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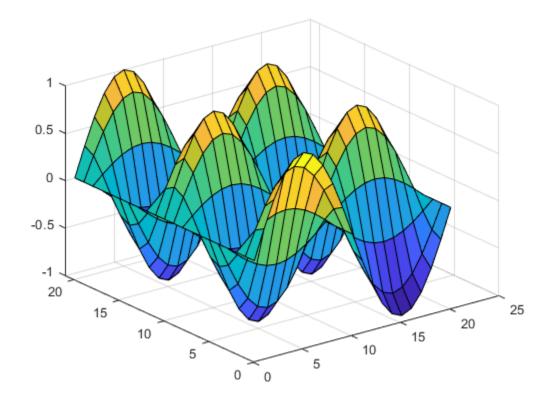
Step 1: Uniform Grids and Grid Functions

```
format long;
h       =    0.05;
hx      =    1;
hy      =    2;
[x,y] =    meshgrid(0:h:hx,hy:-2*h:0);
fun     =    myfunc(x,y);
figure(1);
surf(fun)
```


Step 2: Matrix-Vector Product

```
%********************(I)********************
```

```matlab
h       =   0.005;
hx      =   1;
hy      =   1;
[x,y]   =   meshgrid(0:h:hx,hy:-h:0);
u       =   myfunc(x,y);
u1      =   u(2:end-1,2:end-1);
us      =   u1(:);
n       =   hx/h-1;
e       =   ones(n,1);
T       =   spdiags([-e 2*e -e], -1:1 ,n,n);
A       =   kron(speye(n),T)+kron(T,speye(n));

%*******************(II)*******************

% um is the iteration matrix
um          =   zeros(n+2,n+2);
um(1,:)   =   u(1,:);
um(end,:) =   u(end,:);
um(:,1)   =   u(:,1);
um(:,end) =   u(:,end);

for k = 1:1
    j=2:n+1;
    i=2:n+1;
    um(i,j)=4*u(i,j)-u(i-1,j)-u(i+1,j)-u(i,j-1)-u(i,j+1);
end

uk = um(2:end-1,2:end-1);
um;
u  = myfunc(x,y);

%*******************(III)*******************

B   =   (T*u1)*speye(n)+(speye(n)*u1)*T';
B   =   B(:);

f   =   8*pi^2*sin(2*pi*x).*cos(2*pi*y);
f   =   f(2:end-1,2:end-1);
f   =   f(:);

f(1) = f(1)+u(1,2)+u(2,1);
f(n) = f(n)+u(n+1,1)+u(n+2,2);
f(n*(n-1)+1) = f(n*(n-1)+1)+u(1,n+1)+u(2,n+2);
f(n*n) = f(n*n) + u(n+1,n+2)+u(n+2,n+1);

for i = 2:n-1
    f(i) = f(i)+ u(i+1,1);
end
for i = n*(n-1)+2:n^2-1
    f(i) = f(i)+ u(i+1-n*(n-1),n+2);
end
for i=1:n-2
    f(i*n+1) = f(i*n+1) + u(1,i+2);
```

```matlab
    end
    for i=1:n-2
        f((i+1)*n) = f((i+1)*n) + u(n+2,i+2);
    end

    M    =   zeros((n-2)^2,3);

    MM   =   reshape(A*us/h/h-f,n,n);
    MM   =   MM(2:end-1,2:end-1);
    MM   =   MM(:);

    M(:,1)  = MM;

    MM   =   reshape(uk(:)/h/h-f,n,n);
    MM   =   MM(2:end-1,2:end-1);
    MM   =   MM(:);

    M(:,2)  = MM;

    MM   =   reshape(B/h/h-f,n,n);
    MM   =   MM(2:end-1,2:end-1);
    MM   =   MM(:);

    M(:,3)  = MM;

    M1   = M;

    % down h by half

    h       =   h/2;
    hx      =   1;
    hy      =   1;
    [x,y]   =   meshgrid(0:h:hx,hy:-h:0);
    u       =   myfunc(x,y);
    u1      =   u(2:end-1,2:end-1);
    us      =   u1(:);
    n       =   hx/h-1;
    e       =   ones(n,1);
    T       =   spdiags([-e 2*e -e], -1:1 ,n,n);
    A       =   kron(speye(n),T)+kron(T,speye(n));


    %*****************(II)*****************

    % um is the iteration matrix
    um         =   zeros(n+2,n+2);
    um(1,:)    =   u(1,:);
    um(end,:)  =   u(end,:);
    um(:,1)    =   u(:,1);
    um(:,end)  =   u(:,end);

    for k = 1:1
        j=2:n+1;
        i=2:n+1;
```

```matlab
        um(i,j)=4*u(i,j)-u(i-1,j)-u(i+1,j)-u(i,j-1)-u(i,j+1);
    end

uk = um(2:end-1,2:end-1);
um;
u   = myfunc(x,y);

%******************(III)*******************

B    =   (T*u1)*speye(n)+(speye(n)*u1)*T';
B    =   B(:);

f    =   8*pi^2*sin(2*pi*x).*cos(2*pi*y);
f    =   f(2:end-1,2:end-1);
f    =   f(:);

f(1) = f(1)+u(1,2)+u(2,1);
f(n) = f(n)+u(n+1,1)+u(n+2,2);
f(n*(n-1)+1) = f(n*(n-1)+1)+u(1,n+1)+u(2,n+2);
f(n*n) = f(n*n) + u(n+1,n+2)+u(n+2,n+1);

for i = 2:n-1
    f(i) = f(i)+ u(i+1,1);
end
for i = n*(n-1)+2:n^2-1
    f(i) = f(i)+ u(i+1-n*(n-1),n+2);
end
for i=1:n-2
    f(i*n+1) = f(i*n+1) + u(1,i+2);
end
for i=1:n-2
    f((i+1)*n) = f((i+1)*n) + u(n+2,i+2);
end

M    =   zeros((n-2)^2,3);
MM   =   reshape(A*us/h/h-f,n,n);
MM   =   MM(2:end-1,2:end-1);
MM   =   MM(:);

M(:,1)  = MM;

MM   =   reshape(uk(:)/h/h-f,n,n);
MM   =   MM(2:end-1,2:end-1);
MM   =   MM(:);

M(:,2)  = MM;

MM   =   reshape(B/h/h-f,n,n);
MM   =   MM(2:end-1,2:end-1);
MM   =   MM(:);

M(:,3)  = MM;

M2   = M;
```

```matlab
disp('The maximum norm of three method for h = 0.005');
disp(max(abs(M1)));
disp('The maximum norm of three method for h = 0.0025');
disp(max(abs(M2)));
disp('order of three methods');
disp(sqrt(max(abs(M1))./max(abs(M2))));
```

The maximum norm of three method for h = 0.005
   0.006493725770866   0.006493725788630   0.006493725770866

The maximum norm of three method for h = 0.0025
   0.001623471491001   0.001623471419947   0.001623471491001

order of three methods
   1.999975331546076   1.999975378047876   1.999975331546076

Step 3 Boundary Conditions Dirchlet

```matlab
h      =    0.1;
hx     =    1;
hy     =    1;
[x,y]  =    meshgrid(0:h:hx,hy:-h:0);
u      =    myfunc(x,y);
u1     =    u(2:end-1,2:end-1);
us     =    u1(:);

n  = hx/h-1;
e  = ones(n,1);
T  = spdiags([-e 2*e -e], -1:1 ,n,n);
A  = kron(speye(n),T)+kron(T,speye(n));


f   =   h^2*8*pi^2*sin(2*pi*x).*cos(2*pi*y);
f   =   f(2:end-1,2:end-1);
f   =   f(:);

f(1) = f(1)+u(1,2)+u(2,1);
f(n) = f(n)+u(n+1,1)+u(n+2,2);
f(n*(n-1)+1) = f(n*(n-1)+1)+u(1,n+1)+u(2,n+2);
f(n*n) = f(n*n) + u(n+1,n+2)+u(n+2,n+1);

for i = 2:n-1
    f(i) = f(i)+ u(i+1,1);
end
for i = n*(n-1)+2:n^2-1
    f(i) = f(i)+ u(i+1-n*(n-1),n+2);
end
for i=1:n-2
    f(i*n+1) = f(i*n+1) + u(1,i+2);
end
for i=1:n-2
    f((i+1)*n) = f((i+1)*n) + u(n+2,i+2);
```

```
      end

    disp('By dirchlet method');
    disp(reshape(A\f,n,n));
    disp('Oringal matrix');
    disp(u(2:end-1,2:end-1));
    disp('Maximum diference');
    disp(max(max(reshape(A\f,n,n)-u(2:end-1,2:end-1))));
```

By dirchlet method
  Columns 1 through 3

     0.480695606283314   0.777781829209140   0.777781829209139
     0.181753280475189   0.294082985375649   0.294082985375649
    -0.191179216397811   -0.309334470074230   -0.309334470074230
    -0.493721929352599   -0.798858862683679   -0.798858862683679
    -0.609387575172450   -0.986009808950906   -0.986009808950906
    -0.493721929352599   -0.798858862683679   -0.798858862683679
    -0.191179216397811   -0.309334470074230   -0.309334470074230
     0.181753280475190   0.294082985375649   0.294082985375649
     0.480695606283314   0.777781829209140   0.777781829209140

  Columns 4 through 6

     0.480695606283314   -0.000000000000000   -0.480695606283314
     0.181753280475189   -0.000000000000000   -0.181753280475190
    -0.191179216397811   -0.000000000000000   0.191179216397811
    -0.493721929352599   -0.000000000000000   0.493721929352598
    -0.609387575172450   -0.000000000000000   0.609387575172450
    -0.493721929352599   -0.000000000000000   0.493721929352598
    -0.191179216397811   -0.000000000000000   0.191179216397811
     0.181753280475189   -0.000000000000000   -0.181753280475190
     0.480695606283314   0.000000000000000   -0.480695606283314

  Columns 7 through 9

    -0.777781829209141   -0.777781829209140   -0.480695606283315
    -0.294082985375650   -0.294082985375650   -0.181753280475190
     0.309334470074230   0.309334470074230   0.191179216397811
     0.798858862683679   0.798858862683679   0.493721929352599
     0.986009808950906   0.986009808950906   0.609387575172450
     0.798858862683679   0.798858862683679   0.493721929352599
     0.309334470074230   0.309334470074230   0.191179216397811
    -0.294082985375650   -0.294082985375650   -0.181753280475190
    -0.777781829209140   -0.777781829209140   -0.480695606283315

Oringal matrix
  Columns 1 through 3

     0.475528258147577   0.769420884293813   0.769420884293813
     0.181635632001340   0.293892626146236   0.293892626146236
    -0.181635632001340   -0.293892626146237   -0.293892626146237
    -0.475528258147577   -0.769420884293813   -0.769420884293813
    -0.587785252292473   -0.951056516295154   -0.951056516295154

```
     -0.475528258147577  -0.769420884293813  -0.769420884293813
     -0.181635632001340  -0.293892626146237  -0.293892626146237
      0.181635632001340   0.293892626146237   0.293892626146237
      0.475528258147577   0.769420884293813   0.769420884293813


  Columns 4 through 6

      0.475528258147577   0.000000000000000  -0.475528258147577
      0.181635632001340   0.000000000000000  -0.181635632001340
     -0.181635632001340  -0.000000000000000   0.181635632001340
     -0.475528258147577  -0.000000000000000   0.475528258147577
     -0.587785252292473  -0.000000000000000   0.587785252292473
     -0.475528258147577  -0.000000000000000   0.475528258147577
     -0.181635632001340  -0.000000000000000   0.181635632001340
      0.181635632001340   0.000000000000000  -0.181635632001340
      0.475528258147577   0.000000000000000  -0.475528258147577


  Columns 7 through 9

     -0.769420884293813  -0.769420884293813  -0.475528258147577
     -0.293892626146236  -0.293892626146236  -0.181635632001340
      0.293892626146237   0.293892626146237   0.181635632001340
      0.769420884293813   0.769420884293813   0.475528258147577
      0.951056516295154   0.951056516295154   0.587785252292473
      0.769420884293813   0.769420884293813   0.475528258147577
      0.293892626146237   0.293892626146237   0.181635632001340
     -0.293892626146237  -0.293892626146237  -0.181635632001340
     -0.769420884293813  -0.769420884293813  -0.475528258147577


Maximum diference
   0.034953292655752
```

Step 3 Boundary Conditions Neumann

```
h    =   0.025;
hx   =   1;
hy   =   1;
[x,y] =   meshgrid(0:h:hx,hy:-h:0);
u    =   myfunc(x,y);
n    =   hx/h+1;


ff   =  h^2*8*pi^2*sin(2*pi*x).*cos(2*pi*y);
ff   =  ff(:);


fkx  =  @(x,y)  2*pi*cos(2*pi*x).*cos(2*pi*y);
fky  =  @(x,y) -2*pi*sin(2*pi*x).*sin(2*pi*y);


e  = ones(n,1);
T  = spdiags([-e 2*e -e], -1:1 ,n,n);
A  = kron(speye(n),T)+kron(T,speye(n));


A(1,1)   =  1;
```

```matlab
A(1,2)    =  -0.5;
A(1,n+1) =  -0.5;
sz       = [n n];
[row,col] = ind2sub(sz,1);
ff(1)     = 0.25*ff(1)-h*(fkx(x(row,col),y(row,col))-
fky(x(row,col),y(row,col)))/2;


A(n,n)    =  1;
A(n,n-1) =  -0.5;
A(n,2*n) =  -0.5;
[row,col] = ind2sub(sz,n);
ff(n)     = 0.25*ff(n)-
h*(fkx(x(row,col),y(row,col))+fky(x(row,col),y(row,col)))/2;


A(n*(n-1)+1,n*(n-1)+1) = 1;
A(n*(n-1)+1,n*(n-1)+2) = -0.5;
A(n*(n-1)+1,n*(n-2)+1) = -0.5;
[row,col] = ind2sub(sz,n*(n-1)+1);
ff(n*(n-1)+1)       =
 0.25*ff(n*(n-1)+1)+h*(fkx(x(row,col),y(row,col))+fky(x(row,col),y(row,col)))/2;

A(n^2,n^2)       = 1;
A(n^2,n^2-1)    = -0.5;
A(n^2,n*(n-1)) = -0.5;
[row,col] = ind2sub(sz,n^2);
ff(n^2)           = 0.25*ff(n^2)+h*(fkx(x(row,col),y(row,col))-
fky(x(row,col),y(row,col)))/2;

for i = 2:n-1
    A(i,i) = 2;
    A(i,i-1) = -0.5;
    A(i,i+1) = -0.5;
    A(i,i+n) = -1;
    [row,col] = ind2sub(sz,i);
    ff(i)     = 0.5*ff(i) - h*fkx(x(row,col),y(row,col));
end

for i = n*(n-1)+2:n^2-1
    A(i,i) = 2;
    A(i,i-1) = -0.5;
    A(i,i+1) = -0.5;
    A(i,i-n) = -1;
    [row,col] = ind2sub(sz,i);
    ff(i)     = 0.5*ff(i) + h*fkx(x(row,col),y(row,col));
end

for i=1:n-2
    A(i*n+1,i*n+1)    = 2;
    A(i*n+1,i*n+1-n) = -0.5;
    A(i*n+1,i*n+1+n) = -0.5;
    A(i*n+1,i*n+1+1) = -1;
    [row,col] = ind2sub(sz,i*n+1);
    ff(i*n+1)     = 0.5*ff(i*n+1) - h*fky(x(row,col),y(row,col));
```

```
    end

for i=1:n-2
    A((i+1)*n,(i+1)*n) = 2;
    A((i+1)*n,(i+1)*n-n) = -0.5;
    A((i+1)*n,(i+1)*n+n) = -0.5;
    A((i+1)*n,(i+1)*n-1) = -1;
    [row,col] = ind2sub(sz,(i+1)*n);
    ff((i+1)*n)      = 0.5*ff((i+1)*n) + h*fky(x(row,col),y(row,col));
end

A;
D = u;

BB = reshape(pinv(full(A))*ff,n,n);
BB(1:10,1:10)
u(1:10,1:10)


disp('By Nemann method first 10*10');
disp(BB(1:10,1:10));
disp('Oringal matrix first 10*10');
disp(u(1:10,1:10));
disp('Maximum diference');
disp(max(max(BB(1:10,1:10)-u(1:10,1:10))));
```

ans =

  Columns 1 through 3

  -0.002045069459985   0.155009385020853   0.308160953584346
  -0.002019891261334   0.153100962267918   0.304366980881936
  -0.001944976636195   0.147422685710984   0.293078482974118
  -0.001822170231273   0.138114373364005   0.274573420143759
  -0.001654495947812   0.125405226769476   0.249307448452523
  -0.001446082483156   0.109608187295799   0.217902699976400
  -0.001202061668498   0.091112230482180   0.181132463849260
  -0.000928442106146   0.070372788169934   0.139902145317973
  -0.000631961217815   0.047900534259054   0.095226971660355
  -0.000319919346940   0.024248810221956   0.048206993920262

  Columns 4 through 6

   0.453651090114184   0.587908088892865   0.707635296410565
   0.448065892404051   0.580669964741054   0.698923131658302
   0.431447825377490   0.559133818924187   0.673001159811723
   0.404206080996898   0.523829942826656   0.630507665847785
   0.367011441419092   0.475627635044826   0.572488980615703
   0.320779762112406   0.415713796370571   0.500373716698869
   0.266649420455522   0.345563704354682   0.415937591231748
   0.205953285108330   0.266904687077402   0.321259701850783
   0.140185896362010   0.181673590598401   0.218671332410430

```
    0.070966665596940    0.091969087378789    0.110698549037584

Columns 7 through 9

    0.809892512434606    0.892168581552733    0.952443392523812
    0.799921391666964    0.881184505844938    0.940717233872647
    0.770253551449589    0.848502743119534    0.905827494862037
    0.721619512470544    0.794928026839371    0.848633276658821
    0.655216806176624    0.721779544323558    0.770542890731897
    0.572680487574724    0.630858453977530    0.673479181549917
    0.476042874751164    0.524403534795399    0.559832179768914
    0.367683506455512    0.405036060191059    0.432400251745523
    0.250270549959329    0.275695253547197    0.294321194469994
    0.126695101922809    0.139565914780916    0.148994972590576

Column 10

    0.989237762902659
    0.977058604495374
    0.940821020573819
    0.881417300719783
    0.800310161665715
    0.699496730354292
    0.581459368044990
    0.449104546341420
    0.305691280214385
    0.154750880237285


ans =

Columns 1 through 3

                 0    0.156434465040231    0.309016994374947
                 0    0.154508497187474    0.305212482389889
                 0    0.148778017349658    0.293892626146237
                 0    0.139384128958763    0.275336158073158
                 0    0.126558140723500    0.250000000000000
                 0    0.110615871041237    0.218508012224410
                 0    0.091949871500910    0.181635632001340
                 0    0.071019760960103    0.140290779704295
                 0    0.048340908203385    0.095491502812526
                 0    0.024471741852423    0.048340908203385

Columns 4 through 6

    0.453990499739547    0.587785252292473    0.707106781186547
    0.448401123333710    0.580548640463047    0.698401123333710
    0.431770623113389    0.559016994374947    0.672498511963957
    0.404508497187474    0.523720494614299    0.630036755335050
    0.367286029574068    0.475528258147577    0.572061402817684
    0.321019760960103    0.415626937777453    0.500000000000000
    0.266848920427795    0.345491502812526    0.415626937777453
    0.206107373853763    0.266848920427795    0.321019760960103
```

```
    0.140290779704295    0.181635632001340    0.218508012224410
    0.071019760960103    0.091949871500910    0.110615871041237


  Columns 7 through 9

    0.809016994374947    0.891006524188368    0.951056516295154
    0.799056652687458    0.880036755335050    0.939347432391753
    0.769420884293813    0.847397560890843    0.904508497187474
    0.720839420167342    0.793892626146236    0.847397560890843
    0.654508497187474    0.720839420167342    0.769420884293813
    0.572061402817684    0.630036755335050    0.672498511963957
    0.475528258147577    0.523720494614299    0.559016994374947
    0.367286029574068    0.404508497187474    0.431770623113389
    0.250000000000000    0.275336158073158    0.293892626146236
    0.126558140723500    0.139384128958763    0.148778017349658


  Column 10

    0.987688340595138
    0.975528258147577
    0.939347432391753
    0.880036755335050
    0.799056652687458
    0.698401123333710
    0.580548640463047
    0.448401123333710
    0.305212482389889
    0.154508497187474


By Nemann method first 10*10
  Columns 1 through 3

   -0.002045069459985    0.155009385020853    0.308160953584346
   -0.002019891261334    0.153100962267918    0.304366980881936
   -0.001944976636195    0.147422685710984    0.293078482974118
   -0.001822170231273    0.138114373364005    0.274573420143759
   -0.001654495947812    0.125405226769476    0.249307448452523
   -0.001446082483156    0.109608187295799    0.217902699976400
   -0.001202061668498    0.091112230482180    0.181132463849260
   -0.000928442106146    0.070372788169934    0.139902145317973
   -0.000631961217815    0.047900534259054    0.095226971660355
   -0.000319919346940    0.024248810221956    0.048206993920262


  Columns 4 through 6

    0.453651090114184    0.587908088892865    0.707635296410565
    0.448065892404051    0.580669964741054    0.698923131658302
    0.431447825377490    0.559133818924187    0.673001159811723
    0.404206080996898    0.523829942826656    0.630507665847785
    0.367011441419092    0.475627635044826    0.572488980615703
    0.320779762112406    0.415713796370571    0.500373716698869
    0.266649420455522    0.345563704354682    0.415937591231748
    0.205953285108330    0.266904687077402    0.321259701850783
    0.140185896362010    0.181673590598401    0.218671332410430
```

```
        0.070966665596940    0.091969087378789    0.110698549037584


    Columns 7 through 9

    0.809892512434606    0.892168581552733    0.952443392523812
    0.799921391666964    0.881184505844938    0.940717233872647
    0.770253551449589    0.848502743119534    0.905827494862037
    0.721619512470544    0.794928026839371    0.848633276658821
    0.655216806176624    0.721779544323558    0.770542890731897
    0.572680487574724    0.630858453977530    0.673479181549917
    0.476042874751164    0.524403534795399    0.559832179768914
    0.367683506455512    0.405036060191059    0.432400251745523
    0.250270549959329    0.275695253547197    0.294321194469994
    0.126695101922809    0.139565914780916    0.148994972590576


    Column 10

    0.989237762902659
    0.977058604495374
    0.940821020573819
    0.881417300719783
    0.800310161665715
    0.699496730354292
    0.581459368044990
    0.449104546341420
    0.305691280214385
    0.154750880237285


Oringal matrix first 10*10
  Columns 1 through 3

                   0    0.156434465040231    0.309016994374947
                   0    0.154508497187474    0.305212482389889
                   0    0.148778017349658    0.293892626146237
                   0    0.139384128958763    0.275336158073158
                   0    0.126558140723500    0.250000000000000
                   0    0.110615871041237    0.218508012224410
                   0    0.091949871500910    0.181635632001340
                   0    0.071019760960103    0.140290779704295
                   0    0.048340908203385    0.095491502812526
                   0    0.024471741852423    0.048340908203385


  Columns 4 through 6

    0.453990499739547    0.587785252292473    0.707106781186547
    0.448401123333710    0.580548640463047    0.698401123333710
    0.431770623113389    0.559016994374947    0.672498511963957
    0.404508497187474    0.523720494614299    0.630036755335050
    0.367286029574068    0.475528258147577    0.572061402817684
    0.321019760960103    0.415626937777453    0.500000000000000
    0.266848920427795    0.345491502812526    0.415626937777453
    0.206107373853763    0.266848920427795    0.321019760960103
    0.140290779704295    0.181635632001340    0.218508012224410
    0.071019760960103    0.091949871500910    0.110615871041237
```

```
        Columns 7 through 9

    0.809016994374947    0.891006524188368    0.951056516295154
    0.799056652687458    0.880036755335050    0.939347432391753
    0.769420884293813    0.847397560890843    0.904508497187474
    0.720839420167342    0.793892626146236    0.847397560890843
    0.654508497187474    0.720839420167342    0.769420884293813
    0.572061402817684    0.630036755335050    0.672498511963957
    0.475528258147577    0.523720494614299    0.559016994374947
    0.367286029574068    0.404508497187474    0.431770623113389
    0.250000000000000    0.275336158073158    0.293892626146236
    0.126558140723500    0.139384128958763    0.148778017349658


        Column 10

    0.987688340595138
    0.975528258147577
    0.939347432391753
    0.880036755335050
    0.799056652687458
    0.698401123333710
    0.580548640463047
    0.448401123333710
    0.305212482389889
    0.154508497187474

Maximum diference
    0.001549422307521
```

Step 4 Solve the linear Algebraic Systems

```
tol   =    2*10^(-2);
err   =    1;
h     =    0.1;
hx    =    1;
hy    =    1;
[x,y] =    meshgrid(0:h:hx,hy:-h:0);
u     =    myfunc(x,y);
u1    =    u(2:end-1,2:end-1);
us    =    u1(:);

n  = hx/h-1;
e  = ones(n,1);
T  = spdiags([-e 2*e -e], -1:1 ,n,n);
A  = kron(speye(n),T)+kron(T,speye(n));

f   =   h^2*8*pi^2*sin(2*pi*x).*cos(2*pi*y);
f   =   f(2:end-1,2:end-1);
f   =   f(:);

f(1) = f(1)+u(1,2)+u(2,1);
```

```matlab
f(n) = f(n)+u(n+1,1)+u(n+2,2);
f(n*(n-1)+1) = f(n*(n-1)+1)+u(1,n+1)+u(2,n+2);
f(n*n) = f(n*n) + u(n+1,n+2)+u(n+2,n+1);

for i = 2:n-1
    f(i) = f(i)+ u(i+1,1);
end
for i = n*(n-1)+2:n^2-1
    f(i) = f(i)+ u(i+1-n*(n-1),n+2);
end
for i=1:n-2
    f(i*n+1) = f(i*n+1) + u(1,i+2);
end
for i=1:n-2
    f((i+1)*n) = f((i+1)*n) + u(n+2,i+2);
end
disp('Dierect Solving')
disp(reshape(A\f,n,n));
disp('Original matrix')
disp(u(2:end-1,2:end-1));
disp('Maximum diference');
disp(max(max(reshape(A\f,n,n)-u(2:end-1,2:end-1))));

f = reshape(f,n,n);
um = zeros(n,n);
u2 = u(2:end-1,2:end-1);
um(1,:)= u2(1,:);
um(end,:)=u2(end,:);
um(:,1)=u2(:,1);
um(:,end) = u2(:,end);

 while err > tol
    for j = 2:n-1
        for i = 2:n-1
            um(i,j)  = (f(i,j) + um(i-1,j) + um(i+1,j) + um(i,j-1)
 +um(i,j+1))/4;
        end
    end
    err = norm(f(:)-A*um(:))/norm(f(:));
 end

 disp('when tol is 2*e3')
 disp('GS Solving')
 disp(um);
 disp('Original Matrix')
 disp(u(2:end-1,2:end-1));
 disp('Maximum diference');
 disp(max(max(um-u(2:end-1,2:end-1))));

Dierect Solving
  Columns 1 through 3

    0.480695606283314   0.777781829209140   0.777781829209139
    0.181753280475189   0.294082985375649   0.294082985375649
```

```
    -0.191179216397811    -0.309334470074230    -0.309334470074230
    -0.493721929352599    -0.798858862683679    -0.798858862683679
    -0.609387575172450    -0.986009808950906    -0.986009808950906
    -0.493721929352599    -0.798858862683679    -0.798858862683679
    -0.191179216397811    -0.309334470074230    -0.309334470074230
     0.181753280475190     0.294082985375649     0.294082985375649
     0.480695606283314     0.777781829209140     0.777781829209140


  Columns 4 through 6

     0.480695606283314    -0.000000000000000    -0.480695606283314
     0.181753280475189    -0.000000000000000    -0.181753280475190
    -0.191179216397811    -0.000000000000000     0.191179216397811
    -0.493721929352599    -0.000000000000000     0.493721929352598
    -0.609387575172450    -0.000000000000000     0.609387575172450
    -0.493721929352599    -0.000000000000000     0.493721929352598
    -0.191179216397811    -0.000000000000000     0.191179216397811
     0.181753280475189    -0.000000000000000    -0.181753280475190
     0.480695606283314     0.000000000000000    -0.480695606283314


  Columns 7 through 9

    -0.777781829209141    -0.777781829209140    -0.480695606283315
    -0.294082985375650    -0.294082985375650    -0.181753280475190
     0.309334470074230     0.309334470074230     0.191179216397811
     0.798858862683679     0.798858862683679     0.493721929352599
     0.986009808950906     0.986009808950906     0.609387575172450
     0.798858862683679     0.798858862683679     0.493721929352599
     0.309334470074230     0.309334470074230     0.191179216397811
    -0.294082985375650    -0.294082985375650    -0.181753280475190
    -0.777781829209140    -0.777781829209140    -0.480695606283315

Original matrix
  Columns 1 through 3

     0.475528258147577     0.769420884293813     0.769420884293813
     0.181635632001340     0.293892626146236     0.293892626146236
    -0.181635632001340    -0.293892626146237    -0.293892626146237
    -0.475528258147577    -0.769420884293813    -0.769420884293813
    -0.587785252292473    -0.951056516295154    -0.951056516295154
    -0.475528258147577    -0.769420884293813    -0.769420884293813
    -0.181635632001340    -0.293892626146237    -0.293892626146237
     0.181635632001340     0.293892626146237     0.293892626146237
     0.475528258147577     0.769420884293813     0.769420884293813


  Columns 4 through 6

     0.475528258147577     0.000000000000000    -0.475528258147577
     0.181635632001340     0.000000000000000    -0.181635632001340
    -0.181635632001340    -0.000000000000000     0.181635632001340
    -0.475528258147577    -0.000000000000000     0.475528258147577
    -0.587785252292473    -0.000000000000000     0.587785252292473
    -0.475528258147577    -0.000000000000000     0.475528258147577
    -0.181635632001340    -0.000000000000000     0.181635632001340
```

```
    0.181635632001340    0.000000000000000   -0.181635632001340
    0.475528258147577    0.000000000000000   -0.475528258147577


  Columns 7 through 9

  -0.769420884293813   -0.769420884293813   -0.475528258147577
  -0.293892626146236   -0.293892626146236   -0.181635632001340
   0.293892626146237    0.293892626146237    0.181635632001340
   0.769420884293813    0.769420884293813    0.475528258147577
   0.951056516295154    0.951056516295154    0.587785252292473
   0.769420884293813    0.769420884293813    0.475528258147577
   0.293892626146237    0.293892626146237    0.181635632001340
  -0.293892626146237   -0.293892626146237   -0.181635632001340
  -0.769420884293813   -0.769420884293813   -0.475528258147577


Maximum diference
    0.034953292655752


when tol is 2*e3
GS Solving
  Columns 1 through 3

   0.475528258147577    0.769420884293813    0.769420884293813
   0.181635632001340    0.290000029576655    0.285032281876350
  -0.181635632001340   -0.309547843651024   -0.319496982693206
  -0.475528258147577   -0.795913698517862   -0.809360733328346
  -0.587785252292473   -0.981784141676350   -0.996071992500591
  -0.475528258147577   -0.795846939174758   -0.808093055916763
  -0.181635632001340   -0.309554098498229   -0.317683183138917
   0.181635632001340    0.289858192408024    0.286343168211706
   0.475528258147577    0.769420884293813    0.769420884293813


  Columns 4 through 6

   0.475528258147577    0.000000000000000   -0.475528258147577
   0.169076028081423   -0.013569218149618   -0.193341437800068
  -0.209652498468423   -0.022997574635654    0.168341226757328
  -0.515009507470528   -0.027504674835271    0.465411898271097
  -0.630397945517261   -0.027244263327795    0.581110062977963
  -0.511959781527517   -0.023069568284018    0.470108201966793
  -0.205066053668571   -0.016227526875808    0.175555251994744
   0.172694350230291   -0.008096889214199   -0.187451066622848
   0.475528258147577    0.000000000000000   -0.475528258147577


  Columns 7 through 9

  -0.769420884293813   -0.769420884293813   -0.475528258147577
  -0.301907305189332   -0.297876868558298   -0.181635632001340
   0.290360887060019    0.295608091494463    0.181635632001340
   0.773965452510669    0.778638354539312    0.475528258147577
   0.960550973822759    0.964167753062033    0.587785252292473
   0.777680706800800    0.780565357976041    0.475528258147577
   0.296084496262022    0.298580255496794    0.181635632001340
  -0.297212137985141   -0.295434178825087   -0.181635632001340
```

```
       -0.769420884293813  -0.769420884293813  -0.475528258147577
```

Original Matrix
  Columns 1 through 3

```
   0.475528258147577   0.769420884293813   0.769420884293813
   0.181635632001340   0.293892626146236   0.293892626146236
  -0.181635632001340  -0.293892626146237  -0.293892626146237
  -0.475528258147577  -0.769420884293813  -0.769420884293813
  -0.587785252292473  -0.951056516295154  -0.951056516295154
  -0.475528258147577  -0.769420884293813  -0.769420884293813
  -0.181635632001340  -0.293892626146237  -0.293892626146237
   0.181635632001340   0.293892626146237   0.293892626146237
   0.475528258147577   0.769420884293813   0.769420884293813
```

  Columns 4 through 6

```
   0.475528258147577   0.000000000000000  -0.475528258147577
   0.181635632001340   0.000000000000000  -0.181635632001340
  -0.181635632001340  -0.000000000000000   0.181635632001340
  -0.475528258147577  -0.000000000000000   0.475528258147577
  -0.587785252292473  -0.000000000000000   0.587785252292473
  -0.475528258147577  -0.000000000000000   0.475528258147577
  -0.181635632001340  -0.000000000000000   0.181635632001340
   0.181635632001340   0.000000000000000  -0.181635632001340
   0.475528258147577   0.000000000000000  -0.475528258147577
```

  Columns 7 through 9

```
  -0.769420884293813  -0.769420884293813  -0.475528258147577
  -0.293892626146236  -0.293892626146236  -0.181635632001340
   0.293892626146237   0.293892626146237   0.181635632001340
   0.769420884293813   0.769420884293813   0.475528258147577
   0.951056516295154   0.951056516295154   0.587785252292473
   0.769420884293813   0.769420884293813   0.475528258147577
   0.293892626146237   0.293892626146237   0.181635632001340
  -0.293892626146237  -0.293892626146237  -0.181635632001340
  -0.769420884293813  -0.769420884293813  -0.475528258147577
```

Maximum diference
   0.013111236766880

Step 5 Convergence

```
 lst = [];
for k = [1 8 16 32 64]

    h       =    0.05/k;
    hx      =    1;
    hy      =    1;
    [x,y]   =    meshgrid(0:h:hx,hy:-h:0);
    u       =    myfunc(x,y);
```

```matlab
    u1      =    u(2:end-1,2:end-1);
    us      =    u1(:);
    n       =    hx/h-1;
    e       =    ones(n,1);
    T       =    spdiags([-e 2*e -e], -1:1 ,n,n);
    A       =    kron(speye(n),T)+kron(T,speye(n));
    f    =  8*pi^2*sin(2*pi*x).*cos(2*pi*y);
    f    =  f(2:end-1,2:end-1);
    f    =  f(:);



    f(1) = f(1)+u(1,2)+u(2,1);
    f(n) = f(n)+u(n+1,1)+u(n+2,2);
    f(n*(n-1)+1) = f(n*(n-1)+1)+u(1,n+1)+u(2,n+2);
    f(n*n) = f(n*n) + u(n+1,n+2)+u(n+2,n+1);

    for i = 2:n-1
        f(i) = f(i)+ u(i+1,1);
    end
    for i = n*(n-1)+2:n^2-1
        f(i) = f(i)+ u(i+1-n*(n-1),n+2);
    end
    for i=1:n-2
        f(i*n+1) = f(i*n+1) + u(1,i+2);
    end
    for i=1:n-2
        f((i+1)*n) = f((i+1)*n) + u(n+2,i+2);
    end



    MM  =   reshape(A*us/h/h-f,n,n);
    MM  =   MM(2:end-1,2:end-1);
    MM  =   max(MM(:));
    lst = [lst MM];
end

disp('when I down the divide difference by 1/8 1/16 1/32  1/64');
disp('it equals to around 8 16 32 64');
disp(sqrt((lst(1)./lst(2:end))));
disp('Thus the oreder is 2;')

when I down the divide difference by 1/8 1/16 1/32  1/64
it equals to around 8 16 32 64
  Columns 1 through 3

   7.987058170906883  15.973808472209949  31.947463304055709

  Column 4

  63.894710057943399

Thus the oreder is 2;
```

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