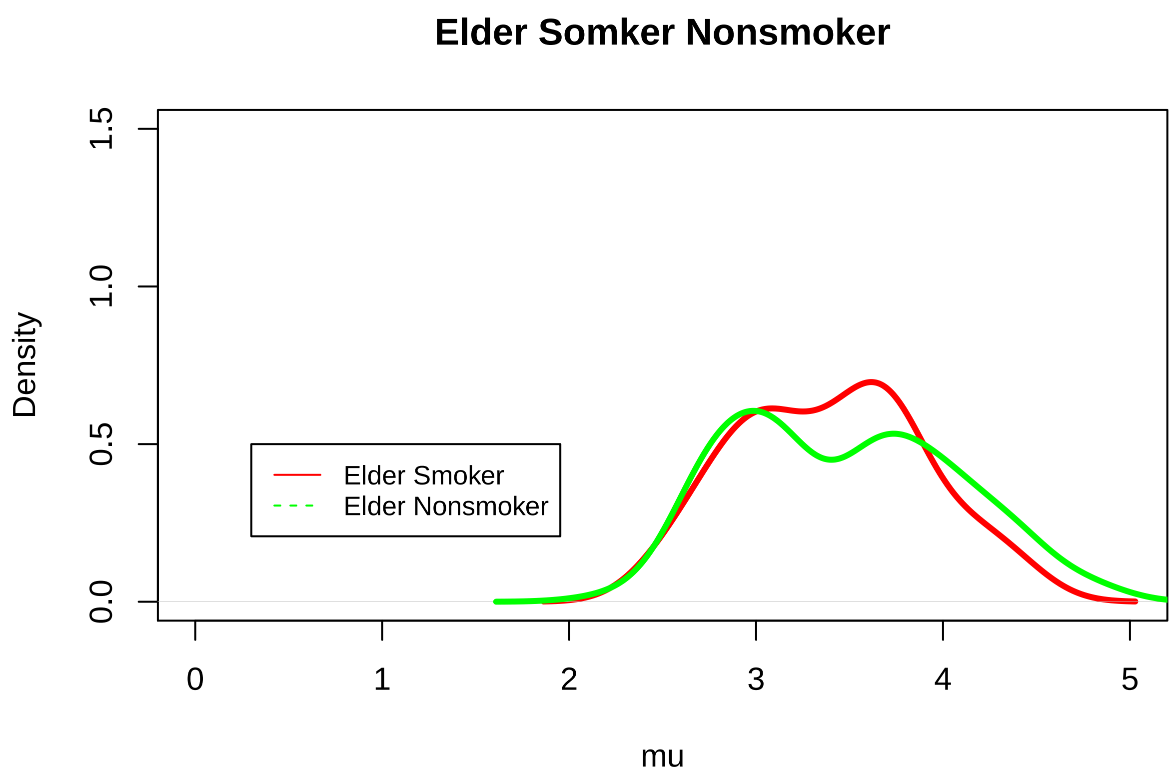

```
plot(main="Women Somker Nonsmoker",density(FEV.fit1$BUGSoutput$mean$mu[which(Male ==0 & Smoke ==1)]),col
lines(density(FEV.fit1$BUGSoutput$mean$mu[which(Male ==0 & Smoke ==0)]),col='green',type='l',xlab ='mu'
legend(0.3,0.5,legend=c("Women Smoker","Women Nonsmoker"),col=c("red","green"),lty=1:2, cex=0.8)
```




```
plot(main="Young Somker Nonsmoker",density(FEV.fit1$BUGSoutput$mean$mu[which(Age <13 & Smoke ==1)]),col=
lines(density(FEV.fit1$BUGSoutput$mean$mu[which(Age<12 & Smoke ==0)]),col='green',type='l',xlab ='mu',lw
legend(0.3,0.5,legend=c("young Smoker","young Nonsmoker"),col=c("red","green"),lty=1:2, cex=0.8)
```




```
plot(main="Elder Somker Nonsmoker",density(FEV.fit1$BUGSoutput$mean$mu[which(Age >=13 & Smoke ==1)]),col
lines(density(FEV.fit1$BUGSoutput$mean$mu[which(Age>=12 & Smoke ==0)]),col='green',type='l',xlab ='mu',l
legend(0.3,0.5,legend=c("Elder Smoker","Elder Nonsmoker"),col=c("red","green"),lty=1:2, cex=0.8)
```




```r
library(MASS)
```

```
## Warning: package 'MASS' was built under R version 3.6.3
##
## Attaching package: 'MASS'
## The following object is masked from 'package:dplyr':
##
##     select
```

```r
fit <- fitdistr(FEV.fit1$BUGSoutput$mean$mu[which(Age >=13 & Smoke ==1)], "normal")
para <- fit$estimate
qnorm(c(0.05,0.95),mean=para[1],sd=para[2])
```

```
## [1] 2.596801 4.203128
```

```r
Lower_lst = c('5%')
Upper_lst = c('95%')
mean_lst = c('mean')

neam_lst =list(FEV.fit1$BUGSoutput$mean$mu[which(Smoke ==1)],
FEV.fit1$BUGSoutput$mean$mu[which(Smoke ==0)],
FEV.fit1$BUGSoutput$mean$mu[which(Male ==1 & Smoke ==1)],
FEV.fit1$BUGSoutput$mean$mu[which(Male ==1 & Smoke ==0)],
FEV.fit1$BUGSoutput$mean$mu[which(Male ==0 & Smoke ==1)],
FEV.fit1$BUGSoutput$mean$mu[which(Male ==0 & Smoke ==1)],
FEV.fit1$BUGSoutput$mean$mu[which(Age <13 & Smoke ==1)],
```

```
FEV.fit1$BUGSoutput$mean$mu[which(Age <13 & Smoke ==0)],
FEV.fit1$BUGSoutput$mean$mu[which(Age >=13 & Smoke ==1)],
FEV.fit1$BUGSoutput$mean$mu[which(Age >=13 & Smoke ==0)])

for (i in 1:10){
  fit <- fitdistr(neam_lst[[i]], "normal")
  para <- fit$estimate
  num = qnorm(c(0.05,0.95),mean=para[1],sd=para[2])
  Lower_lst = c(Lower_lst,num[1])
  Upper_lst = c(Upper_lst,num[2])
  mean_lst = c(mean_lst,para[1])
  }
df <- cbind(c("Group Name","Smoker","NonSmoker","Man Smoker","Man Nonsmoker","Wome Smoker","Women Nonsm
pander(df)
```

Table 2: Table continues below

		Lower_lst	mean_lst
	Group Name	5%	mean
mean	Smoker	2.43253225102686	3.27635363064426
mean	NonSmoker	1.29395274303404	2.56453085359466
mean	Man Smoker	2.84490787889192	3.66583216555558
mean	Man Nonsmoker	1.33542061170634	2.74051786511526
mean	Wome Smoker	2.48620011103649	3.01670127403671
mean	Women Nonsmoker	2.48620011103649	3.01670127403671
mean	Young Smoker	2.25745532303889	3.03475033687851
mean	Young NonSmoker	1.32138029223678	2.4061402953137
mean	Elder Smoker	2.5968010946339	3.39996461815231
mean	Elder,NonSmoker	2.71794283171335	3.66684352271215

	Upper_lst
	95%
mean	4.12017501026166
mean	3.83510896415528
mean	4.48675645221923
mean	4.14561511852418
mean	3.54720243703693
mean	3.54720243703693
mean	3.81204535071813
mean	3.49090029839062
mean	4.20312814167072
mean	4.61574421371095

From the above figures and table, by seeing the subpopulation means distribution we see that when age is young, both men and women who smoke will have larger FEV, as the age gets larger enough,Smoke won't affect too much on FEV. However, we see that only less than 70 out of 675 are somking. Thus, we will see how the the prediction goes in the next few steps.

(d)

```
FEV.fit1$BUGSoutput$summary[7:15,]
```

```
##                       mean         sd        2.5%         25%         50%
## deviance        646.323093 6.58897185 635.853868 641.564415 645.655148
## mean15hgtm66ns    3.580254 0.04139046   3.497497   3.552969   3.580405
## mean15hgtm66s     3.451658 0.05525931   3.344005   3.414680   3.451651
## mean16hgtfm66ns   3.540193 0.04966615   3.443342   3.506138   3.540655
## mean16hgtfm66s    3.653180 0.04897308   3.555868   3.620930   3.653628
## mean17hgtfm70ns   4.215257 0.06724182   4.083033   4.170052   4.215833
## mean17hgtfm70s    4.086661 0.06686450   3.956012   4.041001   4.087014
## mean17hgtm70ns    4.328245 0.06193548   4.204240   4.287285   4.328470
## mean17hgtm70s     4.199649 0.06468008   4.072393   4.156199   4.199915
##                        75%      97.5%
## deviance        650.228214 660.782493
## mean15hgtm66ns    3.608242   3.660278
## mean15hgtm66s     3.488204   3.562505
## mean16hgtfm66ns   3.573568   3.637150
## mean16hgtfm66s    3.686259   3.748452
## mean17hgtfm70ns   4.261191   4.345389
## mean17hgtfm70s    4.132229   4.216914
## mean17hgtm70ns    4.369316   4.449793
## mean17hgtm70s     4.243163   4.328182
```

```
df <- rbind(c('Age 15 Hgt 66 Male Nonsmoker',FEV.fit1$BUGSoutput$summary[8,]),
            c('Age 15 Hgt 66 Male Smoker',FEV.fit1$BUGSoutput$summary[9,]),
            c('Age 16 Hgt 66 Female Nonsmoker',FEV.fit1$BUGSoutput$summary[10,]),
            c('Age 16 Hgt 66 Female Smoker',FEV.fit1$BUGSoutput$summary[11,]),
            c('Age 17 Hgt 70 Female NonSmoker',FEV.fit1$BUGSoutput$summary[12,]),
            c('Age 17 Hgt 70 Female Smoker',FEV.fit1$BUGSoutput$summary[13,]),
            c('Age 17 Hgt 70 Male NonSmoker',FEV.fit1$BUGSoutput$summary[14,]),
            c('Age 17 Hgt 70 Male NonSmoker',FEV.fit1$BUGSoutput$summary[15,]))
pander(df)
```

Table 4: Table continues below

	mean	sd
Age 15 Hgt 66 Male Nonsmoker	3.58025397080158	0.0413904556807518
Age 15 Hgt 66 Male Smoker	3.45165792882221	0.0552593142156521
Age 16 Hgt 66 Female Nonsmoker	3.54019272298573	0.0496661471197968
Age 16 Hgt 66 Female Smoker	3.65318011571949	0.0489730844925996
Age 17 Hgt 70 Female NonSmoker	4.21525741178681	0.067241815330857
Age 17 Hgt 70 Female Smoker	4.08666136980744	0.0668644995782967
Age 17 Hgt 70 Male NonSmoker	4.32824480452057	0.0619354763684468
Age 17 Hgt 70 Male NonSmoker	4.1996487625412	0.0646800772309697

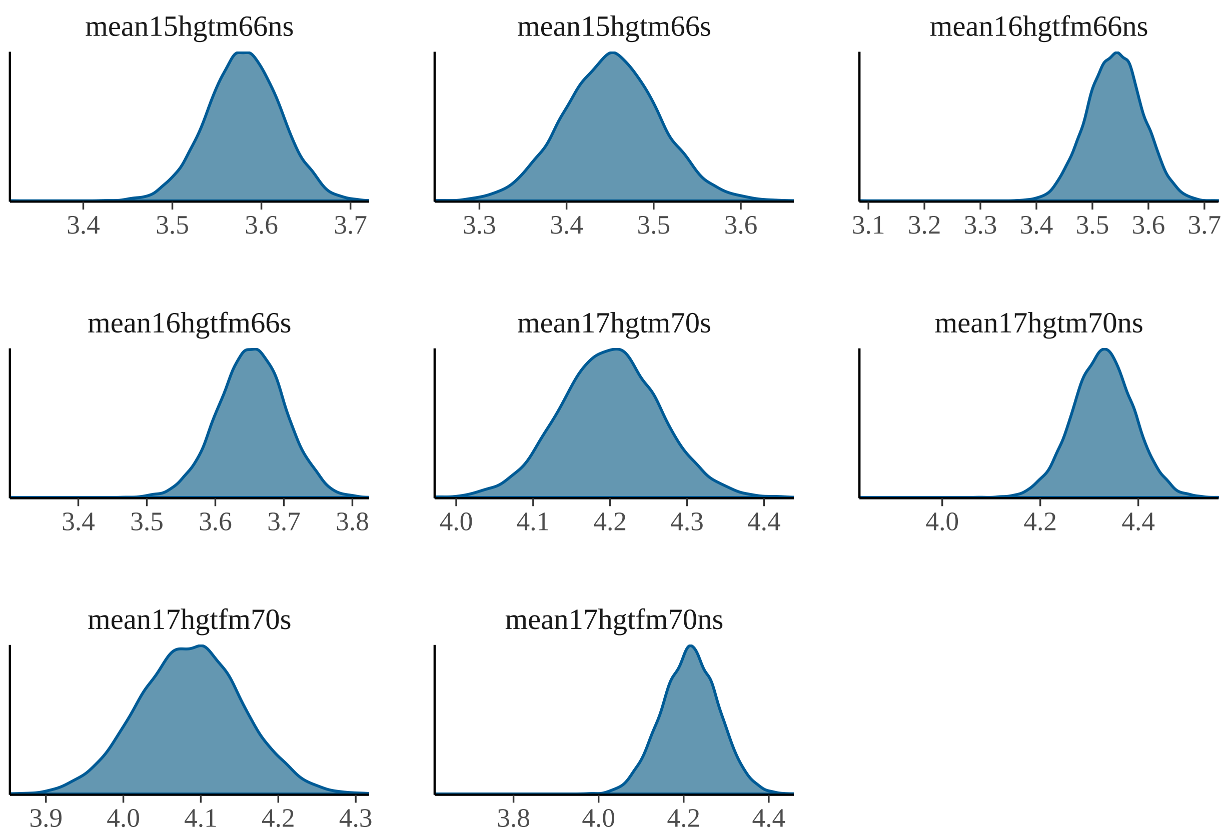
Table 5: Table continues below

2.5%	25%	50%	75%
3.49749715079988	3.55296882516818	3.58040502924773	3.60824200535888
3.34400534039217	3.41467952281119	3.45165071568555	3.48820352260862
3.44334176592521	3.50613777736146	3.54065505556865	3.57356808536838

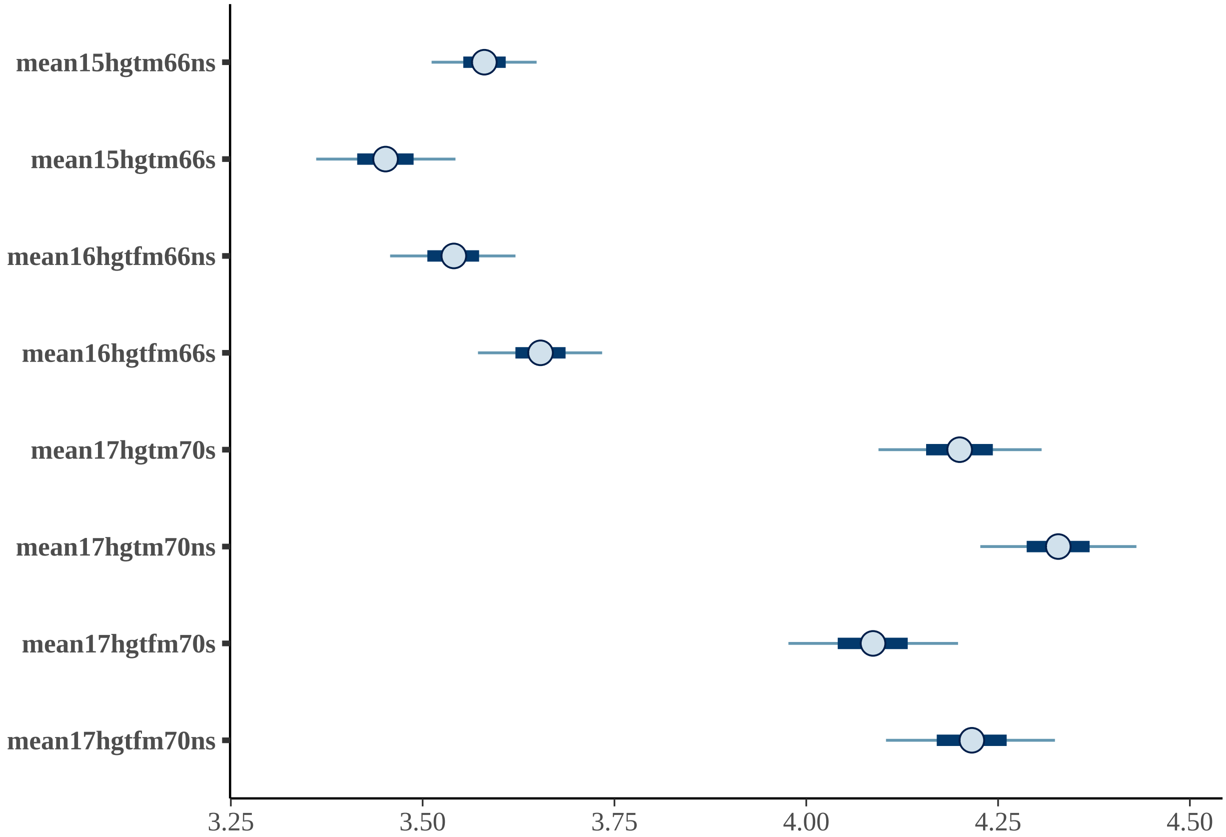
2.5%	25%	50%	75%
3.55586779832752	3.62092996179794	3.65362771127721	3.68625909823313
4.08303279589984	4.17005178796083	4.21583309941435	4.2611906520045
3.95601221383293	4.04100065067145	4.08701435111065	4.13222936501731
4.20423965577081	4.28728450807307	4.3284704619882	4.36931614439207
4.07239258111672	4.15619934557987	4.19991476631687	4.2431632779328

97.5%
3.66027835013261
3.56250540902183
3.63714998913902
3.74845224306454
4.34538854384483
4.21691370602266
4.44979267481496
4.32818209367562

```r
library(bayesplot)
par(mfrow=c(4,2))
jags.mcmc = as.mcmc(FEV.fit1)
mcmc_dens(jags.mcmc,pars=c("mean15hgtm66ns","mean15hgtm66s","mean16hgtfm66ns","mean16hgtfm66s","mean17h
```



```
color_scheme_set("blue")
par(mfrow=c(1,1))
figg = mcmc_intervals(jags.mcmc,pars=c("mean15hgtm66ns","mean15hgtm66s","mean16hgtfm66ns","mean16hgtfm6
figg
```



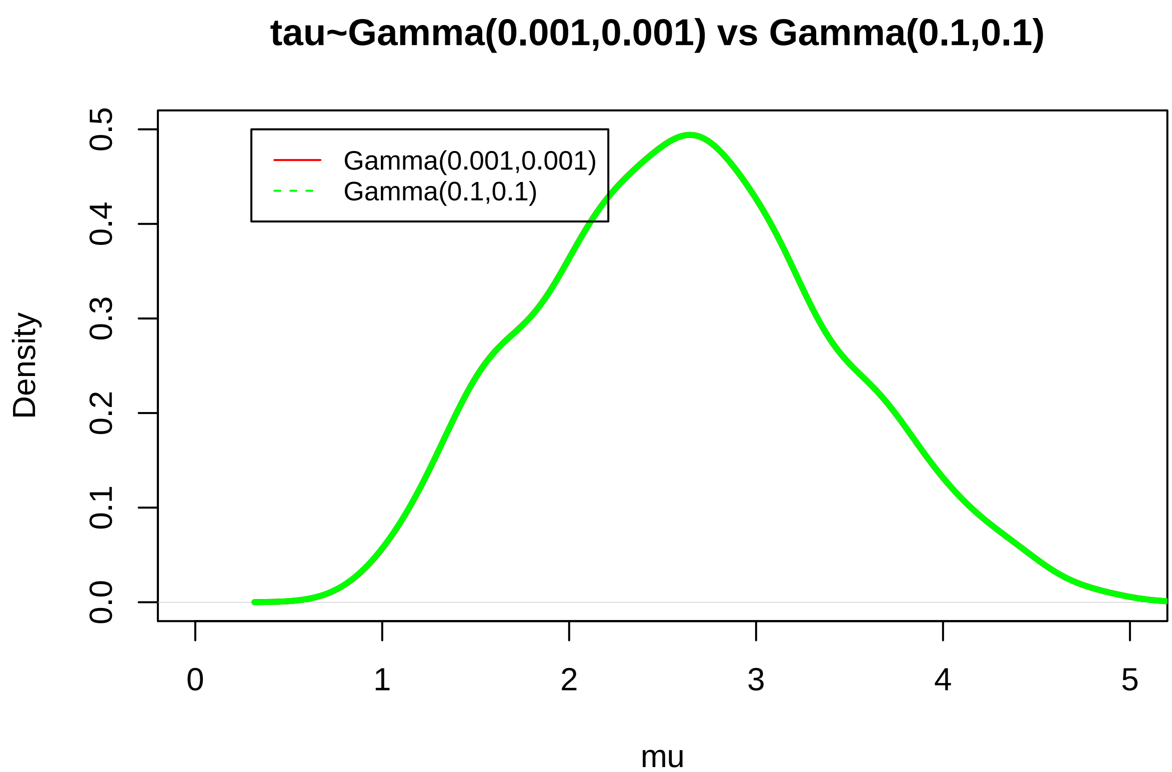
##(e)

```
j = 3
jags.data=list(
  Y=FEV,
  Xmat=X.mat[[j]],
  r=dim(X.mat[[j]])[2],
  n=dim(X.mat[[j]])[1],
  beta0 = mu.lst[[j]],
  Coin = C0inv[[j]],
  a=0.1, b=0.1## diffuse prior
)

model_1 <- "model{
  for(i in 1:n){
  Y[i] ~ dnorm(mu[i], tau)
  mu[i] <- beta[1] + beta[2]*Xmat[i,2] + beta[3]*Xmat[i,3] + beta[4]*Xmat[i,4]+beta[5]*Xmat[i,5]+beta[6
}

beta[1:r] ~ dmnorm(beta0,Coin)
tau ~ dgamma(a, b)
```

```
}"

jags.inits <- list(list(tau=1,beta=c(0,0,0,0,0,0)))
jags.param <- c("beta","tau","mu")

FEV.fit2 <- jags(jags.data, jags.inits, jags.param,model.file=textConnection(model_1),n.chains=1, n.ite
```

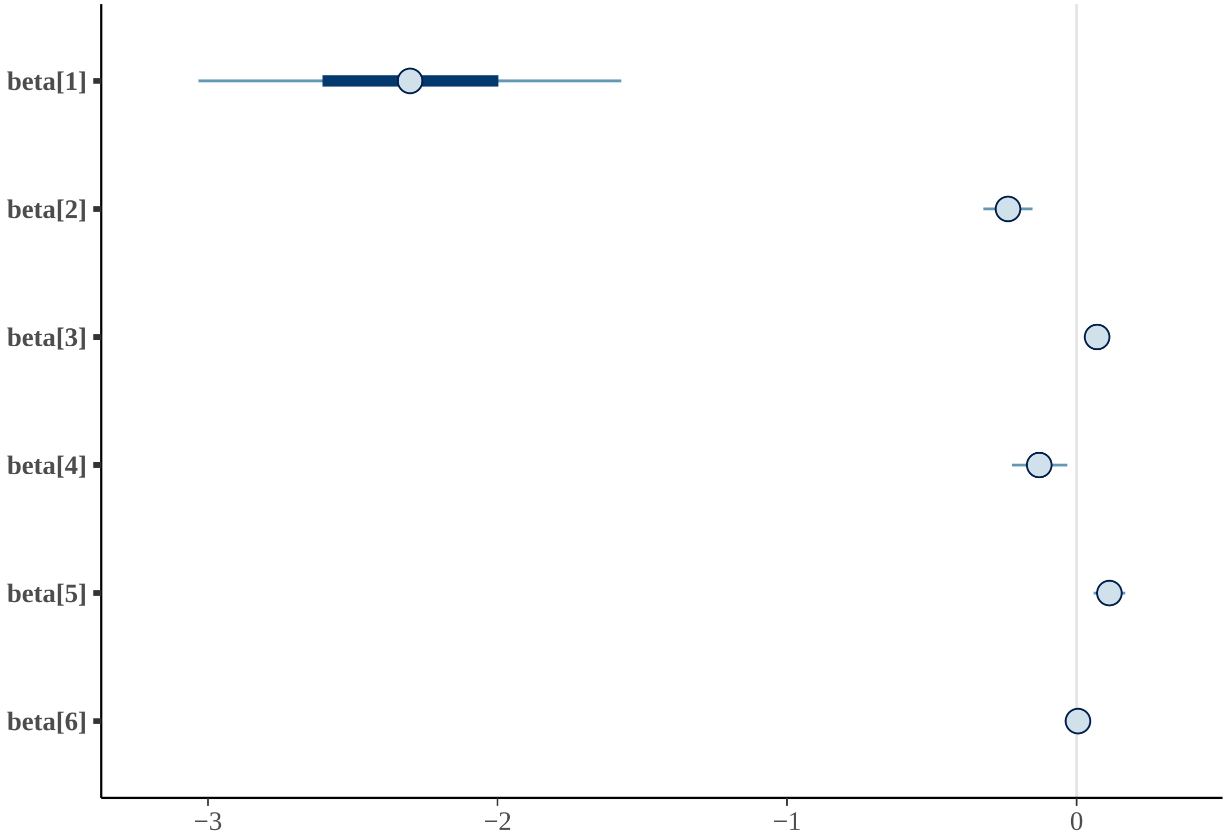
```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 654
##    Unobserved stochastic nodes: 2
##    Total graph size: 5326
##
## Initializing model
```

```
plot(main="tau~Gamma(0.001,0.001) vs Gamma(0.1,0.1) ",density(FEV.fit1$BUGSoutput$mean$mu),col='red',typ
lines(density(FEV.fit2$BUGSoutput$mean$mu),col='green',type='l',xlab ='mu',lwd=3,xlim=c(0,5),ylim=c(0,0


legend(0.3,0.5,legend=c("Gamma(0.001,0.001)","Gamma(0.1,0.1)"),col=c("red","green"),lty=1:2, cex=0.8)
```



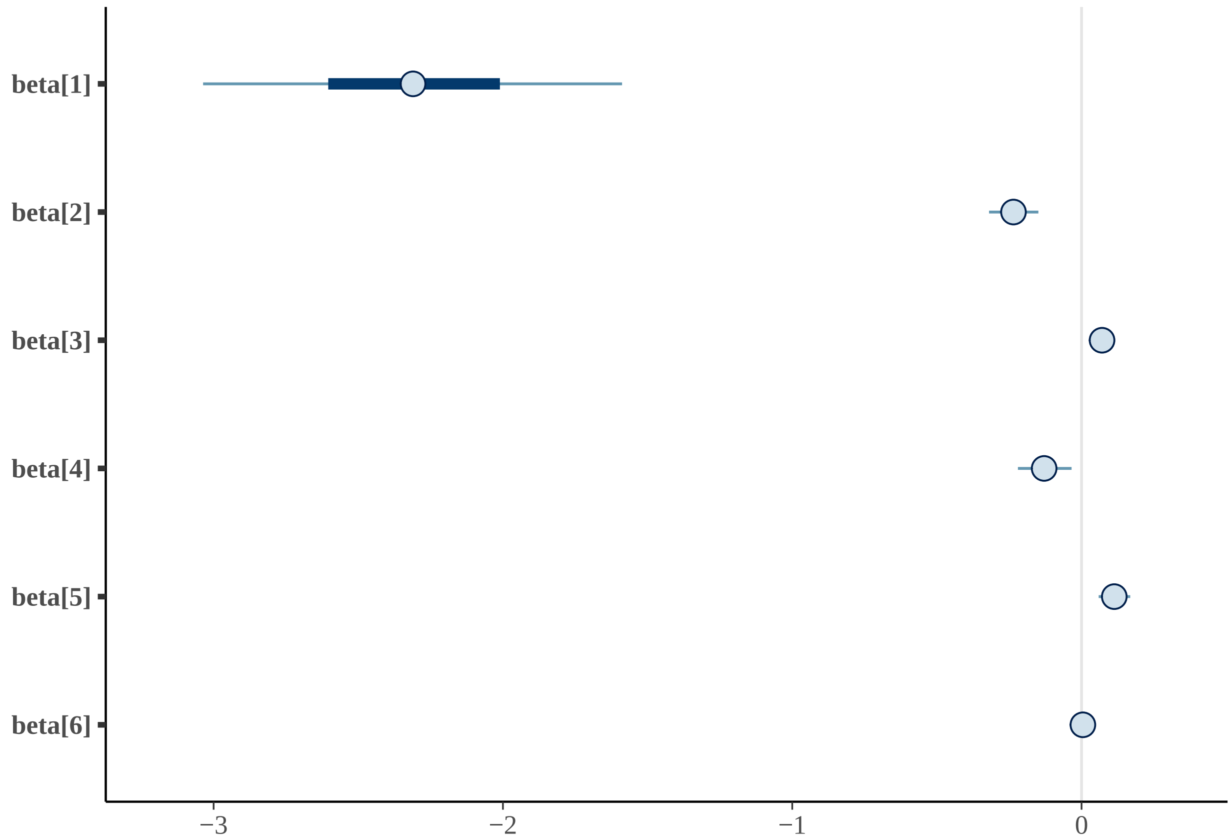

```
# FEV.fit1$BUGSoutput$summary[1:6,]
jags.mcmc2 = as.mcmc(FEV.fit2)
color_scheme_set("blue")
```

```r
par(mfrow=c(2,1))
figg = mcmc_intervals(jags.mcmc,pars=c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]","beta[6]"))
figg2 = mcmc_intervals(jags.mcmc2,pars=c("beta[1]","beta[2]","beta[3]","beta[4]","beta[5]","beta[6]"))
plot(figg)
```



Above is prior gamma(0.001,0.001), below is gamma(0.1,0.1)

```r
plot(figg2)
```



From the above plot we can see that the posterior is pretty much the same for both prior with the beta[1] in prior gamma (0.1,0.1) case a little bit latter than that of prior gamma(0.001,0.001) thus the sensitivity is low for this problem.

##(f)

We compare four models for prediction and find that the model 3 we propose perfroms best, (3) linear full model + age*height. Then we test that this model converges and mixs well. From comparsion of the subpopulations means, we find that when age is young, both men and women who smoke will have larger FEV, as the age gets larger enough,Smoke won't affect too much on FEV.However, since only 60+ out of 650+ adolescents smoke. we need future analysis. Then, we checked for several groups Age 15 Hgt 66 Male Non/smoker,Age 16 Hgt 66 Female Non/smoker,Age 17 Hgt 70 Female Non/Smoker,Age 17 Hgt 70 Male Non/Smoker. Their results distribution are reasonable compre to expert provided prior. The results show that adolescents who smoke will have less FEV which makes sense since smoke may do harm o lung. This is not withour reason, if we see the beta[4] (Smoke coefficent) distribution. We almost 99% say its negative in the model which will consequently reduce the FEV. Finally, by testing sensitivity, we change prior Gamma(0.001,0.001) to Gamma (0.1.0.1) and find that the prosteior mean distribution of mu as well as the distribution of beta don't change much. Thus, we get that the sensitivity is low.

Problem 2

Normal distribution model,

$$Y_i \sim N(\mu_i, \frac{1}{\tau_i})$$

Guess Linear model for X,

$$\mu_i = \beta_0 + \beta_1 x_i$$

Proper reference prior, almost no information from prior,

$$\tau \sim Gamma(0.001, 0.001)$$

$$\beta_i \sim N(0, 0.001)$$

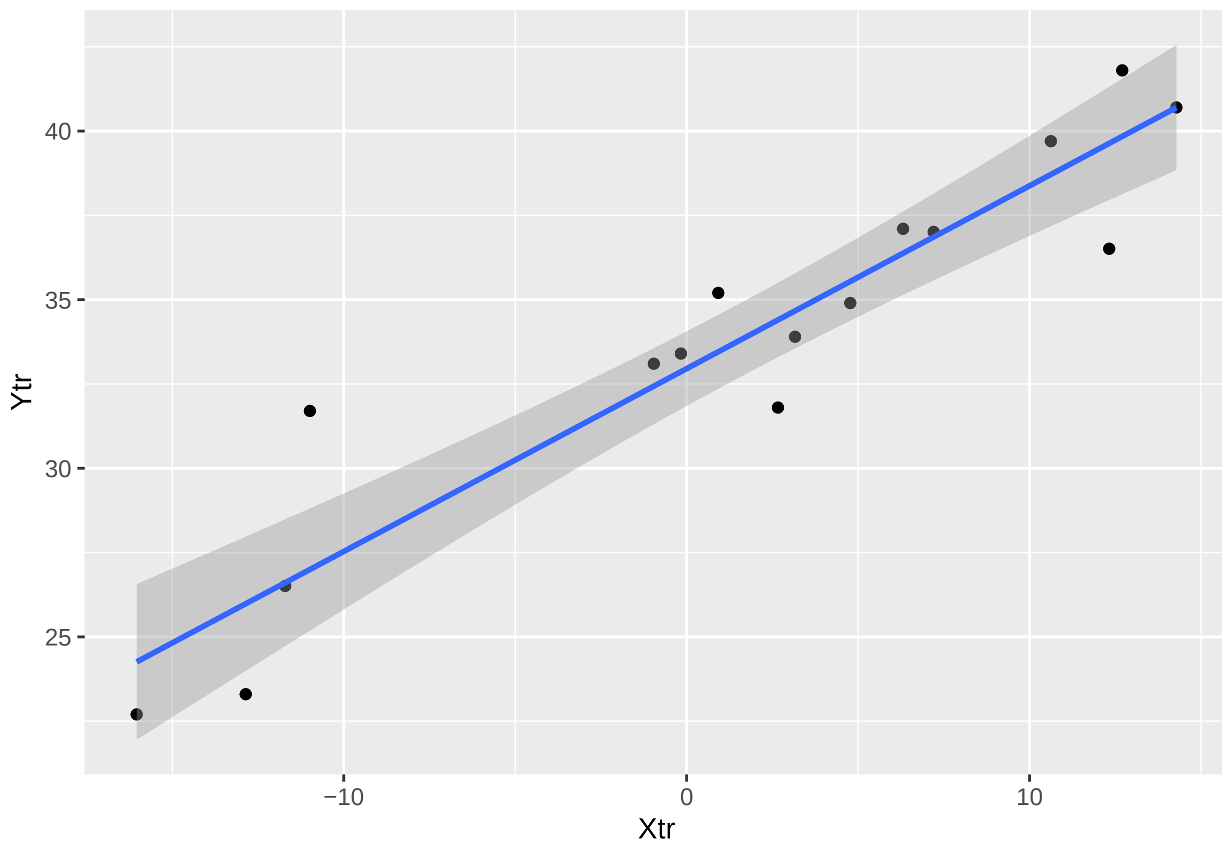
```
#
Ytr = c(37.01,26.51,36.51,40.70,37.10,33.90,41.80,33.40,23.30,35.20,34.90,33.10,22.70,39.70,31.80,31.70
Xtr = c(7.20,-11.71,12.32,14.28,6.31,3.16,12.70,-0.17,-12.86,0.92,4.77,-0.96,-16.04,10.62,2.66,-10.99)

jags.data = list(
  Y = Ytr,
  X = Xtr,
  n = length(Ytr),
  b = 0.001,
  c = 0.001
)

model_reg <-
"model{
for(i in 1:n){
Y[i] ~ dnorm(mu[i],tau)
mu[i] = beta[1]+beta[2]*X[i]
}

beta[1] ~ dnorm(0,b)
beta[2] ~ dnorm(0,b)
tau ~ dgamma(c,c)

pred = beta[1]+beta[2]*(-16.04)
}"
```

```
jags.inits <- list(list(tau=1,beta=c(0,0)))
jags.param = c("beta","tau","pred")
jags.fit <- jags(jags.data,jags.inits,jags.param,
                 model.file = textConnection(model_reg),n.iter=12000,n.chains = 1,n.burnin = 5000,n.thi
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 16
##    Unobserved stochastic nodes: 3
##    Total graph size: 73
##
## Initializing model
```

Estimated Regression Line and Point-wise 95% Probability band.

```
ggplot() +
  aes(x = Xtr, y = Ytr) +
  geom_point(color = "black") +
  geom_smooth(method = "lm",level = 0.95) # linear smooth
```




```r
jags.fit$BUGSoutput$summary
```

```
##                mean         sd        2.5%        25%        50%        75%
## beta[1]  32.9428760 0.56266724 31.83846455 32.5851803 32.9412113 33.2992925
## beta[2]   0.5410619 0.05966755  0.42335958  0.5029002  0.5404859  0.5790436
## deviance 69.4574043 2.73577037 66.35163656 67.4567774 68.7345870 70.6618428
## pred     24.2642436 1.18089715 21.93147854 23.5074641 24.2596982 25.0234412
## tau       0.2376289 0.08936430  0.09588752  0.1733891  0.2268028  0.2891771
##               97.5%
## beta[1]  34.0608380
## beta[2]   0.6611882
## deviance 76.4555970
## pred     26.6117971
## tau       0.4456464
```

```r
df <- rbind(c('beta[1]',jags.fit$BUGSoutput$summary[1,]),
            c('beta[2]',jags.fit$BUGSoutput$summary[2,]),
            c('prediction',jags.fit$BUGSoutput$summary[4,]),
            c('tau',jags.fit$BUGSoutput$summary[5,]))
pander(df)
```

Table 7: Table continues below

	mean	sd	2.5%
beta[1]	32.9428759598246	0.56266724432522	31.8384645545601
beta[2]	0.541061867741863	0.0596675542863946	0.423359584786309
prediction	24.2642436012451	1.18089715099641	21.9314785415893

	mean	sd	2.5%
tau	0.237628883996638	0.0893643034253068	0.0958875163889318

25%	50%	75%	97.5%
32.5851802733294	32.941211251907	33.2992924760897	34.0608379785487
0.502900227252926	0.540485944971818	0.579043577101202	0.661188240189195
23.5074641152	24.2596982286336	25.0234412083571	26.6117971182859
0.173389097130537	0.226802834838906	0.289177109621987	0.445646386618774

The 95% creidble interval for new schools with x=-16.04 is (21.959,26.617) and mean 24.264.

I'm not surprised that higher socioeconomic status were positively associated with higher test scores pn level of verbal since 1. school which has many higher socioeconomic status usually have better teachers 2. the parents in family with higher socioeconomic status are more likely has better education and consequently they can teach their children at home 3. childern in higher socioeconomic status are more likely to participate more social activities which can enhance their verbal skill